1 Concept of Measurement Systems

1.1 INTRODUCTION

Measurement is the act, or the result, of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a unit. The result of the measurement is expressed by a pointer deflection over a predefined scale or a number representing the ratio between the unknown quantity and the standard. A standard is defined as the physical personification of the unit of measurement or its submultiple or multiple values. The device or instrument used for comparing the unknown quantity with the unit of measurement or a standard quantity is called a *measuring instrument.* The value of the unknown quantity can be measured by direct or indirect methods. In direct measurement methods, the unknown quantity is measured directly instead of comparing it with a standard. Examples of direct measurement are current by ammeter, voltage by voltmeter, resistance by ohmmeter, power by wattmeter, etc. In indirect measurement methods, the value of the unknown quantity is determined by measuring the functionally related quantity and calculating the desired quantity rather than measuring it directly. Suppose the resistance as (R) of a conductor can be measured by measuring the voltage drop across the conductor and dividing the voltage (V) by the current (I) through the conductors, by Ohm's $R = \frac{V}{I}$

1.2 FUNDAMENTAL AND DERIVED UNITS

At the time of measuring a physical quantity, we must express the magnitude of that quantity in terms of a unit and a numerical multiplier, i.e.,

Magnitude of a physical quantity = (Numerical ratio) × (Unit)

The numerical ratio is the number of times the unit occurs in any given amount of the same quantity and, therefore, is called the *number of measures.* The numerical ratio may be called *numerical multiplier.* However, in measurements, we are concerned with a large number of quantities which are related to each other, through established physical equations, and therefore the choice of size of units of these quantities cannot be done arbitrarily and independently. In this way, we can avoid the use of awkward numerical constants when we express a quantity of one kind which has been derived from measurement of another quantity.

In science and engineering, two kinds of units are used:

- Fundamental units

- Derived units

The *fundamental units* in mechanics are measures of length, mass and time. The sizes of the fundamental units, whether foot or metre, pound or kilogram, second or hour are arbitrary and can be selected to fit a certain set of circumstances. Since length, mass and time are fundamental to most other physical quantities besides those in mechanics, they are called the *primary fundamental units.* Measures of certain physical quantities in the thermal, electrical and illumination disciplines are also represented by fundamental units. These units are used only when these particular classes are involved, and they may therefore be defined as *auxiliary fundamental units.*

All other units which can be expressed in terms of the fundamental units are called *derived units.* Every derived unit originates from some physical law defining that unit. For example, the area (A) of a rectangle is proportional to its length (l) and breadth (b), or $A = lb$. if the metre has been chosen as the unit of length then the area of a rectangle of 5 metres by 7 metres is 35 m^2. Note that the numbers of measure are multiplied as well as the units. The derived unit for area (A) is then the metre square (m^2).

A derived unit is recognized by its dimensions, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbols for the fundamental units of length, mass and time are L, M and T respectively. The dimensional symbol for the derived unit of area is L^2 and that for volume is L^3 . The dimensional symbol for the unit of force is MLT , which follows from the defining equation for force. The dimensional formulas of the derived units are particularly useful for converting units from one system to another. For convenience, some derived units have been given new names. For example, the derived unit of force in the SI system is called the newton (N), instead of the dimensionally correct kg-m/s^2.

1.3 STANDARDS AND THEIR CLASSIFICATIONS

A standard of measurement is a physical representation of a unit of measurement. A unit is realised by reference to an arbitrary material standard or to natural phenomena including physical and atomic constants. The term ‘standard’ is applied to a piece of equipment having a known measure of physical quantity. For example, the fundamental unit of mass in the SI system is the kilogram, defined as the mass of the cubic decimetre of water at its temperature of maximum of 4°C. This unit of mass is represented by a material standard; the mass of the international prototype kilogram consisting of a platinum–iridium hollow cylinder. This unit is preserved at the International Bureau of Weights and Measures at Sevres, near Paris, and is the material representation of the kilogram. Similar standards have been developed for other units of measurement, including fundamental units as well as for some of the derived mechanical and electrical units.

The classifications of standards are

1. International standards
2. Primary standards

3. Secondary standards
4. Working standards
5. Current standards
6. Voltage standards
7. Resistance standards
8. Capacitance standards
9. Time and frequency standards

1.3.1 International Standards

The international standards are defined by international agreement. They represent certain units of measurement to the closest possible accuracy that production and measurement technology allow. International standards are periodically checked and evaluated by absolute measurements in terms of the fundamental units. These standards are maintained at the International Bureau of Weights and Measures and are not available to the ordinary user of measuring instruments for purposes of comparison or calibration. Table 1.1 shows basic SI Units, Quantities and Symbols.

Table 1.1 *Basic Quantities, SI Units and Symbols*

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Luminous Intensity	Candela	cd
Thermodynamic temperature	Kelvin	K
Electric current	Ampere	A

1.3.2 Primary Standards

The primary standards are maintained by national standards laboratories in different places of the world. The National Bureau of Standards (NBS) in Washington is responsible for maintenance of the primary standards in North America. Other national laboratories include the National Physical Laboratory (NPL) in Great Britain and the oldest in the world, the Physikalisch Technische Reichsanstalt in Germany. The primary standards, again representing the fundamental units and some of the derived mechanical and electrical units, are independently calibrated by absolute measurements at each of the national laboratories. The results of these measurements are compared with each other, leading to a world average figure for the primary standard. Primary standards are not available for use outside the national laboratories. One of the main functions of primary standards is the verification and calibration of secondary standards.

1.3.3 Secondary Standards

Secondary standards are the basic reference standards used in the industrial measurement laboratories. These standards are maintained by the particular involved industry and are checked locally against other reference standards in the area. The responsibility for maintenance and calibration rests entirely with the industrial laboratory itself. Secondary standards are generally sent to the national standards laboratory on a periodic basis for calibration and comparison against the primary standards. They are then returned to the industrial user with a certification of their measured value in terms of the primary standard.

1.3.4 Working Standards

Working standards are the principle tools of a measurement laboratory. They are used to check and calibrate general laboratory instruments for accuracy and performance or to perform comparison measurements in industrial applications. A manufacturer of precision resistances, for example, may use a standard resistor in the quality control department of his plant to check his testing equipment. In this case, the manufacturer verifies that his measurement setup performs within the required limits of accuracy.

1.3.5 Current Standard

The fundamental unit of electric current (Ampere) is defined by the International System of Units (SI) as the constant current which, if maintained in two straight parallel conductors of infinite length and negligible circular cross section placed 1 meter apart in vacuum, will produce between these conductors a force equal to 2×10^{-7} newton per meter length. Early measurements of the absolute value of the ampere were made with a current balance which measured the force between two parallel conductors. These measurements were rather crude and the need was felt to produce a more practical and reproducible standard for the national laboratories. By international agreement, the value of the international ampere was based on the electrolytic deposition of silver from a silver nitrate solution. The *international ampere* was then defined as that current which deposits silver at the rate of 1.118 mg/s from a standard silver nitrate solution. Difficulties were encountered in the exact measurement of the deposited silver and slight discrepancies existed between measurements made independently by the various National Standard Laboratories. Later, the international ampere was superseded by the *absolute ampere* and it is now the fundamental unit of electric current in the SI and is universally accepted by international agreement.

1.3.6 Voltage Standard

In early times, the standard volt was based on an electrochemical cell called the *saturated standard cell* or simply *standard cell*. The saturated cell has temperature dependence, and the output voltage changes about -40 µV/°C from the nominal of 1.01858 volt. The standard cell suffers from this temperature dependence and also from the fact that the voltage is a function of a chemical reaction and not related directly to any other physical constants. In 1962, based on the work of Brian Josephson, a new standard for the volt was introduced. A thin-film junction is cooled to nearly absolute zero and irradiated with microwave energy. A voltage is developed across the junction, which is related to the irradiating frequency by the following relationship:

$$v = \frac{hf}{2e}$$

where, h =Planck's constant = 6.63×10^{-34} J-s

e =charge of an electron = 1.602×10^{-19} C

f =frequency of the microwave irradiation

In Eq. (1.1), the irradiation frequency is the only variable, thus the standard volt is related to the standard of time/frequency. When the microwave irradiating frequency is locked to an atomic clock or a broadcast frequency standard such as WWVB, the accuracy of the standard volt, including all of the system inaccuracies, is one part in 10^8.

The major method of transferring the volt from the standard based on the Josephson junction to secondary standards used for calibration of the standard cell. This device is called the normal or saturated Weston cell. The Weston cell has a positive electrode of mercury and a negative electrode of cadmium amalgam (10% cadmium). The electrolyte is a solution of cadmium sulfate. These components are placed in an H-shaped glass container as shown in Figure 1.1.

Figure 1.1 *Standard cell of emf of 1.0183 volt at 20°C (Courtesy, physics.kenyon.edu)*

1.3.7 Resistance Standard

In the SI system, the absolute value of ohm is defined in terms of the fundamental units of length, mass and time. The absolute measurement of the ohm is carried out by the International Bureau of Weights and Measures in Sevres and also by the national standard laboratories, which preserve a group of primary resistance standards. The NBS maintains a group of those primary standards (1 ohm standard resistors) which are periodically checked against each other and are occasionally verified by absolute measurements. The standard resistor is a coil of wire of some alloy like manganin which has a high electrical resistivity and a low temperature coefficient of resistance. The resistance coil is mounted in a double walled sealed container as shown in Figure 1.2 to prevent changes in resistance due to moisture conditions in the atmosphere. With a set of four or five 1-ohm resistors in this type, the unit resistance can be represented with a precision of a few parts in 10^7 over several years.

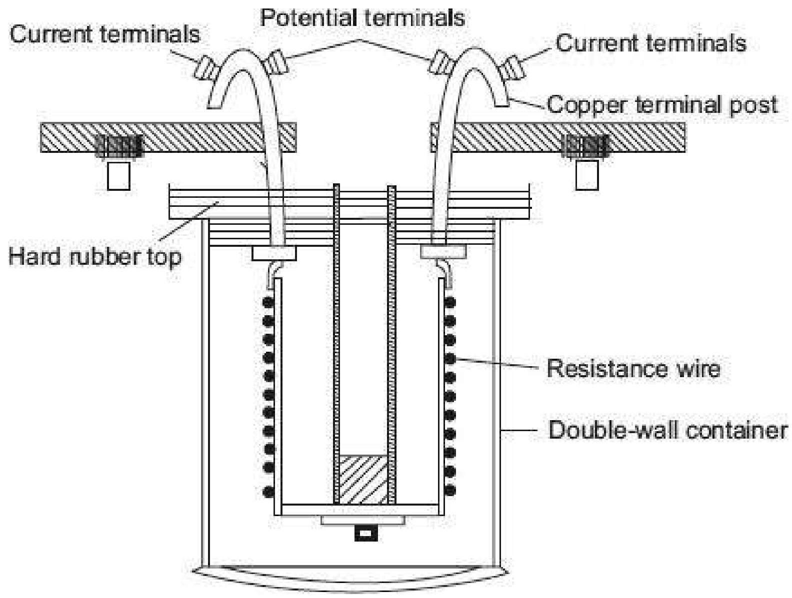


Figure 1.2 *Resistance standard*

Secondary standards and working standards are available from some instrument manufactures in a wide range of values, usually in multiples of 10 ohms. These standard resistors are sometimes called *transfer resistor* and are made of alloy resistance wire, such as manganin or Evanohm. The resistance coil of the transfer resistor is supported between polyester films to reduce stresses on the wire and to improve the stability of the resistor. The coil is immersed in moisture free oil and placed in a sealed container. The connections to the coil are silver soldered, and the terminal hooks are made of nickel-plated oxygen free copper. The transfer resistor is checked for stability and temperature characteristics at its rated power and a specified operating temperature (usually 25°C). A calibration report accompanying the resistor specifies its traceability to NBS standards and includes the *a* and β temperature coefficients. Although the selected resistance wire provides almost constant resistance over a fairly wide temperature range, the exact value of the resistance at any temperature can be calculated from the formula

$$R_t = R_{25°C} + \alpha(t — 25) + \beta(t — 25)^2$$

where R_t = resistance at the ambient temperature *t*

$R_{25°C}$ = reisitance at 25°c

α,β = temperature coefficients

Temperature coefficient *a* is usually less than $10 \times 10^{-}$, and coefficient β lies between $-3 \times 10^{-}$ to -6×10^{-7}. This means that a change in temperature of 10°C from the specified reference temperature of 25°C may cause a change in resistance of 30 to 60 ppm from the nominal value.

1.3.8 Capacitance Standard

Many electrical and magnetic units may be expressed in terms of these voltage and resistance standards since the unit of resistance is represented by the standard resistor and the unit of voltage by standard Weston cell. The unit of capacitance (the farad) can be measured with a Maxwell dc commutated bridge, where the capacitance is computed from the resistive bridge arms and the frequency of the dc commutation. The bridge is shown in Figure 1.3. Capacitor *C* is alternately charged and discharged through the commutating contact and resistor *R*. Bridge balance is obtained by adjusting the resistance R_3, allowing exact determination of the capacitance value in terms of the bridge arm constants and frequency of commutation. Although the exact derivation of the expression for capacitance in terms of the resistances and the frequency is rather involved, it may be seen that the capacitor could be measured accurately by this method. Since both resistance and frequency can be determined very accurately, the value of the capacitance can be measured with great accuracy. Standard capacitors are usually constructed from interleaved metal plates with air as the dielectric material. The area of the plates and the distance between them must be known very accurately, and the capacitance of the air capacitor can be determined from these basic dimensions. The NBS maintains a bank of air capacitors as standards and uses them to calibrate the secondary and working standards of measurement laboratories and industrial users.

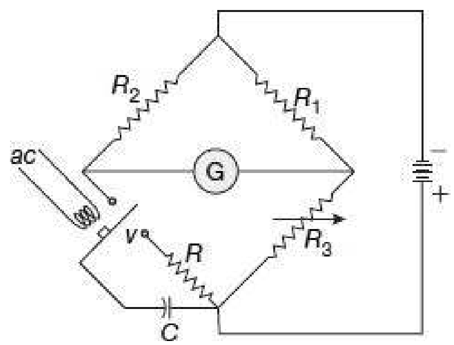


Figure 1.3 *Commutated dc method for measuring capacitance*

1.3.9 Time Standard and Frequency Standard

In early centuries the time reference used was the rotation of the earth around the sun about its axis. Later, precise astronomical observations have shown that the rotation of the earth around the sun is very irregular, owing to secular and irregular variations in the rotational speed of the earth. So the time scale based on this apparent solar time had to be changed. *Mean solar time* was thought to give a more accurate time scale. A *mean solar day* is the average of all the apparent days in the year. A *mean solar second* is then equal to 1/86400 of the mean solar day. The mean solar second is still inappropriate since it is based on the rotation of the earth which is non-uniform.

In the year 1956, the *ephemeris second* has been defined by the International Bureau of Weights and Measures as the fraction 1/31556925.99747 of the tropical year for 1900 January 01 at 12 h ET (Ephemeris Time), and adopted as the fundamental invariable unit of time. A disadvantage of the use of the *ephemeris* second is that it can be determined only several years in arrears and then only indirectly, by observations of the positions of the sun and the moon. For physical measurements, the unit of time interval has now been defined in terms of an atomic standard. The universal second and the *ephemeris* second, however, will continue to be used for navigation, geodetic surveys and celestial mechanics. The atomic units of the time was first related to UT (Universal Time) but was later expressed in terms of ET. The International Committee of Weights and Measures has now defined the second in terms of frequency of the cesium transition, assigning a value of 9192631770 Hz to the hyperfine transition of the cesium atom unperturbed by external fields.

The atomic definition of second realises an accuracy much greater than that achieved by astronomical observations, resulting in a more uniform and much more convenient time base. Determinations of time intervals can now be made in a few minutes to greater accuracy than was possible before in astronomical measurements that took many years to complete. An atomic clock with a precision exceeding 1 μs per day is in operation as a primary frequency standard at the NBS. An atomic time scale, designated NBS-A, is maintained with this clock.

Time and frequency standards are unique in that they may be transmitted from the primary standard at NBS to other locations via radio or television transmission. Early standard time and frequency transmission were in the High Frequency (HF) portion of the radio spectrum, but these transmissions suffered from Doppler shifts due to the fact that radio propagation was primarily ionospheric. Transmission of time and frequency standards via low frequency and very low frequency radio reduces this Doppler shift because the propagation is strictly ground wave. Two NBS operated stations, WWVL and WWVB, operate 20 and 60 kHz, respectively, providing precision time and frequency transmissions.

1.4 METHODS OF MEASUREMENT

As discussed above, the measurement methods can be classified as

- Direct comparison methods
- Indirect comparison methods

1.4.1 Direct Comparison Methods

In direct measurement methods, the unknown quantity is measured directly. Direct methods of measurement are of two types, namely, *deflection methods* and *comparison methods.*

In deflection methods, the value of the unknown quantity is measured by the help of a measuring instrument having a calibrated scale indicating the quantity under measurement directly, such as measurement of current by an ammeter.

In comparison methods, the value of the unknown quantity is determined by direct comparison with a standard of the given quantity, such as measurement of emf by comparison with the emf of a standard cell. Comparison methods can be classified as null methods, differential methods, etc. In null methods of measurement, the action of the unknown quantity upon the instrument is reduced to zero by the counter action of a known quantity of the same kind, such as measurement of weight by a balance, measurement of resistance, capacitance, and inductance by bridge circuits.

1.4.2 Indirect Comparison Methods

In indirect measurement methods, the comparison is done with a standard through the use of a calibrated system. These methods for measurement are used in those cases where the desired parameter to be measured is difficult to be measured directly, but the parameter has got some relation with some other related parameter which can be easily measured.

For instance, the elimination of bacteria from some fluid is directly dependent upon its temperature. Thus, the bacteria elimination can be measured indirectly by measuring the temperature of the fluid.

In indirect methods of measurement, it is general practice to establish an empirical relation between the actual measured quantity and the desired parameter.

The different methods of measurement are summarised with the help of a tree diagram in Figure 1.4.

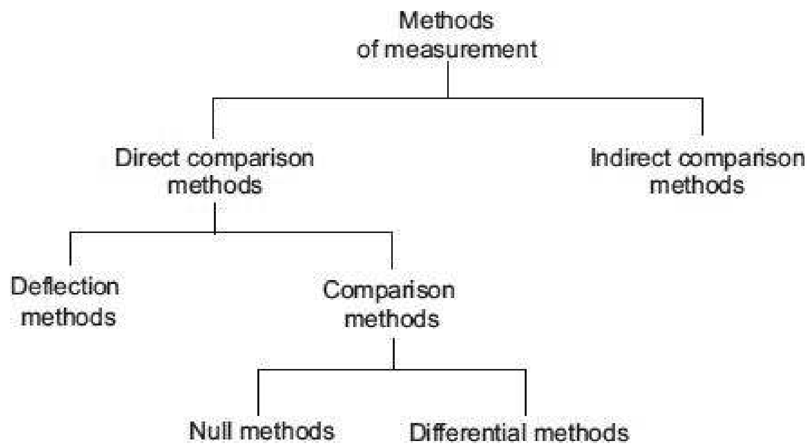


Figure 1.4 *Different methods of measurement*

1.5 MEASUREMENT SYSTEM AND ITS ELEMENTS

A measurement system may be defined as a systematic arrangement for the measurement or determination of an unknown quantity and analysis of instrumentation. The generalised measurement system and its different components/elements are shown in Figure 1.5.

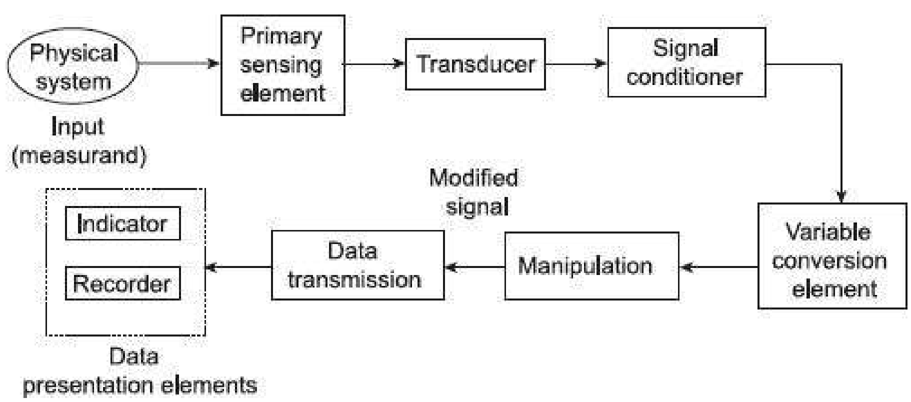


Figure 1.5 *Generalised measurement system*

The operation of a measurement system can be explained in terms of functional elements of the system. Every instrument and measurement system is composed of one or more of these functional elements and each functional element is made of distinct components or groups of components which performs required and definite steps in measurement. The various elements are the following:

1.5.1 Primary Sensing Elements

It is an element that is sensitive to the measured variable. The physical quantity under measurement, called the *measurand*, makes its first contact with the primary sensing element of a measurement system. The measurand is always disturbed by the act of the measurement, but good instruments are designed to minimise this effect. Primary sensing elements may have a non-electrical input and output such as a spring, manometer or may have an electrical input and output such as a rectifier. In case the primary sensing element has a non-electrical input and output, then it is converted into an electrical signal by means of a transducer. The transducer is defined as a device, which when actuated by one form of energy, is capable of converting it into another form of energy.

Many a times, certain operations are to be performed on the signal before its further transmission so that interfering sources are removed in order that the signal may not get distorted. The process may be linear such as amplification, attenuation, integration, differentiation, addition and subtraction or nonlinear such as modulation, detection, sampling, filtering, chopping and clipping, etc. The process is called signal conditioning. So a signal conditioner follows the primary sensing element or transducer, as the case may be. The sensing element senses the condition, state or value of the process variable by extracting a small part of energy from the measurand, and then produces an output which reflects this condition, state or value of the measurand.

1.5.2 Variable Conversion Elements

After passing through the primary sensing element, the output is in the form of an electrical signal, may be voltage, current, frequency, which may or may not be accepted to the system. For performing the desired operation, it may be necessary to convert this output to some other suitable form while retaining the information content of the original signal. For example, if the output is in analog form and the next step of the system accepts only in digital form then an analog-to-digital converter will be employed. Many instruments do not require any variable conversion unit, while some others require more than one element.

1.5.3 Manipulation Elements

Sometimes it is necessary to change the signal level without changing the information contained in it for the acceptance of the instrument. The function of the variable manipulation unit is to manipulate the signal presented to it while preserving the original nature of the signal. For example, an electronic amplifier converts a small low voltage input signal into a high voltage output signal. Thus, the voltage amplifier acts as a variable manipulation unit. Some of the instruments may require this function or some of the instruments may not.

1.5.4 Data Transmission Elements

The data transmission elements are required to transmit the data containing the information of the signal from one system to another. For example, satellites are physically separated from the earth where the control stations guiding their movement are located.

1.5.5 Data Presentation Elements

The function of the data presentation elements is to provide an indication or recording in a form that can be evaluated by an unaided human sense or by a controller. The information regarding measurand (quantity to be measured) is to be conveyed to the personnel handling the instrument or the system for monitoring, controlling or analysis purpose. Such a device may be in the form of analog or digital format. The simplest form of a display device is the common panel meter with some kind of calibrated scale and pointer. In case the data is to be recorded, recorders like magnetic tapes or magnetic discs may be used. For control and analysis purpose, computers may be used.

The stages of a typical measurement system are summarised below with the help of a flow diagram in Figure 1.6.

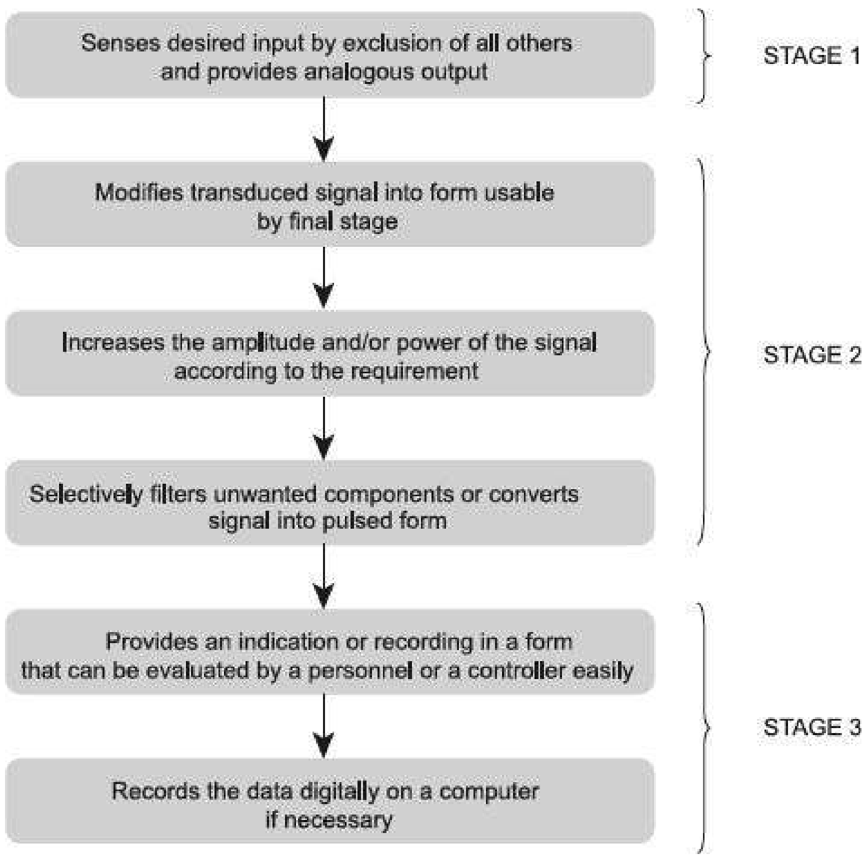


Figure 1.6 *Steps of a typical measurement system*

1.6 CLASSIFICATION OF INSTRUMENTS

The measuring instruments may be classified as follows:

1.6.1 Absolute and Secondary Instruments

1. Absolute Instruments

The instruments of this type give the value of the measurand in terms of instrument constant and its deflection. Such instruments do not require comparison with any other standard. The example of this type of instrument is tangent galvanometer, which gives the value of the current to be measured in terms of tangent of the angle of deflection produced, the horizontal component of the earth's magnetic field, the radius and the number of turns of the wire used. Rayleigh current balance and absolute electrometer are other examples of absolute instruments. Absolute instruments are mostly used in standard laboratories and in similar institutions as standardising. The classification of measuring instruments is shown in Figure 1.7.

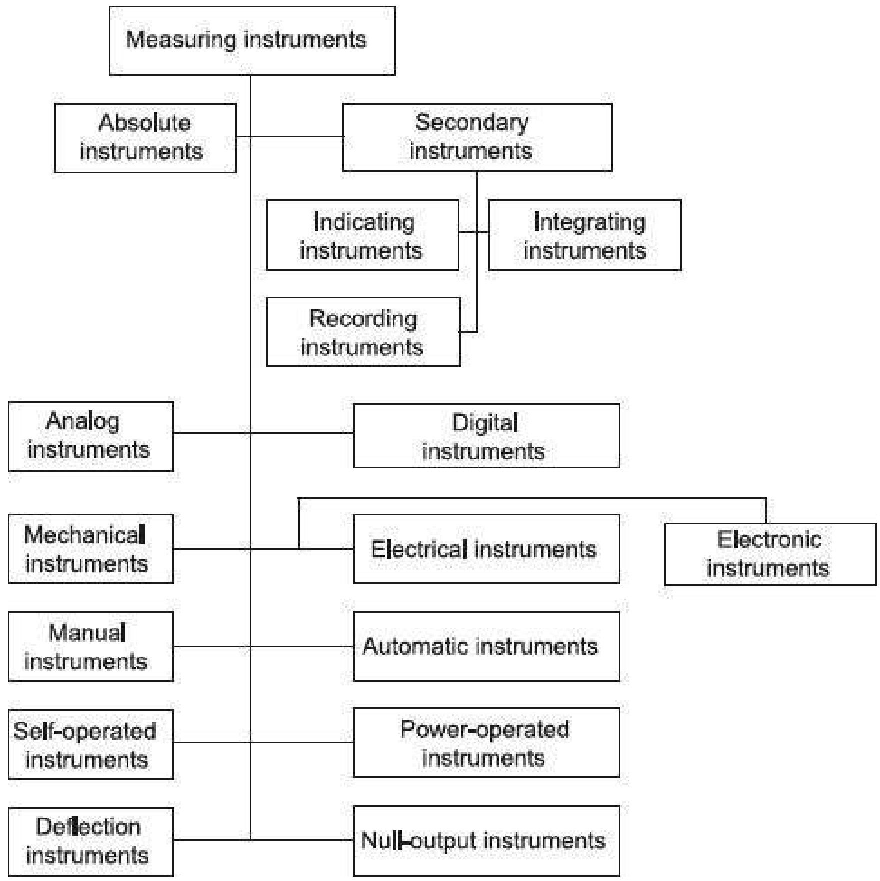


Figure 1.7 *Classification of measuring instruments*

2. Secondary Instruments

These instruments are so constructed that the deflection of such instruments gives the magnitude of the electrical quantity to be measured directly. These instruments are required to be calibrated by comparison with either an absolute instrument or with another secondary instrument, which has already been calibrated before the use. These instruments are generally used in practice.

Secondary instruments are further classified as

- Indicating instruments
- Integrating instruments

- Recording instruments

(i) Indicating Instruments

Indicating instruments are those which indicate the magnitude of an electrical quantity at the time when it is being measured. The indications are given by a pointer moving over a calibrated (pregraduated) scale. Ordinary ammeters, voltmeters, wattmeters, frequency meters, power factor meters, etc., fall into this category.

(ii) Integrating Instruments

Integrating instruments are those which measure the total amount of either quantity of electricity (ampere-hours) or electrical energy supplied over a period of time. The summation, given by such an instrument, is the product of time and an electrical quantity under measurement. The ampere-hour meters and energy meters fall in this class.

(iii) Recording Instruments

Recording instruments are those which keep a continuous record of the variation of the magnitude of an electrical quantity to be observed over a definite period of time. In such instruments, the moving system carries an inked pen which touches lightly a sheet of paper wrapped over a drum moving with uniform slow motion in a direction perpendicular to that of the direction of the pointer. Thus, a curve is traced which shows the variations in the magnitude of the electrical quantity under observation over a definite period of time. Such instruments are generally used in powerhouses where the current, voltage, power, etc., are to be maintained within certain acceptable limit.

1.6.2 Analog and Digital Instruments

1. Analog Instruments

The signals of an analog unit vary in a continuous fashion and can take on infinite number of values in a given range. Fuel gauge, ammeter and voltmeters, wrist watch, speedometer fall in this category.

2. Digital Instruments

Signals varying in discrete steps and taking on a finite number of different values in a given range are digital signals and the corresponding instruments are of digital type. Digital instruments have some advantages over analog meters, in that they have high accuracy and high speed of operation. It eliminates the human operational errors. Digital instruments can store the result for future purposes. A digital multimeter is the example of a digital instrument.

1.6.3 Mechanical, Electrical and Electronics Instruments

1. Mechanical Instruments

Mechanical instruments are very reliable for static and stable conditions. They are unable to respond rapidly to the measurement of dynamic and transient conditions due to the fact that they have moving parts that are rigid, heavy and bulky and consequently have a large mass. Mass presents inertia problems and hence these instruments cannot faithfully follow the rapid changes which are involved in dynamic instruments. Also, most of the mechanical instruments causes noise pollution.

Advantages of Mechanical Instruments

- Relatively cheaper in cost
- More durable due to rugged construction
- Simple in design and easy to use
- No external power supply required for operation
- Reliable and accurate for measurement of stable and time invariant quantity

Disadvantages of Mechanical Instruments

- Poor frequency response to transient and dynamic measurements
- Large force required to overcome mechanical friction
- Incompatible when remote indication and control needed
- Cause noise pollution

2. Electrical Instruments

When the instrument pointer deflection is caused by the action of some electrical methods then it is called an electrical instrument. The time of operation of an electrical instrument is more rapid than that of a mechanical instrument. Unfortunately, an electrical system normally depends upon a mechanical measurement as an indicating device. This mechanical movement has some inertia due to which the frequency response of these instruments is poor.

3. Electronic Instruments

Electronic instruments use semiconductor devices. Most of the scientific and industrial instrumentations require very fast responses. Such requirements cannot be met with by mechanical and electrical instruments. In electronic devices, since the only movement involved is that of electrons, the response time is extremely small owing to very small inertia of the electrons. With the use of electronic devices, a very weak signal can be detected by using pre-amplifiers and amplifiers.

Advantages of Electrical/Electronic Instruments

- Non-contact measurements are possible
- These instruments consume less power
- Compact in size and more reliable in operation
- Greater flexibility
- Good frequency and transient response
- Remote indication and recording possible

- Amplification produced greater than that produced in mechanical instruments

1.6.4 Manual and Automatic Instruments

In case of manual instruments, the service of an operator is required. For example, measurement of temperature by a resistance thermometer incorporating a Wheatstone bridge in its circuit, an operator is required to indicate the temperature being measured.

In an automatic type of instrument, no operator is required all the time. For example, measurement of temperature by mercury-in-glass thermometer.

1.6.5 Self-operated and Power-operated Instruments

Self-operated instruments are those in which no outside power is required for operation. The output energy is supplied wholly or almost wholly by the input measurand. Dial-indicating type instruments belong to this category.

The power-operated instruments are those in which some external power such as electricity, compressed air, hydraulic supply is required for operation. In such cases, the input signal supplies only an insignificant portion of the output power. Electromechanical instruments shown in Figure 1.8 fall in this category.

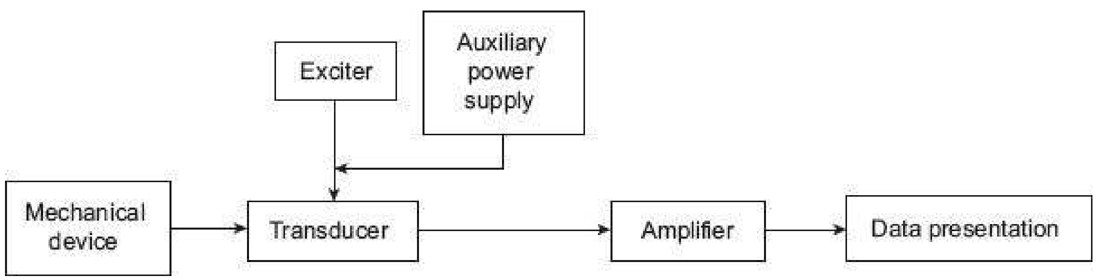


Figure 1.8 *Electromechanical measurement system*

1.6.6 Deflection and Null Output Instruments

In a deflection-type instrument, the deflection of the instrument indicates the measurement of the unknown quantity. The measurand quantity produces some physical effect which deflects or produces a mechanical displacement in the moving system of the instrument. An opposite effect is built in the instrument which opposes the deflection or the mechanical displacement of the moving system. The balance is achieved when opposing effect equals the actuating cause producing the deflection or the mechanical displacement. The deflection or the mechanical displacement at this point gives the value of the unknown input quantity. These type of instruments are suited for measurement under dynamic condition. Permanent Magnet Moving Coil (PMMC), Moving Iron (MI), etc., type instruments are examples of this category.

In null-type instruments, a zero or null indication leads to determination of the magnitude of the measurand quantity. The null condition depends upon some other known conditions. These are more accurate and highly sensitive as compared to deflection-type instruments. A dc potentiometer is a null- type instrument.

1.7 DEFINITIONS OF SOME STATIC CHARACTERISTICS

[redacted]

1. Accuracy

Accuracy is the closeness with which the instrument reading approaches the true value of the variable under measurement. Accuracy is determined as the maximum amount by which the result differs from the true value. It is almost impossible to determine experimentally the true value. The true value is not indicated by any measurement system due to the loading effect, lags and mechanical problems (e.g., wear, hysteresis, noise, etc.).

Accuracy of the measured signal depends upon the following factors:

- Intrinsic accuracy of the instrument itself;
- Accuracy of the observer;
- Variation of the signal to be measured; and
- Whether or not the quantity is being truly impressed upon the instrument.

2. Precision

Precision is a measure of the reproducibility of the measurements, i.e., precision is a measure of the degree to which successive measurements differ from one another. Precision is indicated from the number of significant figures in which it is expressed. Significant figures actually convey the information regarding the magnitude and the measurement precision of a quantity. More significant figures imply greater precision of the measurement.

3. Resolution

If the input is slowly increased from some arbitrary value it will be noticed that the output does not change at all until the increment exceeds a certain value called the resolution or discrimination of the instrument. Thus, the resolution or discrimination of any instrument is the smallest change in the input signal (quantity under measurement) which can be detected by the instrument. It may be expressed as an accrual value or as a fraction or percentage of the full scale value. Resolution is sometimes referred as *sensitivity*. The largest change of input quantity for which there is no output of the instrument is called the *dead zone* of that instrument.

The sensitivity gives the relation between the input signal to an instrument or a part of the instrument system and the output. Thus, the sensitivity is defined as the ratio of output signal or response of the instrument to a change of input signal or the quantity under measurement.

Example 1.1

A moving coil ammeter has a uniform scale with 50 divisions and gives a full-scale reading of 5 A. The instrument can read up to V th of a scale division with a fair degree of certainty. Determine the resolution of the instrument in mA.

Solution Full-scale reading = 5 A

Number of divisions on scale = 50

$$1 \text{ scale division} = \frac{5}{50} \times 1000 = 100 \text{ mA}$$

$$\text{Resolution} = \tfrac{1}{4}\text{th of a scale division} = \frac{100}{4} = 25 \text{ mA}$$

4. Speed of Response

The quickness of an instrument to read the measurand variable is called the speed of response. Alternately, speed of response is defined as the time elapsed between the start of the measurement to the reading taken. This time depends upon the mechanical moving system, friction, etc.

1.8 MEASUREMENT OF ERRORS

In practice, it is impossible to measure the exact value of the measurand. There is always some difference between the measured value and the absolute or true value of the unknown quantity (measurand), which may be very small or may be large. The difference between the true or exact value and the measured value of the unknown quantity is known as the absolute error of the measurement.

If δA be the absolute error of the measurement, A_m and A be the measured and absolute value of the unknown equantity then δA may be expressed as

$$\delta A = A_m - A \quad (1.2)$$

Sometimes, δA is denoted by ε_0.

The relative error is the ratio of absolute error to the true value of the unknown quantity to be measured,

$$\text{i.e., relative error, } \varepsilon_r = \frac{\delta A}{A} = \frac{\varepsilon_0}{A} = \frac{\text{Absolute error}}{\text{True value}} \quad (1.3)$$

When the absolute error ε_0 (=δA) is negligible, i.e., when the difference between the true value A and the measured value A_m of the unknown quantity is very small or negligible then the relative error may be expressed as,

$$\varepsilon_r = \frac{\delta A}{A_m} = \frac{\varepsilon_0}{A_m} \quad (1.4)$$

The relative error is generally expressed as a fraction, i.e., 5 parts in 1000 or in percentage value,

$$\text{i.e., percentage error} = \varepsilon_r \times 100 = \frac{\varepsilon_0}{A_m} \times 100 \quad (1.5)$$

The measured value of the unknown quantity may be more than or less than the true value of the measurand. So the manufacturers have to specify the deviations from the specified value of a particular quantity in order to enable the purchaser to make proper selection according to his requirements. The limits of these deviations from specified values are defined as limiting or guarantee errors. The magnitude of a given quantity having a specified magnitude A_m and a maximum or a limiting error $\pm\delta A$ must have a magnitude between the limits

$$A_m - \delta A \text{ and } A_m + \delta A$$

or,

$$A = A_m \pm \delta A \tag{1.6}$$

For example, the measured value of a resistance of 100 Ω has a limiting error of ±0.5 Ω. Then the true value of the resistance is between the limits 100 ± 0.5, i.e., 100.5 and 99.5 Ω.

Example 1.2

A 0-25 A ammeter has a guaranteed accuracy of 1 percent of full scale reading. The current measured by this instrument is 10 A. Determine the limiting error in percentage.

Solution The magnitude of limiting error of the instrument from Eq. (1.1),

$$\delta A = \varepsilon_r \times A = 0.01 \times 25 = 0.25 \text{ A}$$

The magnitude of the current being measured is 10 A. The relative error at this current is

$$\varepsilon_r = \frac{\delta A}{A} = \frac{0.25}{10} = 0.025$$

Therefore, the current being measured is between the limit of

$$A = A_m(1 \pm \varepsilon_r) = 10(1 \pm 0.025) = 10 \pm 0.25 \text{ A}$$

$$\text{The limiting error} = \frac{0.25}{10} \times 100 = 2.5\%$$

Example 1.3

The inductance of an inductor is specified as 20 H ± 5 percent by a manufacturer. Determine the limits of inductance between which it is guaranteed.

Solution

$$\text{Relative error, } \varepsilon_r = \frac{\text{Percentage error}}{100} = \frac{5}{100} = 0.05$$

Limiting value of inductance, $A = A_m \pm \delta A$

$$= A_m \pm \varepsilon_r A_m = A_m (1 \pm \varepsilon_r)$$

$$= 20(1 \pm 0.05) = 20 \pm 1 \text{ H}$$

Example 1.4

A 0-250 V voltmeter has a guaranteed accuracy of 2% of full-scale reading. The voltage measured by the voltmeter is 150 volts. Determine the limiting error in percentage.

Solution The magnitude of the limiting error of the instrument,

$$\delta A = \varepsilon_r V = 0.02 \times 250 = 5.0 \text{ V}$$

The magnitude of the voltage being measured is 150 V.

The percentage limiting error at this voltage

$$= \frac{5.0}{150} \times 100\% = 3.33\%$$

Example 1.5

The measurand value of a resistance is 10.25 Ω, whereas its value is 10.22 Ω. Determine the absolute error of the measurement.

Solution

Measurand value $A_m = 10.25\ \Omega$

True value $A_m = 10.22\ \Omega$

Absolute error, $\delta A = A_m - A = 10.25 - 10.22 = 0.03\ \Omega$

Example 1.6

The measured value of a capacitor is 205.3 fiF, whereas its true value is 201.4 fiF. Determine the relative error.

Solution

Measured value $A_m = 205.3 \times 10^{-6}$ F

True value, A = 201.4×10^{-6} F

Absolute error, $\varepsilon_0 = A_m - A$

$$= 205.3 \times 10^{-6} — 201.4 \times 10^{-6}$$

$$= 3.9 \times 10^{-6} \text{ F}$$

$$= 3.9 \times 10^{-6} \text{ F}$$

$$\text{Relative error, } \varepsilon_r = \frac{\varepsilon_0}{A} = \frac{3.9 \times 10^{-6}}{201.4 \times 10^{-6}} = 0.0194 \text{ or } 1.94\%$$

Example 1.7 *A wattmeter reads 25.34 watts. The absolute error in the measurement is –0.11 watt. Determine the true value of power.*

Solution

Measured value A_m = 25.34 W

Absolute error δA = –0.11 W

True value A = Measured value - Absolute error

= 25.34 – (–0.11),

= 25.45 W

1.8.1 Types of Errors

The origination of error may be in a variety of ways. They are categorised in three main types.

- Gross error
- Systematic error
- Random error

1. Gross Error

The errors occur because of mistakes in observed readings, or using instruments and in recording and calculating measurement results. These errors usually occur because of human mistakes and these may be of any magnitude and cannot be subjected to mathematical treatment. One common gross error is frequently committed during improper use of the measuring instrument. Any indicating instrument changes conditions to some extent when connected in a complete circuit so that the reading of measurand quantity is altered by the method used. For example, in Figure (1.9)(a) and (b), two possible connections of voltage and current coil of a wattmeter are shown.

In Figure 1.9(a), the connection shown is used when the applied voltage is high and current flowing in the circuit is low, while the connection shown in Figure 1.9(b) is used when the applied voltage is low and current flowing in the circuit is high. If these connections of wattmeter are used in opposite order then an error is liable to enter in wattmeter reading. Another example of this type of error is in the use of a well-calibrated voltmeter for measurement of voltage across a resistance of very high value. The same voltmeter, when connected in a low resistance circuit, may give a more dependable reading because of very high resistance of the voltmeter itself. This shows that the voltmeter has a loading effect on the circuit, which alters the original situation during the measurement.

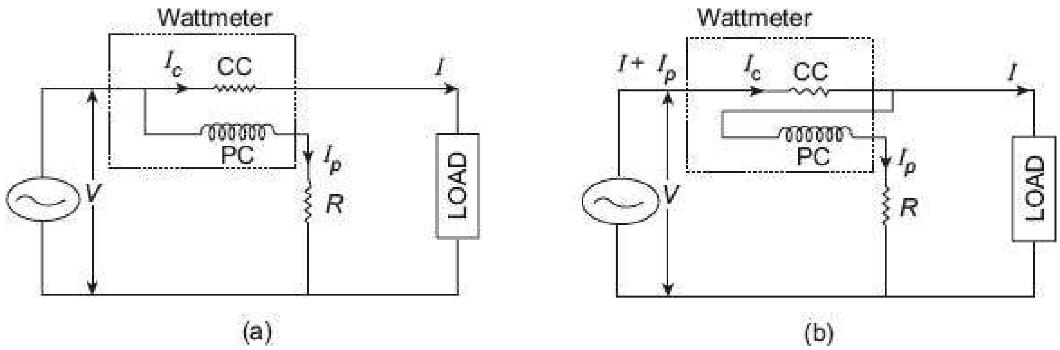
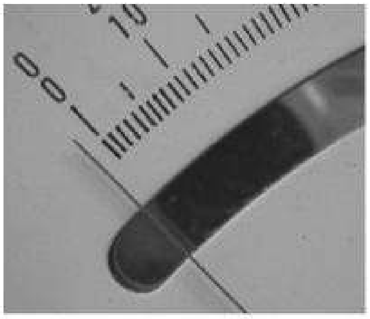
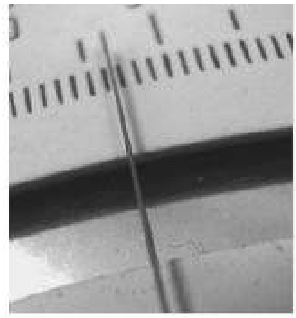


Figure 1.9 *Different connections of wattmeter*

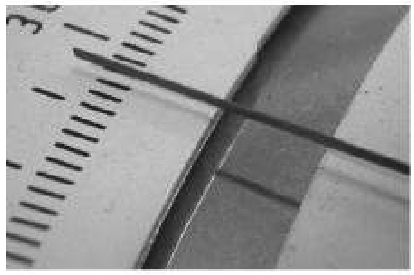
For another example, a multirange instrument has a different scale for each range. During measurements, the operator may use a scale which does not correspond to the setting of the range selector of the instrument. Gross error may also be there because of improper setting of zero before the measurement and this will affect all the readings taken during measurements. The gross error cannot be treated mathematically, so great care should be taken during measurement to avoid this error. Pictorial illustration of different types of gross error is shown in Figure 1.10.



(a) Error will occur in the measurement result if proper zero setting are not there.



(b) Gross error will occur in the measurement result if the pointer deflects between the two scaling points.



(c) Parallax error will occur in the measurement result if the pointer is not set in the vertical position.

Figure 1.10 *Different types of gross errors*

In general, to avoid gross error, at least two, three or more readings of the measurand quantity should be taken by different observers. Then if the readings differ by an unacceptably large amount, the situation can be investigated and the more erroneous readings eliminated.

2. Systematic Error

These are the errors that remain constant or change according to a definite law on repeated measurement of the given quantity. These errors can be evaluated and their influence on the results of measurement can be eliminated by the introduction of proper correction. There are two types of systematic errors:

- Instrumental error
- Environmental error

Instrumental errors are inherent in the measuring instruments because of their mechanical structure and calibration or operation of the apparatus used. For example, in D'Arsonval movement, friction in bearings of various components may cause incorrect readings. Improper zero adjustment has a similar effect. Poor construction, irregular spring tensions, variations in the air gap may also cause instrumental errors. Calibration error may also result in the instrument reading either being too low or too high.

Such instrumental errors may be avoided by

- Selecting a proper measuring device for the particular application
- Calibrating the measuring device or instrument against a standard

- Applying correction factors after determining the magnitude of instrumental errors

Environmental errors are much more troublesome as the errors change with time in an unpredictable manner. These errors are introduced due to using an instrument in different conditions than in which it was assembled and calibrated. Change in temperature is the major cause of such errors as temperature affects the properties of materials in different ways, including dimensions, resistivity, spring effect and many more. Other environmental changes also effect the results given by the instruments such as humidity, altitude, earth's magnetic field, gravity, stray electric and magnetic field, etc. These errors can be eliminated or reduced by taking the following precautions:

- Use the measuring instrument in the same atmospheric conditions in which it was assembled and calibrated.
- If the above precaution is not possible then deviation in local conditions must be determined and suitable compensations are applied in the instrumental reading.
- Automatic compensation, employing sophisticated devices for such deviations, is also possible.

3. Random Errors

These errors are of variable magnitude and sign and do not maintain any known law. The presence of random errors become evident when different results are obtained on repeated measurements of one and the same quantity. The effect of random errors is minimised by measuring the given quantity many times under the same conditions and calculating the arithmetical mean of the results obtained. The mean value can justly be considered as the most probable value of the measured quantity since random errors of equal magnitude but opposite sign are of approximately equal occurrence when making a great number of measurements.

1.9 LOADING EFFECTS

Under ideal conditions, an element used for signal sensing, conditioning, transmission and detection should not change/distort the original signal. The sensing element should not use any energy or take least energy from the process so as not to change the parameter being measured. However, under practical conditions, it has been observed that the introduction of any element in a system results invariably in extraction of the energy from the system, thereby distorting the original signal. This distortion may take the form of attenuation, waveform distortion, phase shift, etc., and consequently, the ideal measurements become impossible.

The incapability of the system to faithfully measure the input signal in undistorted form is called *loading effect.* This results in loading error.

The loading effects, in a measurement system, not only occur in the detector–transducer stage but also occur in signal conditioning and signal presentation stages as well. The loading problem is carried right down to the basic elements themselves. The loading effect may occur on account of both electrical and mechanical elements. These are due to impedances of the various elements connected in a system. The mechanical impedances may be treated similar to electrical impedances.

Sometimes loading effect occurs due to the connection of measuring instruments in an improper way. Suppose a voltmeter is connected with parallel of a very high resistance. Due to the high resistance of the voltmeter itself, the circuit current changes. This is the loading effect of a voltmeter when they are connected in parallel with a very high resistance. Similarly, an ammeter has a very low resistance. So if an ammeter is connected in series with a very low resistance, the total resistance of the circuit changes, and in succession, the circuit current also changes. This is the loading effect of ammeters when they are connected in series with very low resistance.

EXERCISE

Objective-type Questions

1. A null-type instrument as compared to a deflection-type instrument has
   (a) a lower sensitivity
   (b) a faster response
   (c) a higher accuracy
   (d) all of the above
2. In a measurement system, the function of the signal manipulating element is to
   (a) change the magnitude of the input signal while retaining its identity
   (b) change the quantity under measurement to an analogous signal
   (c) to perform non-linear operation like filtering, chopping and clipping and clamping
   (d) to perform linear operation like addition and multiplication
3. The measurement of a quantity
   (a) is an act of comparison of an unknown quantity with a predefined acceptable standard which is accurately known
   (b) is an act of comparison of an unknown quantity with another quantity
   (c) is an act of comparison of an unknown quantity with a known quantity whose accuracy may be known or may not be known
   (d) none of these
4. Purely mechanical instruments cannot be used for dynamic measurements because they have
   (a) large time constant
   (b) higher response time
   (c) high inertia
   (d) all of the above
5. A modifying input to a measurement system can be defined as an input
   (a) which changes the input–output relationship for desired as well as interfering inputs
   (b) which changes the input–output relationship for desired inputs only
   (c) which changes the input–output relationship for interfering inputs only
   (d) none of the above
6. In measurement systems, which of the following static characteristics are desirable?

(a) Sensitivity

(b) Accuracy

(c) Reproducibility

(d) All of the above

7. In measurement systems, which of the following are undesirable static characteristics?

(a) Reproducibility and nonlinearity

(b) Drift, static error and dead zone

(c) Sensitivity and accuracy

(d) Drift, static error, dead zone and nonlinearity

8. In some temperature measurement, the reading is recorded as 25.70°C. The reading has

(a) five significant figures

(b) four significant figures

(c) three significant figures

(d) none of the above

9. In the centre of a zero analog ammeter having a range of -10 A to +10 A, there is a detectable change of the pointer from its zero position on either side of the scale only as the current reaches a value of 1 A (on either side). The ammeter has a

(a) dead zone of 1 A

(b) dead zone of 2 A

(c) resolution of 1 A

(d) sensitivity of 1 A

10. A dc circuit can be represented by an internal voltage source of 50 V with an output resistance of 100 kW. In order to achieve 99% accuracy for voltage measurement across its terminals, the voltage measuring device should have

(a) a resistance of at least 10 W

(b) a resistance of 100 kW

(c) a resistance of at least 10 MW

(d) none of the above

11. In ac circuits, the connection of measuring instruments cause loading errors which may affect

(a) only the phase of the quantity being measured

(b) only the magnitude of the quantity being measured

(c) both the phase and the magnitude of the quantity

(d) magnitude, phase and also the waveform of the quantity being measured

12. A pressure gauge is calibrated form 0–50 kN/m^2. It has a uniform scale with 100 scale divisions. One fifth of the scale division can be read with certainty. The gauge has a

(a) dead zone of 0.2 kN/m^2

(b) resolution of 0.1 kN/m^2

(c) resolution of 0.5 kN/m^2

(d) threshold of 0.1 kN/m^2

13. A pressure measurement instrument is calibrated between 10 bar and 260 bar. The scale span of the instrument is

(a) 10 bar

(b) 260 bar

(c) 250 bar

(d) 270 bar

14. A Wheatstone bridge is balanced with all the four resistances equal to 1 kW each. The bridge supply voltage is 100 V. The value of one of the resistance is changed to 1010 W. The output voltage is measured with a voltage measuring device of infinite resistance. The bridge sensitivity is

(a) 2.5 mV/W

(b) 10 V/W

(c) 25 mV/W

(d) none of the above

15. The main advantage of the null balance technique of measurement is that

(a) it gives a quick measurement

(b) it does not load the medium

(c) it gives a centre zero value at its input

(d) it is not affected by temperature variation

16. The smallest change in a measured variable to which an instrument will respond is

(a) resolution

(b) precision

(c) sensitivity

(d) accuracy

17. The desirable static characteristics of a measurement are

(a) precision

(b) accuracy

(c) sensitivity

(d) all of these

18. The errors mainly caused by human mistakes are

(a) systematic error

(b) instrumental error

(c) observational error

(d) gross error

19. Systematic errors are

(a) environmental error

(b) observational error

(c) instrumental error

(d) all of the above

20. An analog ammeter is

(a) an absolute instrument

(b) an indicating instrument

(c) a controlling instrument

(d) a recording instrument

Answers

1. (c)	2. (a)	3. (a)	4. (d)	5. (a)	6. (d)	7. (d)
8. (b)	9. (b)	10. (c)	11 (d)	12. (b)	13. (c)	14. (c)
15. (b)	16. (a)	17. (d)	18. (d)	19. (d)	20. (b)	

Short-answer Questions

1. Compare the advantages and disadvantages of electrical and mechanical measurement systems.
2. Explain the various classes of measuring instruments with examples.
3. Differentiate clearly between absolute and secondary instruments.
4. Explain analog and digital modes of operation. Why are the digital instruments becoming popular now? What is meant by ADC and DAC?
5. Briefly define and explain all the static characteristics of measuring instruments.
6. Explain *loading effect* in measurement systems.
7. Explain the terms *accuracy*, *sensitivity* and *resolution* as used for indicating instruments.
8. What are the different types of errors in a measuring instrument? Describe their source briefly.
9. What are fundamental and derived units? Briefly explain them.
10. What are the differences between primary and secondary standards?

Long-answer Questions

1. (a) What is measurement? What is meant by the term measurand? What is a measuring instrument?

   (b) Write down the important precautions that should be taken while carrying out electrical measurements.

   (c) With an example, explain the term *loading effect* in a measurement system.

2. (a) Explain the terms:

   (i) Measurement
   (ii) Accuracy
   (iii) Precision
   (iv) Sensitivity
   (v) Reproducibility

   (b) Define *random errors* and explain how they are analysed statistically.

3. (a) What are environmental, instrumental and observational errors? Briefly explain each of them.

   (b) Three resistors have the following ratings:

   R_1 = 47 W ± 4%, R_2 = 65 W ± 4%, R_3 = 55 W ± 4%

   Determine the magnitude and limiting errors in ohms and in percentage of the resistance of these resistors connected in series.

4. (a) What is the necessity of units in measurements? What are various SI units?

   (b) Define the terms *units*, *absolute units*, *fundamental units* and *derived units* with suitable examples.

   (c) What is systematic error and how can we reduce it?

5. (a) Distinguish between international, primary, secondary and working standards.

   (b) What are the primary standards for time and frequency? Briefly discuss each of them.

   (c) Describe the working principle, operation and constructional detail of a primary standard of emf.

2 Analog Meters

2.1 INTRODUCTION

An analog device is one in which the output or display is a continuous function of time and bears a constant relation to its input. Measuring instruments are classified according to both the quantity measured by the instrument and the principle of operation. Three general principles of operation are available: (i) electromagnetic, which utilises the magnetic effects of electric currents; (ii) electrostatic, which utilises the forces between electrically charged conductors; (iii) electro-thermal, which utilises the heating effect.

Electric measuring instruments and meters are used to indicate directly the value of current, voltage, power or energy. In this chapter, we will consider an electromechanical meter (input is as an electrical signal which results in mechanical force or torque as an output) that can be connected with additional suitable components in order to act as an ammeter and a voltmeter. The most common analog instrument or meter is the permanent magnet moving coil instrument and it is used for measuring a dc current or voltage of an electric circuit. On the other hand, the indications of alternating current ammeters and voltmeters must represent the rms values of the current, or voltage, respectively, applied to the instrument.

2.2 CLASSIFICATION OF ANALOG INSTRUMENTS

In a broad sense, analog instruments may be classified into two ways:

1. Absolute instruments
2. Secondary instruments

Absolute instruments give the value of the electrical quantity to be measured in terms of the constants of the instruments and to its deflection, no comparison with another instrument being required. For example, the tangent galvanometer gives the value of the current to be measured in terms of the tangent of the angle of deflection produced by the current, the radius and the number of turns of galvanometer coil, and the horizontal component of the earth's magnetic field. No calibration of the instrument is thus necessary.

Secondary instruments are so constructed that the value of current, voltage or other quantity to be measured can be determined from the deflection of the instruments, only if the latter has been calibrated by comparison with either an absolute instrument or one which has already been calibrated. The deflection obtained is meaningless until such a calibration has been made.

This class of instruments is in most general use, absolute instrument being seldom used except in standard laboratories and similar institutions.

The secondary instruments may be classifies as

1. Indicating instruments
2. Recording instruments
3. Integrating instruments

Indicating instruments are instruments which indicate the magnitude of a quantity being measured. They generally make use of a dial and a pointer for this purpose.

Recording instruments give a continuous record of the quantity being measured over a specified period. The variation of the quantity being measured are recorded by a pen (attached to the moving system of the instrument; the moving system is operated by the quantity being measured) on a sheet of paper that moves perpendicular to the movement of the pen.

Integrating instruments record totalised events over a specified period of time. The summation, which they give, is the product of time and an electrical quantity. Ampere hour and watt hour (energy) meters are examples of this category.

2.3 PRINCIPLE OF OPERATION

Analog instruments may be classified according to the principle of operation they utilise. The effects they utilise are

1. Magnetic effect
2. Heating effect
3. Electrostatic effect
4. Electromagnetic effect
5. Hall effect

The majority of analog instruments including moving coil, moving iron and electrodynamic utilise the magnetic effect. The effect of the heat produced by a current in a conductor is used in thermocouple and hotwire instruments. Electrostatic effect is used in electrostatic voltmeters. The electromagnetic induction effect is used in induction wattmeters and induction energy meters.

2.4 OPERATING TORQUES

Three types of torques are needed for satisfactory operation of any indicating instrument. These are

- Deflecting torque
- Controlling torque
- Damping torque

2.4.1 Deflecting Torque/Force

Any instrument's deflection is found by the total effect of the deflecting torque/force, control torque/ force and damping torque/force. The deflecting torque's value is dependent upon the electrical signal to be measured; this torque/force helps in rotating the instrument movement from its zero position. The system producing the deflecting torque is called the *deflecting system.*

2.4.2 Controlling Torque/Force

The act of this torque/force is opposite to the deflecting torque/force. When the deflecting and controlling torques are equal in magnitude then the movement will be in definite position or in equilibrium. Spiral springs or gravity is usually given to produce the controlling torque. The system which produces the controlling torque is called the *controlling system*. The functions of the controlling system are

- To produce a torque equal and opposite to the deflecting torque at the final steady position of the pointer in order to make the deflection of the pointer definite for a particular magnitude of current
- To bring the moving system back to its zero position when the force causing the instrument moving system to deflect is removed

The controlling torque in indicating instruments is almost always obtained by a spring, much less commonly, by gravity.

2.4.3 Damping Torque/Force

A damping force generally works in an opposite direction to the movement of the moving system. This opposite movement of the damping force, without any oscillation or very small oscillation brings the moving system to rest at the final deflected position quickly. Air friction, fluid friction and eddy currents provide the damping torque/force to act. It must also be noted that not all damping force affects the steady-state deflection caused by a given deflecting force or torque. With the angular velocity of the moving system, the intensity of the damping force rises; therefore, its effect is greatest when it rotates rapidly and zero when the system rotation is zero. In the description of various types of instruments, detailed mathematical expressions for the damping torques are taken into consideration.

When the deflecting torque is much greater than the controlling torque, the system is called underdamped. If the deflecting torque is equal to the controlling torque, it is called *critically damped*. When deflecting torque is much less than the controlling torque, the system is under overdamped condition. Figure 2.1 shows the variation of deflection (d) with time for underdamped, critically damped and overdamped systems.

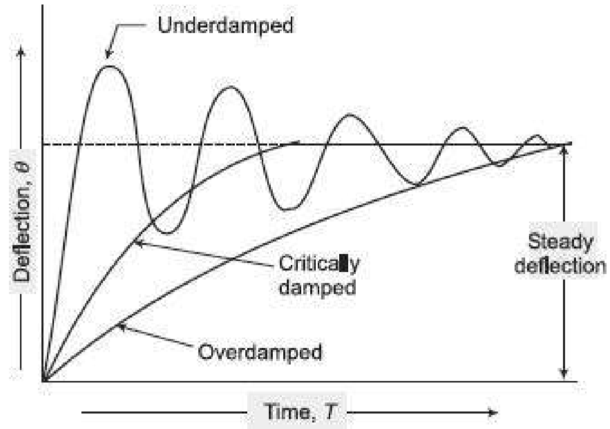


Figure 2.1 *Damping torque curve*

2.5 CONSTRUCTIONAL DETAILS

2.5.1 Moving System

The moving system should have the following properties:

- The moving parts should be light.
- The frictional force should be minimum.

These requirements should be fulfilled in order that power required by the instrument for its operation is small. The power is proportional to the weight of the moving parts and the frictional forces opposing the movement. The moving system can be made light by using aluminium as far as possible. The frictional forces are reduced by using spindle-mounted jewel bearings and by carefully balancing the system.

The force or torque developed by the moving element of an electrical instrument is necessarily small in order that the power consumption of the instrument is kept low so that the introduction of the instrument into a circuit may cause minimum change in the existing circuit conditions. Because of low power levels, the considerations of various methods of supporting the moving elements becomes of vital importance. With the operating force being small, the frictional force must be kept to a minimum in order that the instrument reads correctly and is not erratic in action and is reliable. Supports may be of the following types:

- Suspension
- Taut suspension
- Pivot and jewel bearings

1. *Suspension*

It consist of a fine, ribbon-shaped metal filament for the upper suspension and a coil of fine wire for the lower part. The ribbon is made of a spring material like beryllium copper or μHosphor bronze. This coiling of lower part of suspension is done in order to give negligible restrain on the moving system. The type of suspension requires careful leveling of the instrument, so that the moving system hangs in correct vertical position. This construction is, therefore, not suited to field use and is employed only in those laboratory applications in which very great sensitivity is required. In order to prevent shocks to the suspension during transit, etc., a clamping arrangement is employed for supporting the moving system.

2. *Taut Suspension*

A suspension type of instrument can only be used in vertical position. The taut suspension has a flat ribbon suspension both above and below the moving element, with suspension kept under tension by a spring arrangement (Figure 2.2). The advantage of this type of suspension is that exact levelling is not required if the moving system is properly balanced.

Suspension and taut suspension are customarily used in instruments of galvanometer class which requires a low friction and high sensitivity mechanism. But actually there is no strict line of demarcation between a galvanometer and other indicating instruments. Some sensitive wattmeters and electrostatic voltmeters use flexible suspension.

Ribbon suspension, in addition to supporting the moving element, exerts a controlling torque when twisted. Thus, the use of suspension results in elimination of pivots, jewels, control springs and therefore, pivotless instruments are free from many defects.

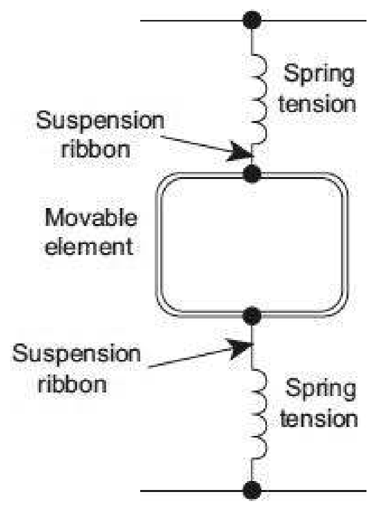


Figure 2.2 *Taut suspension*

3. *Pivot and Jewel Bearings*

The moving system is mounted on a spindle made of hardened steel. The two ends of the spindle are made conical and then polished to form pivots. These ends fit conical holes in jewels located in the fixed part of instruments (Figure 2.3). These jewels, which are preferably made of sapphire, form bearings.

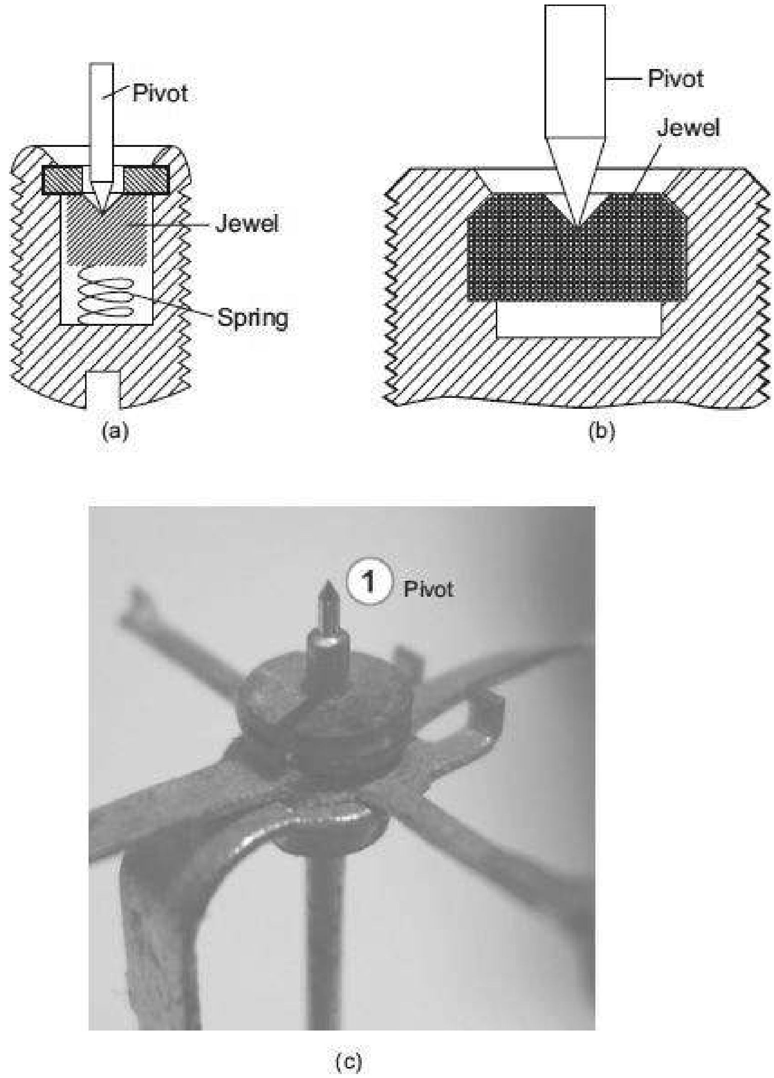


Figure 2.3 (*a) Spring-loaded jewel bearing (b) Jewel bearing (c) Pivot*

It has been found that the frictional torque, for jewel bearings, is proportional to the area of contact between the pivot and jewel. Thus, the contact area between a pivot and jewel should be small. The pivot is ground to a cone and its tip is rounded to a hemispherical surface of small area. The jewel is ground to a cone of somewhat larger angle.

4. *Torque/Weight Ratio*

The frictional torque in an instrument depends upon the weight of moving parts. If the weight of the moving parts is large, the frictional torque will be large. The frictional torque exerts a considerable influence on the performance of an indicating instrument. If the frictional torque is large and is comparable to a considerable fraction of the deflecting torque, the deflection of the moving system will depend upon the frictional torque to an appreciable extent. Also, the deflection will depend on the direction from which the equilibrium position is approached and will be uncertain. On the other hand, if the frictional torque is very small compared with the deflecting torque, its effect on deflection is negligible. Thus, the ratio of deflecting torque to frictional torque is a measure of reliability of the instrument indications and is the inherent quality of the design. Hence (deflecting) torque/weight ratio of an instrument is an index of its performance. The higher the ratio, the better will be its performance.

2.5.2 Controlling System

The controlling torque is provided by a spring or sometimes by gravity.

1. *Spring Control*

A hair-spring, usually of phosphor-bronze attached to the moving system, is used in indicating instruments for control purpose, the schematic arrangement being shown in Figure 2.4(a) and the actual controlling spring used in the instrument is shown in Figure 2.4(b).

To give a controlling torque which is directly proportional to the angle of deflection of the moving system, the number of turns on the spring should be fairly large, so that the deflection per unit length is small. The stress in the spring must be limited to such a value that there is no permanent set.

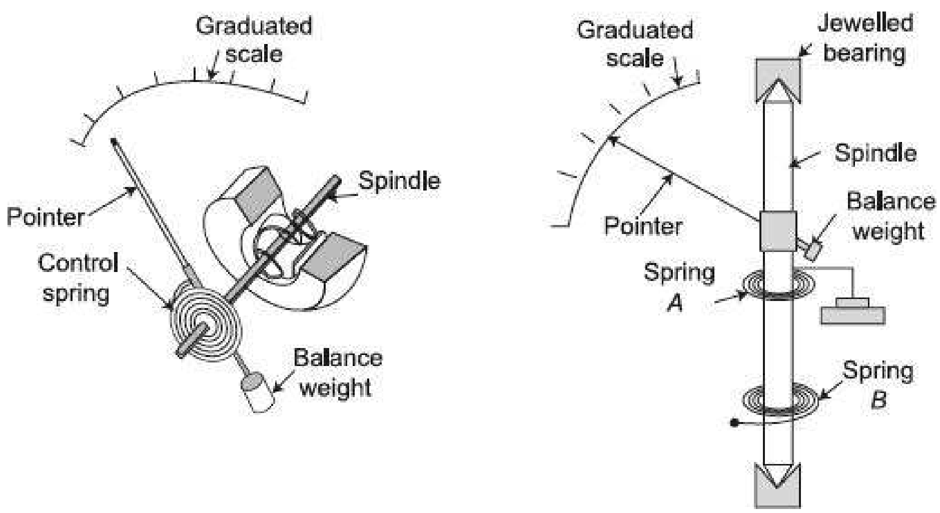


Figure 2.4(a) *Spring control*

Figure 2.4(b) *Spring control in an instrument*

Suppose that a spiral spring is made up of a total length L m of strip whose cross-section is rectangular, the radial thickness being t m and the depth b m. Let E be Young's modulus (N/m^2) for the material of the spring. Then, if θ radians be the deflection of the moving system to which one end of the spring is being attached, the expression for the controlling torque is

$$T_c = \frac{Ebt^3}{12L}\theta \qquad (2.1)$$

Thus, controlling torque ∞ θ ∞ instrument deflection.

2. *Gravity Control*

In a gravity-controlled instrument, a small weight is attached to the moving system in such a way that it produces a restoring or controlling torque when the system is deflected. This is illustrated in Figure 2.5. The controlling torque, when the deflection is θ, is $\omega l \sin\theta$, where W is the control weight and l its distance from the axis of rotation of the moving system, and it is, therefore, proportional only to the *sine* of the angle of deflection, instead of, as with spring control, being directly proportional to the angle of deflection. Gravity-controlled instruments must obviously be used in a vertical position in order that the control may operate.

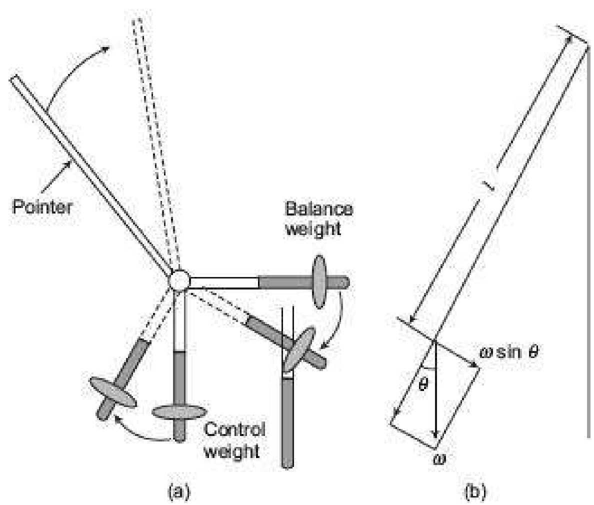


Figure 2.5 *Gravity control*

3. *Comparison of Spring and Gravity Control*

Gravity control has the following advantages when compared with spring control:

- It is cheaper
- Independent of temperature
- Does not deteriorate with time

Consider an instrument in which the deflecting torque T_D is directly proportional to the current (say) to be measured.

Thus, if I is the current,

$$T_D = kI, \text{ (where } k \text{ is a constant)} \qquad (2.2)$$

If the instrument is spring-controlled, the controlling torque being T_C, when the deflection is θ,

$T_C = k_s\theta$ (k_s is spring constant)

Also, $T_C = T_D$

or $k_s\theta = kI$

$$\text{or} \quad k_s\theta = kI$$

$$\therefore \quad \theta = \frac{k}{k_s} \cdot I \tag{2.3}$$

Thus, the deflection is proportional to the current throughout the scale.

Now if the same instrument is gravity controlled,

$T_c = k_g \sin\theta$ (k_g is a constant that depends upon the control weight and its distance from the axis of rotation of the moving system).

And $T_C = T_D = kI$

$$\therefore \quad kg \sin\theta = kI$$

$$\sin\theta = \frac{k}{k_g} \cdot I$$

$$\theta = \sin^{-1}\left(\frac{k}{k_g} \cdot I\right) \tag{2.4}$$

Thus, a gravity-controlled instrument would have a scale which is ‘cramped’ at its lower end instead of being uniformly divided, though the deflecting torque is directly proportional to the quantity to be measured.

2.5.3 Damping System

There are three systems of damping generally used. These are as follows:

- Air-friction damping
- Fluid-friction damping
- Eddy-current damping

1. *Air-Friction Damping*

In this method, a light aluminium piston is attached to the moving system and moves in an air chamber closed at one end, as shown in Figure 2.6. The cross-section of this chamber may be either circular or rectangular. The clearance between the piston and the sides of the chamber should be small and uniform. If the piston is moving rapidly into the chamber, the air in the closed space is compressed and the pressure opposes the motion of the piston (and, therefore, of the whole moving system). If the piston is moving out of the chamber rapidly, the pressure in the closed space falls, and the pressure on the open side of the piston is greater than that on the opposite side. Motion is thus again opposed. Sometimes instead of a piston, a vane, mounted on the spindle of the moving system, moves in a closed-sector-shaped box as shown in Figure 2.7.

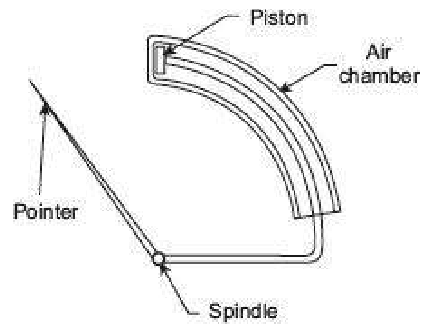


Figure 2.6 *Open-end air friction damping*

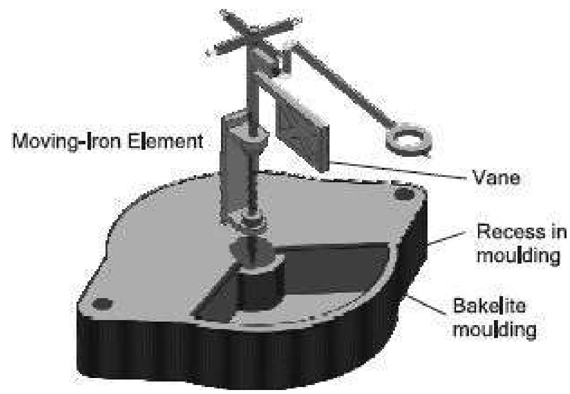


Figure 2.7 *Air-friction damping using vane*

2. *Fluid-Friction Damping*

In this type of damping, a light vane, attached to the spindle of the moving system, dips into a pot of damping oil and should be completely submerged by the oil. This is illustrated in Figure 2.8(a). The frictional drag in the disc is always in the direction opposing motion. There is no friction force when the disc is stationary. In the second system [Figure 2.8(b)], increased damping is obtained by the use of vanes.

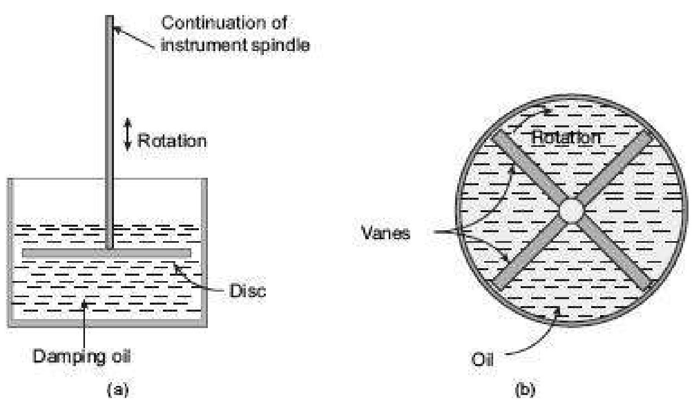


Figure 2.8 *Fluid-friction damping*

3. Eddy-Current Damping

When a sheet of conducting material moves in a magnetic field so as to cut through lines of force, eddy currents are set up in it and a force exists between these currents and the magnetic field, which is always in the direction opposing the motion. The force is proportional to the magnitude of the current and to the strength of the field. The magnitude of the current is proportional to the velocity of movement of the conductor, and thus, if the magnetic field is constant, the damping force is proportional to the velocity of the moving system and is zero when there is no movement of the system.

(i) Eddy-Current Damping Torque of Metal Former Figure 2.9 shows a metallic former moving in the field of a permanent magnet.

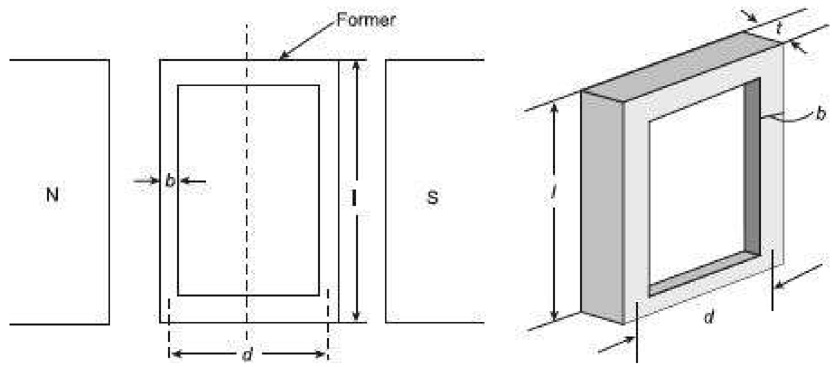


Figure 2.9 *Eddy-current damping on a metal former*

Let,

B = strength of magnetic field; (wb/m^2)

ω = asngular speed of former; (rad/s)

l = length of former; (m)

t = thickness of former; (m)

b = width of former; (m)

d = breadth of former; (m)

ρ = resistivity of material of former; (W m)

Linear velocity of former $v = \left(\frac{d}{2}\right)\omega$ (2.5)

[since linear velocity = radius × angular velocity]

Dynamically generated emf in the former

$$E_e = 2Blv = 2Bl\frac{d}{2}\omega = Bld\omega \quad (2.6)$$

Resistance of path of eddy current $R_e = \frac{\rho 2(d+l)}{bt}$ (2.7)

Eddy current $I_e = \frac{E_e}{R_e} = \frac{Blbtd\omega}{2\rho(d+l)}$ (2.8)

∴ damping force $F_d = BI_e l = \frac{B^2 l^2 btd\omega}{2\rho(d+l)}$ (2.9)

damping torque $T_d = F_d \times d = \frac{B^2 l^2 btd^2\omega}{2\rho(d+l)}$ (2.10)

damping constant $k_d = \frac{T_d}{\omega} = \frac{B^2 l^2 btd^2}{2\rho(d+l)}$ Nm/rad s^{-1} (2.11)

(ii) Eddy-Current Damping Torque of Metal Disc Figure 2.10 shows a metallic disc rotating in the field of a permanent magnet.

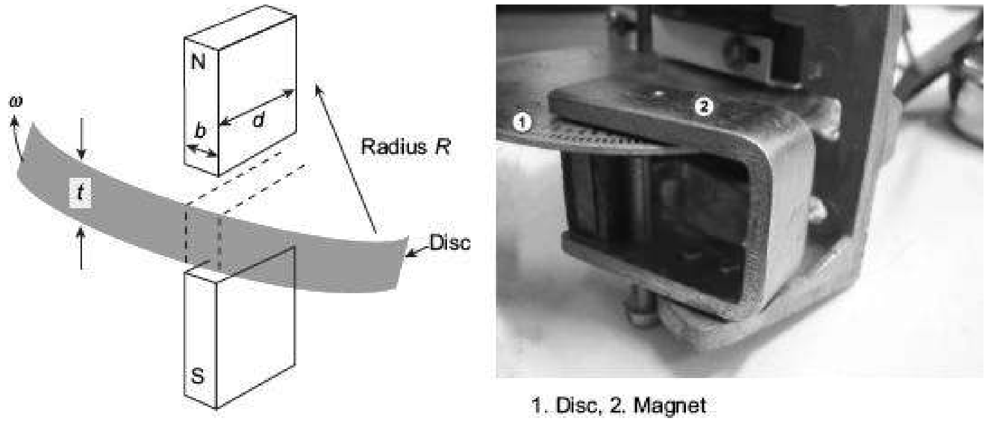


Figure 2.10 *Eddy-current damping on metallic disc* 2

Let, B = flux density of magnetic field; (wb/m^2)

ω = angular speed of disc; (rad/s)

t = thickness of disc; (m)

b = width of permanent magnet; (m)

d = length of permanent magnet; (m)

ρ = resistivity of material of disc; (Ω m)

R = radius measured from centre of pole to centre of disc; (m)

Considering the emf is induced in the disc under the pole face only, therefore, emf induces in the portion below the magnet

$$E_c = Blv = BdR\omega \tag{2.12}$$

[since 'l', length of the portion of the disc under the magnetic field = d]

$$\text{Resistance of eddy-current path under the pole} = \frac{\rho d}{bt} \tag{2.13}$$

Actual path for eddy current is not limited to the portion of the disc under the magnet but is greater than this. Therefore, to take this factor into account, the actual resistance is taken as k times of $\frac{\rho d}{bt}$.

$$\text{Therefore, resistance of eddy-current path } R_c = k\frac{\rho d}{bt} \tag{2.14}$$

where k is a constant which depends upon radial position of the disc and poles.

$$\text{Eddy current} \quad I_e = \frac{E_c}{R_c} = \frac{BRbt\omega}{k\rho} \tag{2.15}$$

$$\text{Damping force} \quad F_D = B \times I_e \times d = \frac{B^2 Rdbt\omega}{k\rho}(N) \tag{2.16}$$

$$\text{Damping torque} \quad T_D = F_D \times R = \frac{B^2 R^2 dbt\omega}{k\rho}(\text{N-m}) \tag{2.17}$$

$$\text{Damping constant} \quad K_D = \frac{T_D}{\omega} = \frac{B^2 R^2 dbt}{k\rho}(\text{N-m/rad s}^{-1}) \tag{2.18}$$

2.6 PERMANENT MAGNET MOVING COIL INSTRUMENT

Basic range: 10 μA-100 mA

Coil resistance: 10 Ω-1 kΩ

Usage:

- dc PMMC ammeters and voltmeters
- ac PMMC ammeters and voltmeters (with rectifiers)

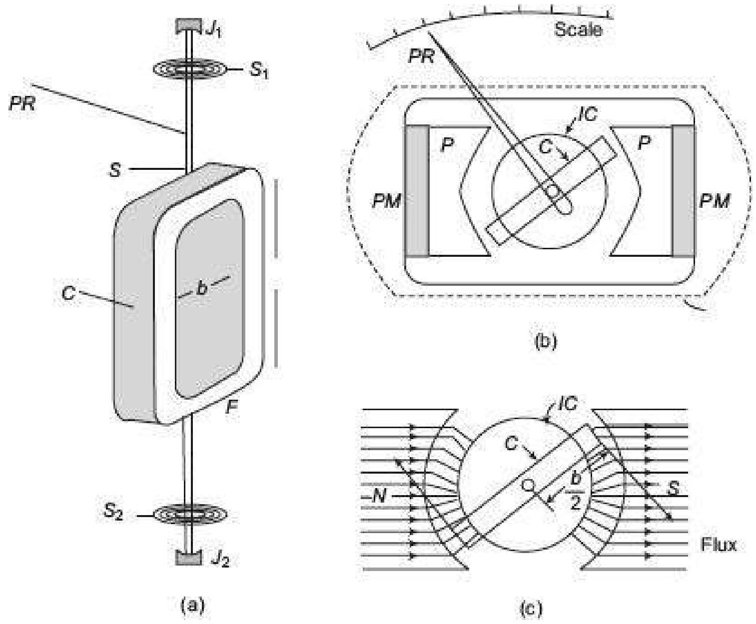
2.6.1 Principle of Operation

The principle on which a Permanent Magnet Moving Coil (PMMC) instrument operates is that a torque is exerted on a current-carrying coil placed in the field of a permanent magnet. A PMMC instrument is shown in Figure 2.11. The coil C has a number of turns of thin insulated wires wound on a rectangular aluminium former F. The frame is carried on a spindle S mounted in jewel bearings J_1, J_2. A pointer PR is attached to the spindle so that it moves over a calibrated scale. The whole of the moving system is made as light in weight as possible to keep the friction at the bearing to a minimum.

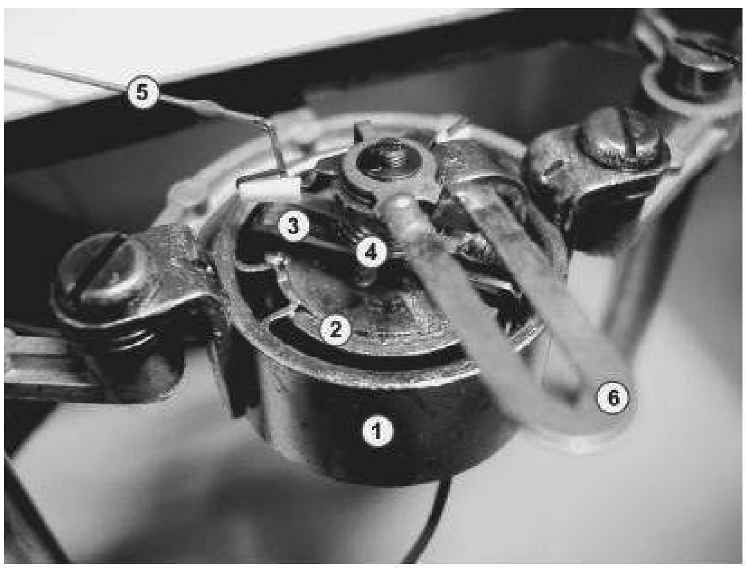
The coil is free to rotate in air gaps formed between the shaped soft-iron pole piece (pp) of a permanent magnet PM and a fixed soft-iron cylindrical core IC [Figure 2.11(b)]. The core serves two purposes; (a) it intensifies the magnetic field by reducing the length of the air gap, and (b) it makes the field radial and uniform in the air gap.

Thus, the coil always moves at right angles to the magnetic field [Figure 2.11(c)]. Modern permanent magnets are made of steel alloys which are difficult to machine. Soft-iron pole pieces (pp) are attached to the permanent magnet PM for easy machining in order to adjust the length of the air gap. Figure 2.11(d) shows the internal parts and Figure 2.11(e) shows schematic of internal parts of a moving-coil instrument.

A soft-iron yoke (Y) is used to complete the flux path and to provide shielding from stray external fields.



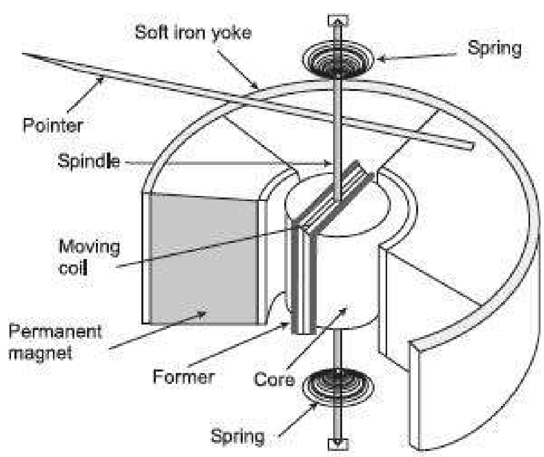
Permanent magnet moving coil instrument



1. External shield
2. Permanent magnet
3. Moving coil
4. Control spring
5. Pointer
6. Arrangement for zero balance of the pointer

(d)

Photograph of different components of a PMMC instrument



(e)

Internal construction of PMMC instruments

Internal construction of PMMC instruments **Figure 2.11**

2.6.2 Deflecting Torque Equation of PMMC Instrument

Let, B = flux density in the air gap (wb/m^2)

i = current in the coil (A)

l = effective axial length of the coil (m)

b = breadth of the coil (m)

n = number of turns of the coil.

Force on one side of the coil is

$$F = Biln \text{ (N)} \tag{2.19}$$

Torque on each side of the coil,

$$\begin{aligned} T &= \text{force} \times \text{distance from axis of rotation} \\ &= F \times b/2 \\ &= Biln \times b/2 \end{aligned} \tag{2.20}$$

Total deflecting torque exerted on the coil,

$$\begin{aligned} T_d &= 2 \times T = 2iln \times b/2 \\ &= Bilnb \text{ (N-m)} \end{aligned} \tag{2.21}$$

For a permanent magnet, B is constant. Also, for a given coil l, b and n are constants and thus the product (*Blnb)* is also a constant, say k_1.

$$\text{Therefore,} \quad T_d = k_1 \times i \tag{2.22}$$

1. Control Torque The control on the movement of the pointer over the scale is provided by two spirally wound, phosphor-bronze springs S_1 and S_2, one at each end of the spindle S. Sometimes these springs also conduct the current into and out of the coil. The control torque of the springs is proportional to the angle θ turned through by the coil.

$$T_c = k_s \times \theta \tag{2.23}$$

where T_C is the control torque and k_s is the spring constant.

At final steady state position, Control torque = Deflecting torque

$$\therefore \quad T_c = T_d$$

$$k_s\theta = k_1 i$$

$$\text{or} \quad \boxed{\theta = \frac{k_1}{k_s} i = ki} \tag{2.24}$$

$$\text{where } k = \frac{k_1}{k_s} = \text{constant}$$

So, angular deflection of the pointer is directly proportional to the current. Thus the scale of the instrument is linear or uniformly divided.

2. *Damping Torque* When the aluminium former (F) moves with the coil in the field of the permanent magnet, a voltage is induced, causing eddy current to flow in it. These current exerts a force on the former. By Lenz’s law, this force opposes the motion producing it. Thus, a damping torque is obtained. Such a damping is called eddy-current damping.

2.6.3 Swamping Resistor

The coil of the instrument is made of copper. Its resistance varies with temperature. A resistor of low temperature coefficients, called the swamping resistor, is connected in series with the coil. Its resistance practically remains constant with temperature. Hence the effect of temperature on coil resistance is swamped by this resistor.

Advantages of PMMC Instruments

1. Sensitive to small current
2. Very accurate and reliable
3. Uniform scale up to 270° or more
4. Very effective built in damping
5. Low power consumption, varies from 25 μW to 200 μW
6. Free from hysteresis and not effected by external fields because its permanent magnet shields the coil from external magnetic fields
7. Easily adopted as a multirange instrument

Disadvantages of PMMC Instruments

1. This type of instrument can be operated in direct current only. In alternating current, the instrument does not operate because in the positive half, the pointer experiences a force in one direction and in the negative half the pointer experiences the force in the opposite direction. Due to the inertia of the pointer, it retains it’s zero position.
2. The moving system is very delicate and can easily be damaged by rough handling.
3. The coil being very fine, cannot withstand prolonged overloading.
4. It is costlier.
5. The ageing of the instrument (permanent magnet and control spring) may introduce some errors.

Example 2.1

The coil of a PMMC instrument has 60 turns, on a former that is 18 mm wide, the effective length of the conductor being 25 mm. It moves in a uniform field of flux density 0.5 Tesla. The control spring constant is 1.5 × 10^{-6} Nm/degree. Calculate the current required to produce a deflection of 100 degree.

Solution Total deflecting torque exerted on the coil,

$$T_d = Bilnb \text{ (N-m)}$$

$$= 0.5 \times i \times 25 \times 10^{-3} \times 60 \times 18 \times 10^{-3}$$

The control torque of the springs is

$T_C = k_s \times \theta$

$= 1.5 \times 10^{-6} \times 100$

At equilibrium, $T_d = T_C$

$= 0.5 \times i\ x\ 18 \times 10^{-3} \times 25 \times 10^{-3} \times 60 = 1.5 \times 10^{-6} \times 100\ 1.5 \times 10^{-6} \times 100$

$$i = \frac{1.5 \times 10^{-6} \times 100}{0.5 \times 18 \times 10^{-3} \times 25 \times 10^{-3} \times 60} = 11.11 \text{ mA}$$

Example 2.2

A PMMC instrument has a coil of dimensions 15 mm × 12 mm. The flux density in the air gap is 1.8 × 10^{-3} wb/m^2 and the spring constant is 0.14 × 10^{-6} N-m/rad. Determine the number of turns required to produce an angular deflection of 90° when a current of 5 mA is flowing through the coil.

Solution Total deflecting torque exerted on the coil,

$T_d = Bilnb$ (N-m)

$= 1.8 \times 10^{-3} \times 5 \times 10^{-3} \times 15 \times 10^{-3} \times 12 \times 10^{-3} \times n$

The control torque of the springs is

$T_C = k_s \times 6$

$= 0.14 \times 10^{-6} \times 90 \times \pi/180$

At equilibrium, $T_d = T_C$

$1.8 \times 10^{-3} \times 5 \times 10^{-3} \times 15 \times 10^{-3} \times 12 \times 10^{-3} \times n = 0.14 \times 10^{-6} \times 90 \times \pi/180$

$$n = \frac{0.14 \times 10^{-6} \times 90 \times \pi/180}{1.8 \times 10^{-3} \times 5 \times 10^{-3} \times 15 \times 10^{-3} \times 12 \times 10^{-3}} = 136$$

Example 2.3

A PMMC voltmeter with a resistance of 20 Ω gives a full-scale deflection of 120° when a potential difference of 100 mV is applied across it. The moving coil has dimensions of 30 mm × 25 mm and is wound with 100 turns. The control spring constant is 0.375 × 10^{-6} N-m/degree. Find the flux density in the air gap. Find also the dimension of copper wire of coil winding if 30% of the instrument resistance is due to coil winding. The specific resistance of copper is 1.7 × 10^{-8} Ωm.

Solution Full-scale deflecting current

$$i = \frac{100}{20} \times 10^{-3} = 5 \times 10^{-3} \text{ A}$$

Total deflecting torque exerted on the coil,

$$T_d = Bilnb \text{ (N-m)}$$

$$= B \times 5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100$$

The control torque of the springs is

$$T_C = k_s \times \theta$$

$$= 0.375 \times 10^{-6} \times 120$$

At equilibrium, $T_d = T_C$

$$B \times 5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100 = 0.375 \times 10^{-6} \times 120$$

$$B = \frac{0.375 \times 10^{-6} \times 120}{5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100} = 0.12 \text{ wb/m}^2$$

Coil winding resistance = 20 × 0.3 = 6 Ω

If the copper wire has a cross-sectional area of *a* m then

$n\rho \frac{l}{a} = R$ [where n be the number of turns, ρ is the resistivity of the copper wire, l is the length of the wire and a is the cross-sectional area]

$$100 \times 1.7 \times 10^{-8} \times \frac{2 \times (30 + 25) \times 10^{-3}}{a} = 6$$

$$a = 100 \times 1.7 \times 10^{-8} \times \frac{2 \times (30 + 25) \times 10^{-3}}{6} = 31.16 \times 10^{-3} \text{ mm}^2$$

If d be the diameter of the copper wire then

$$d = \sqrt{\frac{4 \times 31.16 \times 10^{-3}}{\pi}} = 0.199 \text{ mm}$$

Example 2.4

The coil of a moving-coil voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts a torque of 240 × 10^{-6} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is 1 wb/m^2, estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.

Solution Let the full scale deflecting current be I amp.

Total deflecting torque exerted on the coil,

$$T_d = Bilnb \text{ (N-m)}$$

$$= 1 \times I \times 40 \times 10^{-3} \times 30 \times 10^{-3} \times 100$$

The control torque of the springs is

$$T_C = k_s \times \theta$$

$= 240 \times 10^{-6}$

At equilibrium, $T_d = T_C$

$$1 \times I \times 40 \times 10^{-3} \times 30 \times 10^{-3} \times 100 = 240 \times 10^{-6}$$

$$I = \frac{240 \times 10^{-6}}{1 \times 40 \times 10^{-3} \times 30 \times 10^{-3} \times 100} = 0.002 \text{ A}$$

If R be the series resistance,

$$R = \frac{V}{I} = \frac{100 \times 1}{0.002} = 50 \times 10^3 \ \Omega$$

2.7 EXTENSION OF RANGE OF PMMC INSTRUMENTS

2.7.1 Ammeter Shunts

The moving-coil instrument has a coil wound with very fine wire. It can carry only few mA safely to give full-scale deflection. For measuring higher current, a low resistance is connected in parallel to the instrument to bypass the major part of the current. The low resistance connected in parallel with the coil is called a *shunt*. Figure 2.12 shows a shunt resistance R_{sh} connected in parallel with the basic meter.

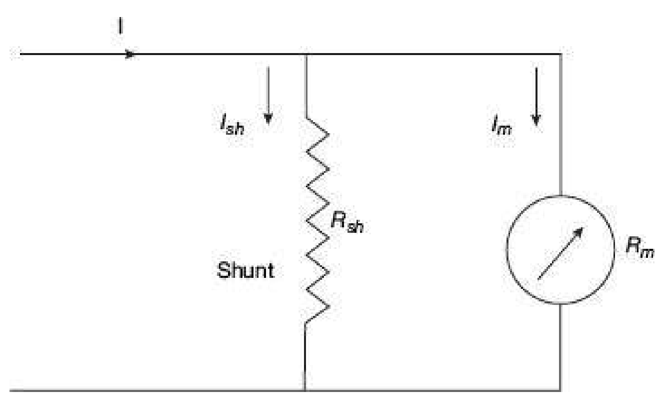


Figure 2.12 Extension of PMMC ammeter using shunt

The resistance of the shunt can be calculated using conventional circuit analysis.

R_{sh} = shunt resistance (Ω)

R_m = coil resistance (Ω)

$I_m = Ifs$ = full-scale deflection current (A)

1_{sh} = shunt current (A)

I = current to be measured (A)

The voltage drop across the shunt and the meter must be same as they are connected in parallel.

$$\therefore \quad I_{sh}R_{sh} = I_mR_m$$

$$\text{Again} \quad I = I_{sh} + I_m \tag{2.25}$$

$$\therefore \quad I_{sh} = I - I_m$$

From Eq. (2.25),

$$R_{sh} = \frac{I_m}{I_{sh}} R_m$$

$$\therefore \quad R_{sh} = \frac{I_m}{I - I_m} R_m \tag{2.26}$$

The ratio of the total current to the current in the meter is called *multiplying power of shunt.* Multiplying power,

$$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$$\therefore \quad R_{sh} = \frac{R_m}{m-1}$$

2.7.2 Voltmeter Multipliers

For measuring higher voltages, a high resistance is connected in series with the instrument to limit the current in the coil to a safe value. This value of current should never exceed the current required to produce the full scale deflection. The high resistance connected in series with the instrument is called a *multiplier.* In Figure 2.13, R_{sc} is the multiplier.

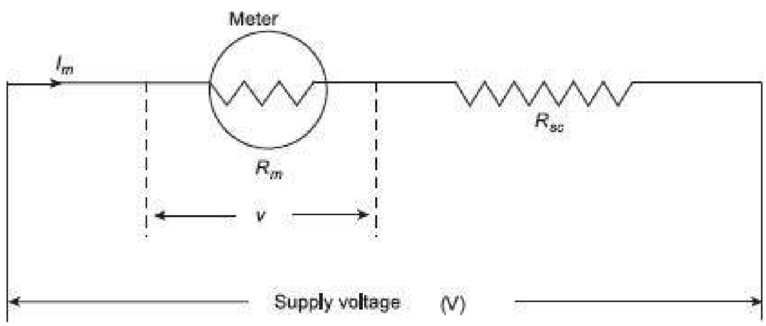


Figure 2.13 *Extension of PMMC voltmeter using multiplier*

The value of multiplier required to extend the voltage range, is calculated as under:

R_{sc} = multiplier resistance (Ω)

R_m = meter resistance (Ω)

$I_m = Ifs$ = full scale deflection current (A)

v = voltage across the meter for producing current I_m (A)

V = voltage to be measured (A)

$$V = I_m R_m$$

$$V = I_m(R_m + R_{sc})$$

$$\therefore \quad R_{sc} = \frac{V - I_mR_m}{I_m} = \frac{V}{I_m} - R_m$$

Now multiplying factor for multiplier

$$m = \frac{V}{v} = \frac{I_m(R_m + R_{sc})}{I_m R_m} = 1 + \frac{R_{sc}}{R_m}$$

$$\therefore \quad R_{sc} = (m-1)R_m$$

Sensitivity The moving-coil instrument is a very sensitive instrument. It is, therefore, widely used for measuring current and voltage.The coil of the instrument may require a small amount of current (in the range of μA) for full-scale deflection. The sensitivity is sometimes expressed in *ohm/volt.* The sensitivity of a voltmeter is given by

$$S = \frac{\text{Total voltmeter resistance in ohm}}{\text{Full scale reading in volts}} \Omega/v = \frac{R_m}{v} = \frac{1}{I_{fs}} \Omega/v$$

where I_{fs} is the full-scale deflecting current. Thus, the sensitivity depends upon on the current to give full-scale deflection.

Example 2.5

A moving-coil voltmeter has a resistance of 100 Ω. The scale is divided into 150 equal divisions. When a potential difference of 1 V is applied to the terminals of the voltmeter a deflection of 100 divisions is obtained. Explain how the instrument could be used for measuring up to 300 V.

Solution Let R_{sc}be the multiplier resistance that would be connected in series with the voltmeter.

Volt/division = 1/100

Voltage across the meter for producing the full-scale deflecting current v = 150 × 1/100 = 1.5 V

Full scale meter current I_m = 1.5/100 amp

Meter resistance R_m = 100 Ω

$$\therefore \quad R_{sc} = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

$$= \frac{300 - 1.5/100 \times 100}{1.5/100} = 19.9 \text{ k}\Omega$$

Example 2.6

A moving coil instrument has a resistance of 5 Ω and gives a full scale deflection of 10 mv. Show how the instrument may be used to measure (a) voltage up to 50 v, and (b) *current up to 10 A.*

Solution Full scale deflection of 10 mv

Full scale deflection current = 10 × 10^{-3}/5 = 2 mA

(a) For measuring the voltage up to 50 V we need to connect a multiplier resistance R_{SC} in series with the instrument

Thus, $R_{SC} = (m-1)R_m$, where $m = \frac{V}{V_m}$

$$\therefore \quad R_{SC} = \left(\frac{50}{10 \times 10^{-3}} - 1\right) \times 5 = 24995\,\Omega$$

(b) For measuring the current up to 10 A we need to connect a shunt resistance in parallel to the instrument.

$$\text{Thus, } R_{Sh} = \frac{R_m}{m-1}, \quad \text{where } m = \frac{I}{I_m} = \frac{10}{2 \times 10^{-3}} = 5 \times 10^3$$

$$\therefore \quad R_{Sh} = \frac{5}{5 \times 10^{-3} - 1} = 1.002 \times 10^{-3}\ \Omega$$

Example 2.7

A moving-coil ammeter has a fixed shunt of 0.02 Ω. With a coil resistance of R = 1000 Ω and a potential difference of 500 mV across it. Full-scale deflection is obtained. (a) To what shunted current does it correspond? (b) Calculate the value of R to give full-scale deflection when shunted current I is (i) 10 A, and (ii) 75 A, (c) With what value of R, 40% deflection obtained with I = 100 A.

Solution

(a) Current through shunt $I_{sh} = 500 \times 10^{-3}/0.02 = 25$ A.

(b) (i) Voltage across shunt for a current of 10 A = 0.02 × 10 = 0.2 V.

Therefore, resistance of meter for a current of 10 A to give full scale deflection = $0.2/(0.5 \times 10^{-3})$ = 400 Ω

(ii) Voltage across shunt for a current of 75 A = 0.02 × 75 = 1.5 V. Therefore, resistance of meter for a current of 75 A to give full scale deflection = $1.5/(0.5 \times 10^{-3})$ = 3000 Ω

(c) Now 40% deflection is obtained with 100 A.
Therefore, current to give full-scale deflection = 100/0.4 = 250 A
Voltage across shunt for a current of 250 A = 0.02 × 250 = 5 V

Resistance of meter for a current of 100 A to give 40% of full scale deflection = $5/(0.5 \times 10^{-3})$ = 10,000 Ω

Example 2.8

A simple shunted ammeter using a basic meter movement with an internal resistance of 1800 Ω and a full-scale deflection current of 100 pA is connected in a circuit and gives reading of 3.5 mA on it's 5 mA scale. The reading is checked with a recently calibrated dc ammeter which gives a reading of 4.1 mA. The implication is that the ammeter has a faulty shunt on it's 5 mA range. Calculate (a) the actual value of faulty shunt, and (b) the current shunt for the 5 mA range.

Solution

(a) 5 mA scale deflection corresponds to 100 µA.

Therefore, 3.5 mA corresponds to $\frac{100 \times 10^{-6} \times 3.5}{5} = 7 \times 10^{-5}$ A

As the shunt and the meter are connected in parallel, the drop across the shunt should be equal to the voltage drop across the meter. Meter current = 7×10^{-5} A Shunt current = $(4.1 \times 10^{-3} - 7 \times 10^{-5})$ A Therefore, actual value of faulty shunt,

$R_{sh} \times (4.1 \times 10^{-3} - 7 \times 10^{-5}) = 1800 \times 7 \times 10^{-5}$

$$R_{sh} = \frac{1800 \times 7 \times 10^{-5}}{(4.1 \times 10^{-3} - 7 \times 10^{-5})} = 31.26\ \Omega$$

(b) 5 mA scale deflection corresponds to 100 µA.

Therefore, 4.1 mA corresponds to $\frac{100 \times 10^{-6} \times 4.1}{5} = 82 \times 10^{-6} A$ = 82×10^{-6} A

As the shunt and the meter are connected in parallel, then the drop across the shunt should be equal to the voltage drop across the meter.

Meter current = 82×10^{-6} A

Shunt current = $(4.1 \times 10^{-3} - 82 \times 10^{-6})$ A
Therefore, actual value of faulty shunt,

$R_{sh} \times (4.1 \times 10^{-3} - 82 \times 10^{-6}) = 1800 \times 82 \times 10^{-6}$

$$R_{sh} = \frac{1800 \times 82 \times 10^{-6}}{(4.1 \times 10^{-3} - 82 \times 10^{-6})} = 36.73\ \Omega$$

Example 2.9

A moving-coil instrument gives the full-scale deflection of 10 mA when the potential difference across its terminals is 100 mV. Calculate (a) the shunt resistance for a full-scale deflection corresponding to 100 A, and (b) the series resistance for full scale reading with 1000 V. Calculate the power dissipation in each case.

Solution

(a) Meter resistance R_m = 100 mV/10 mA = 10Ω

The shunt resistance corresponds to 100 A full-scale deflection

$$R_{sh} = \frac{R_m}{m-1}, \text{ where } m = \frac{I}{I_m} = \frac{100}{10 \times 10^{-3}} = 10 \times 10^3$$

$$= \frac{10}{10 \times 10^3 - 1} = 0.001\ \Omega$$

Now R_m and R_{sh} are connected in parallel, the equivalent resistance is

$$R = \frac{R_{sh} \times R_m}{R_{sh} + R_m} = \frac{0.001 \times 10}{0.001 + 10} = 0.00099\ \Omega$$

Power dissipation P = I^2R = $100^2 \times 0.00099$ = 9.9 W

(b) The series resistance corresponds to 1000 V,

$$R_{sc} = (m-1)R_m, \text{ where } m = \frac{V}{v} = \frac{1000}{100 \times 10^{-3}} = 10^4$$

$$= (10^4 - 1) \times 10 = 99{,}990\ \Omega$$

Now R_m and R_{sc} are connected in series, the equivalent resistance is

R = R_{sh} + R_m = 99,990 +10 = 100,000 Ω

Power dissipation P = V^2 /R = 1000^2 /100,000 = 10 W

Example 2.10

A moving-coil instrument has a resistance of 75 Ω and gives a full-scale deflection of 100-scale divisions for a current of 1 mA. The instrument is connected in parallel with a shunt of 25 Ω resistance and the combination is then connected in series with a load and a supply. What is the current in the load when the instrument gives an indication of 80 scale divisions?

Solution For 80 scale divisions, current through the meter is × 1 = 0.8 mA

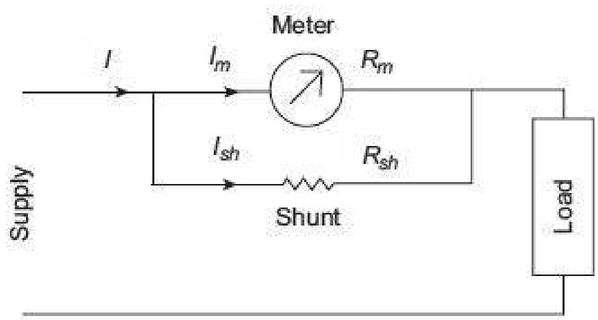
Now,

$$I \times \frac{R_{sh}}{R_{sh} + R_m} = 0.8$$

$$I \times \frac{25}{25 + 75} = 0.8$$

$$I = \frac{0.8 \times 100}{25} = 3.2 \text{ mA}$$

So, current through the load is 3.2 mA.



2.8 MOVING-IRON INSTRUMENTS

Basic range: 10 mA-100 A

Usage:

- dc MI ammeters and voltmeters
- ac MI ammeters and voltmeters

Moving-Iron or MI instruments can be classified as

- Attraction-type moving-iron instruments

- Repulsion-type moving-iron instruments

The current to be measured, in general, is passed through a coil of wire in the moving-iron instruments. In case of voltage measurement, the current which is proportional to the voltage is measured. The number of turns of the coil depends upon the current to be passed through it. For operation of the instrument, a certain number of ampere turns is required. These ampere turns can be produced by the product of few turns and large current or reverse.

2.8.1 Attraction-type Moving-Iron Instruments

The attraction type of MI instrument depends on the attraction of an iron vane into a coil carrying current to be measured. Figure 2.14 shows a attraction-type MI instrument. A soft iron vane IV is attached to the moving system. When the current to be measured is passed through the coil C, a magnetic field is produced. This field attracts the eccentrically mounted vane on the spindle towards it. The spindle is supported at the two ends on a pair of jewel bearings. Thus, the pointer PR, which is attached to the spindle S of the moving system is deflected. The pointer moves over a calibrated scale.

The control torque is provided by two hair springs S_1 and S_2 in the same way as for a PMMC instrument; but in such instruments springs are not used to carry any current. Gravity control can also be used for vertically mounted panel type MI meters. The damping torque is provided by the movement of a thin vane V in a closed sector-shaped box B, or simply by a vane attached to the moving system. Eddy current damping can not be used in MI instruments owing to the fact that any permanent magnet that will be required to produce Eddy current damping can distort the otherwise weak operating magnetic field produced by the coil.

If the current in the fixed coil is reversed, the field produced by it also reverses. So the polarity induced on the vane reverses. Thus whatever be the direction of the current in the coil the vane is always be magnetized in such a way that it is attracted into the coil. Hence such instrument can be used for both direct current as well as alternating current.

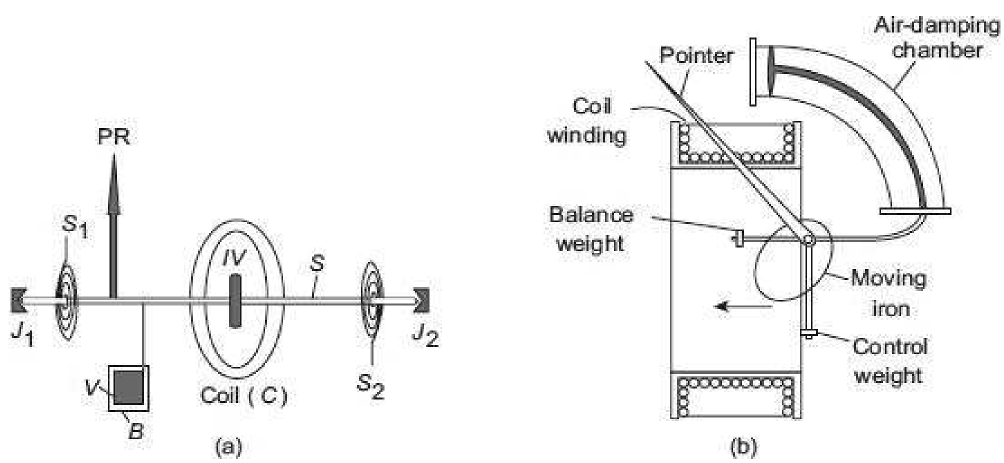


Figure 2.14 *Attraction-type moving iron (MI) instrument*

2.8.2 Repulsion-type Moving-Iron Instruments

In the repulsion type, there are two vanes inside the coil. One is fixed and the other is movable. These are similarly magnetised when the current flows through the coil and there is a force of repulsion between the two vanes resulting in the movement of the moving vane.

Two different designs for moving iron instruments commonly used are as follows:

1. Radial Vane Type In this type, the vanes are radial strips of iron. The strips are placed within the coil as shown in Figure 2.15(a). The fixed vane is attached to the coil and the movable one to the spindle of the instrument. The instrument pointer is attached to the moving vane spindle.

As current flows through the coil, the generated magnetic field induces identical polarities on both the fixed and moving vane. Thus, even when the current through the coil is alternating (for AC measurement), there is always a repulsion force acting between the like poles of fixed and moving vane. Hence deflection of the pointer is always in the same direction irrespective of the polarity of current in the coil. The amount of deflection depends on the repulsion force between the vanes which in turn depends on the amount of current passing through the coil. The scale can thus be calibrated to read the current or voltage directly.

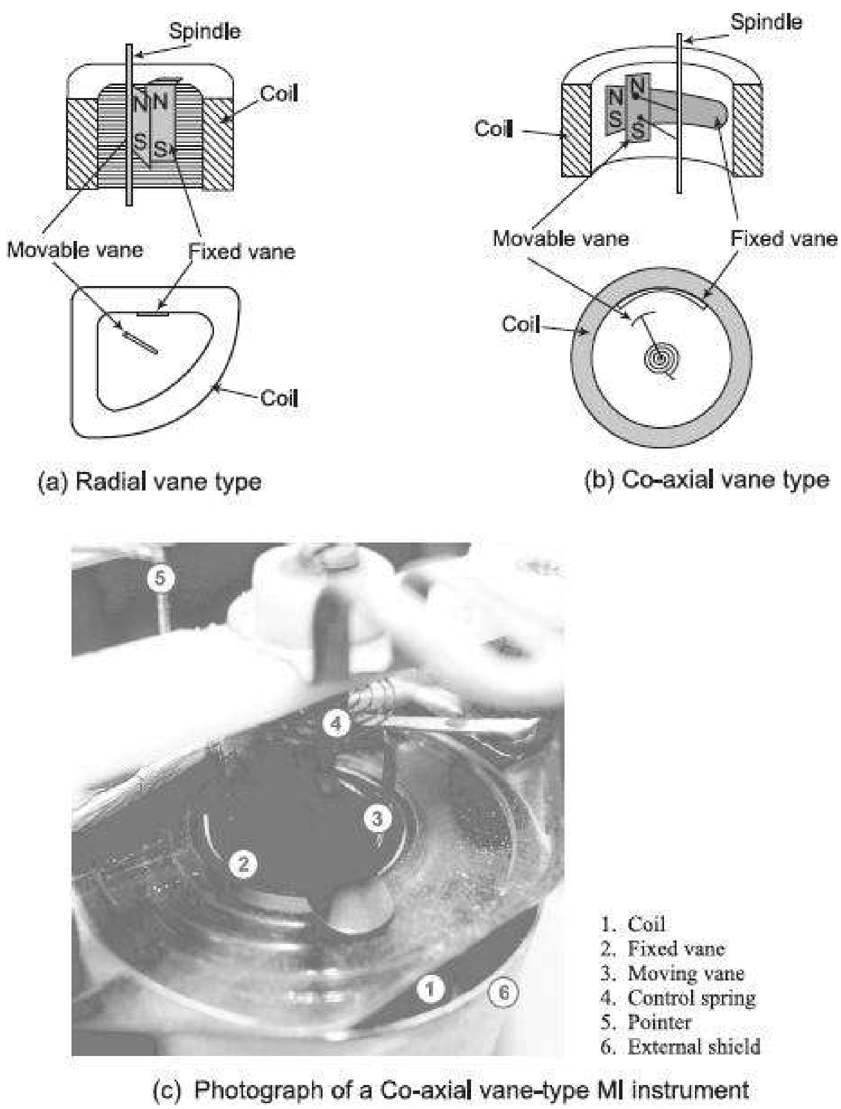


Figure 2.15 *Re-pulsion-type Moving Iron (MI) instruments*

2. *Co-axial Vane Type* I In these type of instruments, the fixed and moving vanes are sections of coaxial cylinders as shown in Figure 2.15(b). Current in the coil magnetizes both the vanes with similar polarity. Thus the movable vane rotates along the spindle axis due to this repulsive force. Coaxial vane type instruments are moderately sensitive as compared to radial vane type instruments that are more sensitive.

Moving iron instruments have their deflection is proportional to the square of the current flowing through the coil. These instruments are thus said to follow a square law response and have non-uniform scale marking. Deflection being proportional to square of the current, whatever be the polarity of current in the coil, deflection of a moving iron instrument is in the same direction. Hence, moving iron instruments can be used for both DC and AC measurements.

2.8.3 Torque Equation of Moving-Iron Instruments

To deduce the expression for torque of a moving iron instrument, energy relation can be considered for a small increment in current supplied to the instrument. This result in a small deflection $d\theta$ and some mechanical work will be done. Let T_d be the deflecting torque.

Therefore mechanical work done = torque × angular displacement

$$= T_d \cdot d\theta \tag{2.27}$$

Due to the change in inductance there will be a change in the energy stored in the magnetic field.

Let I be the initial current, L be the instrument inductance and θ is the deflection. If the current increases by *dl* then it causes the change in deflection $d\theta$ and the inductance by *dL*. In order to involve the increment *dI* in the current, the applied voltage must be increase by:

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(LI) = I\frac{dL}{dt} + L\frac{dI}{dt} \tag{2.28}$$

The electrical energy supplied is $eIdt = I^2dL + ILdI$ (2.29)

[substitute the value of *edt* from equation (2.28)]

The current is changes from I to (I + *dI),* and the inductor L to (L + *dL)*

Therefore the stored energy changes from $= \frac{1}{2}I^2L$ to $\frac{1}{2}(I + dI)^2\ (L + dL)$

Hence the change in stored energy $= \frac{1}{2}(I + dI)^2\ (L + dL) - \frac{1}{2}I^2L$ (2.30)

As *dI* and *dL* are very small, neglecting the second and higher order terms in small quantities, this

becomes $ILdL + \frac{1}{2}I^2dL$

From the principle of conservation of energy,

Electrical energy supplied = Increase in stored energy + Mechanical work done.

$$I^2 dL + ILdI = ILdI + \frac{1}{2} I^2 dL + T_d d\theta$$

$$\therefore \qquad T_d d\theta = \frac{1}{2} I^2 dL \qquad (2.31)$$

$$\text{or deflecting torque } T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \qquad (2.32)$$

where T_d is in newton-metre, I is in ampere, L is in henry and θ is in radians.

The moving system is provided with control springs and in turn the deflecting torque T_d is balanced by the controlling torque $T_C = k\,\theta$

where k is the control spring constant (N-m/rad) and θ is the deflection in radians.

At final steady position, $T_C = T_d$

$$\text{or} \qquad k\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\therefore \text{ deflection} \qquad \boxed{\theta = \frac{1}{2} \frac{I^2}{k} \frac{dL}{d\theta}} \qquad (2.33)$$

Hence, the deflection is proportional to square of the rms value of the operating current. The deflection torque is, therefore, unidirectional whatever may be the polarity of the current.

Advantages of MI Instruments

1. Robust construction and relatively cheap
2. Suitable for measuring both dc and ac
3. Can withstand overload momentarily

Disadvantages of MI Instruments

1. As the deflection is proportional to I^2, hence the scale of the instrument is not uniform. It is cramped in the lower end and expanded in the upper portion.
2. It is affected by stray magnetic fields.
3. There is hysteresis error in the instrument. The hysteresis error may be minimized by using the vanes of nickel-iron alloy.
4. When used for measuring ac the reading may be affected by variation of frequency due to the change in reactance of the coil, which has some inductance. With the increase in frequency iron loses and coil impedance increases.
5. Since large amount of power is consumed to supply I^2R loss in the coil and magnetic losses in the vanes, it is not a very sensitive instrument.

Example 2.11 *The inductance of a moving-iron ammeter with a full-scale deflection of 90° at 1.5 A is given by $L = (200 + 40\theta - 4\theta^2 - \theta^3)$ μH where θ is the deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1 A.*

Solution For an MI instrument,

$$\text{deflection } \theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

Here, $L = (200 + 40\theta - 4\theta^2 - \theta^3)$ μH

Then $\dfrac{dL}{d\theta} = (40 - 8\theta - 3\theta^2)$

For a deflection, $\theta = 90° = \dfrac{\pi}{2}$, current $I = 1.5$ A

$$\frac{\pi}{2} = \frac{1}{2} \times \frac{(1.5)^2}{k} \times \left[40 - 8\left(\frac{\pi}{2}\right) - 3\left(\frac{\pi}{2}\right)^2\right]$$

$$k = \frac{(1.5)^2}{\pi} \times \left[40 - 8\left(\frac{\pi}{2}\right) - 3\left(\frac{\pi}{2}\right)^2\right]$$

$$= 14.348 \text{ N-m/rad}$$

Now for a current of 1 A, the angular deflection θ is

$$\theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

$$\theta = \frac{1}{2} \times \frac{1}{14.348} \times (40 - 8\theta - 3\theta^2)$$

$$40 - 8\theta - 3\theta^2 = 28.696\,\theta$$

$$3\theta^2 + 36.696\,\theta - 40 = 0$$

After solving for θ and taking only the positive value

$$\theta = 1.00712 \text{ rad} = 57.7°$$

Example 2.12 *The law of deflection of a moving-iron ammeter is given by $I = 4\theta^n$ ampere, where θ is the deflection in radian and n is a constant. The self-inductance when the meter current is zero is 10 mH. The spring constant is 0.16 N-m/rad.*

(*a*) *Determine an expression for self-inductance of the meter as a function of θ and n.*

(*b*) *With n = 0.75, calculate the meter current and the deflection that corresponds to a self-inductance of 60 mH.*

Solution

(a) $$\theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

$$\theta = \frac{1}{2}\times\frac{(4\theta^n)^2}{0.16}\times\frac{dL}{d\theta}$$

$$2\theta\cdot d\theta = 100\cdot\theta^{2n}dL$$

$$dL = \frac{1}{50}\theta^{1-2n}d\theta$$

Integrating both sides,

$$L = \frac{1}{100}\frac{\theta^{2-2n}}{(1-n)}+C; \qquad C = \text{Integration constant}$$

at $I = 0$, i.e. $\theta = 0$, $L = 10\times 10^{-13}$

Substituting the value of θ and L, we get

$$C = 10\times 10^{-3}$$

So, $$L = \frac{1}{100}\frac{\theta^{2-2n}}{(1-n)}+10\times10^{-3}$$

(b) Now $n = 0.75$ and $L = 60\times 10^{-3}$

$$60\times 10^{-3} = \frac{1}{100}\frac{\theta^{2-2\times 0.75}}{(1-0.75)}+10\times10^{-3}$$

$$\theta^{0.5} = 25(60\times 10^{-3}) = 1250\times 10^{-3}$$

$$\theta = (1250\times 10^{-3}) = 1.56 \text{ rad} = 89.38 \text{ degree}$$

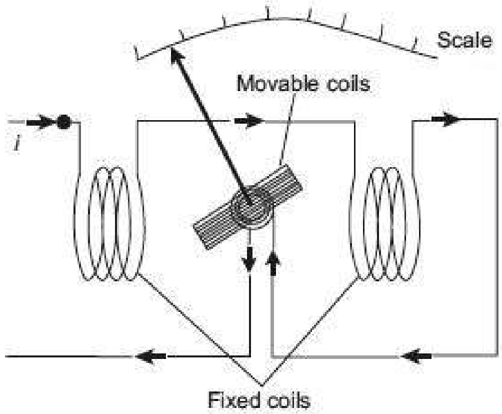
So meter current $I = 4\times(1.56)^{0.75} = 5.58$ Amp

2.9 ELECTRODYNAMOMETER-TYPE INSTRUMENTS

The electrodynamometer is a transfer-type instrument. A transfer-type instrument is one that may be calibrated with a dc source and then used without modification to measure ac. This requires the transfer type instruments to have same accuracy for both dc and ac.

The electrodynamic or dynamometer-type instrument is a moving-coil instrument but the magnetic field, in which the coil moves, is provided by two fixed coils rather than by permanent magnets. The schematic diagram of electrodynamic instrument is shown in Figure 2.16(a) and a practical meter is shown in Figure 2.16(b). It consists of two fixed coils, which are symmetrically situated. It would have a torque in one direction during one half of the cycle and an equal effect in opposite direction during the other half of the cycle. If, however, we were to reverse the direction of the flux each time the current through the movable coil reverses, a unidirectional torque would be produced for both positive half and negative half of the cycle. In electrodynamic instruments, the field can be made to reverse simultaneously with the current in the movable coil if the fixed coil is connected in series with the movable coil.

1. *Controlling Torque* The controlling torque is provided by two control springs. These springs act as leads to the moving coil.



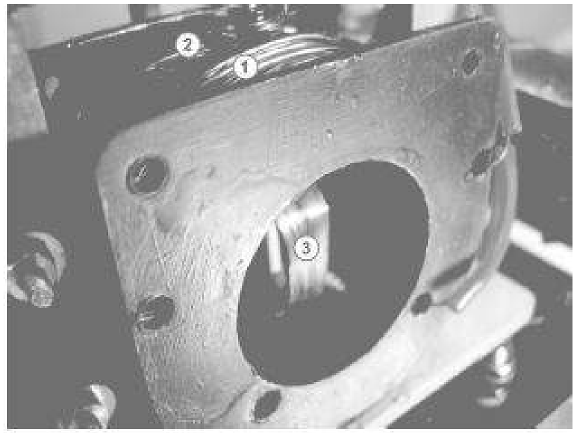


Figure 2.16 Electrodynamometer-type instrument

2. *Damping* Air-friction damping is employed for these instruments and is provided by a pair of aluminium vanes, attached to the spindle at the bottom. These vanes move in a sector-shaped chamber.

2.9.1 Torque Equation of Electrodynamometer-type Instruments

Let, i_1 = instantaneous value of current in the fixed coils, (A)

i_2 = instantaneous value of current in the moving coils, (A)

L_1 = self-inductance of fixed coils, (H)

L_2 = self-inductance of moving coil, (H)

M = mutual inductance between fixed and moving coils (H)

Flux linkage of Coil 1, $\psi_1 = L_1 i_1 + M i_2$

Flux linkage of Coil 2, $\psi_2 = L_2 i_2 + M i_1$

Electrical input energy,

$$= e_1 i_1 dt + e_2 i_2 dt = i_1 d\psi_1 + i_2 d\psi_2$$

As $e_1 = \dfrac{d\psi_1}{dt}$ and $e_2 = \dfrac{d\psi_2}{dt}$

$$= i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \quad (2.34)$$

Energy stored in the magnetic field $= \dfrac{1}{2} i_1^2 L_1 + \dfrac{1}{2} i_2^2 L_2 + i_1 i_2 M$

Change in energy stored $= d\left(\dfrac{1}{2} i_1^2 L_1 + \dfrac{1}{2} i_2^2 L_2 + i_1 i_2 M\right)$

$$= i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + i_2 L_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 M di_2 + i_2 M di_1 + i_1 i_2 dM \quad (2.35)$$

From the principle of conservation of energy,

Total electrical input energy = Change in energy in energy stored + mechanical energy The mechanical energy can be obtained by subtracting Eq. (2.35) from Eq. (2.34).

Therefore, mechanical energy = $\frac{1}{2}i_1^2 dL_1 + \frac{1}{2}i_2^2 dL_2 + i_1 i_2 dM$

Now, the self-inductances L_1 and L_1 are constants and, therefore, dL_2 and dL_2 both are equal to zero. Hence, mechanical energy = $i_1 i_2\, dM$

Suppose T_i is the instantaneous deflecting torque and $d\theta$ is the change in deflection, then, Mechanical energy = work done = $T_i\, d\theta$

Thus we have

$$T_i d\theta = i_1 i_2 dM \quad \text{or} \quad T_i = i_1 i_2 \frac{dM}{d\theta} \tag{2.36}$$

1. ***Operation with dc*** Let, I_1 = current in the fixed coils, I_2 = current in the moving coil

So deflecting torque T_d = $I_1 I_2 \frac{dM}{d\theta}$. This shows that the deflecting torque depends in general on the product of current I_1 and I_2 and the rate of change of mutual inductance.

This deflecting torque deflects the moving coil to such a position where the controlling torque of the spring is equal to the deflecting torque. Suppose θ be the final steady deflection.

Therefore controlling torque $T_C = k\theta$ where k = spring constant (N-m/rad)

At final steady position $T_d = T_C$

$$I_1 I_2 \frac{dM}{d\theta} = k\theta$$

or, the deflection

$$\boxed{\theta = \frac{I_1 I_2}{k}\frac{dM}{d\theta}} \tag{2.37}$$

If the two coils are connected in series for measurement of current, the two currents I_1 and I_2 are equal.

Say, $I_1 = I_2 = I$

Thus, deflection of the pointer is $\theta = \frac{I^2}{k}\frac{dM}{d\theta}$

For dc use, the deflection is thus proportional to square of the current and hence the scale non-uniform and crowded at the ends.

2. ***Operation with ac*** Let, i_1 and i_2 be the instantaneous values of current carried by the coils. Therefore, the instantaneous deflecting torque is:

$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

If the two coils are connected in series for measurement of current, the two instantaneous currents i_1 and i_2 are equal.

Say, $i_i = i_2 = i$

Thus, instantaneous torque on the pointer is $T_i = i_2 \frac{dM}{d\theta}$

Thus, for ac use, the instantaneous torque is proportional to the square of the instantaneous current. As the quantity i^2 is always positive, the current varies and the instantaneous torque also varies. But the moving system due to its inertia cannot follow such rapid variations in the instantaneous torque and responds only to the average torque.

The average deflecting torque over a complete cycle is given by:

$$T_d = \frac{1}{T}\int_0^T T_i \, dt = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i^2 \, dt$$

where T is the time period for one complete cycle.

At final steady position $T_d = T_C$

$$\text{or,} \qquad k\theta = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i^2 \, dt$$

Thus, deflection of the pointer is $\theta = \frac{1}{k}\frac{dM}{d\theta}\frac{1}{T}\int_0^T i^2 \, dt$

Deflection is thus a function of the mean of the square of the current. If the pointer scale is calibrated in terms of square root of this value, i.e. square root of the mean of the square of current value, then rms value of the ac quantity can be directly measured by this instrument.

3. *Sinusoidal Current* If currents i_1 and i_2 are sinusoidal and are displaced by a phase angle j, i.e.

$i_1 = i_{m1} \sin \omega t$ and $i_2 = I_{m1} \sin(wt - j)$

∴ The average deflecting torque

$$T_d = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i_1 i_2 \, dt = \frac{dM}{d\theta}\frac{1}{2\pi}\int_0^{2\pi} I_{m1} \sin \omega t \cdot I_{m2} \sin(\theta t - \varphi) \, d\omega t$$

$$\frac{I_{m1} I_{m2}}{2} \cos \varphi \frac{dM}{d\theta} = I_1 I_2 \cos \varphi \frac{dM}{2\theta}$$

where I_1 and I_2 are the rms values of the currents flowing through the coils. At equilibrium, $T_d = T_C$

$$\text{or} \qquad I_1 I_2 \cos \varphi \frac{dM}{d\theta} = k\theta \qquad (2.38)$$

$$\therefore \qquad \boxed{\theta = \frac{I_1 I_2}{k} \cos \varphi \frac{dM}{d\theta}}$$

As was in the case with ac measurement, with sinusoidal current also the deflection is a function of the mean of the square of the current. If the pointer scale is calibrated in terms of square root of this value, i.e. square root of the mean of the square of current value, then RMS value of the ac quantity can be directly measured by this instrument.

1. *Electrodynamic Ammeter* In an electrodynamic ammeter, the fixed and moving coils are connected in series as shown in Figure 2.17. A shunt is connected across the moving coil for limiting the current. The reactance–resistance ratio of the shunt and the moving coil is kept nearly same for independence of the meter reading with the supply frequency. Since the coil currents are the same, the deflecting torque is proportional to the mean square value of the current. Thus, the scale is calibrated to read the rms value.

2. *Electrodynamic Voltmeter* The electrodynamic instrument can be used as a voltmeter by connecting a large noninductive resistance (R) of low temperature coefficient in series with the instrument coil as shown in Figure 2.18.

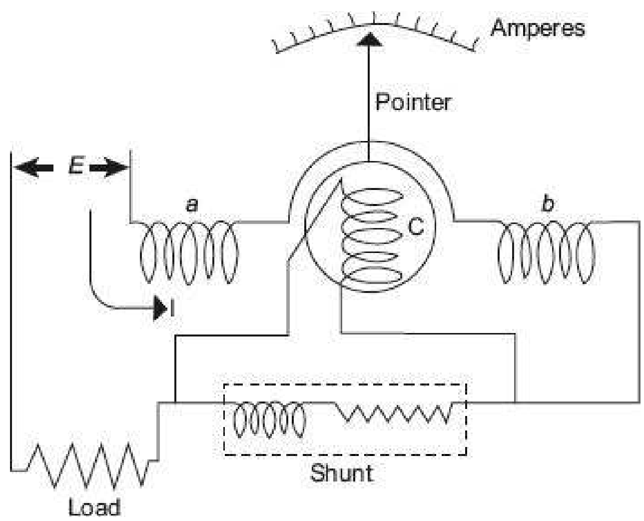


Figure 2.17 *Electrodynamometer ammeter*

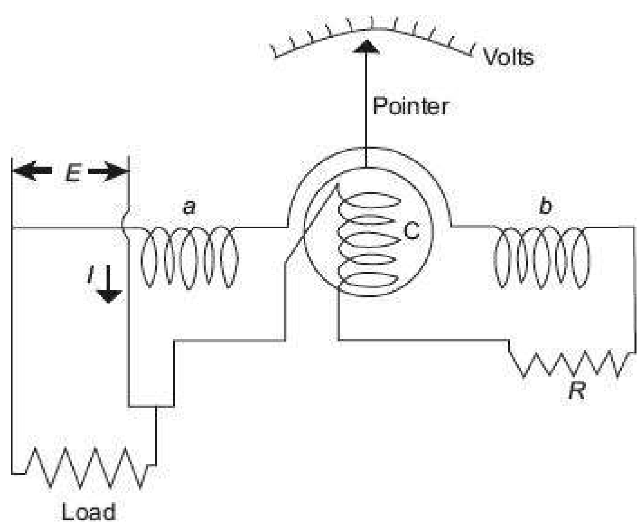


Figure 2.18 *Electrodynamometer voltmeter*

3. *Electrodynamic Wattmeter* The electrodynamic wattmeter consist of two fixed coils *'a'* and *'b'* placed symmetrical to each other and producing a uniform magnetic field. They are connected in series with the load and are called the Current Coils (CC). The two fixed coils can be connected in series or parallel to give two different current ratings. The current coils carry the full-load current or a fraction of full load current. Thus the current in the current coils is proportional to the load current. The moving coil *'c'*, in series with a high non inductive resistance R_v is connected across the supply. Thus the current flowing in the moving coil is proportional to, and practically in phase with the supply voltage. The moving coil is also called the voltage coil or Pressure Coil (PC). The voltage coil is carried on a pivoted spindle which carries the pointer, the pointer moved over a calibrated scale.

Two hair springs are used for providing the controlling torque and for leading current into and out of the moving coil. Damping is provided by air friction. Figure 2.20 shows the basic arrangement of a electrodynamic wattmeter.

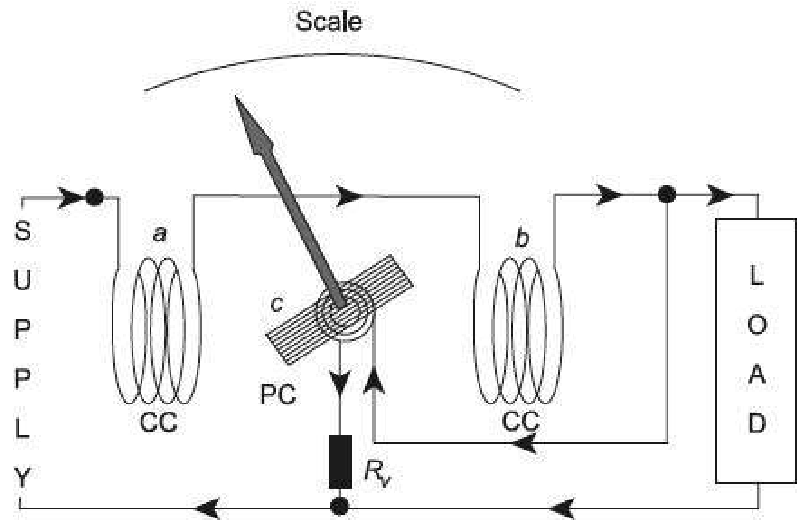


Figure 2.19 *Electrodynamic wattmeter*

4. *Torque Equation*

Let, i_f = current in the fixed coil

i_m = current in the moving coil

i = load current

v = load voltage

T_{in} = instantaneous value of the deflecting torque

p = instantaneous power

T_{in} x i_f i_m

$$T_{in} \propto i_f i_m \tag{2.39}$$

But since $i_f \propto i$ and $i_m \propto v$

$$T_{in} \propto vi \propto p \tag{2.40}$$

Thus, the instantaneous value of the deflecting torque is proportional to the instantaneous power. Owing to the inertia of the moving system, the pointer reads the average power. In dc circuits, the power is given by the product of voltage and current, and hence the torque is directly proportional to the power. Thus, the instrument indicates the power.

For ac, the instrument indicates the average power. This can be proved as follows:

$T_{in} \infty Vi$

Average deflecting torque × average power

Let, $v = V_m \sin d$

$I = I_m \sin(\theta - \Phi)$

Average deflecting torque ∞ average value of $V_m \sin d \times I_m \sin(\theta - \Phi) \infty VI \cos\theta$ If T_d be the average torque, then

$$T_d \propto VI \cos\Phi \propto \text{true power} = kP \qquad (2.41)$$

where P is the true power and k is the constant.

For spring control $T_C = k_s\theta_1$

where T_C is the control torque, k_s is the spring constant and θ_1 is the angle of deflection of the pointer.

For steady deflection,

$$T_c = T_d$$
$$k_s\theta_1 = kP$$
$$\theta_1 = \frac{k}{k_s}P$$
$$\theta_1 \propto P$$

Hence, in case of ac also the deflection is proportional to the true power in the circuit. The scale of the electrodynamometer wattmeter is therefore uniform.

Advantages of Electrodynamometer-type Instruments

1. They can be used on ac as well as dc measurements.
2. These instruments are free from eddy current and hysteresis error.
3. Electrodynamometer-type instruments are very useful for accurate measurement of rms values of voltages irrespective of waveforms.
4. Because of precision grade accuracy and same calibration for ac and dc measurements these instruments are useful as transfer type and calibration instruments.

Disadvantages of Electrodynamometer-type Instruments

1. As the instrument has square law response, the scale is non-uniform.
2. These instruments have small torque/weight ratio, so the frictional error is considerable.
3. More costly than PMMC and MI type of instruments.
4. Adequate screening of the movements against stray magnetic fields is essential.
5. Power consumption is comparably high because of their construction.

The inductance of a 25 A electrodynamic ammeter changes uniformly at the rate of 0.0035 mH/radian. The spring

Example 2.13 *constant is 10^{-6}N-m/radian. Determine the angular deflection at full scale.*

Solution $\frac{dM}{d\theta} = 0.0035 \times 10^{-6}$ H/rad

Now the deflection $\theta = \frac{I^2}{k}\frac{dM}{d\theta}$

Angular deflection at full scale current of I = 25 A is given by:

$$\theta = \frac{25^2}{10^{-6}} \times 0.0035 \times 10^{-6} \times \frac{180°}{\pi} = 125°$$

Example 2.14 *In an electrodynamic instrument the total resistance of the voltage coil circuit is 8200 Ω and the mutual inductance changes uniformly from -173 μH at zero deflection to +175 μH at full scale. The angle of full scale being 95°. If a potential difference of 100 V is applied across the voltage circuit and a current of 3 A at a power factor of 0.75 is passed through the current coil, what will be the deflection. Spring constant of the instrument is 4.63 × 10^{-6} N-m/rad.*

Solution Change in mutual inductance dM = 175 – (–173) = 348 μH

Deflection θ = 95° = 1.66 rad

Rate of change of mutual inductance

$$\frac{dM}{d\theta} = \frac{348}{1.66} = 209.63 \text{ μH/rad}$$

Current through the current coil I_1= 3 A

Current through the voltage coil $I_2 = \frac{100}{8200} = 0.0122$ A

Power factor cos *6* = 0.75

Deflection $\theta = \frac{I_1 I_2}{k}\cos\phi\frac{dM}{d\theta}$

$$\theta = \frac{3 \times 0.0122}{4.63 \times 10^{-6}} \times 0.75 \times 209.63 \times 10^{-6}$$
$$= 1.242 \text{ rad} = 71.2°$$

Example 2.15 *A 50 V range spring-controlled electrodynamic voltmeter has an initial inductance of 0.25 H, the full scale deflection torque of 0.4 × 10^{-4} Nm and full scale deflection current of 50 mA. Determine the difference between dc and 50 Hz ac reading at 50 volts if the voltmeter inductance increases uniformly over the full scale of 90°.*

Solution

Full-scale deflection *6* = 90°

Full-scale deflecting torque, $T_d = 0.4 \times 10^{-4}$ Nm

Full-scae deflection current, I = 50 mA = 0.05 A

Initial induction, M = 0.25 H

Since deflecting torque, $T_d = I^2 \frac{dM}{d\theta}$

So for full-scale deflection 0 4 × 10^{-4} = (0 05)^2 $\frac{dM}{d\theta}$

or, $$\frac{dM}{d\theta} = \frac{0.4 \times 10^{-4}}{(0.05)^2} = 0.016 \text{ H/rad}$$

Total change in inductance for full-scale deflection,

$$dM = 0.016 \times 90 \times \frac{\pi}{180} = 0.0251\ H$$

Total mutual inductance, M = 0.25 + 0.0251 = 0.2751 H

The resistance of voltmeter, $R = \frac{\text{Voltage}}{\text{Current}} = \frac{50}{0.05} = 1000\ \Omega$

The impedance while measuring the voltage of 50 V at 50 Hz AC

$$Z = \sqrt{(1000)^2 + (2\pi \times 50 \times 0.2751)^2} = 1004\ \Omega$$

And voltmeter reading, = $\frac{50}{1004} \times 1000 = 49.8\ V$

Therefore, difference in reading = 50 – 49.8 = 0.2 V

2.10 ELECTROSTATIC INSTRUMENTS

In electrostatic instruments, the deflecting torque is produced by action of electric field on charged conductors. Such instruments are essentially voltmeters, but they may be used with the help of external components to measure the current and power. Their greatest use in the laboratory is for measurement of high voltages.

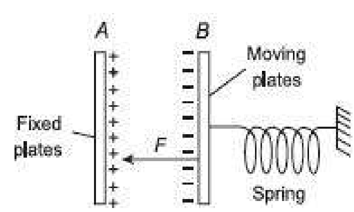


Figure 2.20 Linear motion of electrostatic instruments

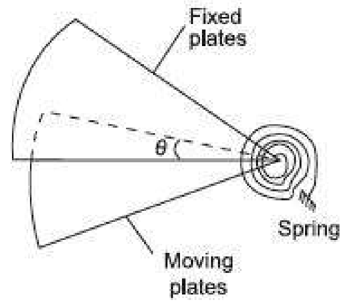


Figure 2.21 Rotary motion of electrostatic instruments

Rotary motion of electrostatic instruments

There are two ways in which the force acts:

1. One type involves two oppositely charged electrodes. One of them is fixed and the other is movable. Due to force of attraction, the movable electrode is drawn towards the fixed one.
2. In the other type, there is force of attraction or repulsion between the electrodes which causes rotary motion of the moving electrode.

In both the cases, the mechanism resembles a variable capacitor and the force or torque is due to the fact that the mechanism tends to move the moving electrode to such a position where the energy stored is maximum.

2.10.1 Force and Torque Equation

1. *Linear Motion* Referring to Figure 2.20, plate *A* is fixed and *B* is movable. The plates are oppositely charged and are restrained by a spring connected to the fixed point. Let a potential difference of V volt be applied to the plates; then a force of attraction F Newton exists between them. Plate B moves towards A until the force is balanced by the spring. The capacitance between the plates is then C farad and the stored energy is ½ CV joules.

Now let there be a small increment *dV* in the applied voltage, then the plate *B* moves a small distance *dx* towards *A*. when the voltage is being increased a capacitive current flows. This current is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt} \qquad (2.42)$$

The input energy is $Vidt = V^2 dC + CVdV$ (2.43)

Change in stored energy = $= \frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$

$$= \frac{1}{2}V^2 dC + CVdV \qquad (2.44)$$

(neglecting the higher order terms as they are small quantities)

From the principle of conservation of energy,

Input electrical energy = increase in stored energy + mechanical work done

$$V^2 dC + CVdV = \frac{1}{2}V^2 dC + CVdV + Fdx$$

$$\therefore \quad \boxed{F = \frac{1}{2}V^2 \frac{dC}{dx}} \tag{2.45}$$

2. *Rotational Motion* The forgoing treatment can be applied to the rotational motion by writing an angular displacement θ in place of linear displacement *x* and deflecting torque T_d instead of force *F* (Figure 2.21).

Deflecting torque $T_d = \frac{1}{2}V^2 \frac{dC}{d\theta}$ (2.46)

If the instrument is spring controlled or has a suspension then

Controlling torque $T_C = k\&$, where k = spring constant

θ = deflection

Hence, deflection

$$\boxed{\theta = \frac{1}{2}\frac{V^2}{k}\frac{dC}{d\theta}} \tag{2.47}$$

Since the deflection is proportional to the square of the voltage to be measured, the instrument can be used on both ac and dc. The instrument exhibits a square law response and hence the scale is non-uniform.

Advantages of Electrostatic Instrumentss

1. These instruments draws negligible amount of power from the mains.
2. They may be used on both ac and dc.
3. They have no frequency and waveform errors as the deflection is proportional to square of voltage and there is no hysteresis.
4. There are no errors caused by the stray magnetic field as the instrument works on the electrostatic principle.
5. They are particularly suited for high voltage.

Disadvantages of Electrostatic Instruments

1. The use of electrostatic instruments is limited to certain special applications, particularly in ac circuits of relatively high voltage, where the current drawn by other instruments would result in erroneous indication. A protective resistor is generally used in series with the instrument in order to limit the current in case of a short circuit between plates.
2. These instruments are expensive, large in size and are not robust in construction.
3. Their scale is not uniform.
4. The operating force is small.

2.11 INDUCTION-TYPE INSTRUMENTS

Induction-type instruments are used only for ac measurement and can be used either as ammeter, voltmeter or wattmeter. However, the induction principle finds its widest application as a watt-hour or energy meter (for details, refer Chapter 8). In such instruments, the deflecting torque is produced due to the reaction between the flux of an ac magnet and the eddy currents induced by another flux.

2.11.1 Principle of Operation

The operations of induction-type instruments depend on the production of torque due to the interaction between a flux Φ_1 (whose magnitude depends on the current or voltage to be measured) and eddy current induced in a metal disc or drum by another flux Φ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy current also depends on the flux producing them, the instantaneous value of the torque is proportional to the square of current or voltage under measurement and the value of mean torque is proportional to the mean square value of this current or voltage.

Consider a thin aluminium or copper disc D free to rotate about an axis passing through its centre as shown in Figure 2.22. Two electromagnets P_1 and P_2 produce alternating fluxes Φ_1 and Φ_2 respectively which cuts this disc. Consider any annular portion of the disc around P_1 with centre of the axis of P_l. This portion will be linked by flux $\Phi_1$and so an alternating emf Φ_1 be induced in it. Φ_2 will induce an emf e_2 which will further induce an eddy current i_2 in an annular portion of the disc around P_l. This eddy currents i_2 flows under the pole P_l.

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both Φ_1 and Φ_2 are increasing. By applying Lenz's law, the direction of the induced currents i_1 and i_2 can be found as indicated in Figure 2.22(b).

The portion of the disc which is traversed by flux Φ_1 and carries eddy currents i_2 experiences a force F_1 along the direction as indicated. As $F = Bil$, force $F_1 \propto \Phi_1 i_2$. Similarly, the portion of the disc lying under flux Φ_2 and carrying eddy current i_1 experiences a force $\Phi_2 \propto F_2$ij.

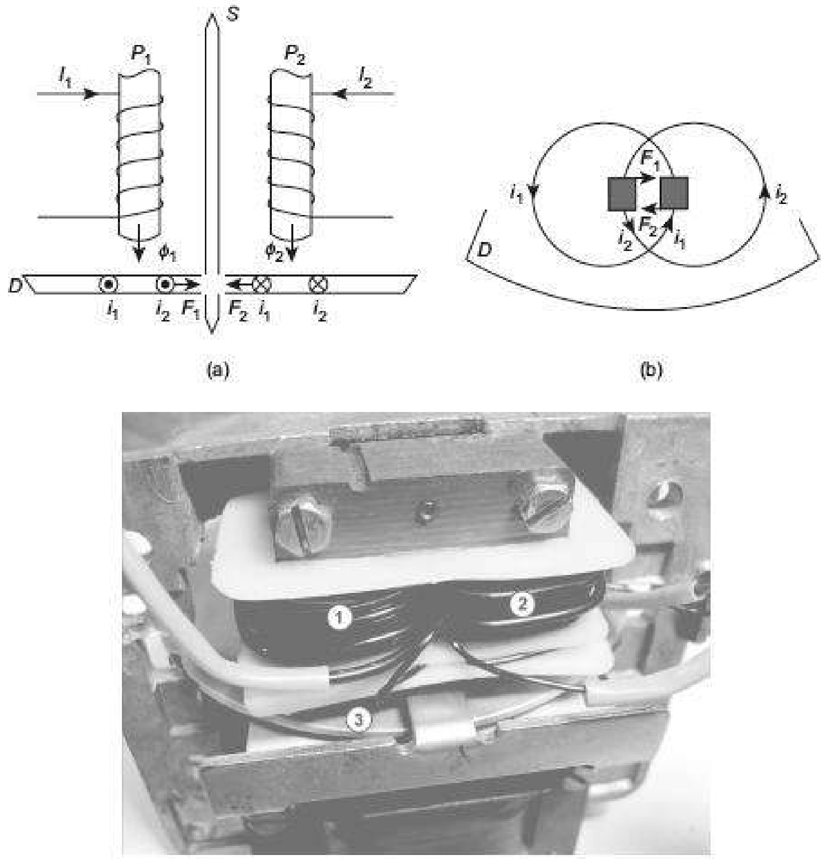
$$\therefore \quad F_1 \propto \varphi_1 i_2 = k\varphi_1 i_2 \tag{2.48}$$

$$F_2 \propto \varphi_2 i_1 = k\varphi_2 i_1 \tag{2.49}$$

It is assumed that the constant k is the same in both the cases due to the symmetrical position of P_1 and P_2 with respect to the disc.

If r be the effective radius at which these forces acts, then net instantaneous torque T acting on the disc being equal to the different of the two torques, it is given by

$$T = r(k\varphi_1 i_2 - k\varphi_2 i_1) = k_1(\varphi_1 i_2 - \varphi_2 i_1) \tag{2.50}$$



(a) (b)

1/2 - Electromagnetic coils, 3 – Aluminium rotating disc

(c) Photograph of Induction type instrument

Figure 2.22 *Principle of operation of induction-type instrument*

Let the alternating flux φ_1 be given by $\varphi_1 = \varphi_{1m} \sin \omega t$. The flux $\varphi_2$which is assumed to lag φ_1 by an angle *a* radian is given by $\varphi_2 = \varphi_{2m} \sin (\omega t - a)$

Induced emf $e_1 = \frac{d\varphi_1}{dt} = \frac{d}{dt} (\varphi_{1m} \sin \omega t) = \omega\varphi_{1m} \cos \omega t$

Assuming the eddy current path to be purely resistive and of value R, then the value of eddy current is

$$i_1 = \frac{e_1}{R} = \frac{\omega \varphi_{1m}}{R} \cos\omega t$$

similarly, $e_2 = \omega \varphi_{2m} \cos(\omega t - \alpha)$ and $i_2 = \frac{e_2}{R} = \frac{\omega\varphi 2m}{R}\cos(\omega t - \alpha)$

Substituting these values of i_1 and i_2 in Eq. (2.48), we get *kw*

$$T = \frac{k_1\omega}{R}\left[\varphi_{1m} \sin\omega t \cdot \varphi_{2m} \cos(\omega t - \alpha) - \varphi_{2m} \sin(\omega t - \alpha) \cdot \varphi_{1m} \cos\omega t\right]$$

$$= \frac{k_1\omega}{R}\varphi_{1m}\varphi_{2m}\left[\sin\omega t \cdot \cos(\omega t - \alpha) - \sin(\omega t - \alpha) \cdot \cos\omega t\right]$$

$$= \frac{k_1\omega}{R}\varphi_{1m}\varphi_{2m} \sin\alpha = k_2\omega\varphi_{1m}\varphi_{2m} \sin\alpha \quad \left[\text{putting } \frac{k_1}{R} = k_2\right] \qquad (2.51)$$

The following is observed:

1. If $\alpha = 0$, i.e., if two fluxes are in phase, then net torque is zero. If, on the other hand, $a = 90°$, the net torque is maximum for a given values of φ_{1m} and φ_{2m}.

2. The net torque is such a direction as to rotate the disc from the pole with leading flux, towards the pole with lagging flux.
3. Since the expression for torque does not involve t, it is independent of time, i.e., it has a steady value at all times.
4. The torque T is inversely proportional to R; the resistance of the eddy current path. Hence, it is made of copper or more often, of aluminium.

2.12 ELECTROTHERMAL INSTRUMENTS

Mainly there are two types of thermal instruments:

- Hot-wire type
- Thermocouple instrument

Hot-wire and thermocouple meter movements use the heating effect of current flowing through a resistance to cause meter deflection. Each uses this effect in a different manner. Since their operation depend only on the heating effect of current flow, they may be used to measure both direct and alternating currents of any frequency on a single scale.

2.12.1 Hot-wire Instrument

The hot-wire meter movement deflection depends on the expansion of a high resistance wire caused by the heating effect of the wire itself as current flows through it. A resistance wire is stretched between the two meter terminals, with a thread attached at a right angles to the centre of the wire. A spring connected to the opposite end of the thread exerts a constant tension on the resistance wire. Current flow heats the wire, causing it to expand. This motion is transferred to the meter pointer through the thread and a pivot. Figure 2.23 shows the basic arrangement of a hot wire type instrument.

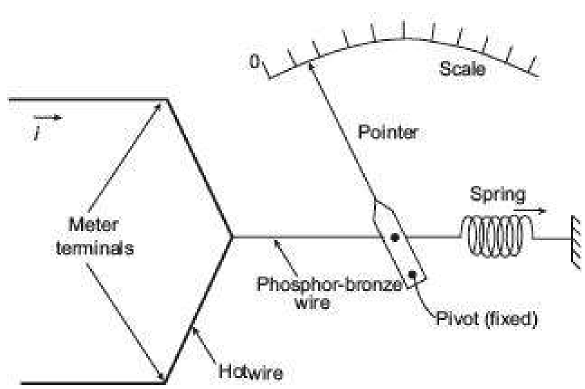


Figure 2.23 *Hot-wire instruments*

Advantages of Hot-wire-type Instruments

1. The deflection depends upon only the rms value of the current flowing through the wire, irrespective if its waveform and frequency. Hence, the instrument can used for ac as well as dc system.
2. The calibration is same for ac as well as dc measurement. So it is a transfer-type instrument.
3. They are free from stray magnetic fields because no magnetic field is used to cause their operation.
4. It is cheap in cost and simple in construction.
5. With suitable adjustments, error due to temperature variation can be made negligible.

6. This type of instruments are quite suitable for very high frequency measurement.

Disadvantages of Hot-wire-type Instruments

1. Power consumption is relatively high.
2. Nonuniform scale.
3. These are very sluggish in action as time is taken in heating up the wire.
4. The deflection of the instrument is not the same for ascending and descending values.
5. The reading depends upon the atmospheric temperature.

2.12.2 Thermocouple-Type Instrument

When two metals having different work functions are placed together, a voltage is generated at the junction which is nearly proportional to the temperature of the junction. This junction is called a thermocouple. This principle is used to convert heat energy to electrical energy at the junction of two conductors as shown in Figure 2.24.

The heat at the junction is produced by the electrical current flowing in the heater element while the thermocouple produces an emf at its output terminals, which can be measured with the help of a PMMC meter. The emf produced is proportional to the temperature and hence to the rms value of the current. Therefore, the scale of the PMMC instrument can calibrate to read the current passing through the heater. The thermocouple type of instrument can be used for both ac and dc applications. The most effective feature of a thermocouple instrument is that they can be used for measurement of current and voltages at very high frequency. In fact, these instruments are very accurate well above a frequency of 50 MHz.

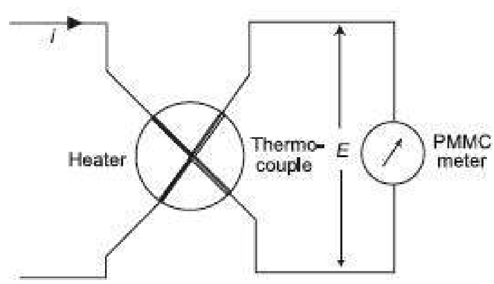


Figure 2.24 *Circuit diagram of thermocouple instrument*

Advantages of Thermocouple-type Instruments

1. These are not affected by stray magnetic fields.
2. They have very high sensitivity.
3. The indication of these instruments are practically unaffected by the frequency and waveform of the measuring quantity. Hence these instruments can be used for measurement of currents upto frequencies of 50 MHz and give accuracy as high as 1%.
4. These instruments are very useful as transfer instruments for calibration of dc instruments by potentiometer and a standard cell.

Disadvantages of Thermocouple-Type Instruments

1. Considerable power losses due to poor efficiency of thermal conversion.

2. Low accuracy of measurement and sensitivity to overloads, as the heater operates at temperatures close to the limit values. Thus, the overload capacity of such instrument is approximately 1.5 times of full-scale current.

3. The multi-voltmeters used with thermo-elements must be necessarily more sensitive and delicate than those used with shunts, and therefore, requires careful handling.

2.13 RECTIFIER-TYPE INSTRUMENTS

The basic arrangement of a rectifier type of instrument using a full-wave rectifier circuit is shown in Figure 2.25. If this instrument is used for measuring ac quantity then first the ac signal is converted to dc with the help of the rectifier. Then this dc signal is measured by the PMMC meter. The multiplier resistance R_s, is used to limit the value of the current in order that it does not exceed the current rating of the PMMC meter.

These types of instruments are used for light current work where the voltage is low and resistances high.

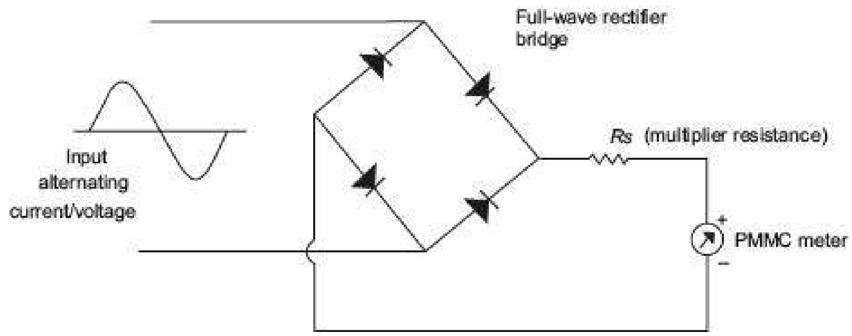


Figure 2.25 *Rectifier-type instrument*

2.13.1 Sensitivity of Rectifier-Type Instrument

The dc sensitivity of a rectifier-type instrument is

$S_{dc} = \frac{1}{I_{fs}} \Omega/v$ where I_{fs} is the current required to produce full-scale deflection.

1. *Sensitivity of a Half-wave Rectifier Circuit*

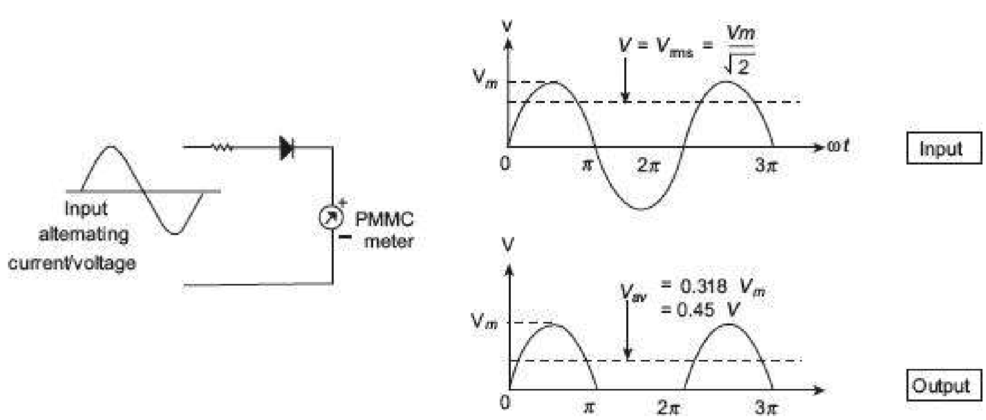


Figure 2.26 *Half-wave rectifier*

Figure 2.26 shows a simple half-wave rectifier circuit along with the input and output waveform. The average value of voltage/current for half-wave rectifier,

$$V_{av} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{V_m}{\pi} = 0.318V_m = 0.45 \text{ V} \qquad (2.52)$$

Hence, the sensitivity of a half-wave rectifier instrument with ac is 0.45 times its sensitivity with dc and the deflection is 0.45 times that produces with dc of equal magnitude V.

$$S_{ac} = 0.45S_{dc} \qquad (2.53)$$

2. *Sensitivity of a Full-wave Rectifier Circuits*

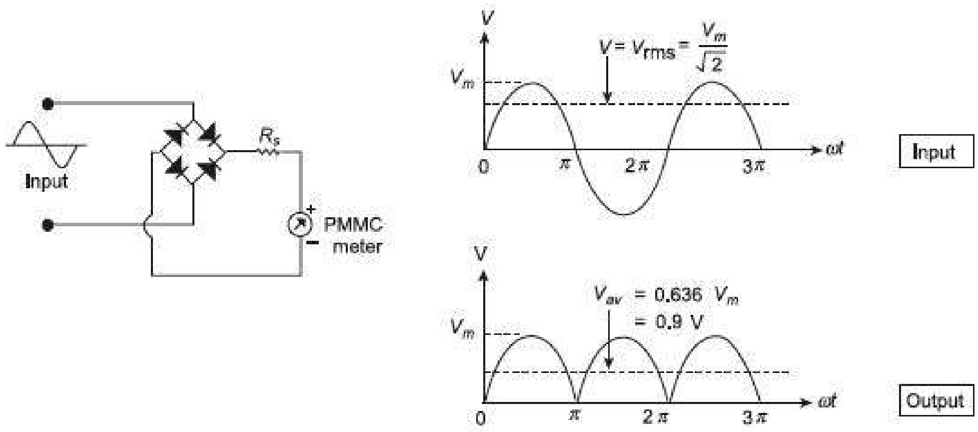


Figure 2.27 *Full-wave rectifier*

Figure 2.27 *Full-wave rectifier*

Figure 2.27 shows a full-wave rectifier circuit along with the input and output waveform. Average value of voltage/current for full-wave rectifier,

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{2V_m}{\pi} = 0.636V_m = 0.9 \text{ V} \qquad (2.54)$$

So the deflection is 0.9 times in a full-wave rectifier instrument with an ac than that produced with dc of equal magnitude V.

Sensitivity of a full-wave rectifier instrument with an ac is 0.9 times its sensitivity with dc.

$$S_{ac} = 0.9S_{dc} \qquad (2.55)$$

2.13.2 Extension of Range of Rectifier Instrument as Voltmeter

Suppose it is intended to extend the range of a rectifier instrument which uses a PMMC instrument having a dc sensitivity of S_{dc}.

Let, v = voltage drop across the PMMC instrument

V = applied voltage

Therefore, for dc operation, the values of series resistance (multiplier) needed can be calculated from Figure 2.28 as

$$V = R_S \cdot I_{fs} + R_d \cdot I_{fs} + R_m \cdot I_{fs}$$

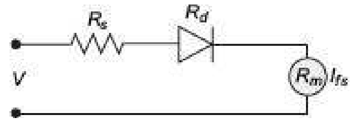


Figure 2.28 *Range extension of rectifier voltmeter*

$$
\begin{aligned}
R_s &= \left(\frac{V}{I_{fs}}\right) - R_m - R_d \\
&= S_{dc}V - R_m - R_d \text{ (for half-wave rectification)} \\
&= S_{dc}V - R_m - 2R_d \text{ (for full-wave rectification)}
\end{aligned}
\tag{2.56}
$$

where R_m = meter resistance

R_d = diode forward resistance

For ac voltmeter,

$$
\begin{aligned}
R_s &= S_{ac}V - R_m - R_d = 0.45S_{dc}V - R_m - R_d \text{ (for half-wave)} \\
&= S_{ac}V - R_m - 2R_d = 0.9\,S_{dc}V - R_m - R_d \text{ (for full-wave)}
\end{aligned}
\tag{2.57}
$$

Limitations

1. Rectifier instruments are only accurate on the waveforms on which they are calibrated. Since calibration assumes pure sine waves, the presence of harmonics gives erroneous readings.
2. The rectifier is temperature sensitive, and therefore, the instrument readings are affected by large variations of temperature.

Applications

1. The rectifier instrument is very suitable for measuring alternating voltages in the range of 50–250 V.
2. The rectifier instrument may be used as a micrometer or low milliammeter (up to 10–15 mA). It is not suitable for measuring large currents because for larger currents the rectifier becomes too bulky and providing shunts is impracticable due to rectifier characteristics.
3. Rectifier instruments find their principal application in measurement in high-impedance circuits at low and audio frequencies. They are commonly used in communications circuits because of their high sensitivity and low power consumption.

2.14 TRUE rms VOLTMETER

The commonly available multimeters are average or peak reading instruments, and the rms values they display are based on the signal mean value. They multiply the average value with some factor to convert it to the rms reading. For this reason, conventional multimeters are only suited for sinusoidal signals. For measuring rms value of a variety of signals over a wide range of frequencies, a new kind of voltmeter—called the True RMS (TRMS) voltmeter has been developed. Since these voltmeters do not measure rms value of a signal based on its average value, they are suited for any kind of waveforms (such as sine wave, square wave or sawtooth wave).

The conventional moving-iron voltmeter has its deflection proportional to the square of the current passing through its coil. Thus, if the scale is calibrated in terms of square root of the measured value, moving-iron instruments can give true rms value of any signal, independent of its wave shape. However, due to large inertia of the mechanical moving parts present in such a moving iron instrument, the frequency bandwidth of such a true rms voltmeter is limited. Similar is the case for electrodynamometer type instruments which once again have their deflecting torque proportional to the current through their operating coil. But once again, though electrodynamometer-type instruments can give true rms indication of a signal of any waveform, their frequency bandwidth is also limited due to their mechanical moving parts.

Modern-day true rms reading voltmeters are made to respond directly to the heating value of the input signal. To measure rms value of any arbitrary waveform signal, the input signal is fed to a heating element and a thermocouple is placed very close to it. A thermocouple is a junction of two dissimilar metals whose contact potential is a function of the temperature of the junction. The heating value is proportional to the square of the rms value of the input signal. The heater raises the temperature of the heater and the thermocouple produces an output voltage that is proportional to the power delivered to the heater by the input signal. Power being proportional to the square of the current (or voltage) under measurement, the output voltage of the thermocouple can be properly calibrated to indicate true rms value of the input signal. This way, such a thermal effect instrument permits the determination of true rms value of an unknown signal of any arbitrary waveform. Bandwidth is usually not a problem since this kind of principle can be used accurately even beyond 50 MHz. Figure 2.28 shows such an arrangement of thermocouple based true rms reading voltmeter.

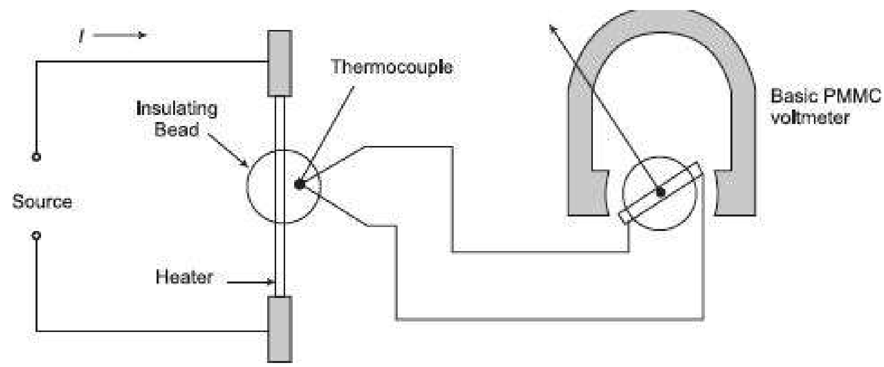


Figure 2.28 *Thermocouple based true rms reading voltmeter*

2.15 COMPARISON BETWEEN DIFFERENT TYPES OF INSTRUMENTS

Sl. No.	Type of Instruments	Suitability for type of measurement	Type of control	Type of damping	Specialty
1.	*Moving Coil* (i) PMMC	dc measurement (current and voltage only)	Spring	Eddy current	It is most accurate type for dc measurements and most widely used for measurement of dc voltage, current and resistance.
	(ii) Dynamometer	dc or ac measurement (current, voltage and power)	Spring	Air friction	Mainly used as wattmeter. Also used as standard meter for calibration and as transfer instrument.
2.	*Moving Iron*	dc or ac measurement (current, voltage)	Spring or gravity control	Air friction	It is cheaper to manufacture and mostly used as an indicating instrument. It is very accurate for ac and dc, if properly designed.
3.	*Electrostatic*	dc or ac (voltage only)	Gravity or spring	Air friction	These instruments have very low power consumption and can be made to cover a large range of voltage. Usually, range is above 500 volts.
4.	*Induction*	ac measurement (current, voltage, Power and energy) only.	Spring	Eddy current	Ammeters and voltmeters of this type are expensive and not of high degree of accuracy. These instruments are mainly used for measurement of power and energy in ac circuits.
5.	*Thermal* (i) Hot wire	dc or ac measurement (current, voltage and power)	Spring	Eddy current	These instrument have same calibration for both ac and dc. These are free from errors due to frequency, wave form and external field when used on ac, therefore, these are particularly used for ac measurement.
	(ii) Thermocouple	dc or ac measurement (current and voltage)			These are free from errors due to frequency, wave form and external field when used on ac and are used for measurement of current and voltage at power frequencies upto 100 MHz.
6.	*Rectifier*	dc or ac measurement (current and voltage)	Spring	Eddy current	These instruments are nothing but permanent magnet moving coil instruments used in conjunction with rectifying device for AC measurements (current and voltage) from about 20 Hz to 20 kHz.

EXERCISE

Objective-type Questions

1. A spring produces a controlling torque of $16x10^{-6}$ Nm for a deflection of 120Y. If the width and length become two times their original values and the thickness is halved, the value of controlling torque for the same deflection will be

   (a) 16×10^{-6} N

   (b) $8x10^{-6}$ Nm (c)$2x10^{-6}$ Nm

   (d) $32x10^{-6}$ Nm

2. The shunt resistance in an ammeter is usually

   (a) less than meter resistance

   (b) equal to meter resistance

   (c) more than meter resistance

   (d) of any value

3. A voltage of 200 V produces a deflection of 90° in a PMMC spring-controlled instrument. If the same instrument is provided with gravity control, what would be the deflection?

(a) 45°

(b) 65°

(c) 90°

(d) cannot be determined by the given data

4. A current-carrying conductor is shown in Figure (a). If it is brought in a magnetic field shown in Figure (b)

   (a) it will experience a force from left to right.

   (b) it will experience a force from right to left.

   (c) it will experience a force from top to bottom.

   (d) it will experience no force.

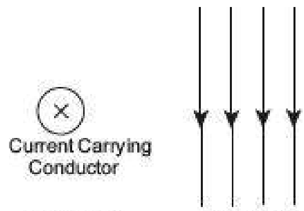


Figure (a) Figure (b)

5. The high torque to weight ratio in an analog indicating instrument indicates

   (a) high friction loss

   (b) nothing as regards friction loss

   (c) low friction loss

   (d) none of the above

6. Swamping resistance is connected

   (a) in series with the shunt to reduce temperature error in shunted ammeter

   (b) in series with the ammeters to reduce errors on account of friction

   (c) in series with meter and have a high resistance temperature coefficient in order to reduce temperature errors in ammeters.

   (d) in series with the meter and have a negligible resistance co-efficient in order to reduce temperature errors in shunted ammeters

7. Moving-iron instruments when measuring voltages or currents

   (a) indicate the same values of the measurement for both ascending and descending values

   (b) indicate higher value of measurand for ascending values

   (c) indicate higher value of measurand for descending values

   (d) none of the above

8. A moving-iron type of instrument can be used as

   (a) standard instruments for calibration of other instruments

   (b) transfer-type instruments

   (c) indicator-type instruments as on panels

   (d) all of the above

9. In spring-controlled moving iron instruments, the scale is

   (a) uniform

   (b) cramped at the lower end and expanded at the upper end

   (c) expanded at the lower end and cramped at the upper end

   (d) cramped both at the lower and the upper ends

10. Thermocouple instruments can be used for a frequency range

(a) up to 500 Hz

(b) up to 5 MHz

(c) up to 100 Hz

(d) up to 1 MHz

11. The reason why eddy-current damping cannot he used in a moving-iron instrument, is

(a) they have a strong operating magnetic field

(b) they are not normally used in vertical position

(c) they need a large damping force which can only he provided by air friction

(d) they have a very weak operating magnetic field and introduction of a permanent magnet required for eddy current damping would distort the operating magnetic field

12. An electrodynamometer type of instrument finds its major use as

(a) standard instrument only

(b) both as standard and transfer instrument

(c) transfer instrument only

(d) indicator-type instrument

13. The frequency range of moving-iron instruments is

(a) audio-frequency band 20 Hz to 20 kHz

(b) very low-frequency band 10 Hz to 30 kHz

(c) low-frequency band 30 Hz to 300 kHz

(d) power frequencies 0 to 125 Hz.

14. A voltage of 200 V at 5 Hz is applied to an electrodynamometer type of instrument which is spring controlled. The indication on the instruments is

(a) 200 V

(b) 0 V

(c) the instrument follows the variations in voltage and does not give a steady response

(d) none of the above

15. Spring-controlled moving-iron instruments exhibit a square law response resulting in a non-linear scale. The shape of the scale can be made almost linear by

(a) keeping rate of change of inductance, L, with deflection, θ, as constant

(b) keeping $\frac{1}{\theta}.\frac{dL}{d\theta}$ as constant

(c) keeping $\theta.\frac{dL}{d\theta}$ as constant

(d) keeping $\frac{1}{k\theta}$ as constant, where k is the spring constant

16. Electrostatic-type instruments are primarily used as

(a) ammeters

(b) voltmeters

(c) wattmeters

(d) ohmmeters

17. The sensitivity of a PMMC instrument is 10CkZ/V. If this instrument is used in a rectifier-type voltmeter with half wave rectification. What would be the sensitivity?

(a) 10 kΩ/V

(b) 4.5 kΩ/V

(c) 9 kΩ/V

(d) 22.2 kΩ/V

18. The heater wire of thermocouple instrument is made very thin in order

(a) to have a high value of resistance

(b) to reduce skin effects at high frequencies

(c) to reduce the weight of the instrument

(d) to decrease the over-ranging capacity of the instrument

19. Which instrument has the highest frequency range with accuracy within reasonable limits?

(a) PMMC

(b) Moving iron

(c) Electrodynamometer

(d) Rectifier

20. Which meter has the highest accuracy in the prescribed limit of frequency range?

(a) PMMC

(b) Moving iron

(c) Electrodynamometer

(d) Rectifier

Answers

1. (c)	2. (a)	3. (c)	4. (d)	5. (c)	6. (d)	7. (c)
8. (c)	9. (b)	10. (d)	11. (d)	12. (b)	13. (d)	14. (c)
15. (c)	16. (b)	17. (b)	18. (b)	19. (d)	20. (a)	

Short-answer Questions

1. Describe the various operating forces needed for proper operation of an analog indicating instrument.
2. Sketch the curves showing deflection versus time for analog indicating instruments for underdamping, critical damping and overdamping.
3. What are the difference between recording and integrating instruments? Give suitable examples in each case.
4. Derive the equation for deflection of a PMMC instrument if the instrument is spring controlled.
5. How can the current range of a PMMC instrument be extended with the help of shunts?
6. Derive the equation for deflection of a spring-controlled moving-coil instrument.
7. Describe the working principle of a rectifier-type instrument. What is the sensitivity of such an instrument?
8. What are the advantages and disadvantages of a PMMC instrument?
9. Describe the working principle and constructional details of an attraction-type moving iron instrument.
10. Derive the expression for deflection for a rotary-type electrostatic instrument using spring control.
11. What is swamping resistance? For what purpose is swamping resistance used?
12. How many ways can the damping be provided in an indicating instrument?

Long-answer Questions

1. (a) How many operating forces are necessary for successful operation of an indicating instrument? Explain the methods of providing these forces.

   (b) A moving-coil instrument has the following data: number of turns = 100, width of coil = 20 mm, depth of coil = 30 mm, flux density in the gap = 0.1 Wb/m^2. Calculate the deflecting torque when carrying a current of 10 mA. Also calculate the deflection if the control spring constant is 2×10^{-6} N-m/degree.

[Ans. 60×10^{-6} Nm, 30°]

2. (a) What are the advantages and disadvantages of moving-coil instruments?

(b) A moving-coil voltmeter has a resistance of 200 W and the full scale deflection is reached when a potential difference of 100 mV is applied across the terminals. The moving coil has effective dimensions of 30 mm × 25 mm and is wound with 100 turns. The flux density in the gap is 0.2 Wb/m^2. Determine the control constant of the spring if the final deflection is 100° and a suitable diameter of copper wire for the coil winding if 20% of the total instrument resistance is due to the coil winding. Resistivity of copper is 1.7×10^{-8} Ωm.

[Ans. 0.075×10^{-6} Nm/degree; 0.077 mm]

3. (a) Derive the expression for the deflection of a spring controlled permanent magnet moving coil instrument. Why not this instrument able to measure the ac quantity?

(b) The coil of a moving coil voltmeter is 40 mm × 30 mm wide and has 100 turns wound on it. The control spring exerts a torque of 0.25×10^{-3} Nm when the deflection is 50 divisions on the scale. If the flux density of the magnetic field in the air-gap is 1 Wb/m^2, find the resistance that must be put in series with the coil to give 1 volt per division. Resistance of the voltmeter is 10000 Ω.

[Ans. 14000 Q]

4. (a) A moving-coil instrument has at normal temperature a resistance of 10 W and a current of 45 mA gives full scale deflection. If its resistance rises to 10.2 Ω due to temperature change, calculate the reading when a current of 2000 A is measured by means of a 0.225×10^{-3}. A shunt of constant resistance. What is the percentage error?

[Ans. 44.1 mA, -1.96%]

(b) The inductance of a certain moving-iron ammeter is $(8 + 4\theta - \frac{1}{2}\theta^2)$ pH, where *0* is the deflection in radian from the zero position. The control spring torque is 12×10^{-6} Nm/rad. Calculate the scale position in radian for current of 5 A.

[2.04 rad]

5. (a) Discuss the constructional details of a thermocouple-type instrument used at very high frequencies. Write their advantages and disadvantages.

(b) The control spring of a moving-iron ammeter exerts a torque of $0.5 \times 10^{-}$ Nm/degree when the deflection is 52°. The inductance of the coil varies with pointer deflection according to

deflection (degree)	20	40	60	80
inductance (|mH)	659	702	752	792

Determine the current passing through the meter.

[0.63 A]

6. (a) Describe the constructional details of an attraction-type moving iron instrument with the help of a neat diagram. Derive the equation for deflection if spring control is used and comment upon the shape of scale.

(b) Derive a general equation for deflection for a spring-controlled repulsion-type moving-iron instrument. Comment upon the share of the scale. Explain the methods adopted to linearise the scale.

3 Instrument Transformers

3.1 INTRODUCTION

Instrument transformers are used in connection with measurement of voltage, current, energy and power in ac circuits. There are principally two reasons for use of instrument transformers in measurement: first, to extend (multiply) the range of the measuring instrument and second, to isolate the measuring instrument from a high-voltage line.

In power systems, levels of currents and voltages handled are very high, and, therefore, direct measurements with conventional instruments is not possible without compromising operator safety, and size and cost of instrument. In such a case, instrument transformers can be effectively used to step down the voltage and current within range of the existing measuring instruments of moderate size. Instrument transformers are either (a) current transformer or CT, or (b) voltage or potential transformers or PT. The former is used to extend current ranges of instruments and the latter for increasing the voltage ranges.

Instrument transformers have their primary winding connected to the power line and secondary windings to the measuring instrument. In this way, the measuring instruments are isolated from the high power lines. In most applications, it is necessary to measure the current and voltage of large alternators, motors, transformers, buses and other power transmission equipments for metering as well as for relaying purposes. Voltages in such cases may range from 11,000 to even 330,000 V. It would be out of question to bring down these high-voltage lines directly to the metering board. This will require huge insulation and pose great danger for operating personnel otherwise. In such a case, instrument transformers can greatly solve this problem by stepping down the high voltage to safe levels for measurement.

3.2 ADVANTAGES OF INSTRUMENT TRANSFORMERS

Shunt and multipliers used for extension of instrument ranges are suitable for dc circuits and to some extent, for low power, low accuracy ac circuits. Instrument transformers have certain distinguishing characteristics as compared to shunts and multipliers, as listed below.

1. Using shunts for extension of range on ammeters in ac circuits will require careful designing of the reactance and resistance proportions for the shunt and the meter. Any deviation from the designed time constants of the shunt and the meter may lead to errors in measurement. This problem is not present with CT being used with ammeter.

2. Shunts cannot be used for circuits involving large current; otherwise the power loss in the shunt itself will become prohibitably high.

3. Multipliers, once again, due to inherent leakage current, can introduce errors in measurement, and can also result in unnecessary heating due to power loss.

4. Measuring circuits involving shunts or multipliers, being not electrically isolated from the power circuit, are not only safe for the operator, but also insulation requirements are exceedingly high in high-voltage measurement applications.

5. High voltages can be stepped down by the PT to a moderate level as can be measured by standard instruments without posing much danger for the operator and also not requiring too much insulation for the measuring instrument.

6. Single range moderate size instruments can be used to cover a wide range of measurement, when used with a suitable multi-range CT or PT.

7. Clamp-on type or split-core type CT’s can be very effectively used to measure current without the need for breaking the main circuit for inserting the CT primary winding.

8. Instrument transformers can help in reducing overall cost, since various instruments, including metering, relaying, diagnostic, and indicating instruments can all be connected to the same instrument transformer.

3.3 CURRENT TRANSFORMERS (CT)

The primary winding of a current transformer is connected in series with the line carrying the main current. The secondary winding of the CT, where the current is many times stepped down, is directly connected across an ammeter, for measurement of current; or across the current coil of a wattmeter, for measurement of power; or across the current coil of a watt-hour meter for measurement of energy; or across a relay coil. The primary winding of a CT has only few turns, such that there is no appreciable voltage drop across it, and the main circuit is not disturbed. The current flowing through the primary coil of a CT, i.e., the main circuit current is primarily determined by the load connected to the main circuit and not by the load (burden) connected to the CT secondary. Uses of CT for such applications are schematically shown in Figure 3.1.

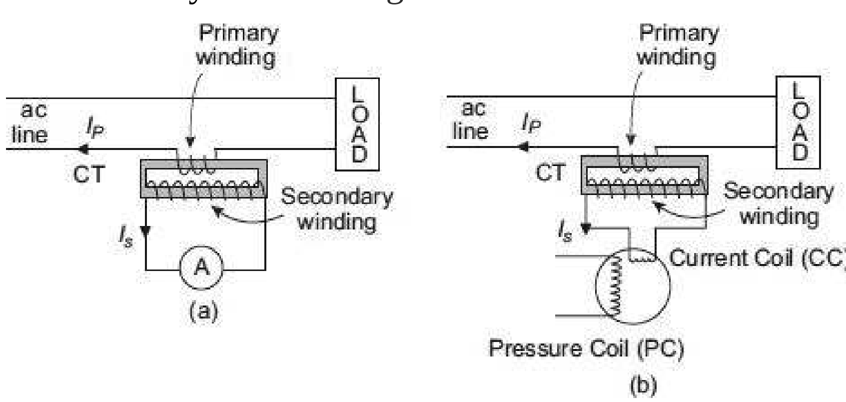


Figure 3.1 *CT for (a) current, and (b) power measurement*

One of the terminals of the CT is normally earthed to prevent any accidental damage to the operating personnel in the event of any incumbent insulation breakdown.

When a typical name plate rating of a CT shows 500/1 A 5 VA 5P20 it indicates that the CT rated primary and secondary currents are 500 A and 1 A respectively, its rated secondary burden is 5 VA, it is designed to have 5% accuracy and it can carry up to 20 times higher current than its rated value while connected in line to detect fault conditions, etc.

3.4 THEORY OF CURRENT TRANSFORMERS

Figure 3.2 represents the equivalent circuit of a CT and Figure 3.3 plots the phasor diagram under operating condition of the CT.

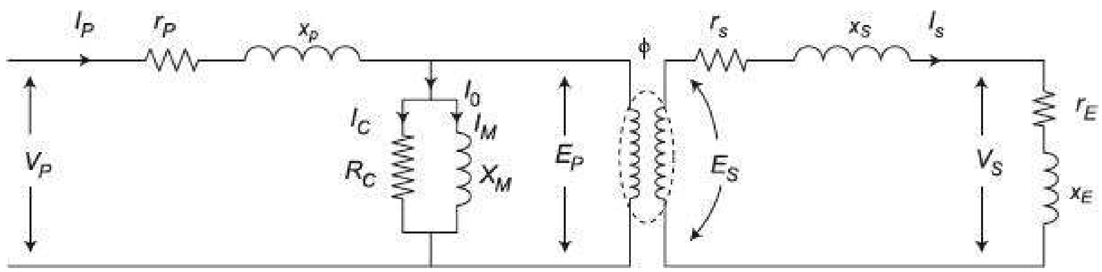


Figure 3.2 *Equivalent circuit of a CT*

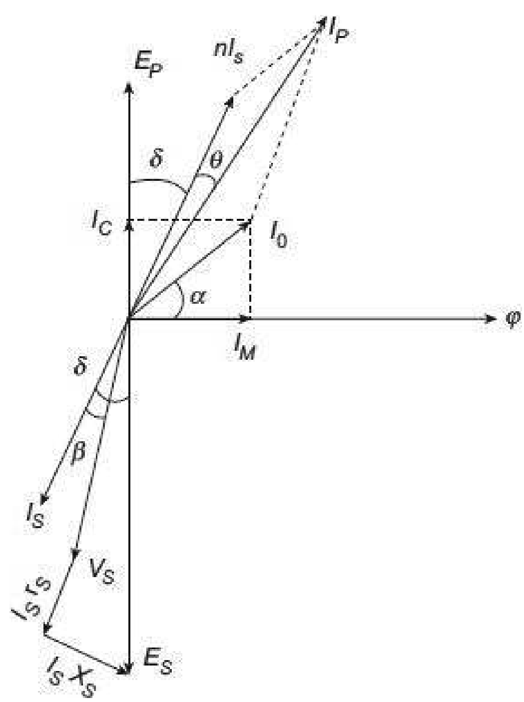


Figure 3.3 *Phasor diagram of a CT*

V_P = primary supply voltage

E_P = primary winding induced voltage

V_S = secondary terminal voltage

E_S = secondary winding induced voltage

$I_{\boldsymbol{P}}$ = primary current

$I_{\boldsymbol{S}}$ = secondary current

I_0 = no-load current

I_C = core loss component of current

I_M = magnetising component of current

r_P = resistance of primary winding

x_P = reactance of primary winding

r_S = resistance of secondary winding

x_S = reactance of secondary winding

R_C = imaginary resistance representing core losses

X_M = magnetising reactance

r_e = resistance of external load (burden) including resistance of meters, current coils, etc.

x_E = reactance of external load (burden) including reactance of meters, current coils, etc.

N_P = primary winding number of turns

N_S = secondary winding number of turns

n = turns ratio = N_S / N_P

φ = working flux of the CT

θ = the "phase angle" of the CT

δ = phase angle between secondary winding induced voltage and secondary winding current (i.e. phase angle of total burden, including secondary winding)

β = phase angle of secondary load (burden) circuit only

α = phase angle between no-load current I_0 and flux φ

The flux φ is plot along the positive x-axis. Magnetising component of current I_M is in phase with the flux. The core loss component of current I_c, leads by I_M 90°. Summation of I_C and I_M produces the no-load current I_0, which is α angle ahead of flux φ.

The primary winding induced voltage E_P is in the same phase with the resistive core loss component of the current I_C. As per transformer principles, the secondary winding induced voltage E_S will be 180° out of phase with the primary winding induced voltage E_P. The secondary current I_S lags the secondary induced voltage E_S by angle $\boldsymbol{\delta}$.

The secondary output terminal voltage V_S is obtained by vectorically subtracting the secondary winding resistive and reactive voltage drops $I_S\ r_S$ and $I_S\ x_S$ respectively from the secondary induced voltage E_S. The phase angle difference between secondary current I_S and secondary terminal voltage V_S is β, which is the phase angle of the load (burden).

The secondary current I_S, when reflected back to primary, can be represented by the 180° shifted phasor indicated by nI_S, where n is the turns ratio. The primary winding current I_P is the phasor summation of this reflected secondary current (load component) nI_S and the no-load current I_0.

The phase angle difference θ between the primary current I_P and the reflected secondary current nI_S is called phase angle of the CT.

3.4.1 Current Transformation Ratio of CT

Redrawing expanded view of the phasor diagram of Figure 3.3, we obtain the phasor diagram of Figure 3.4.

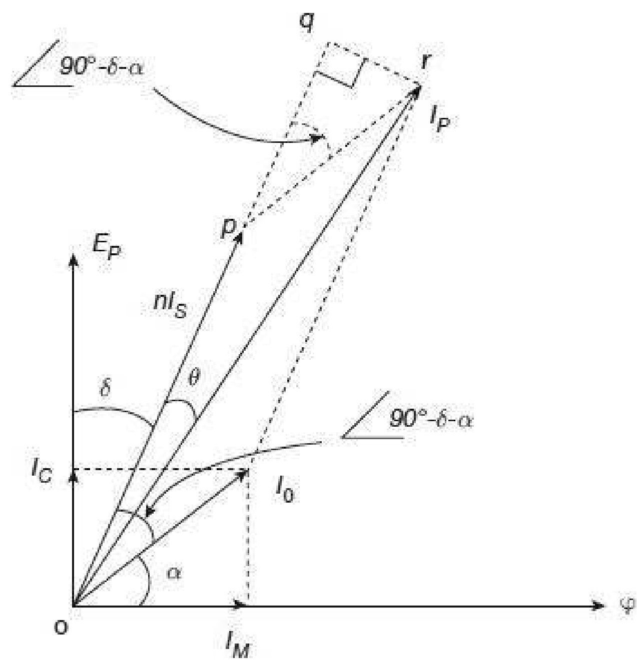


Figure 3.4 *Expanded view of a section of Figure 3.3*

From the right-angle triangle *pqr,* we get

$$pr = I_0$$

$$pq = I_0 \cdot \cos(90^\circ - \delta - \alpha) = I_0 \cdot \sin(\delta + \alpha)$$

$$qr = I_0 \cdot \sin(90^\circ - \delta - \alpha) = I_0 \cdot \cos(\delta + \alpha)$$

Now, $$(or)^2 = (op + pq)^2 + (qr)^2$$

Or, $$(Ip)^2 = (nI_S + I_0 \cdot \sin(\delta + \alpha))^2 + (I_0 \cdot \cos(\delta + \alpha))^2$$

$$= n^2 I_S^2 + I_0^2 \cdot \sin^2(\delta + \alpha) + 2nI_S I_0 \,.\, \sin(\delta + \alpha) + I_0^2 \,.\, \cos^2(\delta + \alpha)$$

$$= n^2 I_S^2 + 2nI_S I_0 .\ \sin(\delta + \alpha) + I_0^2 (\sin^2(\delta + a) + \cos^2(\delta + \alpha))$$

$$= n^2 I_S^2 + 2nI_S I_0 \cdot \sin(\delta + a) + I_0^2$$

$$\therefore \qquad I_P = \sqrt{n^2 I_S^2 + 2nI_S I_0 \cdot \sin(\delta+\alpha) + I_0^2} \tag{3.1}$$

In a well-designed CT, the no-load current I_0 is much less as compared to the primary current I_p or even the reflected secondary current (which is nominally equal to the primary current) nI_S.

i.e., $I_0 << nI_s$

Equation (3.1) can thus now be approximated as

$$I_P = \sqrt{n^2 I_S^2 + 2nI_S I_0 \cdot \sin(\delta+\alpha) + (I_0 \cdot \sin(\delta+\alpha))^2}$$

Hence, $I_p = nI_S + I_0 \,.\, \sin(\delta + \alpha)$

CT transformation ratio can now be expressed as

$$R = \frac{I_P}{I_S} = \frac{nI_S + I_0 \cdot \sin(\delta+\alpha)}{I_S} = n + \frac{I_0}{I_S}\sin(\delta+\alpha)$$

Though approximate, Eq. (3.2) is sufficiently accurate for practical estimation of CT transformation ratio. The above equation, however, is true for only when the power factor of the burden is lagging, which is mostly true in all practical inductive meter coils being used as burden.

Equation (3.2) can be further expanded as

$$R = n + \frac{I_0}{I_S}\sin(\delta+\alpha) = n + \frac{I_0}{I_S}(\sin\delta\cos\alpha + \cos\delta\sin\alpha)$$

or, transformation ratio $R \approx n + \dfrac{I_M \sin\delta + I_C \cos\delta}{I_S}$

[since, $I_M = I_0 \cos\alpha$ and $I_C = I_0 \sin\alpha$]

3.4.2 Phase Angle of CT

As can be seen from the phasor diagram in Figure 3.3, the secondary current of a CT is almost 180° out of phase from the primary current. If the angle was exactly 180° then there would have been no phase angle error introduced in the CT when it is to be used along with wattmeter for power measurements. In reality, however, due to presence of the parallel circuit branches, namely, the magnetising and the core loss branches, the phase angle difference is usually less than 180°. This causes some error in phase to be introduced while CT operation in practice.

The angle by which the secondary current phasor, when reversed, i.e., the reflected secondary current phasor nI_S, differs in phase from the primary current I_P, is called the **phase angle of the CT**. This angle is taken as positive when the reversed secondary current **leads** the primary current, in other cases when the reversed secondary current lags the primary current, the CT phase angle is taken as negative.

From the phasor diagram in Figure 3.4,

$$\tan\theta = \frac{qr}{oq} = \frac{qr}{po+qp} = \frac{I_0 \cdot \sin[90^\circ - (\delta+\alpha)]}{nI_S + I_0 \cdot \cos[90^\circ - (\delta+\alpha)]}$$

$$= \frac{I_0 \cdot \cos(\delta+\alpha)}{nI_S + I_0 \cdot \sin(\delta+\alpha)} a$$

For very small angles, $\theta \approx \frac{I_0 \cdot \cos(\delta+\alpha)}{nI_S + I_0 \cdot \sin(\delta+\alpha)}$

This expression can still be simplified with the assumption $I_0 << nI_S$.

$\theta \approx \frac{I_0 \cos(\delta+\alpha)}{nI_S}$ rad

Or, $\theta \approx \frac{I_0(\cos\delta\cos\alpha - \sin\delta\sin\alpha)}{nI_S} \approx \frac{I_M\cos\delta - I_C\sin\delta}{nI_S}$ rad

Or, phase angle of CT $\theta \approx \frac{180}{\pi}\left(\frac{I_M\cos\delta - I_C\sin\delta}{nI_S}\right)$ degree

3.5 ERRORS INTRODUCED BY CURRENT TRANSFORMERS

When used for current measurement, the only essential requirement of a CT is that its secondary current should be a pre-defined fraction of the primary current to be measured. This ratio should remain constant over the entire range of measurement, such that no errors are introduced in the measurement. However, from Eq. (3.3), it is clear that the transformation ratio *R* of the CT differs from the turns ratio *n* . This difference is not constant, but depends on the magnitude of magnetising and loss components of no-load current, and also on the secondary winding load current and its phase angle. The secondary winding current thus is never a constant fraction of the primary winding current under all conditions of load and of frequency. This introduces considerable amount of error in current measurement.

While power measurements, it is required that the secondary current of CT is displaced exactly by 180° from the primary current. As seen from Eq. (3.6), this condition is not fulfilled, but the CT has a phase angle error θ. This will introduce appreciable error during power measurements.

3.5.1 Ratio Error

Ratio error is defi ned as

$$\text{Percentage ratio error} = \frac{\text{Nominal ratio} - \text{Actual ratio}}{\text{Actual ratio}} \times 100\%$$

$$= \frac{Kn - R}{R} \times 100\%$$

where, $\text{Nominal ratio} = \frac{\text{Rated primary winding current}}{\text{Rated secondary winding current}} = Kn \approx n$

In practice, the CT burden is largely resistive with a small value of inductance, thus the secondary phase angle δ is positive and generally small. The nominal ratio *Kn,* is sometimes loosely taken equal to the turns ratio *n* . This assumption, as will be described in later sections, is true in the case when turns compensation is not used in CT.

Thus, sin $\delta \approx 0$ and cos $\delta \approx 1$. Therefore, Eq. (3.3) can be approximated as

$$R \approx n + \frac{I_M \sin\delta + I_C \cos\delta}{I_S} \approx n + \frac{I_C}{I_S}$$

Accordingly, percentage ratio error can be approximated as

$$\text{Percentage ratio error} = \frac{Kn - \left(n + \frac{I_C}{I_S}\right)}{\left(n + \frac{I_C}{I_S}\right)} \times 100\%$$

3.5.2 Phase-Angle Error

Error in phase angle is given following Eq. (3.6) as

$$\theta \approx \frac{180}{\pi}\left(\frac{I_M \cos\delta - I_C \sin\delta}{nI_S}\right)$$

In practice, the CT burden is largely resistive with a small value of inductance, thus the secondary phase angle δ is positive and generally small.

Thus, sin $\delta \approx 0$ and cos $\delta \approx 1$. Therefore, Eq. (3.10) can be approximated as

$$\theta \approx \frac{180}{\pi} \times \frac{I_M}{nI_S} \text{ degree}$$

3.5.3 Causes of Errors in CT

In an ideal CT, the actual transformation ratio should have been exactly equal to the turns ratio and the phase angle should have been zero. However, due to inherent physical limitations inherent to the electric and magnetic circuits of the CT, practical performance deviates from these ideal behaviors and errors are introduced in measurement. The reasons for these errors are given here.

1. Primary winding always needs some magnetising MMF to produce flux and, therefore, the CT draws the magnetising current I_M.
2. CT no-load current must have a component I_c that has to supply the core losses, i.e., the eddy current loss and the hysteresis loss.
3. Once the CT core becomes saturated, the flux density in the core no longer remains a linear function of the magnetising force, this may introduce further errors.
4. Primary and secondary flux linkages differ due to unavoidable flux leakages.

3.5.4 Reducing Errors in CT

Errors are produced in the ratio and phase angle of a CT owing to the presence of the no-load component of the primary current. Improvement of accuracy, then, depends upon minimising this component or nullifying in some way its effects in introducing errors. The most obvious idea would be to attempt to keep the magnetising current component as small as possible. This can be achieved by a combination of the following schemes:

1. *Low Flux Density*

The magnetising component of current may be restricted by using low values of flux density. This may be achieved by using large cross-section for core. For this reason, CTs are normally designed with much lower flux densities as compared to a normal power transformer.

2. *High Permeability Core Material*

The magnetising component of current may be made small by the use of high permeability core material. Some special core materials, such as Permalloy, are even better than the highest grade silicon steel with respect to permeability, particularly at lower flux densities. Hipernik (50% Fe + 50% Ni) has high permeability at low flux densities along with reasonably high saturation density value. It is used frequently as core material for manufacturing CTs.

3. *Modification of Turns Ratio*

The accuracy of current transformers may be improved, at least in terms of transformation ratio, by suitably modifying the actual number of turns. Instead of using the number of turns in exact accordance with the desired nominal ratio, a change in few numbers of turns may be made in the secondary winding. Primary number of turns itself being so less, any change in number of turns in the primary may result in wide variation of the turns ratio. For normal operating conditions, the usual secondary current is found to be less than the nominal value due to the no-load current. Correction in such cases, thus, may be made by a small reduction in the secondary number of turns. This correction can, however, be exact only for particular value of current and burden impedance. CTs in such cases are normally marked as ‘compensated’ for that particular operating condition.

4. *Use of Shunts*

If the secondary current is found to be too high, it may be reduced by a shunt placed across primary or secondary. This method can make an exact correction, once again, only for a particular value and type of burden. Use of shunts can also help in reducing the phase-angle error.

5. *Wound-Core Construction*

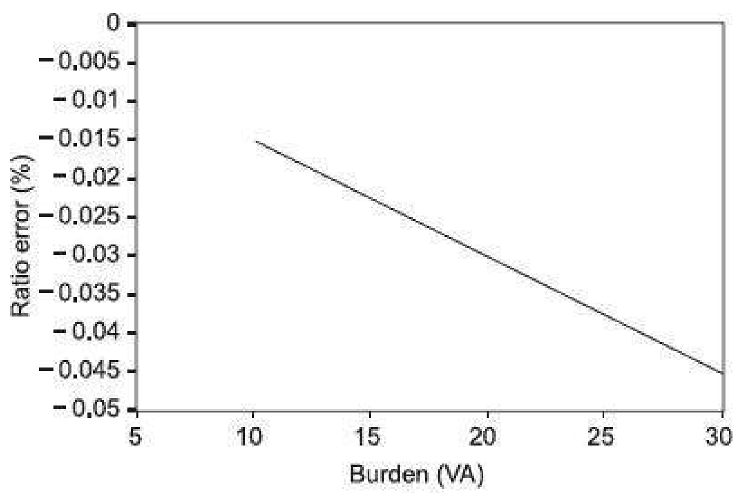
An improvement in the magnetisation characteristics of the CT core may be achieved by the use of wound-core construction. This type of construction for the core has been in use for some time in distribution transformers. By special treatment of the silicon steel to be used as core material, and by using it to carry flux always in the direction of grain orientation (rolling the sheet steel in proper way), magnetic properties of the core can be largely improved. This improvement may be utilised in CTs to reduce the ratio and phase angle errors.

3.6 OPERATIONAL CHARACTERISTICS OF CURRENT TRANSFORMERS

Characteristics of current transformers under different operating conditions may be estimated from the phasor diagram as shown in Figure 3.3.

3.6.1 Effect of Change in Burden on Secondary Circuit

Burden connected with the CT secondary may include ammeters, wattmeter current coils, relay coils, and so forth. All these being connected in series, carry the same current through them. In a current transformer, since the current depends solely on the primary current, an increase in the burden impedance will not change the secondary current, but will demand more voltage in the secondary terminals. This will increase volt-ampere burden of the secondary. With more instruments in the circuit, a higher voltage is required to make the current flow and this, in turn, requires more flux to flow through the core and hence a greater magnetising component of the primary current. This will result in an increase in the ratio and phase angle error of the current transformer. Figure 3.5 summarises the variations of ratio and phase angle errors in a typical current transformer at different values of the secondary burden VA.



(a)

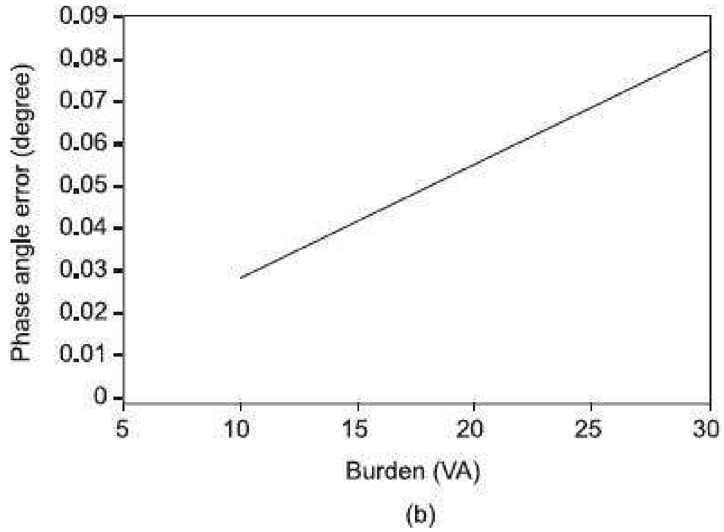


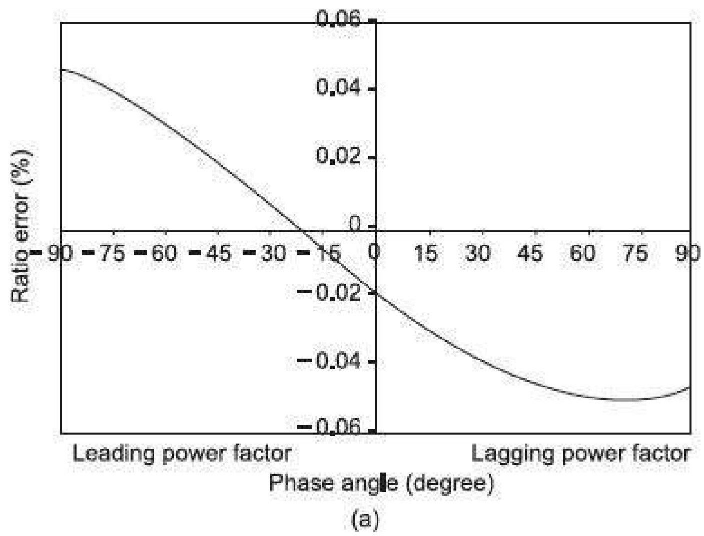
Figure 3.5 *Effects of secondary burden variation on (a) CT ratio error, and (b) CT phase-angle error*

3.6.2 Effect of Change in Power Factor of Secondary Burden

1. *Ratio Error*

As can be observed from the phasor diagram of a current transformer in Figure 3.3, for all inductive burdens, the secondary winding current I_S lags behind the secondary induced voltage $\boldsymbol{E_S}$, so that the phase angle difference δ is positive. Under these conditions, from Eq. (3.2), the actual transformation ratio is always greater than the turns ratio, and thus according to Eq. (3.9), ratio error is always negative for inductive (lagging power factor) burdens.

For highly capacitive burdens, the he secondary winding current I_S leads the secondary induced voltage E_S, so that the phase angle difference δ is negative. In such a case, the actual transformation ratio may even become less than the turns ratio and ratio error may thus become positive.



2. *Phase-Angle Error*

From Eq. (3.5) it is observed that for inductive burdens, the phase angle error remains positive till a certain low value of power factor is reached at highly inductive burdens when the phase angle error crosses over the axis to turn negative. For capacitive burdens, however, the phase-angle error always remains positive.

Figure 3.6 shows the variations of ratio and phase-angle errors in a typical current transformer at different values of the secondary burden power factor. These variations are described with the assumption of secondary burden VA to remain constant.

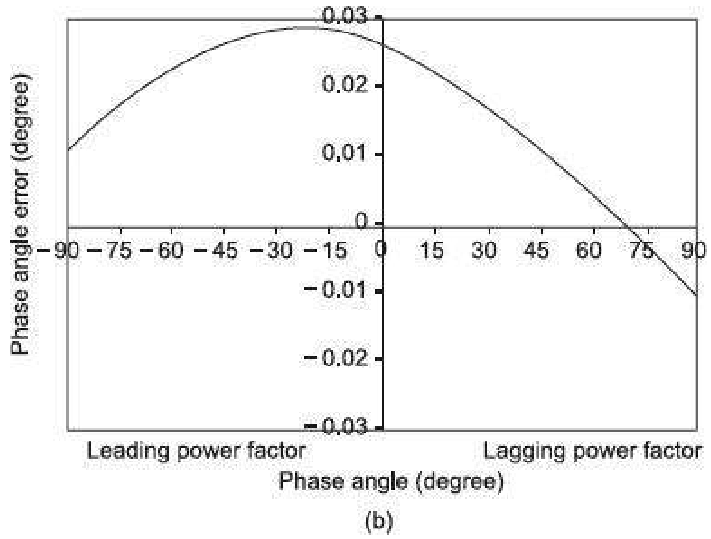
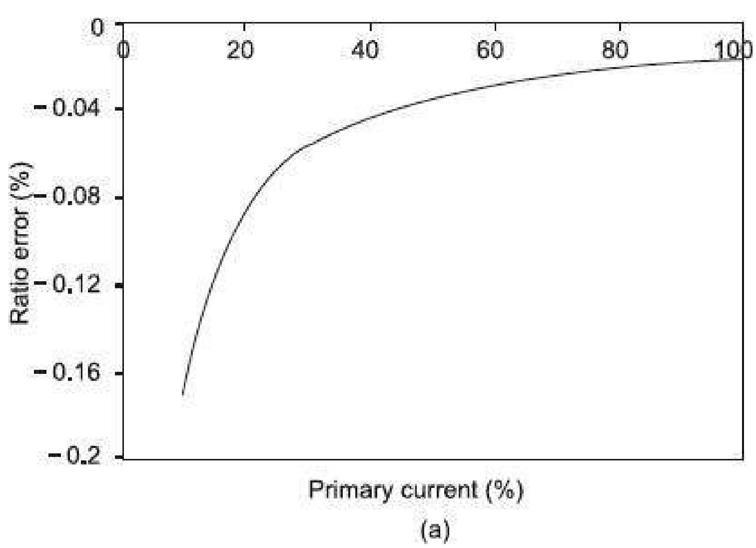


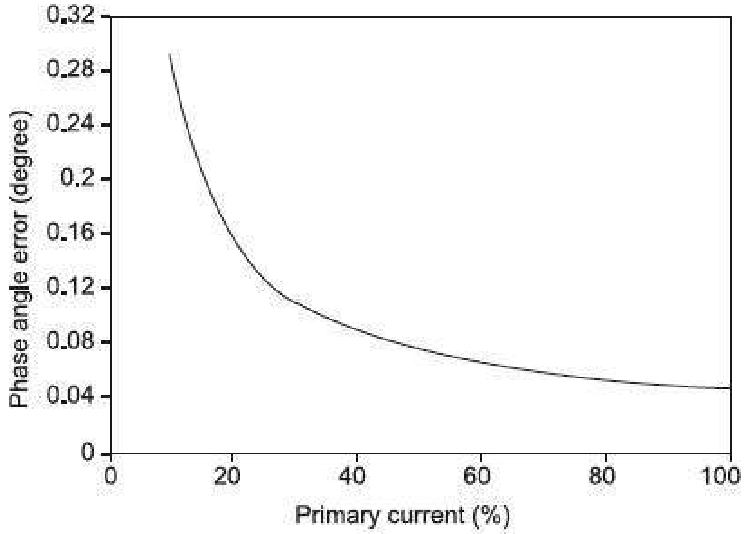
Figure 3.6 *Effects of secondary burden power factor variation on (a) CT ratio error, and (b) CT phase-angle error*

3.6.3 Effect of Change in Primary Winding Current

As the primary winding current ***Ip*** changes, the secondary winding current ***Is*** also changes proportionately. With lower values of ***Ip*** (and Is), the magnetising and loss components of no-load current become comparatively higher, and, therefore, both of ratio and phase angle errors become higher. As the primary current I_p increases, the secondary current I_S also increases, thereby reducing the proportions of magnetising and loss components of currents, which reduces the ratio and phase angle errors. These observations can be verified following Eqs (3.2) and (3.5).

Such variations of CT errors with changing primary (and hence secondary) current under a typical unity power factor burden are shown in Figure 3.7.





(b)

Figure 3.7 *Effects of primary current variation on (a) CT ratio error, and (b) CT phase-angle error*

3.7 DESIGN AND CONSTRUCTIONAL FEATURES OF CURRENT TRANSFORMERS

3.7.1 Design Features

1. *Number of Primary Ampere-Turns (AT)*

One of the primary conditions for restricting ratio and phase-angle errors is that the magnetising ampere-turns in the primary shall be only a small proportion of the total primary ampere turns. To satisfy this condition, in most practical cases, the number of primary ampere-turns may be estimated as to be in the range 5000–10000. In the case of CTs having a single bar as their primary winding, the number of primary ampere turns is, of course, determined by the primary current. For most practical purposes, with commercially available magnetic materials at the core of the CT, primary currents not less than 100 A have proved to be necessary for producing satisfactory amount of ampere turns.

2. *Core*

To satisfy the condition of achieving low magnetising ampere-turns, the core material must have a low reluctance and low iron loss. The flux density in the core also needs to be restricted to low values. Core materials such as Mumetal (an alloy of iron and nickel containing copper) has properties of high permeability, low loss, and low retentivity—all of which are advantageous for being used in CTs. Mumetal, however, has the disadvantage of having low saturation flux densities.

The length of magnetic path in the core should be as small as permissible from the point of view of mechanical construction and with proper insulation requirements in order to reduce the core reluctance. For similar reasons, core joints should be avoided as far as practicable, or in other case, core joints must be as efficient as possible by careful assembly.

3. *Windings*

Primary and secondary windings should be placed close together to reduce the leakage reactance; otherwise the ratio error will go up. Thin SWG wires are normally used for secondary winding, whereas copper strips are generally used for primary winding, dimensions of which depend, obviously, upon the primary current.

The windings need to be designed for proper robustness and tight bracing with a view to withstand high mechanical forces without damage. Such mechanical forces my get developed due to sudden short circuits in the system where the CT is connected.

4. *Insulation*

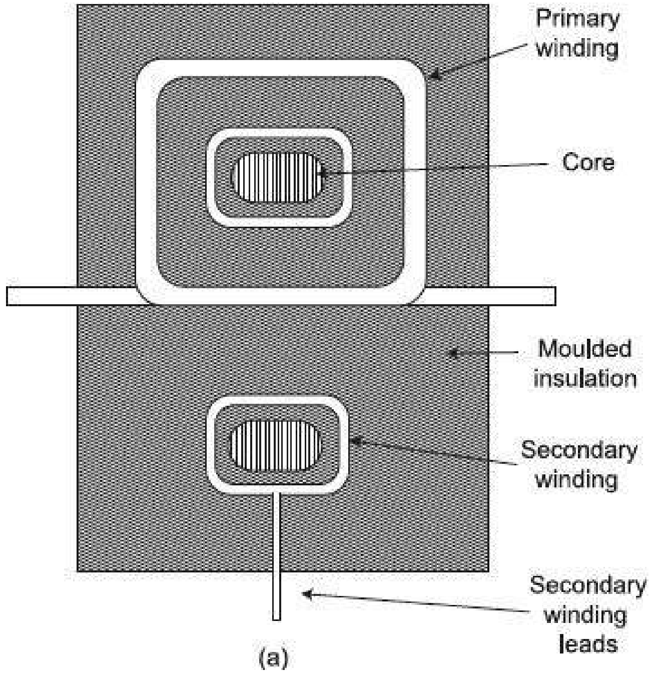
Lower voltage rating applications allow windings to be insulated with tape and varnish. Higher voltage applications, however, require oil-immersed insulation arrangements for the winding. Still high voltages may require use of solid compound insulation systems.

3.7.2 Constructional Features

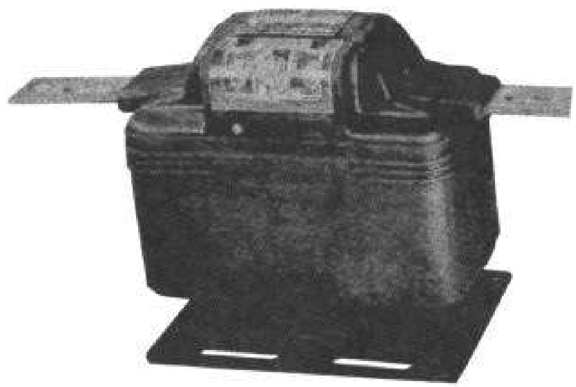
1. *Indoor Type CTs*

For indoor table/panel mounted applications, winding construction can be of two types in a CT: (i) wound type, and (ii) bar type.

In wound-type winding, the primary winding consists of a few turns of heavy conductor to whose projected ends, the primary conductor, cable or bus bar is bolted. The secondary winding, which composes of a large number of turns, is wound over a Bakelite former around a central core. The heavy primary conductor is either wound directly on top of the secondary winding, suitable insulation being first applied over the secondary winding, or the primary is wound entirely separately, insulated with suitable tape and then assembled with the secondary winding on the core. The entire system is housed, normally, within a molded insulation cover. Figure 3.8(a) shows a schematic diagram of the cross section of a wound-type CT. Figure 3.8 (b) shows a photograph of such a wound-type CT.



(a)



(b)

Figure 3.8 *(a)Cross section of wound-type CT (b) Butyl-molded wound primary type indoor CT (Courtesy of General Electric Company)*

The bar-type CT includes the laminated core and secondary winding but no primary winding as such. The primary consists of the bus-bar or conductor, which is passed through the opening in the insulating sleeve through the secondary winding. The primary winding (bar) here forms an integral part of the CT. Such CTs are sometimes termed as single-turn primary-type CT. The external diameter of the bar type primary must be large enough to keep the voltage gradient in the dielectric at its surface, to an acceptable value such that corona effect can be avoided. Figure 3.9 shows a schematic diagram of the cross-section of a bar-type CT. Figure 3.10 show photograph of such bar-type CTs.

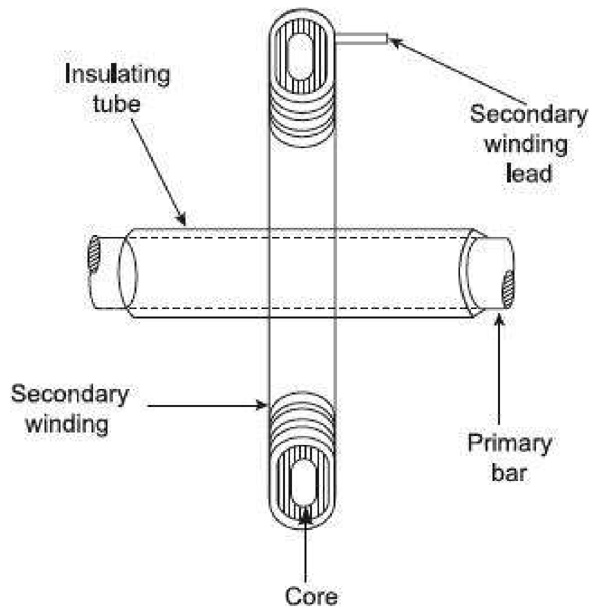


Figure 3.9 *Cross section of bar-type CT*

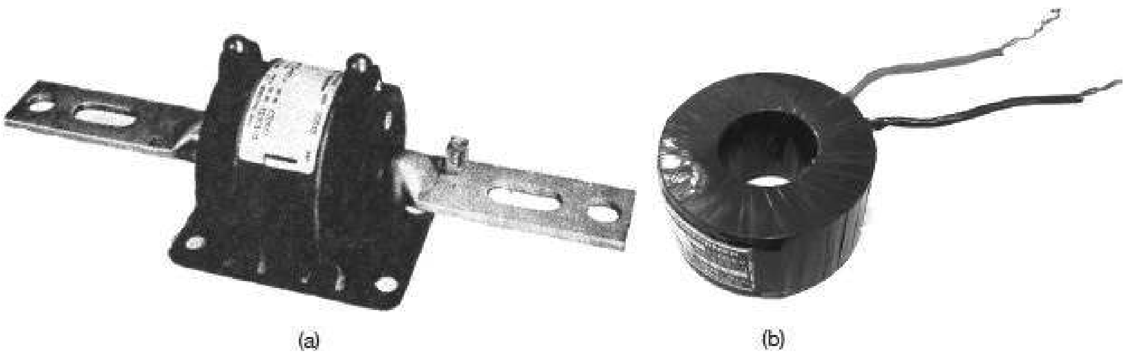


Figure 3.10 *(a) Butyl-molded bar primary-type indoor CT for 200–800 A and 600 V range circuit (Courtesy of General Electric Company) (b) Single conductor (bar type) primary CT*

2. Clamp-on Type or Portable Type CTs

By the use of a construction with a suitably split and hinged core, upon which the secondary winding is wound, it is possible to measure the current in a heavy-current conductor or bus-bar without breaking the current circuit. The split core of the CT along with the secondary winding is simply clamped around the main conductor, which acts as the primary winding of the CT. When used with range selectable shunts and a calibrated ammeter, clamp on type CTs can be very conveniently used for direct and quick measurement of current. Figure 3.11 shows photograph of such a clamp-on type CT.

Figure 3.11 *Multi-range clamp-on type CT (Courtesy of Metravi)*

A portable-type CT in which high and largely adjustable current ranges can be obtained by actually winding the primary turns through the core opening is illustrated in Figure 3.12. For example, if one turn of the primary conductor through the opening gives a current ratio of 800/5, then two primary turns through the same opening will give a current ratio of 400/5, and so on.

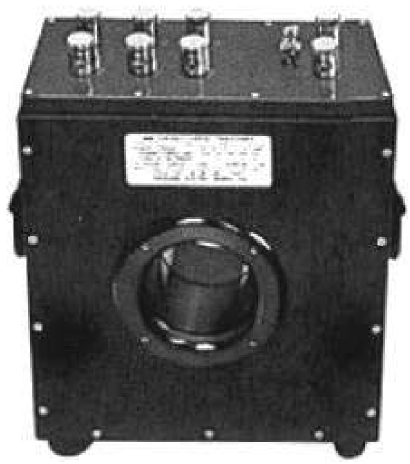


Figure 3.12 *Multi-range portable type CT (Courtesy of Yokogawa)*

3. *Bushing-type CTs*

The bushing-type CT is similar in concept to the bar type in the sense that core and secondary winding are mounted around the single primary conductor. It has a circular core that carries the secondary wound over it and forming a unit that may be installed in the high-voltage bushing of a circuit breaker or a power transformer. The 'primary winding' in such a case is simply the main conductor in the bushing. Figure 3.13 shows the view of such a bushing-type CT.

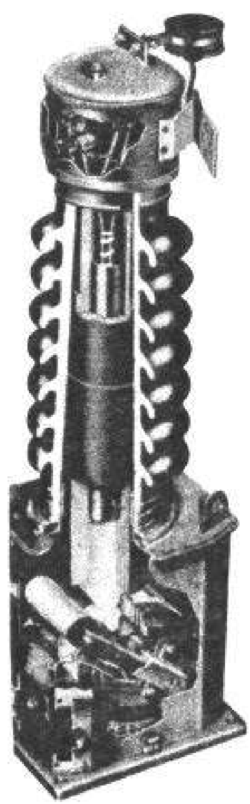


Figure 3.13 *Bushing-type CT (Courtesy of Westinghouse Electric Corporation)*

3.8 PRECAUTIONS IN USE OF CURRENT TRANSFORMER

3.8.1 Open Circuiting of CT Secondary

Current transformers are always used with its secondary circuit closed through very low resistance loads such as ammeters, wattmeter current coils, or relay coils. In such a case, a current transformer should never have its secondary terminals open circuited while the primary circuit is still energised.

One difference between a normal power transformer and a current transformer is that in a CT, the primary current is independent of the current flowing in the secondary, primary winding being connected in series with the main load, carries the line current. Thus, primary current is in no way controlled by the conditions of the secondary winding circuit of the CT.

Under normal operating conditions, the secondary current produces a so-called 'back-mmf' that almost balances the primary winding ampere turns and restricts the flux in the core. This small mmf is responsible for producing flux in the core and supplies the iron losses. This resultant flux being small, the voltage induced in the secondary winding is also low.

If by any reason whatsoever, the secondary winding gets open circuited, then the secondary reverse mmf vanishes. The demagnetising effect of the secondary mmf is now absent and the core carries the high flux created due to the primary ampere-turns only. This large flux greatly increases flux density in the core and pushes it towards saturation. This large flux, when links with the large number of secondary turns, produces a very high voltage, that could be damaging for the winding insulation as well as dangerous for the operator. The transformer insulation may get damaged under such high voltage stress.

In addition to this, increased hysteresis and eddy current losses at higher flux densities may overheat the transformer core.

Moreover, the high magnetising forces acting on the core while secondary condition tend to magnetise the magnetic material to high values. If the open circuit condition is suddenly removed, the accuracy of the CT may be seriously impaired by the residual magnetism remaining in the core in case the primary circuit is broken or the secondary circuit is re-energised. This residual effect causes the transformer to operate at a different operating point on the magnetisation curve, which affects the permeability and hence the calibration. This may lead to an altogether different values of ratio and phase-angle errors, obtained after such an open circuit, as compared to the corresponding values before it occurred. A transformer so treated, must first be demagnetised and then recalibrated before can be used reliably.

For these reasons, care must be taken to ensure that, even when not in use for measurement purposes, the secondary circuit is closed at any time when the primary circuit is energised. In those idle periods, the secondary circuit could (and should) be safely short circuited quite safely, since while being used for measurement it is practically short circuited by the ammeter, or wattmeter current coils, impedances of which are merely appreciable.

3.8.2 Permanent Magnetisation of Core

Core material of a CT may undergo permanent magnetisation due to one or more of the following reasons:

1. As discussed earlier, if by any chance the secondary winding is open circuited with the primary winding still energised, then the large flux will tend to magnetise the core to high values. If such a condition is abruptly removed, then there is a good possibility that a large residual magnetism will remain in the core.
2. A switching transient passing through the CT primary may leave behind appreciable amount of residual magnetism in the core.
3. Permanent magnetisation may result from flow of dc current through either of the winding. The dc current may be flown through the winding for checking resistance or checking polarity.
4. Permanent magnetisation may also result from transient short circuit current flowing through the line to which the CT is connected. Such transient currents are found to have dc components along with ac counterparts.

The presence of permanent magnetisation in the core may alter the permeability of the material resulting in loss of calibration and increase in both ratio error and phase angle error. Thus, for reliable operation of the CT, the residual magnetism must be removed and the CT be restored back to its original condition. There are several methods of demagnetising the core as described below:

1. One method is to pass a current through the primary winding equal to the current that was flowing during the period when the CT secondary was open circuited. The CT secondary circuit is left open. The voltage supply to the primary is then gradually reduced to zero. The CT core thus undergoes several cycles of gradually reducing magnetisation till it finishes down to zero.
2. In the second method, the primary winding is supplied from a source so that rated current flows in the primary winding. A variable resistor of value certain hundred ohms is connected across the secondary winding. This simulates almost the CT secondary open circuit condition. The variable resistance is then gradually and uniformly reduced down to zero. In this way, the magnetisation of the CT core gradually reduced down from its initial high value to normal original values.

Example 3.1

A current transformer has single-turn primary and a 100-turn secondary winding. The secondary winding of purely resistive burden of 1.5 W draws a current of 6 A. The magnetising ampere-turns is 60 A. Supply frequency is 50 Hz and core cross-sectional area is 800 mm^2. Calculate the ratio and phase angle of the CT. Also find the flux density in *the core. Neglect flux leakage, iron losses and copper losses*.

Solution Neglecting copper losses and flux leakage implies that secondary winding resistance and reactance are not be considered. The burden of 1.5 Ω thus can be assumed to be the burden of the entire secondary circuit.

Secondary induced voltage

ES = Secondary current × Secondary burden = 6 × 1.5 = 9 V

As the secondary burden is purely resistive, the secondary current is in phase with the secondary induced voltage, i.e., the phase angle difference between E_S and I_S, $\delta = 0$, and power factor is unity.

Given that iron losses, and hence the loss component of current can be neglected, i.e., $I_C = 0$.

No-load current in this case is thus simply equal to the magnetising component of current, i.e., $I_0 = I_M$

Now, magnetising component of no-load current is given by

$$I_M = \frac{\text{Magnetising mmf}}{\text{Primary winding turns}} = \frac{60}{1} = 60 \text{ A}$$

Secondary winding current $I_S = 6$ A

Turns ratio = n = 100/1 = 100

Reflected secondary winding current = $nI_S = 100 \times 6 = 600$ A

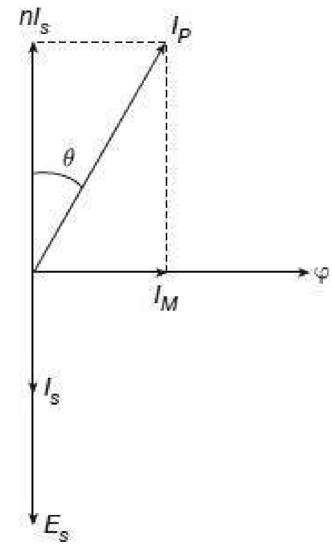
Referring to the phasor diagram shown in the fi gure, the primary current can be calculated as

$$I_P = \sqrt{(nI_S)^2 + (I_M)^2} = \sqrt{600^2 + 60^2} = 603 \text{ A}$$

∴ actual transformation ratio

$$R = \frac{I_P}{I_S} = \frac{603}{6} = 100.5$$

$$\text{Phase angle} = \theta = \tan^{-1}\left(\frac{I_M}{nI_S}\right) = \tan^{-1}\frac{60}{600} = 5.7°$$



From the relation $E_S = 4.44f\varphi_m N_S$, the maximum flux φ_M in the core can be calculated as

$$\phi_m = \frac{E_S}{4.44 fN_S} = \frac{9}{4.44 \times 50 \times 100} = 0.41 \times 10^{-3} \text{ Wb}$$

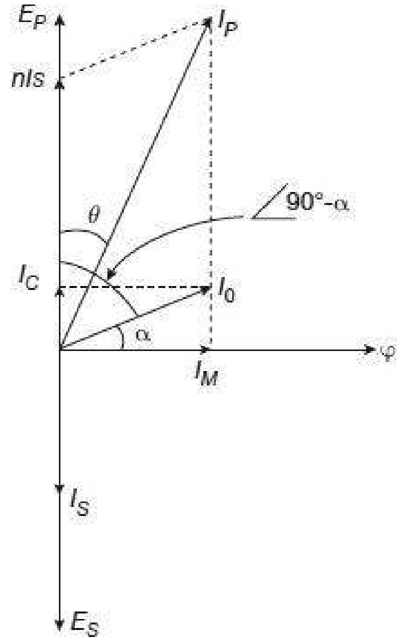
Given, area of core = 800 mm^2 = 800 × 10^{-6} m^2

∴ maximum flux density $B_m = \dfrac{0.41 \times 10^{-3}}{800 \times 10^{-6}} = 0.51 \text{ Wb/m}^2$

Example 3.2

A ring core type CT has a ratio of 2000/10. When operating at rated primary current with a secondary burden of noninductive resistance value of 2 Ω, takes a no-load current of 2 A at power factor of 0.3. Calculate (i) the phase angle difference between primary and secondary currents, and (ii) the ratio error at full load.

Solution Phasor diagram for the corresponding situation with purely resistive burden is shown in the figure.



The burden being purely resistive, the secondary current I_S and secondary induced voltage E_S will be in the same phase as shown in the figure. Therefore, secondary winding power factor will be unity and the phase angle difference between I_S and E_S, $\delta = 0$.

Give, no-load current $I_0 = 2$ A, at power factor of 0.3.

∴ $\cos(90° - \alpha) = 0.3$ or $\alpha = 17.46°$

Nominal ratio $K_n = \dfrac{2000}{10} = 200$

Without any turns compensation, nominal ratio equals the turns ratio, thus $n = K_n$

Under full load rated condition, primary current = 2000 A and secondary current = 10 A.

∴ transformation ratio

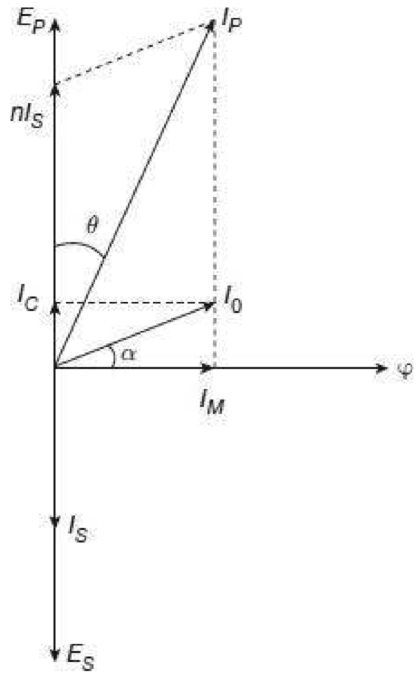
$$R = n + \frac{I_0}{I_S}\sin(\delta + \alpha) = 200 + \frac{2}{10}\sin(0° + 17.46°) = 200.06$$

$$\therefore \quad \text{ratio error} = \frac{\text{Nominal Ratio} - \text{Actual Ratio}}{\text{Actual Ratio}} \times 100\% = \frac{K_n - R}{R} \times 100\%$$

Example 3.3

A 1500/5 A, 50 Hz single-turn primary type CT has a secondary burden comprising of a pure resistance of 1.5 Ω. Calculate flux in the core, ratio error and phase-angle error at full load. Neglect leakage reactance and assume the iron loss in the core to be 3 W at full load. The magnetising ampere-turns is 150.

Solution Phasor diagram for the corresponding situation with purely resistive burden is shown in the figure.



The burden being purely resistive, the secondary current I_S and secondary induced voltage E_S will be in the same phase as shown in the figure. Therefore, secondary winding power factor will be unity and the phase angle difference between I_S and $\boldsymbol{E_S}$, $\boldsymbol{\delta} = 0$.

Nominal ratio $K_n = \frac{1500}{5} = 300$

Without any turns compensation, nominal ratio equals the turns ratio, thus $n = K_n = 300$

Given, primary number of turns $N_P = 1$

Hence, secondary number of turns $N_S = n. N_P = 300$

Given, secondary burden resistance = 1.5 Ω.

Neglecting leakage flux and hence neglecting secondary leakage reactance, the total impedance of secondary circuit = 1.5 Ω

Under full load rated condition, primary current = 1500 A and secondary current = 5 A.

∴ secondary induced voltage $E_S = 5 \times 1.5 = 7.5$ V

And, primary induced voltage $E_P = E_S / n = 7.5/300 = 0.025$ V

From the relation, $E_S = 4.44\, f\varphi_m\, N_S$, the maximum flux can be calculated as

$$\phi_m = \frac{E_S}{4.44 f N_S} = \frac{7.5}{4.44 \times 50 \times 300} = 0.113 \times 10^{-3} \text{ Wb}$$

Given, iron loss at full load = 3 W

∴ loss component of current $I_C = \frac{\text{Iron loss}}{E_p} = \frac{3}{0.025} = 120$ A

Magnetising component of current $I_M = \frac{\text{Magnetising mmf}}{N_p} = \frac{150}{1} = 150$ A

Noting the fact that for purely resistive burden $\delta = 0$

∴ transformation ratio $R \approx n + \dfrac{I_M \sin\delta + I_C \cos\delta}{I_S} \approx 300 + \dfrac{120}{5} = 324$

Percentage ratio error $= \dfrac{K_n - R}{R} \times 100\% = \dfrac{300 - 324}{324} \times 100\% = -7.41\%$

Phase-angle error $\theta \approx \dfrac{180}{\pi}\left(\dfrac{I_M \cos\delta - I_C \sin\delta}{nI_S}\right) = \dfrac{180}{\pi}\left(\dfrac{120}{300 \times 5}\right) = 4°35'$

Example 3.4 *A bar-type CT has 400 turns in the secondary winding. The impedance of the secondary circuit is (2 + j1.5) W. With 4 A flowing in the secondary, the magnetising mmf is 80 A and the iron loss is 1 W. Determine ratio and phase-angle errors.*

Solution Phasor diagram for the corresponding situation with resistive plus inductive burden in the whole secondary circuit is shown in the figure.

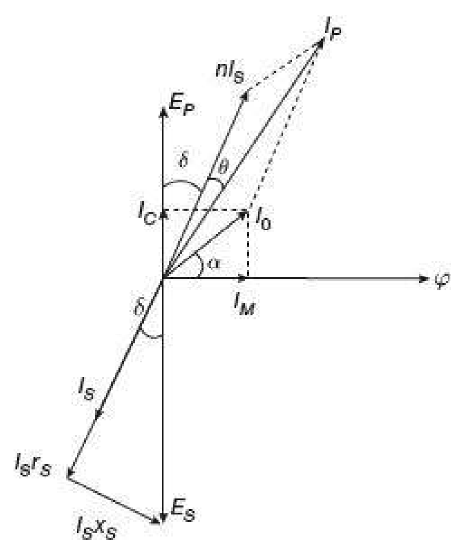
Given, for bar-type CT, primary number of turns $N_P = 1$

Secondary number of turns $N_S = 400$

∴ turns ratio $n = N_S / N_P = 400$

Secondary circuit burden impedance $Z_S = \sqrt{(2)^2 + (1.5)^2} = 2.5\ \Omega$

Secondary circuit power factor $= \cos\delta = \dfrac{2}{2.5} = 0.8$ and $\sin\delta = \dfrac{1.5}{2.5} = 0.6$



Secondary induced voltage $E_S = I_S \times Z_S = 4 \times 2.5 = 10$ V

Primary induced voltage $E_P = E_S/n = 10/400 = 0.025$ V

Loss component of current $= I_C = \dfrac{\text{Iron loss}}{E_P} = \dfrac{1}{0.025} = 40$ A

Magnetising mmf 80 Magnetising current $I_M = \dfrac{\text{Magnetising mmf}}{N_P} = \dfrac{80}{1} = 80$ A

∴ transformation ratio $R \approx n + \dfrac{I_M \sin\delta + I_C \cos\delta}{I_S}$

or, $$R = 400 + \frac{80 \times 0.6 + 40 \times 0.8}{4} = 420$$

Percentage ratio error $= \frac{K_n - R}{R} \times 100\% = \frac{400 - 420}{420} \times 100\% = -4.76\%$

Phase-angle error $\theta \approx \frac{180}{\pi}\left(\frac{I_M \cos\delta - I_C \sin\delta}{nI_S}\right)$

or, $$\theta = \frac{180}{\pi}\left(\frac{80 \times 0.8 - 40 \times 0.6}{400 \times 4}\right) = 3°9'$$

Example 3.5

A bar-type CT has 300 turns in the secondary winding. An ammeter connected to the secondary has a resistance of 1 Ω and reactance of 0.8 W, and the secondary winding impedance is (0.5 + j0.6)Ω. The magnetising MMF requirement for the core is 60 A and to supply the iron loss the current required is 25 A. (a) Find the primary winding current and also determine the ratio error when the ammeter in the secondary winding shows 5 A. (b) How many turns should be reduced in the secondary to bring down ratio error to zero at this condition?

Solution

(a) Total resistance of secondary circuit = 1 + 0.5 = 1.5 Ω

Total reactance of secondary circuit = 0.8 + 0.6 = 1.4 Ω

∴ secondary circuit phase angle $\delta = \tan^{-1}\left(\frac{1.4}{1.5}\right) = 43°$

∴ for secondary circuit, cos δ = 0.73 and sin δ = 0.68

Given, for bar-type CT, primary number of turns $N_P = 1$

Secondary number of turns $N_S = 300$

∴ turns ratio $n = N_S / N_P = 300$

Without any turns compensation, nominal ratio equals the turns ratio, thus $K_n = n$ = 300 Given, loss component of current = I_c = 25 A

Magnetising current $I_M = \frac{\text{Magnetising mmf}}{N_P} = \frac{60}{1} = 60$ A

∴ transformation ratio $R \approx n + \frac{I_M \sin\delta + I_C \cos\delta}{I_S}$

or, $$R = 300 + \frac{60 \times 0.68 + 25 \times 0.73}{5} = 311.8$$

∴ ratio error $= \frac{K_n - R}{R} \times 100\% = \frac{300 - 311.8}{311.8} \times 100\% = -3.78\%$

Primary current I_p = Transformation ratio × Secondary current = $R \times I_s$

or, $I_p = 311.8 \times 4 = 1247.2$ A

(b) To achieve zero ratio error, we must have the transformation ratio and nominal ratio to be equal.

$\therefore$ $R = Kn$

Thus, $n + \frac{I_M \sin\delta + I_C \cos\delta}{I_S} = 300$, with n = the desired turns ratio

or, $$300 = n + \frac{60 \times 0.68 + 25 \times 0.73}{5} = n + 11.8$$

or, $$n = 288.2$$

Thus, secondary number of turns required = $n \times N_P = 288.2 \times 1 = 288.2$

$\therefore$ reduction in secondary winding turns = 300 - 288.2 ≈ 12

Example 3.6

A bar-type CT with turns ratio 1:199 is rated as 2000:10 A, 50 VA. The magnetising and core loss components of primary current are 15 A and 10 A respectively under rated condition. Determine the ratio and phase angle errors for the rated burden and rated secondary current at 0.8 p.f. lagging and 0.8 p.f. leading. Neglect impedance of secondary winding.

Solution Given, for bar-type CT. primary number of turns $N_P = 1$

Turns ratio $n = N_S / N_P = 199$

$\therefore$ secondary number of turns $N_S = 199$

Nominal ratio $K_n = 2000/10 = 200$

Given, loss component of current = $I_c = 10$ A

Magnetising current $I_m = 15$ A

Power Factor = 0.8 lagging

For lagging p.f., the secondary phase angle δ is positive.

Thus, $\cos\delta = 0.8$ and $\sin\delta = \sqrt{1^2 - 0.8^2} = 0.6$

Transformation ratio $R \approx n + \frac{I_M \sin\delta + I_C \cos\delta}{I_S}$

or, $$R = 199 + \frac{15 \times 0.6 + 10 \times 0.8}{10} = 200.7$$

$\therefore$ ratio error $= \frac{K_n - R}{R} \times 100\% = \frac{200 - 200.7}{200.7} \times 100\% = -0.35\%$

Phase-angle error $\theta \approx \frac{180}{\pi}\left(\frac{I_M \cos\delta - I_C \sin\delta}{nI_S}\right)$

or, $$\theta = \frac{180}{\pi}\left(\frac{15 \times 0.8 - 10 \times 0.6}{199 \times 10}\right) = 10'37''$$

Power Factor = 0.8 leading

For lagging p.f., the secondary phase angle δ is negative.

Thus, $\cos \delta = 0.8$ and $\sin \delta = -0.6$

Transformation ratio $R \approx n + \dfrac{I_M \sin\delta + I_C \cos\delta}{I_S}$

or, $$R = 199 + \frac{-15\times 0.6 + 10\times 0.8}{10} = 198.9$$

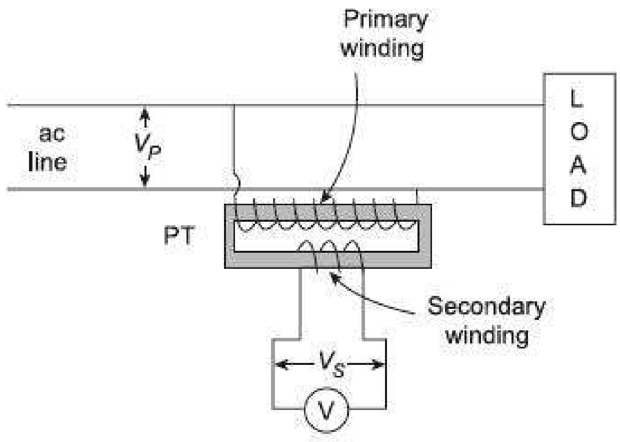
$\therefore$ $$\text{ratio error} = \frac{K_n - R}{R}\times 100\% = \frac{200 - 198.9}{198.9}\times 100\% = +0.55\%$$

Phase-angle error $\theta \approx \dfrac{180}{\pi}\left(\dfrac{I_M \cos\,\delta - I_C \sin\,\delta}{nI_S}\right)$

or, $$\theta = \frac{180}{\pi}\left(\frac{15\times 0.8 + 10\times 0.6}{199\times 10}\right) = 31'\,1''$$

3.9 POTENTIAL TRANSFORMERS (PT)

Measurement of voltage, power, etc., of high voltage lines requires the high level of voltage being stepped down before being applied to the measuring instrument. This is essential from the point of view of safety of operating personnel, reduction in size of instrument and saving in insulation cost. Potential transformers or PTs are used in such cases to operate voltmeters, potential coils of wattmeters, relays and other devices to be operated with high-voltage lines. The primary winding of the PT is connected across the high-voltage line whose voltage is to be measured and the measuring instruments are connected across the secondary of the PT. For all these purposes, it is essential that the secondary voltage be a definite fraction of the primary voltage, and in some applications they need to be in the same phase as well. Uses of PT for such applications are schematically shown in Figure 3.14.



(a)

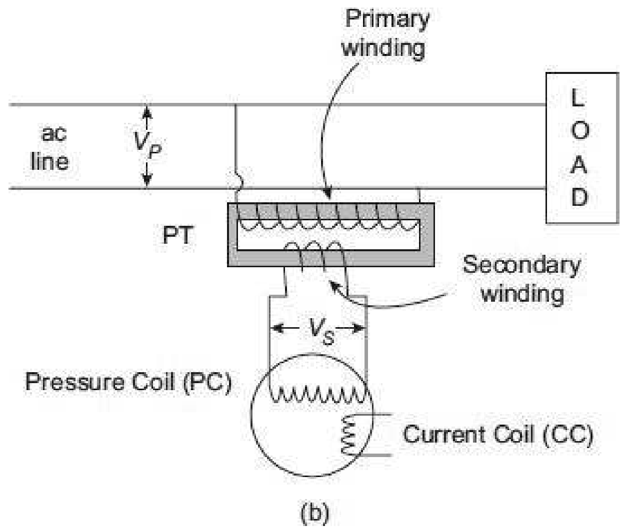


Figure 3.14 *Use of PT for (a) voltage, and (b) power measurement*

There is essentially no difference in theory between a PT used for measurement purposes and a power transformer for regular use. The main differences between a PT and a power transformer are actually special requirements for the measurement system. These are the following:

1. Attenuation ratio must be accurately maintained in a PT since it is being used for measurement purposes.
2. Voltage drops in the windings must be minimised in a PT in order to reduce effects of phase shift and ratio error. Voltage drops in windings can be reduced by proper design to minimise leakage reactance and using large copper conductors.
3. Loading in a PT is always small, only of the order of few volt-amperes. Loading of a PT is actually limited by accuracy considerations; whereas in a power transformer, load limitation is on a heating basis.
4. Overload capacity for PTs are designed to be up to 2-3 times their normal rated values. Whereas, high capacity power transformers, under special circumstances, can take up overloads up to only 20% above their normal rating.

3.10 THEORY OF POTENTIAL TRANSFORMERS

Figure 3.15 represent the equivalent circuit of a PT and Figure 3.16 plots the phasor diagram under operating condition of the PT.

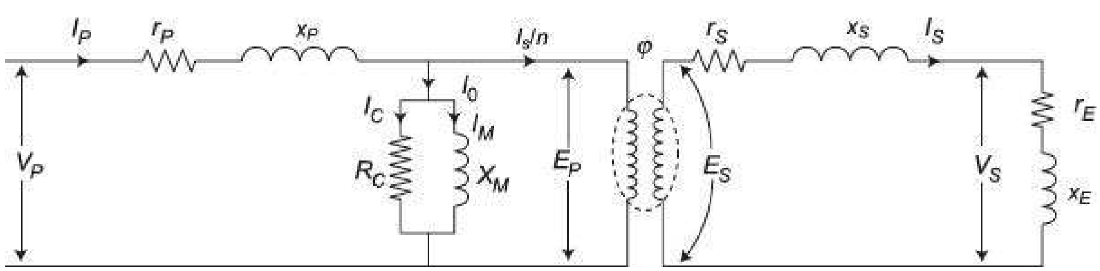


Figure 3.15 *Equivalent circuit of a PT*

V_P = primary supply voltage

E_P = primary winding induced voltage

V_S = secondary terminal voltage

E_S = secondary winding induced voltage

$I_{\boldsymbol{P}}$ = primary current

$I_{\boldsymbol{S}}$ = secondary current

1_0 = no-load current

I_C = core loss component of current

I_M = magnetising component of current

r_P = resistance of primary winding

x_P = reactance of primary winding

r_S = resistance of secondary winding

x_S = reactance of secondary winding

R_C = imaginary resistance representing core losses

X_M = magnetising reactance

r_E = resistance of external load (burden) including resistance of meters, current coils etc.

x_E = reactance of external load (burden) including reactance of meters, current coils, etc.

N_P = primary winding number of turns

N_S = secondary winding number of turns

n = turns ratio $= N_P / N_S$

φ = working flux of the PT

θ = the ‘phase angle’ of the PT

δ = phase angle between secondary winding terminal voltage and secondary winding current (i.e., phase angle of load circuit)

β = phase angle between primary load current and secondary terminal voltage reversed

α = phase angle between no-load current I_0 and flux φ

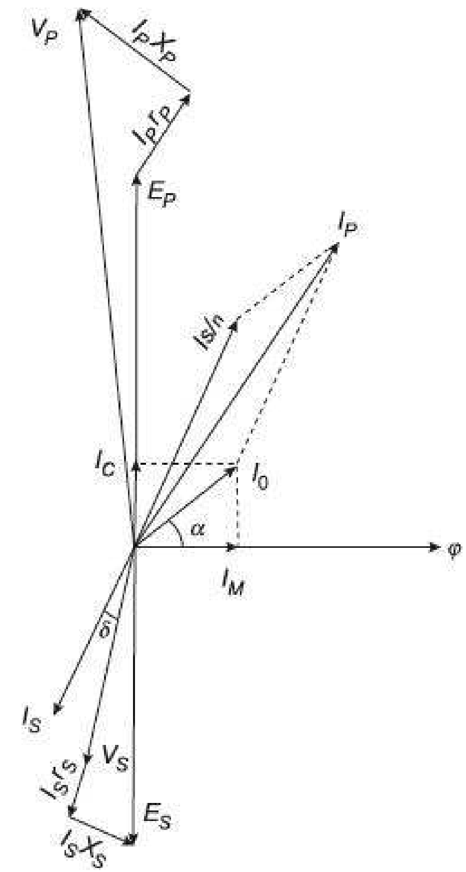


Figure 3.16 Phasor diagram of a PT

The flux ***j*** is plotted along the positive x-axis. Magnetising component of current I_m is in phase with the flux. The core loss component of current $I_{C,}$ leads by I_M90°. Summation of I_c and I_m produces the no-load current I_0.

The primary winding induced voltage E_P is in the same phase with the resistive core loss component of current I_C. As per transformer principles, the secondary winding induced voltage E_S will be 180° out of phase with the primary winding induced voltage E_P. Secondary output terminal voltage V_S is obtained by vectorically subtracting the secondary winding resistive and reactive voltage drops $I_S\ r_S$ and I_Sx_S respectively from the secondary induced voltage E_S.

Secondary voltages when referred to primary side need to be multiplied by the turns ratio n, whereas, when secondary currents are to be referred to primary side, they need to be divided by n. Secondary current I_S, when reflected back to primary, can be represented by the 180° shifted phasor indicated by I_S/n. Primary winding current I_p is the phasor summation of this reflected secondary current (load component) I_S/n and the no-load current I_0.

Vectorically adding the primary winding resistive and reactive voltage drops with the primary induced voltage will give the primary line voltage V_P.

3.10.1 Voltage Transformation Ratio of PT

Redrawing the expanded view of the phasor diagram of Figure 3.14, we obtain the phasor diagram of Figure 3.17.

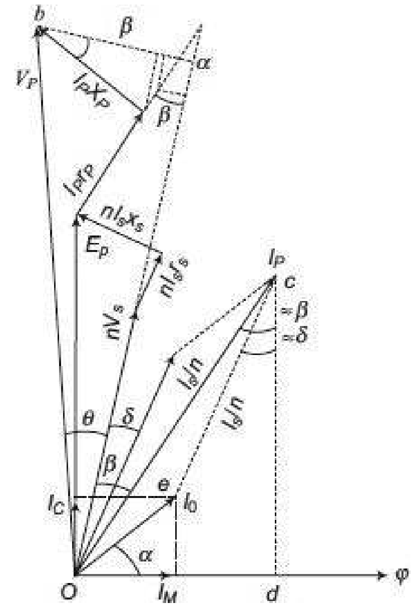


Figure 3.17 *Expanded view of a section of Figure 3.16*

The phase-angle difference θ between the primary voltage V_P and the reflected secondary voltage nV_S is called phase angle of the PT. Ideally, without any no-load current and without any voltage drop in winding impedances, these two phasors must have been in the same phase, i.e., ideally $\theta = 0$.

From the equivalent circuit of the PT shown in Figure 3.15 and the phasor diagram of Figure 3.16, we have

$$\overline{V_P} = \overline{E_P} + \overline{I_P r_P} + \overline{I_P x_P} = n\overline{E_S} + \overline{I_P r_P} + \overline{I_P x_P}$$

or, $$\overline{V_P} = n\left(\overline{V_S} + \overline{I_S r_S} + \overline{I_S x_S}\right) + \overline{I_P r_P} + \overline{I_P x_P} = n\overline{V_S} + n\overline{I_S r_S} + n\overline{I_S x_S} + \overline{I_P r_P} + \overline{I_P x_P}$$

From the phasor diagram in Figure 3.17, we have:

$$oa = V_P \cos\theta$$

and

$$oa = nV_s + nI_s r_s \cos\delta + nI_s x_s \sin\delta + I_P r_P \cos\beta + I_P x_P \sin\theta$$

or, $$V_P \cos\theta = nV_S + nI_S r_S \cos\delta + nI_S x_S \sin\delta + I_P r_P \cos\beta + I_p x_P \sin\beta$$

or, $$V_P \cos\theta = nV_S + nI_S(r_S \cos\delta + x_S \sin\delta) + I_P r_P \cos\beta + I_p x_P \sin\beta \qquad (3.12)$$

In reality, the phase angle difference θ is quite small, thus for the sake of simplicity, both V_P and V_S reversed can be approximated to perpendicular to the flux ***j***, and hence:

$$\angle ocd \approx \beta \text{ and } \angle ecd \approx \delta$$

Thus, $$I_P \cos\beta = I_C + \frac{I_S}{n}\cos\delta \text{ and } I_P \sin\beta = I_M + \frac{I_S}{n}\sin\delta \qquad (3.13)$$

In reality, once again, since θ is very small, sometimes even less than 1°, then we can approximate as

$\cos\theta = 1$, and $V_P \cos\theta = V_P$

Substituting the above values in Eq. (3.12) we have,

$$V_P = nV_S + nI_S(r_S\cos\delta + x_S\sin\delta) + \left(I_C + \frac{I_S}{n}\cos\delta\right)r_P + \left(I_M + \frac{I_S}{n}\sin\delta\right)x_P$$

or, $$V_P = nV_S + I_S\cos\delta\left(nr_S + \frac{r_P}{n}\right) + I_S\sin\delta\left(nx_S + \frac{x_P}{n}\right) + (I_C r_P + I_M x_P) \quad (3.14)$$

or, $$V_P = nV_S + \frac{I_S}{n}\cos\delta(n^2 r_S + r_P) + \frac{I_S}{n}\sin\delta(n^2 x_S + x_P) + (I_C r_P + I_M x_P)$$

or, $$V_P = nV_S + \frac{I_S}{n}\cos\delta\, R_P + \frac{I_S}{n}\sin\delta\, X_P + (I_C r_P + I_M x_P)$$

or, $$V_P = nV_S + \frac{I_S}{n}(R_P\cos\delta + X_P\sin\delta) + (I_C r_P + I_M x_P) \quad (3.15)$$

Here, R_P = equivalent resistance of the PT referred to primary side

X_P = equivalent reactance of the PT referred to primary side

Thus, actual voltage transformation ratio:

$$R = \frac{V_P}{V_S} = n + \frac{\frac{I_S}{n}(R_P\cos\delta + X_P\sin\delta) + (I_C r_P + I_M x_P)}{V_S} \quad (3.16)$$

Equation (3.14) may be re-written as

$$V_P = nV_S + nI_S\cos\delta\left(r_S + \frac{r_P}{n^2}\right) + nI_S\sin\delta\left(x_S + \frac{x_P}{n^2}\right) + (I_C r_P + I_M x_P)$$

or, $$V_P = nV_S + nI_S\cos\delta\, R_S + nI_S\sin\delta\, X_S + (I_C r_P + I_M x_P)$$

or, $$V_P = nV_S + nI_S(R_S\cos\delta + X_S\sin\delta) + (I_C r_P + I_M x_P) \quad (3.17)$$

Here, R_S = equivalent resistance of the PT referred to secondary side

X_S = equivalent reactance of the PT referred to secondary side

Thus, actual voltage transformation ratio can again be written as

$$R = \frac{V_P}{V_S} = n + \frac{nI_S(R_S\cos\delta + X_S\sin\delta) + (I_C r_P + I_M x_P)}{V_S} \quad (3.18)$$

Following Eqs (3.16) and (3.18), the error in ratio, i.e., the difference between actual transformation ratio and turns ratio can be expressed in either of the following two forms:

$$R - n = \frac{\frac{I_S}{n}(R_P\cos\delta + X_P\sin\delta) + (I_C r_P + I_M x_P)}{V_S} \quad (3.19)$$

$$= \frac{nI_S(R_S\cos\delta + X_S\sin\delta) + (I_C r_P + I_M x_P)}{V_S} \quad (3.20)$$

3.10.2 Phase Angle of PT

From the phasor diagram of Figure 3.17,

$$\tan\theta = \frac{ab}{oa} = \frac{I_P x_P\cos\beta - I_P r_P\sin\beta + nI_S}{nV_S + nI_S r_S\cos\delta + nI_S x_S\sin\delta -}$$

To simplify the computations, here we can make the assumption that in the denominator, the terms containing I_p and I_S being much less compared to the large voltage

nV_S, those terms can be neglected; thus we get a simplified form:

$$\tan\theta = \frac{I_P x_P \cos\beta - I_P r_P \sin\beta + nI_S x_S \cos\delta - nI_S r_S \sin\delta}{nV_S} \quad (3.21)$$

Following Eq. (3.12), we get

$$\tan\theta = \frac{x_P\left(I_C + \frac{I_S}{n}\cos\delta\right) - r_P\left(I_M + \frac{I_S}{n}\sin\delta\right) + nI_S x_S \cos\delta - nI_S r_S \sin\delta}{nV_S}$$

$$\text{or,} \qquad \tan\theta = \frac{I_S \cos\delta\left(\frac{x_P}{n} + nx_S\right) - I_S \sin\delta\left(\frac{r_P}{n} + nr_S\right) + I_C x_P - I_M r_P}{nV_S}$$

$$\text{or,} \qquad \tan\theta = \frac{\frac{I_S\cos\delta}{n}(x_P + n^2 x_S) - \frac{I_S\sin\delta}{n}(r_P + n^2 r_S) + I_C x_P - I_M r_P}{nV_S}$$

$$\text{or,} \qquad \tan\theta = \frac{\frac{I_S\cos\delta}{n}X_P - \frac{I_S\sin\delta}{n}R_P + I_C x_P - I_M r_P}{nV_S}$$

$$\text{or,} \qquad \tan\theta = \frac{\frac{I_S}{n}(X_P\cos\delta - R_P\sin\delta) + I_C x_P - I_M r_P}{nV_S}$$

Since ***0*** is small, we can assume tan *0* = 0; thus,

$$\theta = \frac{\frac{I_S}{n}(X_p\cos\delta - R_p\sin\delta) + I_C x_P - I_M r_P}{nV_S} \quad (3.22)$$

$$\text{or,} \qquad \theta = \frac{\frac{I_S}{n}(n^2 X_S\cos\delta - n^2 R_S\sin\delta) + I_C x_P - I_M r_P}{nV_S}$$

$$\text{or,} \qquad \theta = \frac{nI_S(X_S\cos\delta - R_S\sin\delta) + I_C x_P - I_M r_P}{nV_S}$$

$$\text{Thus, phase angle } \theta = \frac{I_S}{V_S}(X_S\cos\delta - R_S\sin\delta) + \frac{I_C x_P - I_M r_P}{nV_S} \quad (3.23)$$

3.11 ERRORS INTRODUCED BY POTENTIAL TRANSFORMERS

3.11.1 Ratio Error and Phase-Angle Error

It can be seen from the above section that, like current transformers, potential transformers also introduce errors in measurement. This error may be in terms of magnitude or phase, in the measured value of voltage. The ratio error (difference between nominal ratio and actual transformation ratio) only is important when measurements of voltage are to be made; the phase angle error is of importance only while measurement of power.

In presence of these errors, the voltage applied to the primary circuit of the PT can not be obtained accurately by simply multiplying the voltage measured by the voltmeter connected across the secondary by the turns ratio *n* of the PT.

These errors depend upon the resistance and reactance of the transformer winding as well as on the value of no-load current of the transformer.

3.11.2 Reducing Errors in PT

As discussed in the previous section, errors are introduced in the ratio and phase angle of a PT owing to the presence of the no-load component of the primary current and voltage drops across winding impedances. Improvement of accuracy, then, depends upon minimising these components or nullifying in some way their effects in introducing errors. This can be achieved by a combination of the following schemes:

1. Reducing the loss component and magnetising components, i.e., the no-load component of the primary current can be achieved by reducing the length of magnetic path in the core, using good quality core magnetic materials, designing with appropriate value of flux densities in the core, and adopting precautionary measures while assembling and interleaving of core laminations.
2. Winding resistance can be reduced by using thick conductors and taking care to reduce the length of mean turn of the windings.
3. Winding leakage flux and hence leakage reactance can be reduced by keeping the primary and secondary windings as close as permissible from the point of view of insulation requirements.
4. Sufficiently high flux densities in the core will reduce the core cross-section, thereby reducing the length of winding wound over the core. This, in turn, will reduce the winding resistance. An optimisation in the core flux density value to be used needs to done, since too high a flux density will increase the no-load current, which is also not desirable.
5. From (3.18) it is clear that at no load, the actual PT transformation ratio exceeds the turns ratio by an amount $(\boldsymbol{I_C\, r_P} + \boldsymbol{I_M}\, X_P)/V_S$. With increased loading, this difference grows due to further voltage drops in winding resistance and reactance. If the turns ratio can be set at a value less than the nominal ratio, then the difference between nominal ratio and actual transformation ratio under operating condition can be brought down. This can be achieved by reducing the number of turns in the primary winding or increasing the number of turns in the secondary winding. This makes it possible to make the actual transformation ratio to be equal to the nominal ratio, at least for a particular value and type of burden.

3.12 OPERATIONAL CHARACTERISTICS OF POTENTIAL TRANSFORMERS

Characteristics of potential transformers under different operating conditions may be estimated from the phasor diagram as shown in Figure 3.20 and expressions for ratio error (3.19) and phase-angle error (3.22).

3.12.1 Effect of Change in Secondary Burden (VA or Current)

With increase in PT secondary burden, the secondary current is increased. This in turn will increase the primary current as well. Both primary and secondary voltage drops are increased and hence, for a given value of the primary supply voltage V_P, secondary terminal voltage V_S is reduced with increase of burden. The effect is therefore to increase the actual transformation ratio $\boldsymbol{V_P/V_S}$ with resulting increase in the ratio error as per Eq. (3.19). This increase in ratio error with increasing secondary burden is almost linear as shown in Figure 3.18.

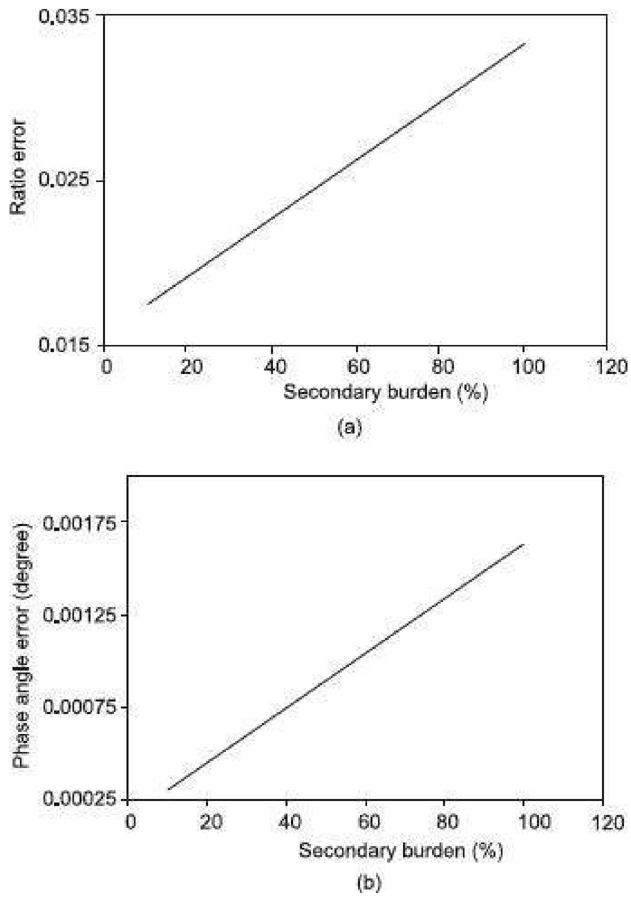


Figure 3.18 *Effects of secondary burden variation on (a) PT ratio error, and (b) PT phase angle error*

With regard to the phase angle, with increasing voltage drops due to increased burden, the phase difference between V_P and V_S reversed increases with resulting increase in phase angle error. Such a variation of phase-angle error with varying secondary burden in a typical potential transformer is shown in Figure 3.18.

3.12.2 Effect of Change in Power Factor of Secondary Burden

As can be observed from the phasor diagram of potential transformer in Figure 3.18, for all inductive burdens, the secondary winding current I_S lags behind the secondary terminal voltage $\boldsymbol{V}_S$, so that the phase angle difference δ is positive. At lower power factors, this phase angle difference δ increases as I_S moves further away from $\boldsymbol{V}_S$. Thus, from the phasor diagram, it is apparent that I_p will now become closer to I_0. This will, in turn, move V_P and V_S more towards to be in phase with E_P and E_S respectively. It is to be kept in mind that change in power factor does not affect the magnitudes of resultant voltage drops in primary and secondary windings substantially. Under these conditions, with primary supply voltage V_P being considered to remain the same, there is a reduction of E_P relative to $\boldsymbol{V}_P$, and V_S relative to $\boldsymbol{E_S}$. The actual transformation ratio $\boldsymbol{V_P / V_S}$ of the potential transformer will thus increase with reduction in burden power factor.

Further, since V_S is advanced in phase and V_P is retarded in phase, the phase angle of the transformer (between V_S and $\boldsymbol{V}_P$) is reduced with reduction in burden (inductive) power factor.

Figure 3.19 shows the variations of ratio and phase-angle errors in a typical current transformer at different values of the secondary burden power factor.

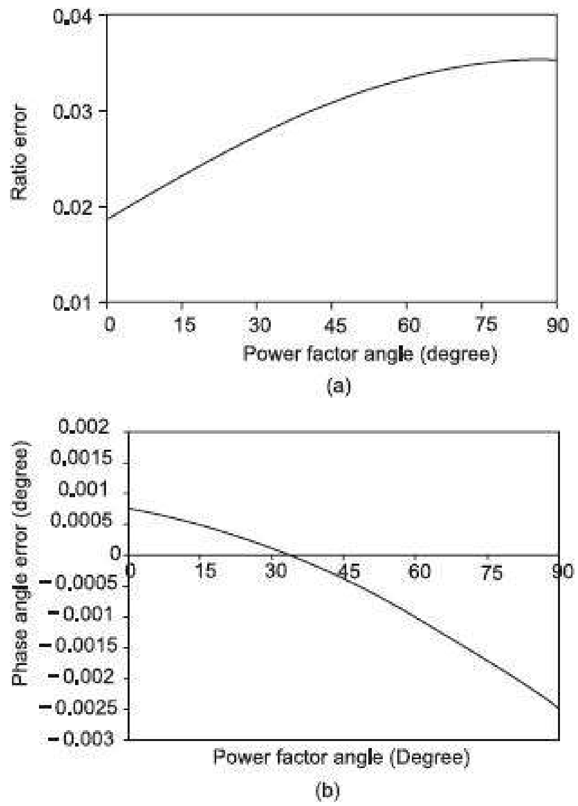


Figure 3.19 *Effects of secondary burden power factor variation on (a) PT ratio error, and (b) PT phase angle error*

3.13 DESIGN AND CONSTRUCTIONAL FEATURES OF POTENTIAL TRANSFORMERS

1. *Core*

The core construction of a PT may be of shell type or core type. Core-type construction is only used for low voltage applications. Special care is taken during interleaving and assembling of the core laminations so that minimal air gap is present in the stack joints.

2. *Winding*

The primary and secondary windings are made coaxial to restrict the leakage reactance to a minimum. In simplifying assembly and reducing insulation requirement, the low-voltage secondary winding is placed nearer to the core, with the high-voltage primary being wound over the secondary. For lower voltage rating the high-voltage primary winding can be made of a single coil, but for higher voltages, however, a number of separate coils can be assembled together to reduce insulation complicacies.

3. *Insulation*

Cotton tape and varnish is the most common insulation applied over windings during coil construction. At low voltages, PTs are usually filled with solid compounds, but at higher voltages above 7-10 kV, they are oil-immersed.

4. *Bushings*

Oil-filled bushings are normally used for oil filled potential transformers as this reduces the overall height. Some potential transformers connected between line and neutral of a grounded neutral system have only one bushing for the high voltage terminal. Some potential transformers can have two bushings when neither side of the line is at ground potential. A view of such a two-bushing potential transformer is shown in Figure 3.20.

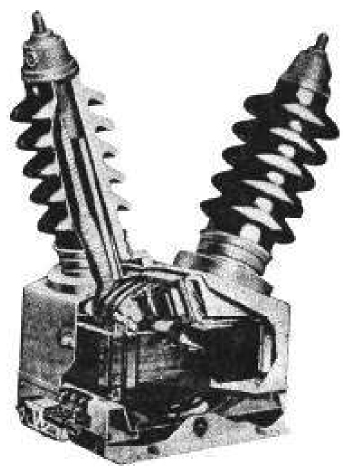


Figure 3.20 *Cutaway view of two-bushing type potential transformer (Courtesy of Westinghouse Electric Corp.)*

5. *Cost and Size*

Power transformers are designed keeping in view the efficiency, regulation and cost constraints. The cost in such cases is reduced by using smaller cores and conductor sizes. In potential transformers, however, cost cannot be compromised with respect to desired performance and accuracy. Accuracy requirements in potential transformers in terms of ratio and phase angle obviate the use of good quality and amount of magnetic material for the core and also thick conductors for the winding.

6. *Overload Capacity*

Potential transformers are normally designed for very low power ratings and relatively large weight/ power ratio as compared to a power transformer. Theoretically, this can enable a potential transformer to run on appreciable overloads without causing much heating. Overload capacities of potential transformers are, however, limited by accuracy requirements, rather than heating.

Example 3.1 *A potential transformer with nominal ratio 1100/110 V has the following parameters:*

Primary resistance = 82 Φ secondary resistance = 0.9 Φ

Primary reactance = 76 Φ secondary reactance = 0.72 Φ

No load current = 0.02 a at 0.4 power factor

Calculate (a) phase angle error at no load

(b) burden in VA at unity power factor at which phase angle error will be zero

Solution No load power factor = cos (90° + α) = 0.4

α = 23.6° and sina = 0.4, cosa = 0.917

$I_C = I_0 \sin a$ = 0.02 ¥0.4 = 0.008 A

$I_M = I_0 \cos a$ = 0.02 ¥0.0.917 = 0.0183 A

Turns ratio n = 1100/110 = 10

From Eq. (3.21), $\theta = \dfrac{\frac{I_S}{n}(X_P \cos\delta - R_P \sin\delta) + I_C x_P - I_M r_P}{nV_S}$ rad

(a) At no load, $I_S = 0$

$$\therefore \qquad \theta = \frac{I_C x_P - I_M r_P}{nV_S} = \frac{0.008 \times 76 - 0.0183 \times 82}{10 \times 100} = -3'06''$$

(b) At unity power factor, δ = 0; thus cos δ = 1 and sin δ = 0

$$\text{From the equation, } \theta = \frac{\frac{I_S}{n}(X_P \cos\ \delta - R_P \sin\ \delta) + I_C x_P - I_M r_P}{nV_S} \quad \text{we have}$$

$$\theta = \frac{\frac{I_S}{n}(X_P \cos\delta) + I_C x_P - I_M r_P}{nV_S}$$

For zero phase-angle error, θ = 0;

$$\text{Thus,} \qquad \theta = \frac{\frac{I_S}{n}(X_P \cos\delta) + I_C x_P - I_M r_P}{nV_S} = 0$$

$$\text{or,} \qquad I_S = (I_M r_P - I_C x_P)\frac{n}{X_P}$$

X_P - Total impedance referred to primary - $x_P + n \times X_S$

or, x_P - 76 + 10^2 × 0.72 - 148 Ω

Thus, or, $I_S = (0.0183 \times 82 - 0.008 \times 76)\dfrac{10}{148} = 0.06$ A

secondary burden - $V_S \times I_X$ - 110 × 0.06 - 6.6 VA

A potential transformer rated at 6600/110 V has 24000

Example 3.8

turns in the primary and 400 turns in the secondary winding. With rated voltage applied to the primary and secondary circuit opened, the primary winding draws a current of0.004 A, lagging the voltage by 75°. In another operating condition with a certain burden connected to the secondary, the primary draws 0.015 A at an angle 60° lagging with respect to the voltage. The following parameters are given for the transformer:

Primary resistance = 600 Ω *secondary resistance = 0.6Ω*

Primary reactance = 1500 Ω *secondary reactance = 0.96 Ω*

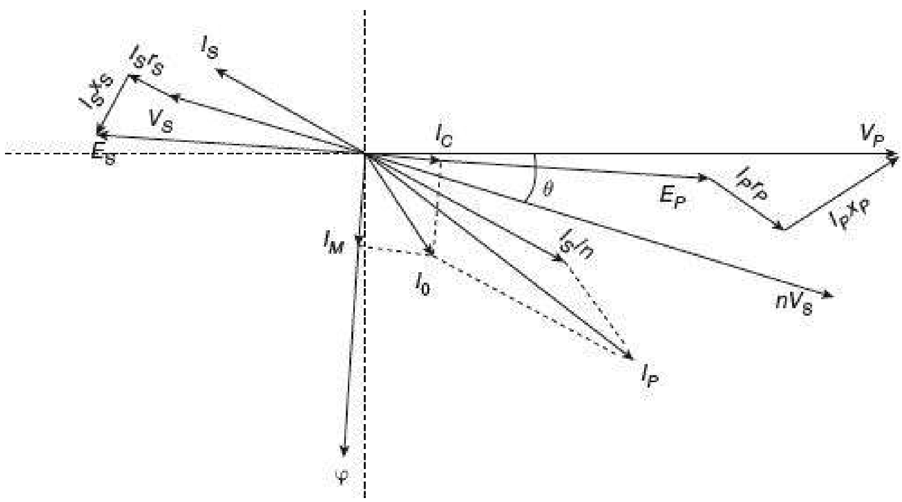
Calculate (a) Secondary load current and terminal voltage, using rated applied voltage as the reference

(b) The load burden in this condition

(c) Actual transformation ratio and phase angle

(d) How many turns should be changed in the primary winding to make the actual ratio equal to the nominal ratio under such operating condition?

Solution The corresponding phasor diagram is shown in the figure.



Nominal ratio = 6600/110 = 60 Turns ratio = 24000/400 = 60 No load current = 0.004 A

No load power factor = cos 75° = 0.26 and sin 75° = 0.966 Primary current = 0.015 A

Primary power factor = cos 60° = 0.5 and sin 60° = 0.866

Primary voltage V_P is taken as reference

Thus, $V_P = 6600 + j0$

$$I_p = 0.015(0.5 - j0.866) = 0.0075 - j0.013$$

$$I_0 = 0.004(0.26 - j0.966) = 0.00104 - j0.0039$$

As seen from Figure 3.21, the phasor $\frac{I_S}{n}$ is the phasor difference of I_p and I_q.

Thus, $\frac{I_S}{n} = (0.0075 - j0.013) - (0.00104 - j0.0039) = 0.00646 - j0.0091$

Thus, I_S (reversed) = 60 × (0.00646- j0.0091)= 0.388 - j0.546

secondary current phasor I_S = - (0.388 - j0.546) = -0.388 + j0.546

(a) Secondary current magnitude $I_S = \sqrt{(0.388)^2 + (0.546)^2} = 0.67$ A Primary voltage

$E_P = V_P - I_P \times Z_P$

or, E_P = 6600 + j0 - (0.0075 - j0.013)(600 + j1500)

or, E_P = 6576 - j3.45

Secondary induced voltage (reversed)

$$E_S(\text{reversed}) = \frac{E_P}{n} = \frac{6576 - j3.45}{60} = 109.6 - j0.0575$$

secondary induced voltage phasor = E_S = -(-109.6 - j0.0575)

or, E_S = -109.6 + j0.575

secondary terminal voltage

$V_S = E_S - I_S \times Z_S$

= -109.6 + j 0.0575 - (-0.388 + j0.546)(0.6 + j0.96)

= -109.9 + j0.76

magnitude of secondary terminal voltage

$$V_S = \sqrt{(109.9)^2 + (0.76)^2} = 109.9 \text{ A}$$

(b) Secondary load burden = $V_S \times I_S$ = 109.9 × 0.67 = 73.63 VA

(c) Actual transformation ratio $R = \frac{V_P}{V_S} = \frac{6600}{109.9} = 60.055$

V_S (reversed) - - (-109.9 + j0.76) - 109.9 - j0.76 V

Phase angle by which V_S(reversed) lags V_P

$$\theta = \frac{180}{\pi} \times \tan^{-1}\left(\frac{0.76}{109.9}\right) = 23.7'$$

(d) In order to make actual ratio equal to the nominal ratio, the primary number of turns should be reduced to

3.14 DIFFERENCES BETWEEN CT AND PT

In summary, the following differences between a current transformer (CT) and a potential transformer (PT) can be tabulated:

CT	PT
Reduce the main power line current to be measured by normal range instruments, i.e. current is stepped down from primary to secondary	Reduce the main power line voltage to be measured by normal range instruments, i.e. voltage is stepped down from primary to secondary
Primary winding of CT is connected in series with the main power line to sense current	Primary winding of PT is connected across (in parallel with) the main power line to sense voltage
Primary winding has less number of turns as compared to the secondary winding	Primary winding has more number of turns as compared to the secondary winding
CT secondary side should never be open circuited while energised, to restrict accidental over-voltage	PT secondary can be safely open circuited even if the PT is energized, since secondary voltage is always restricted by the turns ratio
In many cases, such as in bar type, in single primary winding type and in clamp-on type CT, the primary winding is nothing but the main power line conductor itself	In all the PTs, separate primary as well as secondary windings are necessary. Primary winding terminals are connected across the main power line (in parallel)
While using CT for measurement of power, secondary winding is connected in series with the current coil of the wattmeter	While using PT for measurement of power, secondary winding is connected in parallel with the pressure coil of the wattmeter

EXERCISE

Objective-type Questions

1. The disadvantages of using shunts for high current measurements are
   (a) power consumption by the shunts themselves is high
   (b) it is difficult to achieve good accuracy with shunts at high currents
   (c) the metering circuit is not electrically isolated from the power circuit
   (d) all of the above
2. The disadvantages of using multipliers with voltmeters for measuring high voltages are
   (a) power consumption by multipliers themselves is high at high voltages
   (b) multipliers at high voltage need to be shielded to prevent capacitive leakage
   (c) the metering circuit is not electrically isolated from the power circuit
   (d) all of the above
3. The advantages of instrument transformers are
   (a) the readings of instruments used along with instrument transformers rarely depend on the impedance of the instrument
   (b) due to availability of standardised instrument transformers and associated instruments, there is reduction in cost and ease of replacement
   (c) the metering circuit is electrically isolated from the power circuit
   (d) all of the above
4. Nominal ratio of a current transformer is
   (a) ratio of primary winding current to secondary winding current
   (b) ratio of rated primary winding current to rated secondary winding current
   (c) ratio of number of turns in the primary to number of turns in the secondary
   (d) all of the above
5. Burden of a CT is expressed in terms of
   (a) secondary winding current
   (b) VA rating of the transformer

(c) power and power factor of the secondary winding circuit

(d) impedance of secondary winding circuit

6. Ratio error in a CT is due to

(a) secondary winding impedance (b) load impedance

(c) no load current (d) all of the above

7. Phase-angle error in a CT is due to

(a) primary winding impedance

(b) primary circuit phase angle

(c) leakage flux between primary and secondary

(d) all of the above

8. Errors in instrument transformers can be aggravated by

(a) leakage flux (b) core saturation

(c) transients in main power line (d) all of the above

9. Phase-angle error in a CT can be reduced by

(a) reducing number of secondary turns

(b) using thin conductors for the primary winding

(c) using good quality, low loss steel for core

(d) all of the above

10. Ratio error in a CT can be reduced by

(a) using good quality, low loss steel for core

(b) placing primary and secondary windings closer to each other

(c) using thick conductors for secondary winding

(d) all of the above

11. Flux density in instrument transformers must be designed to be

(a) sufficiently low to reduce core losses

(b) sufficiently high to reduce core section and hence reduce length of winding

(c) sufficiently low to prevent core saturation

(d) properly optimized to have a balance among (a)-(c)

12. Current in the primary winding of CT depends on

(a) burden in the secondary winding of the transformer

(b) load connected to the system in which the CT is being used for measurement

(c) both burden of the secondary and load connected to the system

(d) none of the above

13. Turns compensation is used in CT to reduce

(a) phase-angle error

(b) both ratio and phase angle error

(c) primarily ratio error, reduction in phase angle error is incidental

(d) none of the above

14. Secondary winding of CT should never be open circuited with primary still energised because that will

(a) increase power loss in the secondary winding

(b) increase terminal voltage in the secondary winding

(c) increase the leakage flux manifolds

(d) all of the above

15. Open circuiting the secondary winding of CT with primary still energised will result in
    (a) unrestricted primary flux to generate high voltages across secondary terminals
    (b) possible insulation damage due to high voltage being generated
    (c) injury to careless operator
    (d) all of the above
16. A short-circuiting link is provided on the secondary side of a CT to
    (a) allow high current to flow in the primary when the secondary winding of the CT is short circuited with the link
    (b) allow adjustments to be made in the secondary side, like replacing the ammeter, with the primary energized but the short circuiting link in use
    (c) enable primary current to drop down to zero when the secondary is open circuited with the short circuiting link in use
    (d) all of the above
17. Clamp-on type and split-core type CTs are used because
    (a) their accuracy is high
    (b) it is possible to insert the CT in the circuit without breaking the main line
    (c) they are cheaper
    (d) all of the above
18. Transformation ratio of a PT is defined as
    (a) ratio of primary winding voltage to secondary winding voltage
    (b) ratio of rated primary winding voltage to rated secondary winding voltage
    (c) ratio of primary number of turns to secondary number of turns
    (d) all of the above
19. When the secondary winding of a PT is suddenly open circuited with the primary winding still open circuited then
    (a) large voltages will be produced across the secondary terminals that may be dangerous for the operating personnel
    (b) large voltages thus produced may damage the insulation
    (c) the primary winding draws only no-load current
    (d) none of the above
20. The size of a PT as compared to a power transformer of same VA
    (a) is smaller (b) is bigger
    (c) is the same (d) there is no relation as such

Answers

1. (d)	2. (d)	3. (a)	4. (b)	5. (b)	6. (d)	7. (c)
8. (d)	9. (d)	10. (d)	11. (d)	12. (b)	13. (c)	14. (b)
15. (d)	16. (d)	17. (b)	18. (a)	19. (c)	20. (a)	

Short-answer Questions

1. Discuss the advantages of instrument transformers as compared to shunts and multipliers for extension of instrument range.
2. Describe with clear schematic diagrams, how high voltage and currents are measured with the help of instrument transformers.
3. Draw and explain the nature of equivalent circuit the and corresponding phasor diagram of a current transformer.

4. Discuss the major sources of error in a current transformer.
5. Describe the design and constructional features of a current transformer for reducing ratio error and phase-angle error.
6. Explain with the help of a suitable example, the method of turns compensation in a CT to reduce ratio error.
7. Why should the secondary winding of a CT never be open circuited with its primary still energised?
8. Explain how the core of a CT may get permanent magnetisation induced in it. What are the bad effects of such permanent magnetisation? What are the ways to de-magnetise the core in such situations?
9. Draw and explain the constructional features of wound-type, bar type, clamp type and bushing type CTs.
10. Draw the equivalent circuit and phasor diagram of a potential transformer being used for measurement of high voltages.
11. What are the differences between a potential transformer and a regular power transformer?
12. Describe the methods employed for reducing ratio error and phase angle error in PTs.

Long-answer Questions

1. Draw and explain the nature of equivalent circuit and corresponding phasor diagram of a current transformer. Derive expressions for the corresponding ratio error and phase angle error.
2. (a) What are the sources of error in a current transformer?

   (b) A ring-core type CT with nominal ratio 1000/5 and a bar primary has a secondary winding resistance of 0.8 Ω and negligible reactance. The no load current is 4 A at a power factor of 0.35 when full load secondary current is flowing in a burden if 1.5 Ω no-inductive resistance. Calculate the ratio error and phase-angle error at full load. Also calculate the flux in the core at 50 Hz.
3. (a) Describe the design and constructional features of a current transformer for reducing ratio error and phase-angle error.

   (b) A 1000/10 A, 50 Hz single-turn primary type CT has a secondary burden comprising of a pure resistance of 1.0 Ω. Calculate flux in the core, ratio error and phase-angle error at full load. Neglect leakage reactance and assume the iron loss in the core to be 5 Ω at full load. The magnetising ampere-turns is 180.
4. (a) Why the secondary winding of a CT should never be open circuited with its primary still energised?

   (b) A bar-type CT has 300 turns in the secondary winding. The impedance of the secondary circuit is (1.5 + j2)Ω. With 5 A flowing in the secondary, the magnetising mmf is 1000 A and the iron loss is Determine ratio and phase-angle errors.
5. (a) Explain the method of turns compensation in a CT to reduce ratio error.

   (b) A bar-type CT has 400 turns in the secondary winding. An ammeter connected to the secondary has resistance of 1.5 Ω and reactance of 1.0 Ω, and the secondary winding impedance is (0.6 + j0.8) Ω. The magnetising mmf requirement for the core is 80 A and to supply the iron loss the current required is 30 A. (i) Find the primary winding current and also determine the ratio error when the ammeter in the secondary winding shows 4 A. (ii) How many turns should be reduced in the secondary to bring down ratio error to zero at this condition?
6. Draw and explain the nature of equivalent circuit and corresponding phasor diagram of a potential transformer. Derive expressions for the corresponding ratio error and phase-angle error.
7. (a) What are the differences between a potential transformer and a regular power transformer?

   (b) A potential transformer with nominal ratio 1000/100 V has the following parameters:

   Primary resistance = 96 Ω secondary resistance = 0.8Ω

   Primary reactance = 80 Ω secondary reactance = 0.65Ω

   No load current = 0.03 a at 0.35 power factor Calculate

   (i) phase angle error at no load

   (ii) burden in VA at unity power factor at which phase angle error will be zero.

   (a) Describe the methods employed for reducing ratio error and phase angle error in PTs?

(b) A potential transformer rated at 6000/100 V has 24000 turns in the primary and 400 turns in the secondary winding. With rated voltage applied to the primary and secondary circuit opened, the primary winding draws a current of 0.005 A lagging the voltage by 700. In another operating condition with a certain burden connected to the secondary, the primary draws 0.012 A at an angle 540 lagging with respect to the voltage. The following parameters are given for the transformer:

Primary resistance = 600 Ω	secondary resistance = 0.6 Ω
Pimary reactance = 1500 Ω	secondary reactance = 0.96 Ω

Calculate

(i) Secondary load current and terminal voltage, using rated applied voltage as the reference

(ii) The load burden in this condition

(iii) Actual transformation ratio and phase angle

(iv) How many turns should be changed in the primary winding to make the actual ratio equal to the nominal ratio under such operating condition?

4 Measurement of Resistance

4.1 INTRODUCTION

Resistors are used in many places in electrical circuits to perform a variety of useful tasks. Properties of resistances play an important role in determining performance specifications for various circuit elements including coils, windings, insulations, etc. It is important in many cases to have reasonably accurate information of the magnitude of resistance present in the circuit for analysing its behaviour. Measurement of resistance is thus one of the very basic requirements in many working circuits, machines, transformers, and meters. Apart from these applications, resistors are used as standards for the measurement of other unknown resistances and for the determination of unknown inductance and capacitance.

From the point of view of measurement, resistances can be classified as follows:

1. Low Resistances

All resistances of the order less than 1 Ω may be classified as low resistances. In practice, such resistances can be found in the copper winding in armatures, ammeter shunts, contacts, switches, etc.

2. Medium Resistances

Resistances in the range 1 Ω to 100 kΩ may be classified as medium resistances. Most of the electrical apparatus used in practice, electronic circuits, carbon resistance and metal-film resistors are found to have resistance values lying in this range.

3. High Resistances

Resistances higher than 100 kΩ are classified as high resistances. Insulation resistances in electrical equipment are expected to have resistances above this range.

The above classifications are, however, not rigid, but only form a guideline for the method of measurement to be adopted, which may be different for different cases.

4.2 MEASUREMENT OF MEDIUM RESISTANCES

The different methods for measurement of medium range resistances are (i) ohmmeter method, (ii) voltmeter–ammeter method, (iii) substitution method, and (iv) Wheatstone-bridge method.

4.2.1 Ohmmeter Method for Measuring Resistance

Ohmmeters are convenient direct reading devices for measurement of approximate resistance of circuit components without concerning too much about accuracy. This instrument is, however, very popular in the sense that it can give quick and direct readings for resistance values without any precise adjustments requirements from the operator. It is also useful in measurement laboratories as an adjunct to a precision bridge. Value of the unknown resistance to be measured is first obtained by the ohmmeter, and this can save lot of time in bridge balancing for obtaining the final precision value using the bridge.

Series-type Ohmmeter

Figure 4.1 shows the elements of a simple single-range series-type ohmmeter.

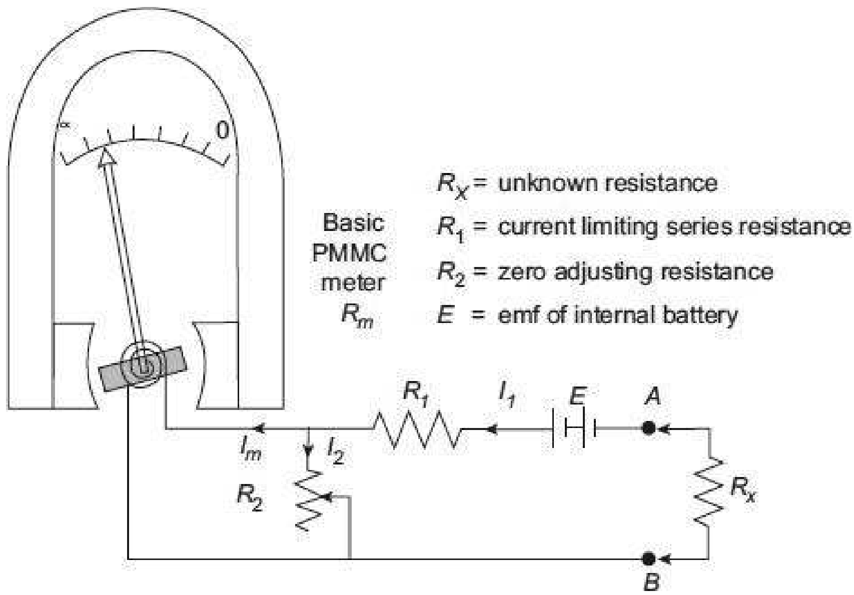


Figure 4.1 *Single-range series ohmmeter*

The series-type ohmmeter consists basically of a sensitive dc measuring PMMC ammeter connected in parallel with a variable shunt R_2. This parallel circuit is connected in series with a current limiting resistance R_1 and a battery of emf E. The entire arrangement is connected to a pair of terminals *(A–B)* to which the unknown resistance R_x to be measured is connected.

Before actual readings are taken, the terminals *A–B* must be shorted together. At this position with $R_x = 0$, maximum current flows through the meter. The shunt resistance R_2 is adjusted so that the meter deflects corresponding to its right most full scale deflection (FSD) position. The FSD position of the pointer is marked 'zero-resistance', i.e., 0 Ω on the scale. On the other hand, when the terminals A–B are kept open $(R_x \to \infty)$, no current flows through the meter and the pointer corresponds to the left most zero current position on the scale. This position of the pointer is marked as '∞Ω' on the scale. Thus, the meter will read infinite resistance at zero current position and zero resistance at full-scale current position. Series ohmmeters thus have '0' mark at the extreme right and '∞' mark at the extreme left of scale (opposite to those for ammeters and voltmeters).

The main difficulty is the fact that ohmmeters are usually powered by batteries, and the battery voltage gradually changes with use and age. The shunt resistance R_2 is used in such cases to counteract this effect and ensure proper zero setting at all times.

For zero setting, $R_x = 0$, where R_m = internal resistance of the basic PMMC meter coil

$\therefore$ equivalent resistance of the circuit $R_{eq} = R_1 + \frac{R_2 R_m}{R_2 + R_m}$

And, total current $I_1 = \frac{E}{R_{eq}} = I_2 + I_m$

The current I_2 can be adjusted by varying R_2 so that the meter current I_m can be held at its calibrated value when the main current I_1 changes due to drop in the battery emf E.

> If R_2 were not present, then it would also have been possible to bring the pointer to full scale by adjustment of the series resistance R_1, But this would have changed the calibration all along the scale and cause large error..

(i) Design of R_1 and R_2 The extreme scale markings, i.e., 0 and ∞, in an ohmmeter do not depend on the circuit constants. However, distributions of the scale markings between these two extremes are affected by the constants of the circuit. It is thus essential to design for proper values of the circuit constants, namely, R_1 and R_2 in particular to have proper calibration of the scale. The following parameters need to be known for determination of R_1 and R_2.

- Meter current I_m at full scale deflection (= I_{FSD}.)
- Meter coil resistance, R_m
- Ohmmeter battery voltage, E
- Value of the unknown resistance at half-scale deflection, (R_h), i.e., the value of R_x when the pointer is at the middle of scale

With terminals A–B shorted, when $R_x = 0$

Meter carries maximum current, and current flowing out of the battery is given as $I_{1MAX} = \frac{E}{R_i}$

where Ri = internal resistance of the ohmmeter $= R_1 + \frac{R_2 R_m}{R_2 + R_m}$

At half-scale deflection, $R_x = R_h$, and $I_h = \frac{I_{1MAX}}{2} = \frac{E}{R_i + R_h}$

$$\therefore \qquad \frac{E}{R_i + R_h} = \frac{E}{2R_i}$$

$$\text{or,} \qquad R_i = R_h$$

$$\therefore \qquad I_h = \frac{E}{2R_h} \text{ and } R_h = R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

For full-scale deflection,

$$I_h = I_{FSD} \text{ and } I_1 = 2I_h = 2\times\frac{E}{2R_h} = \frac{E}{R_h}$$

Also, $I_m R_m = I_2 R_2$ and $I_2 = I_1 - I_m$

$$\therefore \text{ At FSD, } I_{FSD}R_m = R_2(I_1 - I_m) = R_2\left(\frac{E}{R_h} - I_{FSD}\right)$$

Thus,

$$R_2 = \frac{I_{FSD}R_m R_h}{(E - I_{FSD}R_h)} \quad (4.1)$$

Again, since $R_h = R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m}$, putting the value of R_2 from (4.1), we get

$$R_1 = R_h - \frac{I_{FSD}R_m R_h}{E} \quad (4.2)$$

(ii) . Shape of Scale in Series Ohmmeters Electrical equivalent circuit of a series-type ohmmeter is shown in Figure 4.2.

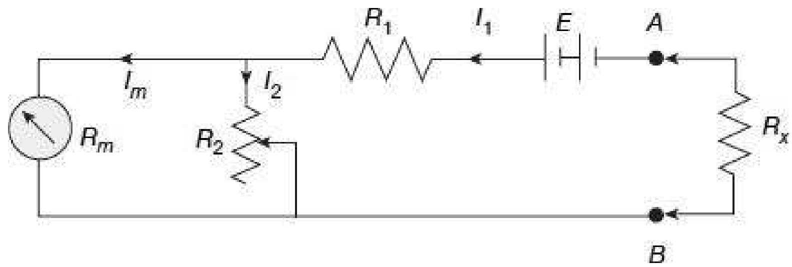


Figure 4.2 *Electrical equivalent circuit of a series-type ohmmeter*

Internal resistance of the ohmmeter

$$R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

Thus, $$I_1 = \frac{E}{R_i + R_x}$$

Meter current $$I_m = I_1 \times \frac{R_2}{R_2 + R_m} = \frac{E}{R_i + R_x} \times \frac{R_2}{R_2 + R_m}$$

With the terminals *A*–*B* short circuited $R_x = 0$; thus, from (4.3) we have

$$I_{FSD} = \frac{E}{R_i} \times \frac{R_2}{R_2 + R_m} \quad (4.4)$$

From (4.3) and (4.4), the meter can be related to the FSD as

$$\frac{I_m}{I_{FSD}} = \frac{\frac{E}{R_i + R_x} \times \frac{R_2}{R_2 + R_m}}{\frac{E}{R_i} \times \frac{R_2}{R_2 + R_m}} = \frac{R_i}{R_i + R_x}$$

Thus,

$$I_m = \frac{R_i}{R_i + R_x} \times I_{FSD} \quad (4.5)$$

From Eq. (4.5), it can be observed that the meter current I_m is not related linearly with the resistance R_x to be measured. The scale (angle of deflection) in series ohmmeter if thus non-linear and cramped.

The above relation (4.5) also indicates the fact that the meter current and hence graduations of the scale get changed from the initial calibrated values each time the shunt resistance R_2 is adjusted. A superior design is found in some ohmmeters where an adjustable soft-iron shunt is placed across the pole pieces of the meter, as indicated in Figure 4.3.

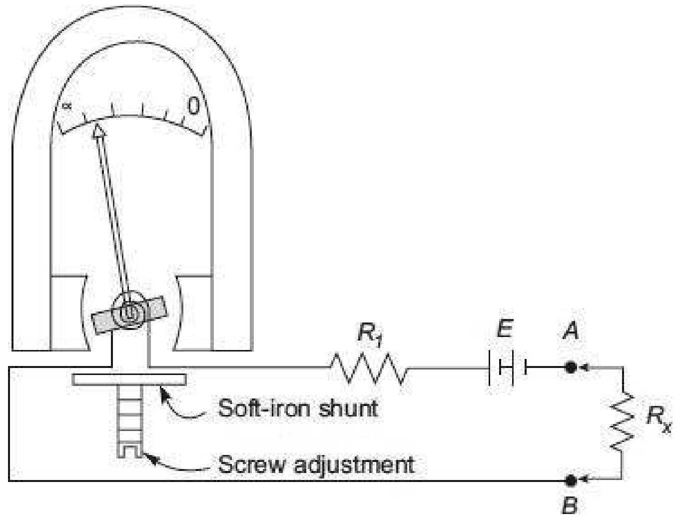


Figure 4.3 *Series ohmmeter with soft-iron magnetic shunt*

The soft-iron magnetic shunt, when suitably positioned with the help of screw adjustment, modifies the air gap flux of main magnet, and hence controls sensitivity of movement. The pointer can thus be set at proper full scale marking in compensation against changes is battery emf, without any change in the electrical circuit. The scale calibrations thus do not get disturbed when the magnetic shunt is adjusted.

1. Multi-range Series Ohmmeter

For most practical purposes, it is necessary that a single ohmmeter be used for measurement of a wide range of resistance values. Using a single scale for such measurements will lead to inconvenience in meter readings and associated inaccuracies. Multi-range ohmmeters, as shown schematically in Figure 4.4, can be used for such measurements. The additional shunt resistances R_3, R_4…, R_7 are used to adjust the meter current to correspond to 0 to FSD scale each time the range of the unknown resistance R_x is changed. In a practical multi-range ohmmeter, these shunt resistances are changed by rotating the range setting dial of the ohmmeter. The photograph of such a laboratory grade analog multi-range ohmmeter is provided in Figure 4.5.

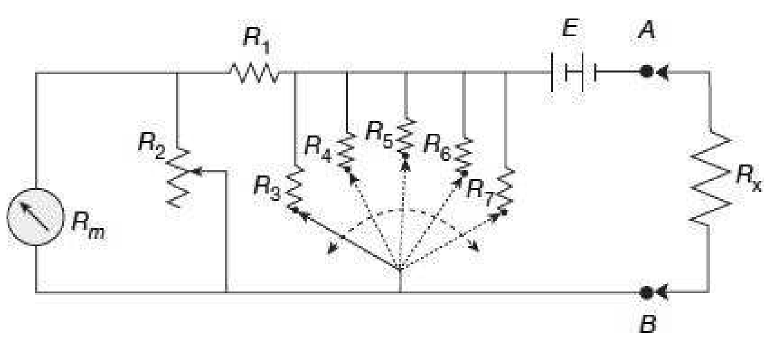


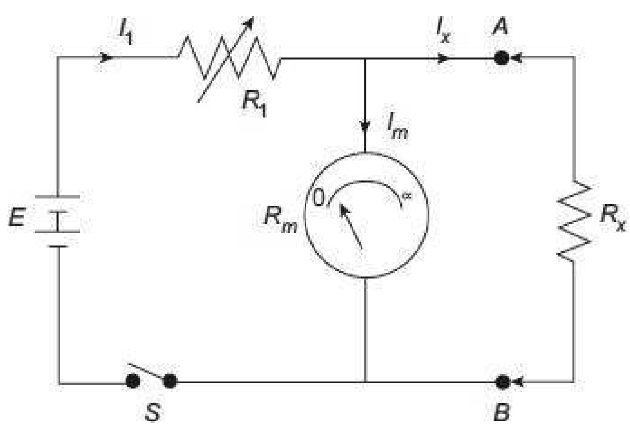
Figure 4.4 *Multi-range series-type ohmmeter*

Figure 4.5 *Photograph of multi-range ohmmeter (Courtesy, SUNWA)*

2. Shunt-type Ohmmeter

Figure 4.6 shows the schematic diagram of a simple shunt-type ohmmeter.

The shunt-type ohmmeter consists of a battery in series with an adjustable resistance R_1 and a sensitive dc measuring PMMC ammeter. The unknown resistance R_x to be measured is connected across terminals A–B and parallel with the meter.



R_x = unknown resistance
R_1 = scale adjusting resistance
E = emf of internal battery
R_m = PMMC meter resistance
S = switch

Figure 4.6 *Shunt-type ohmmeter*

When the terminals A–B are shorted ($R_x = 0$), the meter current is zero, since all the current in the circuit passes through the short circuited path A–B, rather than the meter. This position of the pointer is marked ‘zero-resistance’, i.e., ‘0 Ω’ on the scale. On the other hand, when R_x is removed, i.e., the terminals *A–B* open circuited ($R_x \rightarrow \infty$), entire current flows through the meter. Selecting proper value of R_1, this maximum current position of the pointer can be made to read full scale of the meter. This position of the pointer is marked as '∞Ω' on the scale. Shunt type ohmmeters, accordingly, has '0 Ω' at the left most position corresponding to zero current, and '∞Ω' at the rightmost end of the scale corresponding to FSD current.

When not under measurement, i.e., nothing is connected across the terminals *A–B* ($R_x \to \infty$) the battery always drives FSD current through the meter. It is thus essential to disconnect the battery from rest of the circuit when the meter is idle. A switch S, as shown in Figure 4.6, is thus needed to prevent the battery from draining out when the instrument is not in use.

Shape of Scale in Shunt Ohmmeters Internal resistance of the ohmmeter

$$R_i = \frac{R_1 R_m}{R_1 + R_m}$$

With terminals A–B open, the full-scale current through the meter is

$$I_{FSD} = \frac{E}{R_1 + R_m} \tag{4.6}$$

With R_x connected between terminals *A-B*, the current out of the battery is

$$I_1 = \frac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}}$$

Thus, meter current $I_m = I_1 \times \dfrac{R_x}{R_x + R_m} = \dfrac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} \times \dfrac{R_x}{R_x + R_m}$ (4.7)

From Eqs (4.6) and (4.7), the meter can be related to the FSD as

$$\frac{I_m}{I_{FSD}} = \frac{\dfrac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} \times \dfrac{R_x}{R_x + R_m}}{\dfrac{E}{R_1 + R_m}} = \frac{R_x(R_1 + R_m)}{R_1(R_m + R_x) + R_m R_x} = \frac{R_x(R_1 + R_m)}{R_1 R_m + R_x(R_1 + R_m)}$$

or, $$\frac{I_m}{I_{FSD}} = \frac{R_x}{\dfrac{R_1 R_m}{(R_1 + R_m)} + R_x} = \frac{R_x}{R_x + R_i}$$

$$I_m = \frac{R_x}{R_i + R_x} \times I_{FSD} \tag{4.8}$$

From Eq (4.8), it can be observed that the meter current I_m increases almost linearly with the resistance R_x to be measured for smaller values of R_x when $R_x << R_i$. The scale (angle of deflection) in shunt type ohmmeters is thus almost linear in the lower range, but progressively becomes more cramped at higher values of R_x. Shunt-type ohmmeters are thus particularly suitable for measurement of low resistances when the meter scale is nearly uniform.

Design a single-range series-type ohmmeter using a PMMC ammeter that has internal resistance of 50 Ω and requires a current of 1 mA for full-scale deflection.

Example 4.1

The internal battery has a voltage of 3 V. *It is desired to read half scale at a resistance value of 2000 Ω. Calculate (a) the values of shunt resistance and current limiting series resistance, and (b) range of values of the shunt resistance to accommodate battery voltage variation in the range 2.7 to 3.1 V.*

Solution Schematic diagram of the series ohmmeter with the given values is shown in the following figure

Given, R_m = meter internal resistance = 50 Ω

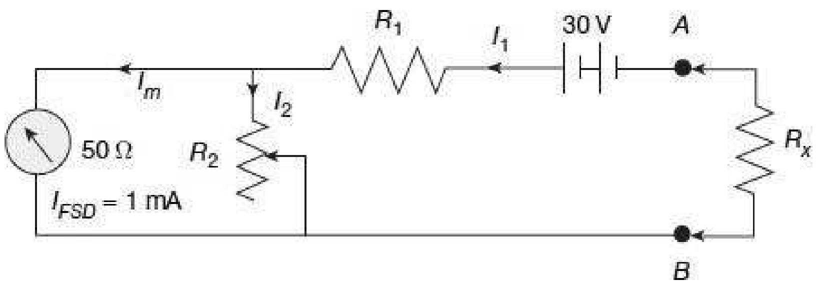
I_{FSD} = meter full scale deflection current = 1 mA

R_h = half-scale deflection resistance = 2000 W

E = battery voltage = 3 V

(a) With terminals A–B shorted, when $R_x = 0$

Meter carries maximum current, and current flowing out of the battery is given as $I_{1MAX} = \frac{E}{R_i}$,



where E = battery voltage = 3 V, and

R_i = internal resistance of the ohmmeter $= R_1 + \frac{R_2 R_m}{R_2 + R_m}$

At half-scale deflection, $R_x = R_h$, and battery current $I_h = \frac{I_{1MAX}}{2} = \frac{E}{R_i + R_h}$

$$\therefore \qquad \frac{E}{R_i + R_h} = \frac{E}{2R_i}$$

$$\text{or,} \qquad R_i = R_h$$

$$\therefore \qquad I_h = \frac{E}{2R_h} = \frac{3}{2 \times 2000} = 0.75 \text{ mA}$$

$$\text{and} \qquad R_h = 2000 = R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m} = R_1 + \frac{50R_2}{R_2 + 50}$$

For full-scale deflection,

$$\therefore \quad \text{At } FSD,\ R_2 = \frac{I_{FSD}R_m}{I_2} = \frac{1\times10^{-3}\times 50}{0.5\times10^{-3}} = 100\ \Omega$$

Again, since $R_1 + \dfrac{50R_2}{R_2+50} = 2000$, putting the value of R_2

$$R_1 = 1966.7\ \Omega$$

(b) For a battery voltage of E = 2.7 V, battery current at half scale is

$$I_h = \frac{E}{2R_h} = \frac{2.7}{2\times 2000} = 0.675 \text{ mA}$$

For full-scale deflection,

$$I_m = I_{FSD} = 1\text{mA and } I_1 = 2I_h = 1.35 \text{ mA}$$

Also, $I_mR_m = I_2R_2$ and $I_1 - I_m = (1.35\text{-}1)$ mA = 0.35 mA

$$\therefore \text{At } FSD,\ R_2 = \frac{I_{FSD}R_m}{I_2} = \frac{1\times10^{-3}\times 50}{0.35\times10^{-3}} = 142.86\ \Omega$$

For a battery voltage of E = 3.1 V, battery current at half scale is

$$I_h = \frac{E}{2R_h} = \frac{3.1}{2\times 2000} = 0.775 \text{ mA}$$

For full-scale deflection,

$$I_m = I_{FSD} = 1\text{mA and } I_1 = 2I_h = 1.55 \text{ mA}$$

Also, $I_mR_m = I_2R_2$ and $I_1 - I_m = (1.55\text{-}1)$ mA = 0.55 mA

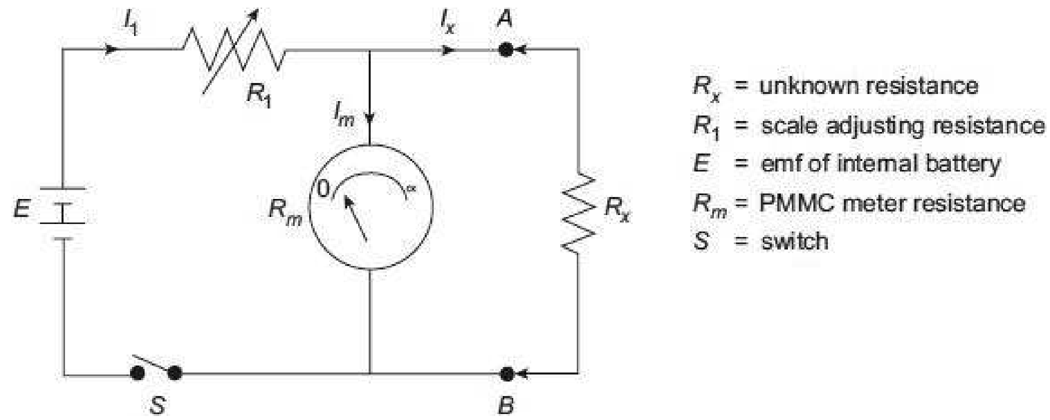
$$\therefore \text{At FSD},\ R_2 = \frac{I_{FSD}R_m}{I_2} = \frac{1\times10^{-3}\times 50}{0.55\times10^{-3}} = 90.0\ \Omega$$

∴ range of R_2 to accommodate the given change in battery voltage is 142.86 Ω > R_2 > 90.9 Ω.

Example 4.2

A shunt-type ohmmeter uses a 2 mA basic d'Arsonval movement with an internal resistance of 25 Ω. The battery emf is 1.5 V.
Calculate (a) value of the resistor in series with the battery to adjust the FSD, and (b) at what point (.percentage) of full-scale will 100 Ω be marked on the scale?

Solution Schematic diagram of a shunt type-ohmmeter under the condition as stated in Example 4.2 is shown below:



At FSD when terminals A–B is opened, meter FSD current is

$$I_m = I_{FSD} = \frac{E}{R_1 + R_m} = \frac{1.5}{R_1 + 25} = 2 \times 10^{-3}$$

$$\boxed{\text{Thus, } R_1 = 725\,\Omega}$$

When $R_x = 100\Omega$, battery output current will be

$$I_1 = \frac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} = \frac{1.5}{725 + \dfrac{25 \times 100}{25 + 100}} = 2.013 \text{ mA}$$

∴ meter current is $I_m = I_1 \times \dfrac{R_x}{R_m + R_x} = 2.013 \times \dfrac{100}{25 + 100} = 1.6104 \text{ mA}$

Thus, percentage of full scale at which the meter would read 100 Ω is

$$\boxed{\frac{I_m}{I_{FSD}} \times 100\% = \frac{1.6104}{2} \times 100\% = 80.52\%}$$

4.2.2 Voltmeter–Ammeter Method for Measuring Resistance

The voltmeter–ammeter method is a direct application of ohm's law in which the unknown resistance is estimated by measurement of current *(I)* flowing through it and the voltage drop (V) across it. Then measured value of the resistance is

$$R_m = \frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{V}{I}$$

This method is very simple and popular since the instruments required for measurement are usually easily available in the laboratory.

Two types of connections are employed for voltmeter–ammeter method as shown in Figure 4.7.

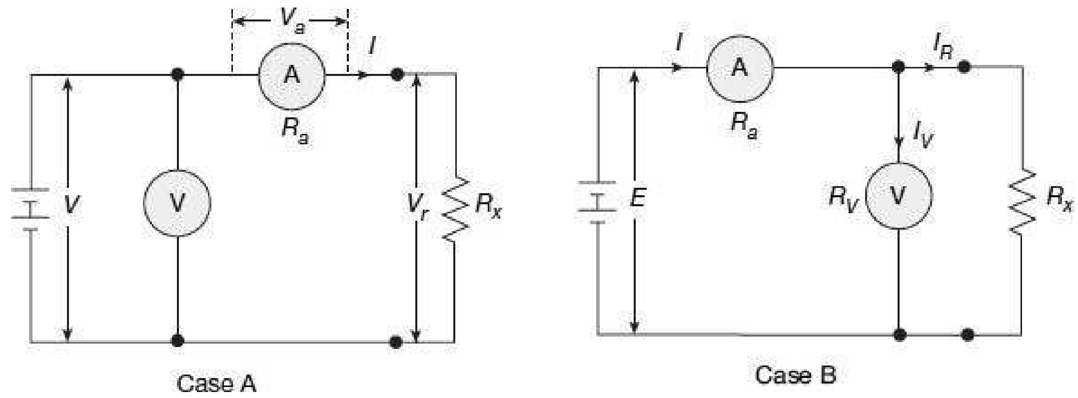


Figure 4.7 *Measurement of resistance by voltmeter–ammeter method*

R_x = true value of unknown resistance

R_m = measured value of unknown resistance

R_a = internal resistance of ammeter

R_V = internal resistance of voltmeter

It is desired that in both the cases shown in Figure 4.7, the measured resistance R_m would be equal to the true value R_x of the unknown resistance. This is only possible, as we will see, if the ammeter resistance is zero and the voltmeter resistance is infinite.

Case A

In this circuit, the ammeter is connected directly with the unknown resistance, but the voltmeter is connected across the series combination of ammeter and the resistance R_x. The ammeter measures the true value of current through the resistance but the voltmeter does not measure the true value of voltage across the resistance. The voltmeter measures the sum of voltage drops across the ammeter and the unknown resistance R_x.

Let, voltmeter reading = V

And, ammeter reading = I

$\therefore$ measured value of resistance $= R_m = \frac{V}{I}$

However, $\quad V = V_a + V_r$

or, $\quad V = I \times R_a + I \times R_x = I \times (R_a + R_x)$

Thus, $\frac{V}{I} = R_m = (R_a + R_x)$ (4.9)

The measured value R_m of the unknown resistance is thus higher than the true value R, by the quantity R_a, internal resistance of the ammeter. It is also clear from the above that true value is equal to the measured value only if the ammeter resistance is zero.

Error in measurement is $\varepsilon = \frac{R_m - R_x}{R_x} = \frac{R_a}{R_x}$

Equation (4.10) denotes the fact that error in measurement using connection method shown in Case A will be negligible only if the ratio $\frac{R_a}{R_x} \to 0$. In other words, if the resistance under measurement is much higher as compared to the ammeter resistance (R_x R_a.), then the connection method shown in Case A can be employed without involving much error.

Therefore, circuit shown in Case A should be used for measurement of high resistance values.

Case B

In this circuit, the voltmeter is connected directly across the unknown resistance, but the ammeter is connected in series with the parallel combination of voltmeter and the resistance R_x. The voltmeter thus measures the true value of voltage drop across the resistance but the ammeter does not measure the true value of current through the resistance. The ammeter measures the summation of current flowing through the voltmeter and the unknown resistance R_x.

Let, voltmeter reading = V

And, ammeter reading = I

Thus, $V = I_R \times R_X = I_V \times R_V$

However, $I = I_V + I_R$

∴ measured value of resistance

$$= R_m = \frac{V}{I} = \frac{V}{I_V + I_R} = \frac{V}{\frac{V}{R_V} + \frac{V}{R_x}} = \frac{R_V R_x}{R_V + R_x} = \frac{R_x}{1 + \frac{R_x}{R_V}}$$

or $$R_m = \frac{R_x}{1 + \frac{R_x}{R_V}} \qquad (4.11)$$

The measured value R_m of the unknown resistance is thus lower than the true value R_x by a quantity related to internal resistance of the voltmeter. It is also clear from Eq. (4.11) that true value is equal to the measured value only if the quantity $\frac{R_x}{R_V} \to 0$, i.e., if voltmeter resistance is infinite. In other words, if the voltmeter resistance is much higher as compared to the resistance under measurement ($R_V >> R_A$) then the connection method shown in Case B can be emploted without involving much error.

Therefore, circuit shown in Case B should be used for measurement of low resistance values.

Example 4.3

A voltmeter of 600 Ω resistance and a milliammeter of 0.8 Ω resistance are used to measure two unknown resistances by voltmeter–ammeter method.
If the voltmeter reads 40 V and milliammeter reads 120 mA in both the cases, calculate the percentage error in the values of measured resistances if (a) in the first case, the

voltmeter is put across the resistance and the milliammeter connected in series with the supply, and (b) in the second case, the voltmeter is connected in the supply side and milliammeter connected directly in series with the resistance.

Solution The connections are shown in the following figure.

Voltmeter reading $V = 40$ V

Ammeter reading $I = 120$ mA

∴ measured resistance from voltmeter and I

ammeter readings is given by

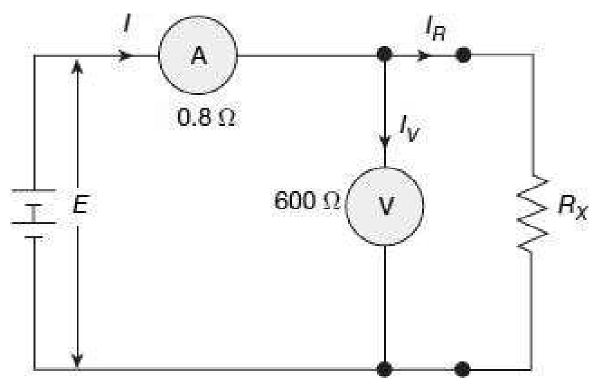
$$= R_m = \frac{V}{I} = \frac{40}{120 \times 10^{-3}} = 333\ \Omega$$

The ammeter reads the current flowing I_R through the resistance R_x and also the current I_V through the voltmeter resistance R_V.

Thus, $\qquad I = I_V + I_R$

Now, the voltmeter and the resistance R_x being in parallel, the voltmeter reading is given by

$$V = I_R \times R_X = I_V \times R_V$$



Current through voltmeter

$$I_V = \frac{V}{R_V} = \frac{40}{600} = 66.67 \text{ mA}$$

∴ true current throught resistance $I_R = I - I_V = 120\text{-}66.67 = 55.33$ mA

∴ true value of resistance $= R_x = \frac{V}{I_R} = \frac{40}{53.33 \times 10^{-3}} = 750\ \Omega$

Thus, percentage error $\varepsilon = \frac{R_m - R_x}{R_x} = \frac{333 - 750}{750} \times 100\% = 55.5\%$

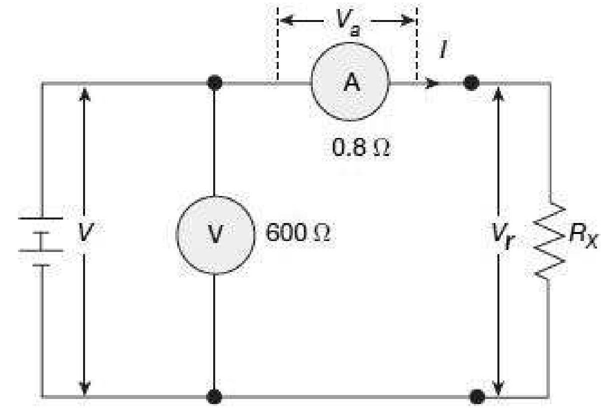
The connections are shown in the following fi gure.

Voltmeter reading V = 40V

Ammeter reading I = 120 mA

∴ measured resistance from voltmeter and ammeter readings is given by

$$= R_m = \frac{V}{I} = \frac{40}{120 \times 10^{-3}} = 333\ \Omega$$



Voltmeter reads the voltage drop V_r across the resistance R_x and also the voltage drop V_a across the ammeter resistance R_a.

Thus, $V = V_a + V_r$

Voltage drop across ammeter

$$V_a = I \times R_a = 120 \times 10^{-3} \times 0.8 = 0.096\ \text{V}$$

∴ true voltage drop across the resistance

$$V_r = V - V_a = 40 – 0.096 \times 39.904\ \text{V}$$

∴ true value of resistance $= R_x = \frac{V_r}{I} = \frac{39.904}{120 \times 10^{-3}} = 332.53\ \Omega$

Percentage error in measurement is $\varepsilon = \frac{333 - 332.53}{332.53} \times 100\% = 0.14\%$

4.2.3 Substitution Method for Measuring Resistance

The connection diagram for the substitution method is shown in Figure 4.8.

In this method the unknown resistance R_x is measured with respect to the standard variable resistance *S*. The circuit also contains a steady voltage source *V*, a regulating resistance *r* and an ammeter. A switch is there to connect R_x and *S* in the circuit alternately.

To start with, the switch is connected in position 1, so that the unknown resistance R_x gets included in the circuit. At this condition, the regulating resistance *r* is adjusted so that the ammeter pointer comes to a specified location on the scale. Next, the switch is thrown to position 2, so that the standard resistance *S* comes into circuit in place of R_x. Settings in the regulating resistance are not changed. The standard variable resistance *S* is varied till ammeter pointer reaches the same location on scale as was with R_x. The value of the standard resistance *S* at this position is noted from its dial. Assuming that the battery emf has not changed and also since the value of *r* is kept same in both the cases, the current has been kept at the same value while substituting one resistance with another one. The two resistances thus, must be equal. Hence, value of the unknown resistance R_x can be estimated from dial settings of the standard resistance S.

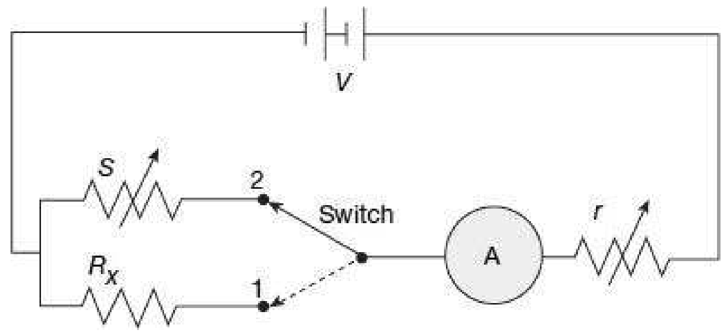


Figure 4.8 *Substitution method*

Accuracy of this method depends on whether the battery emf remains constant between the two measurements. Also, other resistances in the circuit excepting *R* and *S* should also not change during the course of measurement. Readings must be taken fairly quickly so that temperature effects do not change circuit resistances appreciably. Measurement accuracy also depends on sensitivity of the ammeter and also on the accuracy of the standard resistance *S*.

4.2.4 Wheatstone Bridge for Measuring Resistance

The Wheatstone bridge is the most commonly used circuit for measurement of medium-range resistances. The Wheatstone bridge consists of four resistance arms, together with a battery (voltage source) and a galvanometer (null detector). The circuit is shown in Figure 4.9.

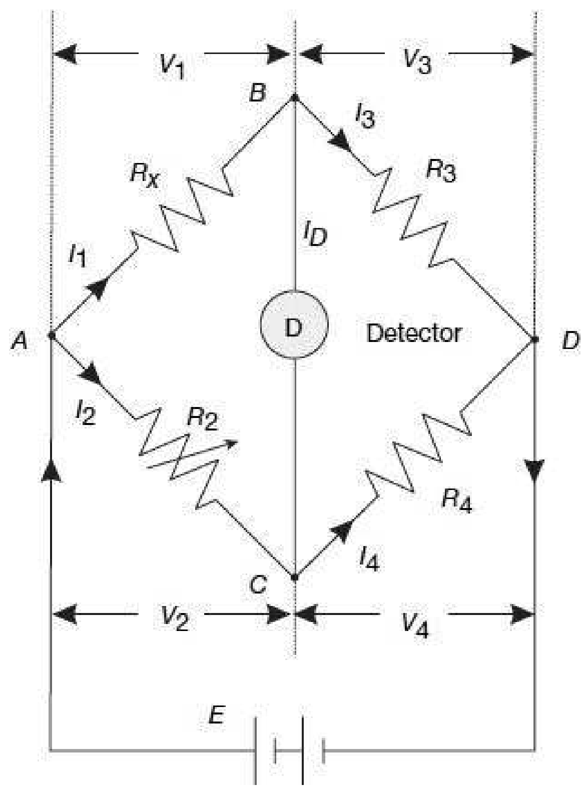


Figure 4.9 *Wheatstone bridge for measurement of resistance*

In the bridge circuit, R_3 and R_4 are two fixed known resistances, R_2 is a known variable resistance and R_X is the unknown resistance to be measured. Under operating conditions, current I_D through the galvanometer will depend on the difference in potential between nodes B and C. A bridge balance condition is achieved by varying the resistance R_2 and checking whether the galvanometer pointer is resting at its zero position. At balance, no current flows through the galvanometer. This means that at balance, potentials at nodes B and C are equal. In other words, at balance the following conditions are satisfied:

1. The detector current is zero, i.e., $I_D = 0$ and thus $I_t = I_3$ and $I_2 = I_4$
2. Potentials at node B and C are same, i.e., $V_B = V_C$, or in other words, voltage drop in the arm AB equals the voltage drop across the arm AC, i.e., $V_{AB} = V_{AC}$ and voltage drop in the arm BD equals the voltage drop across the arm CD, i.e., $V_{BD} = V_{CD}$

From the relation $V_{AB} = V_{AC}$ we have $I_1 \times R_x = I_2 \times R_2$ (4.12)

At balanced 'null' position, since the galvanometer carries no current, it as if acts as if open circuited, thus

$$I_1 = I_3 = \frac{E}{R_X + R_3} \text{ and } I_2 = I_4 = \frac{E}{R_2 + R_4}$$

Thus, from Eq. (4.12), we have

$$\frac{E}{R_X + R_3} \times R_X = \frac{E}{R_2 + R_4} \times R_2$$

or, $$\frac{R_X + R_3}{R_X} = \frac{R_2 + R_4}{R_2}$$

or, $$\frac{R_X + R_3}{R_X} - 1 = \frac{R_2 + R_4}{R_2} - 1$$

or, $$\frac{R_X + R_3 - R_X}{R_X} = \frac{R_2 + R_4 - R_2}{R_2}$$

or, $$\frac{R_3}{R_X} = \frac{R_4}{R_2}$$

or, $$\frac{R_X}{R_2} = \frac{R_3}{R_4}$$

or, $$R_X = R_2 \times \frac{R_3}{R_4} \quad (4.13)$$

Thus, measurement of the unknown resistance is made in terms of three known resistances. The arms BD and CD containing the fixed resistances R_3 and R_4 are called the **ratio arms**. The arm AC containing the known variable resistance R_2 is called the **standard arm**. The range of the resistance value that can be measured by the bridge can be increased simply by increasing the ratio R_3/R_4.

Errors in a Wheatstone Bridge

A Wheatstone bridge is a fairly convenient and accurate method for measuring resistance. However, it is not free from errors as listed below:

1. Discrepancies between the true and marked values of resistances of the three known arms can introduce errors in measurement.

2. Inaccuracy of the balance point due to insufficient sensitivity of the galvanometer may result in false null points.

3. Bridge resistances may change due to self-heating (I^2R) resulting in error in measurement calculations.

4. Thermal emfs generated in the bridge circuit or in the galvanometer in the connection points may lead to error in measurement.

5. Errors may creep into measurement due to resistances of leads and contacts. This effect is however, negligible unless the unknown resistance is of very low value.

6. There may also be personal errors in finding the proper null point, taking readings, or during calculations.

Errors due to inaccuracies in values of standard resistors and insufficient sensitivity of galvanometer can be eliminated by using good quality resistors and galvanometer.

Temperature dependent change of resistance due to self-heating can be minimised by performing the measurement within as short time as possible.

Thermal emfs in the bridge arms may cause serious trouble, particularly while measuring low resistances. Thermal emf in galvanometer circuit may be serious in some cases, so care must be taken to minimise those effects for precision measurements. Some sensitive galvanometers employ all-copper systems (i.e., copper coils as well as copper suspensions), so that there is no junction of dissimilar metals to produce thermal emf. The effect of thermal emf can be balanced out in practice by adding a reversing switch in the circuit between the battery and the bridge, then making the bridge balance for each polarity and averaging the two results.

Example 4.4

Four arms of a Wheatstone bridge are as follows: AB = 100 Ω, BC = 10 Ω, CD = 4 Ω, DA = 50 Ω. A galvanometer with internal resistance of 20 Ω is connected between BD, while a battery of 10-V dc is connected between AC. Find the current through the galvanometer. Find the value of the resistance to be put on the arm DA so that the bridge is balanced.

Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out Thevenin equivalent voltage across nodes *BD* and also the Thevenin equivalent resistance between terminals *BD*.

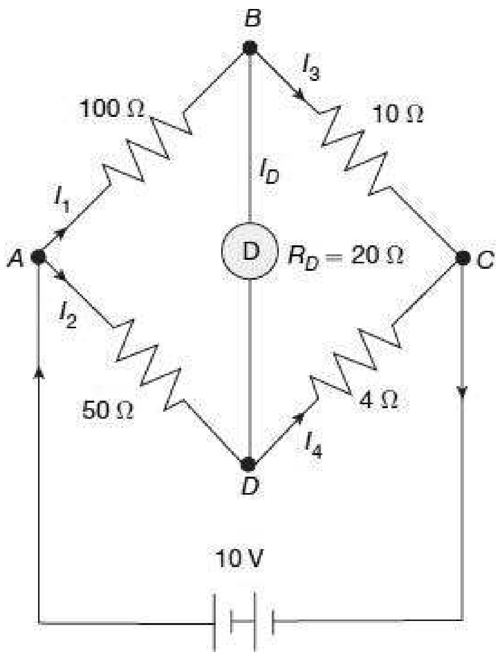
To find out Thevenin's equivalent voltage across *BD*, the galvanometer is open circuited, and the circuit then looks like the figure given below.

At this condition, voltage drop across the arm BC is given by

$$V_{BC} = 10 \times \frac{10}{100+10} = 0.91 \text{ V}$$

Voltage drop across the arm DC is given by:

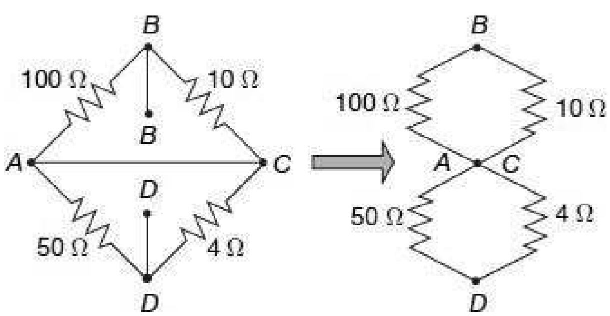
$$V_{DC} = 10 \times \frac{4}{50+4} = 0.74 \text{ V}$$



Hence, voltage difference between the nodes *B* and *D*, or the Thevenin equivalent voltage between nodes *B* and *D* is

$V_{TH} = V_{BD} = V_B - V_D = V_{BC} - V_{DC} = 0.91 - 0.74 = 0.17$ V

To obtain the Thevenin equivalent resistance between nodes *B* and D, the 10 V source need to be shorted, and the circuit looks like the figure given below.



The Thevenin equivalent resistance between the nodes B and D is thus

$$R_{Th} = \frac{100 \times 10}{100+10} + \frac{50 \times 4}{50+4} = 12.79 \ \Omega$$

Hence, current through galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.17}{20+12.79} = 5.18 \text{ mA}$$

In order to balance the bridge, there should be no current through the galvanometer, or in other words, nodes *B* and *D* must be at the same potential.

Balance equation is thus

$$\frac{100}{10} = \frac{R_{DA}}{4} \quad \text{or, } R_{DA} = 40\ \Omega$$

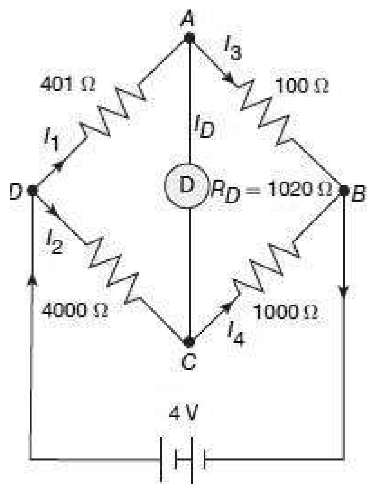
Example 4.5

The four arms of a Wheatstone bridge are as follows: AB = 100 Ω, BC = 1000 Ω, CD = 4000 Ω, DA = 400 Ω. A galvanometer with internal resistance of 100 Ω and sensitivity of 10 mm/μA is connected between AC, while a battery of 4 V dc is connected between BD. Calculate the current through the galvanometer and its deflection if the resistance of arm DA is changed from 400 Ω to 401 Ω.

Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out the Thevenin equivalent voltage across nodes *AC* and also the Thevenin equivalent resistance between terminals AC.

To find out Thevenin equivalent voltage across AC, the galvanometer is open circuited. At this condition, voltage drop across the arm *AB* is given by



$$V_{AB} = 4 \times \frac{100}{100+401} = 0.798\ \text{V}$$

Voltage drop across the arm *CB* is given by

$$V_{CB} = 4 \times \frac{1000}{4000+1000} = 0.8\ \text{V}$$

Hence, voltage difference between the nodes *A* and *C*, or the Thevenin equivalent voltage between nodes *A* and *C* is

$$V_{TH} = V_{AC} = V_A - V_C = V_{AB} - V_{CB} = 0.798 - 0.8 = -0.002\ \text{V}$$

To obtain the Thevenin equivalent resistance between nodes *A* and C, the 10 V source need to be shorted. Under this condition, the Thevenin equivalent resistance between the nodes *A* and *C* is thus

$$R_{Th} = \frac{100 \times 401}{100 + 401} + \frac{1000 \times 4000}{1000 + 4000} = 880.04\ \Omega$$

Hence, current through the galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.002}{100 + 880.04} = 2.04\ \mu\text{A}$$

Deflection of the galvanometer

= Sensitivity × Current = 10 mm/μA = 2.04 μA = 20.4 mm

4.3 MEASUREMENT OF LOW RESISTANCES

The methods used for measurement of medium resistances are not suitable for measurement of low resistances. This is due to the fact that resistances of leads and contacts, though small, are appreciable in comparison to the low resistances under measurement. For example, a contact resistance of 0.001 Ω causes a negligible error when a medium resistance of value say, 100 Ω is being measured, but the same contact resistance would cause an error of 10% while measuring a low resistance of value 0. 01 Ω. Hence special type of construction and techniques need to be used for measurement of low resistances to avoid errors due to leads and contacts. The different methods used for measurement of low range resistances are (i) voltmeter–ammeter method, (iii) Kelvin's double-bridge method, and (iv) potentiometer method.

4.3.1 Voltmeter–Ammeter Method for Measuring Low Resistance

In principle, the voltmeter–ammeter method for measurement of low resistance is very similar to the one used for measurement of medium resistances, as described in Section 4.2.2. This method, due to its simplicity, is very commonly used for measurement of low resistances when accuracy of the order of 1% is sufficient. The resistance elements, to be used for such measurements, however, need to of special construction. Low resistances are constructed with four terminals as shown in Figure 4.10.

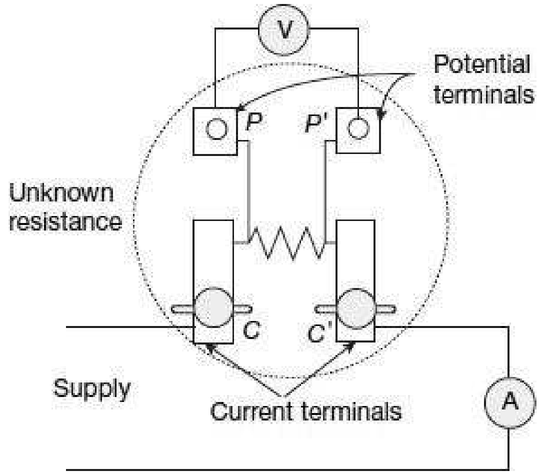


Figure 4.10 *Voltmeter–ammeter method for measuring*

One pair of terminals *CC'*, called the current terminals, is used to lead current to and from the resistor. The voltage drop across the resistance is measured between the other pair of terminals *PP'*, called the potential terminals. The voltage indicated by the voltmeter is thus simply the voltage drop of the resistor across the potential terminals *PP'* and does not include any contact resistance drop that may be present at the current terminals *CC'*.

Contact drop at the potential terminals PP' are, however, less itself, since the currents passing through these contacts are extremely small (even zero under 'null' balance condition) owing to high resistance involved in the potential circuit. In addition to that, since the potential circuit has a high resistance voltmeter in it, any contact resistance drop in the potential terminals PP' will be negligible with respect to the high resistances involved in the potential circuit.

Value of the unknown resistance R_X in this case is given by

$$R_X = \frac{\text{Voltmeter reading}}{\text{Ammeter reading}}$$

Precise measurement in this method requires that the voltmeter resistance to be appreciably high, otherwise the voltmeter current will be an appreciable fraction of the current actually flowing through the ammeter, and a serious error may be introduced in this account.

4.3.2 Kelvin's Double-Bridge Method for Measuring Low Resistance

Kelvin's double-bridge method is one of the best available methods for measurement of low resistances. It is actually a modification of the Wheatstone bridge in which the errors due to contacts and lead resistances can be eliminated. The connections of the bridge are shown in Figure 4.11.

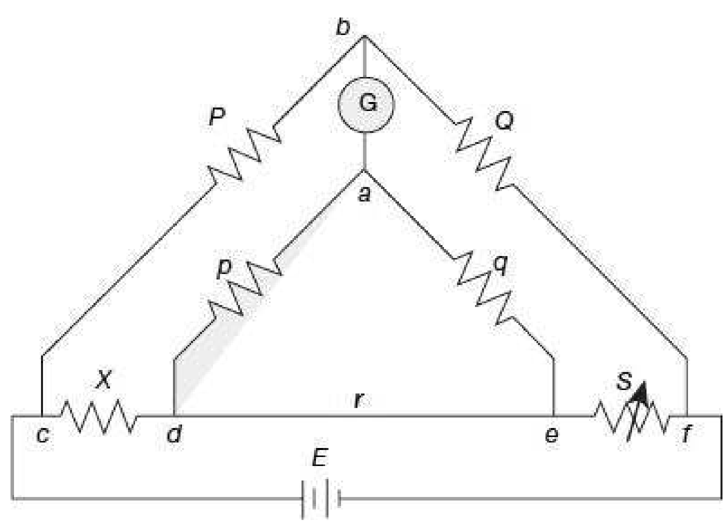


Figure 4.11 *Kelvin's double bridge*

Kelvin's double bridge incorporates the idea of a second set of ratio arms, namely, *p* and q, and hence the name **'double bridge'**.

X is the unknown low resistance to be measured, and *S* is a known value standard low resistance. ‘r’ is a very low resistance connecting lead used connect the unknown resistance *X* to the standard resistance S. All other resistances P, *Q*, *p*, and *q* are of medium range. Balance in the bridge is achieved by adjusting S.

Under balanced condition, potentials at the nodes *a* and *b* must be equal in order that the galvanometer *G* gives “null” deflection. Since at balance, no current flows through the galvanometer, it can be considered to be open circuited and the circuit can be represented as shown in Figure 4.12.

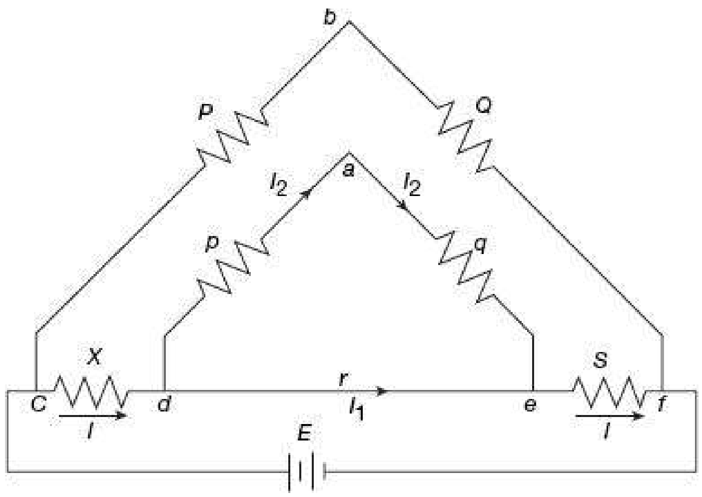


Figure 4.12 *Kelvin’s double-bridge under balanced condition*

Since under balanced condition, potentials at the nodes *a* and *b* are equal, the we must have

$$V_{cb} = V_{cda}$$

Now, $$V_{cb} = E \times \frac{P}{P+Q} \tag{4.14}$$

and $$V_{cda} = V_{cd} + V_{da} = X \times I + p \times I_2$$

where, $$I_2 = I \times \frac{r}{r+p+q}$$

$\therefore$ $$V_{cda} = I \times X + I \times \frac{pr}{r+p+q} = I\left(X + \frac{pr}{r+p+q}\right) \tag{4.15}$$

Supply voltage $$E = V_{cd} + V_{de} + V_{ef} = I \times X + I \times \frac{(p+q)}{p+q+r} \times r + I \times S$$

or, $$E = I\left(X + S + \frac{(p+q)}{p+q+r} \times r\right) \tag{4.16}$$

From (4.14) and (4.16), we have

$$V_{cb} = \frac{P}{P+Q} \times I\left(X + S + \frac{(p+q)}{p+q+r} \times r\right)a \tag{4.17}$$

$\therefore$ the balance equation $V_{cb} = V_{cda}$ can now be re-written as

$$\frac{P}{P+Q}\times I\left(X+S+\frac{(p+q)}{p+q+r}\times r\right)=I\left(X+\frac{pr}{r+p+q}\right)a$$

or, $$\left(X+S+\frac{(p+q)}{p+q+r}\times r\right)=\left(1+\frac{Q}{P}\right)\times\left(X+\frac{pr}{r+p+q}\right)$$

or, $$X+S+\frac{(p+q)}{p+q+r}\times r=X+\frac{pr}{r+p+q}+\frac{Q}{P}\times X+\frac{Q}{P}\times\frac{pr}{r+p+q}a$$

or, $$S+\frac{(p+q)}{p+q+r}\times r=\frac{pr}{r+p+q}+\frac{Q}{P}\times X+\frac{Q}{P}\times\frac{pr}{r+p+q}a$$

or, $$\frac{Q}{P}\times X= S+\frac{(p+q)}{p+q+r}\times r-\frac{pr}{r+p+q}-\frac{Q}{P}\times\frac{pr}{r+p+q}$$

or, $$\frac{Q}{P}\times X= S+\frac{pr}{p+q+r}+\frac{qr}{p+q+r}-\frac{pr}{r+p+q}-\frac{Q}{P}\times\frac{pr}{r+p+q}$$

or, $$\frac{Q}{P}\times X=S+\frac{qr}{p+q+r}-\frac{Q}{P}\times\frac{pr}{r+p+q}$$

or, $$\frac{Q}{P}\times X=S+\frac{qr}{p+q+r}\left(1-\frac{Q}{P}\times\frac{p}{q}\right)a$$

or, $$X= \frac{P}{Q}\times S+\frac{qr}{p+q+r}\left(\frac{P}{Q}-\frac{p}{q}\right) \tag{4.18}$$

The second quantity of the Eq. (4.18), $\frac{qr}{p+q+r}\left(\frac{P}{Q}-\frac{p}{q}\right)$, can be made very small by making the ratio P/Q as close as possible to p/q. In that case, there is no effect of the connecting lead resistance ‘r’ on the expression for the unknown resistance. Thus, the expression for the unknown resistance *X* can now be simply written as

or, $$X=\frac{P}{Q}\times S \tag{4.19}$$

However, in practice, it is never possible to make the ratio *p/q* exactly equal to P/Q. Thus, there is always a small error.

$\Delta=\left(\frac{P}{Q}-\frac{p}{q}\right)$and hence, the resistance value becomes

or, $$X= \frac{P}{Q}\times S+\frac{q}{p+q+r}\times\Delta\times r \tag{4.20}$$

It is thus always better to keep the value of ‘r’ as small as possible, so that the product Δ × *r* is extremely small and therefore the error part can be neglected, and we can assume, under balanced condition,

$$\boxed{X= \frac{P}{Q}\times S}$$

In order to take into account the effects of thermoelectric emf, two measurements are normally done with the battery connections reversed. The final value of resistance is taken as the average of these two readings.

A 4-terminal resistor was measured with the help of a Kelvin’s double bridge having the following components: Standard resistor = 98.02 nW, inner ratio arms = 98.022 Ω

Example 4.6 *and 202 W, outer ratio arms = 98.025 Ω and 201.96 W, resistance of the link connecting the standard resistance and the unknown resistance = 600 nW. Calculate the value of the unknown resistance.*

Solution From Eq. (4.18), value of the unknown resistance is

$$X = \frac{P}{Q} \times S + \frac{qr}{p+q+r}\left(\frac{P}{Q} - \frac{p}{q}\right)$$

or, $$X = \frac{98.025}{201.96} \times 98.02 \times 10^{-6} + \frac{202 \times 600 \times 10^{-6}}{98.022 + 202 + 600 \times 10^{-6}}\left(\frac{98.025}{201.96} - \frac{98.022}{202}\right)$$

or, $$X = 47.62\ \mu\Omega$$

4.3.3 Potentiometer Method for Measuring Low Resistance

The circuit for measurement of low value resistance with a potentiometer is shown in Figure 4.13.

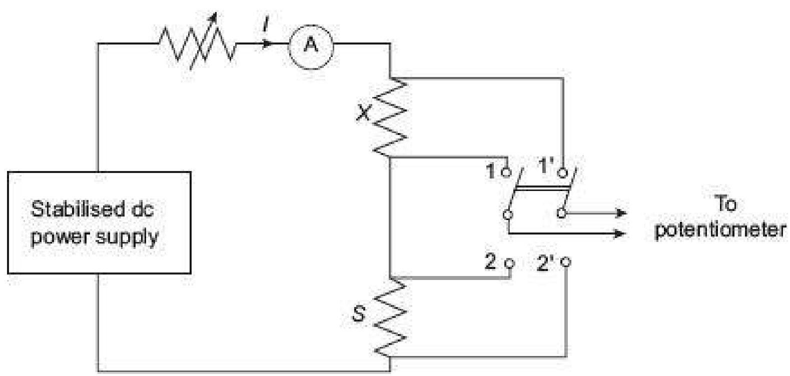


Figure 4.13 *Measurement of low resistance using potentiometer*

The unknown resistance *X* is connected in series with a standard known resistance S. Current through the ammeter in the circuit is controlled by a rheostat. A two-pole double throw switch is used. When the switch is in the position 1-1', the unknown resistance *X* gets connected to the potentiometer, whereas when the switch is at position 2-2', the standard resistance *S* gets connected to the potentiometer.

Potentiometers are believed to give reasonably accurate values of potentials.

Thus, with the switch in position 1-1', the potentiometer reading is the voltage drop across the unknown resistance, given by

$$V_X = I \times X \tag{4.21}$$

Without changing any of the circuit parameters, now if the switch is thrown to position 2-2', potentiometer now reads the voltage drop across the standard resistance, given by

$$V_S = I \times S \tag{4.22}$$

From Eqs (4.21) and (4.22), we get

or, $$X = \frac{V_R}{V_S} \times S \tag{4.23}$$

Knowledge of accurate value of the standard resistance *S* can thus give reasonably accurate values of the unknown resistance *X*.

Accuracy of this method however, depends on the assumption that the value of current remains absolutely constant during the two sets of measurements. Therefore, an extremely stabilised dc power supply is required in this method.

Value of the standard resistor S should be of the same order as the unknown resistance X. The ammeter inserted in the circuit has no other function rather than simply indicating whether there is any current is flowing in the circuit is not. Exact value of the current is not required for final calculations. It is however, desired that the current flowing through the circuit be so adjusted that the voltage drop across each resistor is of the order of 1 V to be suitable for accurate measurement by commercially available potentiometers.

4.4 MEASUREMENT OF HIGH RESISTANCES

High resistances of the order of several hundreds and thousands of meghoms (MW) are often encountered in electrical equipments in the form of insulation resistance of machines and cables, leakage resistance of capacitors, volume and surface resistivity of different insulation materials and structures.

4.4.1 Difficulties in Measurement of High Resistance

1. Since the resistance under measurement has very high value, very small currents are encountered in the measurement circuit. Adequate precautions and care need to be taken to measure such low value currents.

2. Surface leakage is the main difficulty encountered while measurement of high resistances. The resistivity of the resistance under measurement may be high enough to impede flow of current through it, but due to moisture, dust, etc., the surface of the resistor may provide a lower resistance path for the current to pass between the two measuring electrodes. In other words, there may thus be a leakage through the surface. Leakage paths not only pollute the test results, but also are generally variable from day to day, depending on temperature and humidity conditions.
   The effect of leakage paths on measurements can be eliminated by the use of guard circuits as described by Figure 4.14.
   Figure 4.14(a) shows a high resistance R_X being mounted on a piece of insulation block. A battery along with a voltmeter and a micro-ammeter are used to measure the resistance by voltmeter–ammeter method. The resistance R_X under measurement is fitted on the insulating block at the two binding posts A and B. I_X is the actual current flowing through the high resistance and I_L is the surface leakage current flowing over the body of the insulating block. The micro-ammeter, in this case, thus reads the actual current through the resistor, and also the leakage current ($I = I_X + I_L$.). Measured value of the resistance, thus computed from the ratio E/I, will not be the true value of R_X, but will involve some error. To avoid this error, a guard arrangement has been added in Figure 4.14(b). The guard arrangement, at one end is connected to the battery side of the micro-ammeter, and the other end is wrapped over the insulating body and surrounds the resistance terminal A. The surface leakage current now, flows through this guard and bypasses the micro-ammeter. The

micro-ammeter thus reads the true of current I_X through the resistance R_X. This arrangement thus allows correct determination of the resistance value from the readings of voltmeter and micro-ammeter.

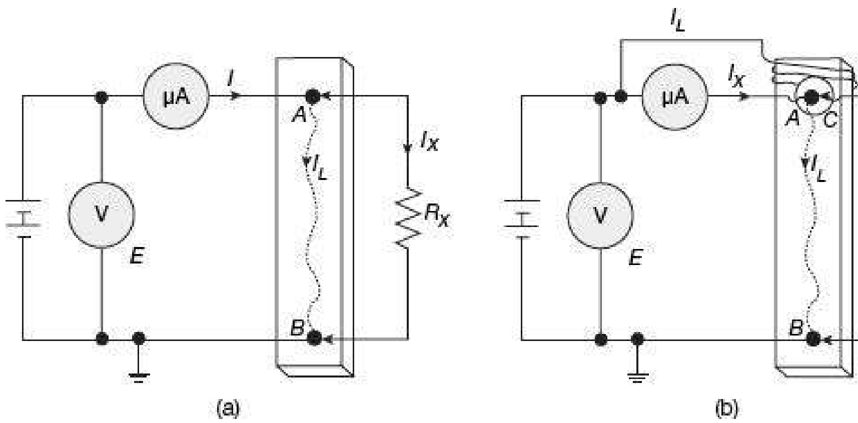


Figure 4.14 *Guard circuit for measurement of high resistance: (a) Circuit without guard (b) Circuit with guard*

3. Due to electrostatic effects, stray charges may be induced in the measuring circuit. Flow of these stray charges can constitute a current that can be comparable in magnitude with the low value current under measurement in high resistance circuits. This may thus, cause errors in measurement. External alternating electromagnetic fields can also affect the measurement considerably. Therefore, the measuring circuit needs to be carefully screened to protect it against such external electrostatic or electromagnetic effects.

4. While measuring insulation resistance, the test object often has considerable amount of capacitance as well. On switching on the dc power supply, a large charging current may flow initially through the circuit, which gradually decays down. This initial transient current may introduce errors in measurement unless considerable time is provided between application of the voltage supply and reading the measurement, so that the charging current gets sufficient time to die down.

5. High resistance measurement results are also affected by changes in temperature, humidity and applied voltage inaccuracies.

6. Reasonably high voltages are used for measurement of high resistances in order to raise the current to substantial values in order to be measured, which are otherwise extremely low. So, the associated sensitive galvanometers and micro-ammeters need to be adequately protected against such high voltages.

Taking these factors into account, the most well-known methods of high resistance measurements are (i) direct deflection method, (ii) loss of charge method, and (iii) megohmmeter or meggar.

4.4.2 Direct Deflection Method for High Resistance Measurement

The direct deflection method for measuring high resistances is based on the circuit described in Figure 4.14, which in effect is the voltmeter–ammeter method. For measurement of high resistances, a sensitive galvanometer is used instead of a micro-ammeter as shown in Figure 4.14. A schematic diagram for describing the direct deflection method for measurement of insulation resistance of a metal sheathed cable is given in Figure 4.15.

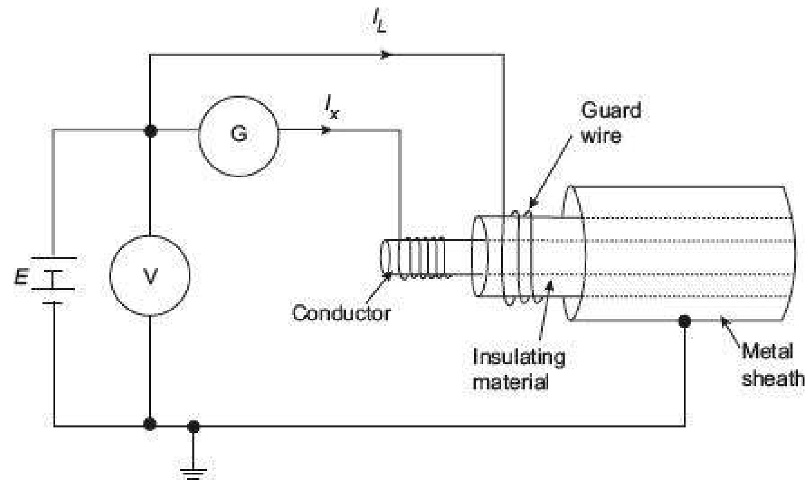


Figure 4.15 *Measurement of cable insulation resistance*

The test specimen, cable in this case, is connected across a high voltage stable dc source; one end of the source being connected to the inner conductor of the cable, and the other end, to the outer metal sheath of the cable. The galvanometer G, connected in series as shown in Figure 4.15, is intended to measure the current I_X flowing through the volume of the insulation between the central conductor

and the outer metal sheath. Any leakage current I_L flowing over the surface of the insulating material is bypassed through a guard wire wound on the insulation, and therefore does not flow through the galvanometer.

A more detailed scheme for measurement of insulation resistance of a specimen sheet of solid insulation is shown in Figure 4.16.

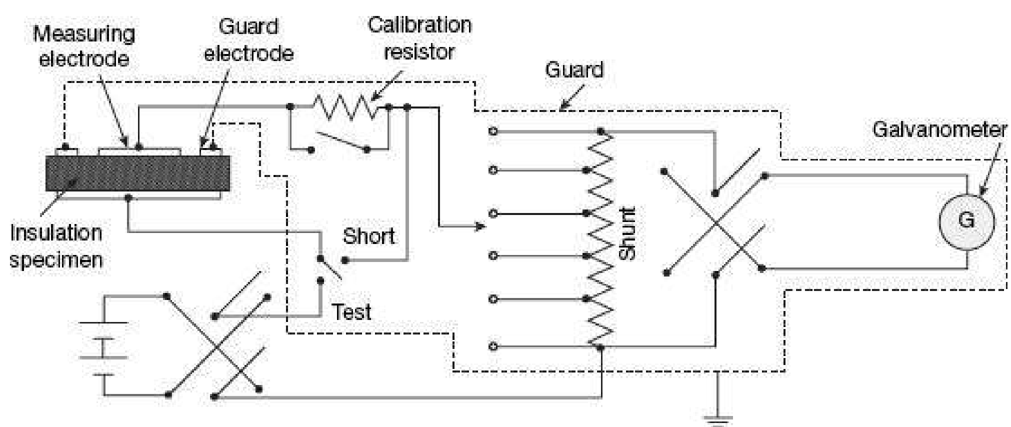


Figure 4.16 *Measurement of high resistance by direct deflection method*

A metal disk covering almost the entire surface is used as electrode on one side of the insulation sheet under measurement. On the other side of the insulating sheet, the second electrode is made of a smaller size disk. A guard ring is placed around the second electrode with a small spacing in between them. This guarding arrangement bypasses any surface leakage current on the insulator or any other parts of the circuit from entering the actual measuring circuit. The galvanometer thus reads specifically the volume resistance of the insulation specimen, independent of any surface leakage.

A calibrated Ayrton shunt is usually included along with the galvanometer to provide various scale ranges and also to protect it.

The galvanometer scale is graduated directly in terms of resistance. After one set of measurement is over, the galvanometer is calibrated with the help of a high value (≈ 1 MΩ) calibrating resistor and the shunts.

In case the insulation under measurement has high inherent capacitance values (like in a cable), there will be an initial inrush of high capacitive charging current when the dc source is first switched on. This charging current will, however, decay down to a steady dc value with time. To protect the galvanometer from such initial rush of high current, the Ayrton shunt connected across the galvanometer should be placed at the highest resistance position (lower most point in Figure 4.16). Thus, initially the galvanometer is bypassed from the high charging current.

After the test is complete, it is required that the test specimen should be discharged, especially if it is of capacitive in nature. The 'test-short' switch is placed in the 'short' position so that any charge remaining in the insulation specimen is discharged through the short circuited path.

The change-over switch across the battery enables tests at different polarities. The switch across the galvanometer enables reversal of the galvanometer connections.

A special technique, Price's guard-wire method is employed for measurement of insulation resistance of cables which do not have metal sheath outside. The schematic diagram of such a measurement system is provided in Figure 4.17.

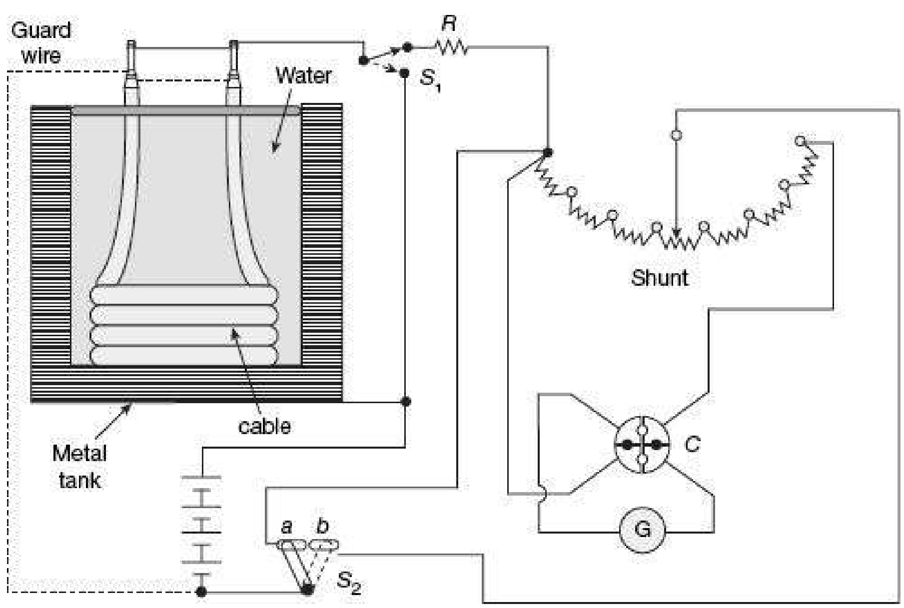


Figure 4.17 *Measurement of high resistance by Price's guard-wire method*

The unsheathed cable, except at the two ends where connections are made, is immersed in water in a tank. For testing of the cable insulation, the cable core conductor acts as one electrode and in the absence of the metal sheath outside, the water and the tank act as the other electrode for measurement. The cable is immersed in slightly saline water for about a day and at nearly constant ambient temperature.

The two ends of the cables are trimmed as shown in Figure 4.17, thus exposing the core conductor as well as some portion of the insulation. The core conductors are connected together to form one electrode of the measuring system. A guard circuit is formed by twisting a bare wire around the exposed portion of the insulation at the two stripped ends of the cable. This guard wire is connected to the negative terminal of the supply battery. The positive terminal of the battery is connected to the metal tank. This enables any surface leakage current to bypass the galvanometer and pass directly to the battery. Thus, the galvanometer will read only true value of the current flowing through volume of the insulation, and not the additional surface leakage current.

The D'Arsonval galvanometer to be used is normally of very high resistance and very sensitive to record the normally extremely low insulation currents. An Ayrton universal shunt is usually included along with the galvanometer to provide various scale ranges and also to protect it. The galvanometer scale is graduated directly in terms of resistance. After one set of measurement is over, the galvanometer is calibrated with the help of the high value ($\approx$ 1 MΩ) calibrating resistor R and the shunt. The resistance R and the shunt also serve the purpose of protecting the galvanometer from accidental short circuit current surges. The 4-terminal commutator C, as shown in Figure 4.17 is used for reversal of galvanometer connections.

Since the cable will invariably have high capacitance value, there will be an initial inrush of high capacitive charging current when the dc source is first switched on. This charging current will, however, decay down to a steady dc value with time. To protect the galvanometer from such initial rush of high current, the switch S_2 is placed on position a so that initially the galvanometer is bypassed from the high charging current. Once the capacitor charging period is over and the current settles down, the switch S_2 is pushed over to position b to bring the galvanometer back in the measurement circuit. The contacts a and b are sufficiently close enough to prevent the circuit from breaking while the switch S_2 is moved over.

After the test is complete, it is required that the test specimen should be discharged. The switch S_1 is used for this purpose, so that any charge remaining in the insulation specimen is discharged through itself.

4.4.3 Loss of Charge Method for High Resistance Measurement

In this method, the resistance to be measured is connected directly across a dc voltage source in parallel with a capacitor. The capacitor is charged up to a certain voltage and then discharged through the resistance to be measured. The terminal voltage across the resistance-capacitance parallel combination is recorded for a pre-defined period of time with a help of a high-resistance voltmeter (electrostatic voltmeter or digital electrometers). Value of the unknown resistance is calculated from the discharge time constant of the circuit. Operation of the loss of charge method can be described by the schematic circuit diagram of Figure 4.18.

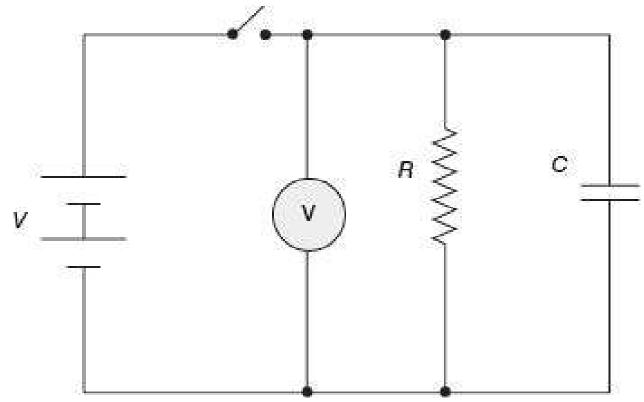


Figure 4.18 *Loss of charge method for measurement of high resistance*

In Figure 4.18, the unknown resistance *R* to be measured is connected across the capacitor *C* and their parallel combination is connected to the dc voltage source.

Let the capacitor is initially charged up to a voltage of *V* while the switch is kept ON. Once the switch is turned OFF, the capacitor starts to discharge through the resistance *R*.

During the discharge process, the voltage *v* across the capacitor at any instant of time *t* is given by

Thus, the insulation resistance can be calculated as

$$R = \frac{t}{C \times \log_e\left(\frac{V}{v}\right)} \tag{4.24}$$

With known value of *C* and recorded values of t, *V* and v, the unknown resistance *R* can be estimated using (4.24).

The pattern of variation of voltage *v* with time is shown in Figure 4.19.

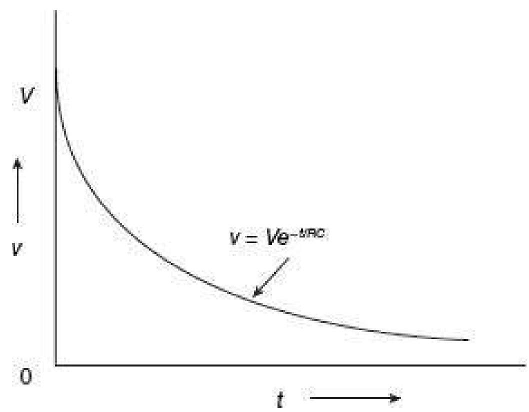


Figure 4.19 *Capacitor discharge pattern*

Great care must be taken to record the voltages *V* and *v* and also the time *t* very precisely, otherwise large errors may creep in to the calculation results.

This method, though simple in principle, require careful choice of the capacitor. The capacitor *C* itself must have sufficiently high value of its own leakage resistance, at least in the same range as the unknown resistance under measurement. The resistance of the voltmeter also needs to be very high to have more accurate results.

4.4.4 Megohmmeter, or Meggar, for High Resistance

Measurement

One of the most popular portable type insulation resistance measuring instruments is the megohmmeter or in short, meggar. The meggar is used very commonly for measurement of insulation resistance of electrical machines, insulators, bushings, etc. Internal diagram of a meggar is shown in Figure 4.20.

The traditional analog deflecting-type meggar is essentially a permanent magnet crossed-coil shunt type ohmmeter.

The instrument has a small permanent magnet dc generator developing 500 V dc (some other models also have 100 V, 250 V, 1000 or 2500 V generators). The generator is hand driven, through gear arrangements, and through a centrifugally controlled clutch switch which slips at a predefined speed so that a constant voltage can be developed. Some meggars also have rectified ac as power supply.

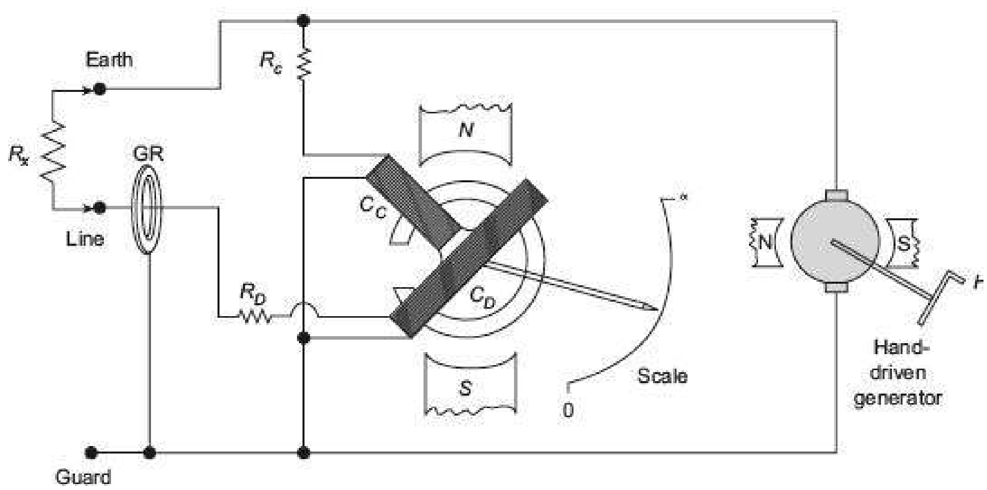


Figure 4.20 *Meggar for high resistance measurement*

The moving system in such instruments consists of two coils, the control coil C_C and the deflecting coil C_D. Both the coils are mounted rigidly on a shaft that carries the pointer as well. The two coils move in the air gap of a permanent magnet. The two coils are arranged with such numbers of turns, radii of action, and connected across the generator with such polarities that, for external magnetic fields of uniform intensity, the torque produced by the individual coils are in opposition thus giving an astatic combination. The deflecting coil is connected in series with the unknown resistance R_X under measurement, a fixed resistor R_D and then the generator. The current coil or the compensating coil, along with the fixed resistance R_C is connected directly across the generator. For any value of the unknown, the coils and the pointer take up a final steady position such that the torques of the two coils are equal and balanced against each other. For example, when the resistance R_X under measurement is removed, i.e., the test terminals are open-circuited, no current flows through the deflecting coil C_D, but maximum current will flow through the control coil C_C. The control coil C_C thus sets itself perpendicular to the magnetic axis with the pointer indicating '∞ Ω' as marked in the scale shown in Figure 4.20. As the value of R_X is brought down from open circuit condition, more and more current flows through the deflecting coil *CD*, and the pointer moves away from the '∞ Ω' mark clockwise (according to Figure 4.20) on the scale, and ultimately reaches the '0 Ω' mark when the two test terminals are short circuited.

The surface leakage problem is taken care of by the guard-wire arrangement. The guard ring *(GR* in Figure 4.20) and the guard wire diverts the surface leakage current from reaching the main moving system and interfering with its performance.

Photographs of some commercially available meggars are shown in Figure 4.21.

(a)

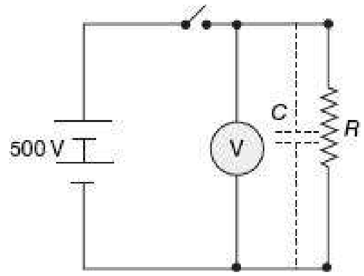
(b)

Figure 4.21 Commercial meggars: (a) Analog type (Courtesy, WACO) (b) Digital type (Courtesy, Yokogawa)

Example 4.7

A cable is tested for insulation resistance by loss of charge method. An electrostatic voltmeter is connected between the cable conductor and earth. This combination is found to form a capacitance of 800 pF between the conductor and earth. It is observed that after charging the cable with 500 V or sufficiently long time, when the voltage supply is withdrawn, the voltage drops down to 160 V in 1 minute. Calculate the insulation resistance of the cable.

Solution The arrangement can be schematically shown by the following figure.



While the cable is charged with 500 V for a long period of time, it is expected that the capacitance of the system is charged up to a steady voltage of 500 V before it is discharged.

Thus, the capacitor discharges from 500 V to 160 V in 1 minute.

During the discharge process, the voltage *v* across the capacitor at any instant of time *t* is given by

$$v = Ve^{-t/RC}$$

where *V* is the initial voltage in the capacitor *C*, connected across the unknown resistance *R*.

Thus, the insulation resistance is

$$R = \frac{t}{C \times \log_e\left(\frac{V}{v}\right)} = \frac{60}{800 \times 10^{-12} \times \log_e\left(\frac{500}{160}\right)} = 65 \times 10^9\ \Omega = 65{,}000\ \text{M}\Omega$$

4.5 LOCALISATION OF CABLE FAULTS

Underground cables during their operation can experience various fault conditions. Whereas routine standard tests are there to identify and locate faults in high-voltage cables, special procedures, as will be described in this section are required for localisation of cable faults in low distribution voltage level cables. Determination of exact location of fault sections in underground distribution cables is extremely important from the point of view of quick restoration of service without loss of time for repair.

The faults that are most likely to occur are *ground faults* where cable insulation may break down causing a current to flow from the core of the cable to the outer metal sheath or to the earth; or there may be *short-circuit faults* where a insulation failure between two cables, or between two cores of a multi-core cable results in flow of current between them.

Loop tests are popularly used in localisation of the aforesaid types of faults in low voltage cables. These tests can be carried out to localise a ground fault or a short-circuit fault, provided that an unfaulty cable runs along with the faulty cable. Such tests have the advantage that fault resistance does not affect the measurement sensitivity, that the fault resistance is not too high. Loop tests work on the simple principles of a Wheatstone bridge for measurement of unknown resistances.

4.5.1 Murray Loop Test

Connections for this test are shown in Figure 4.22. Figure 4.22(a) shows the connection diagram for localisation of ground faults and Figure 4.22(b) relates to localisation of short circuit faults. The general configuration of a Wheatstone bridge is given in Figure 4.23 for ready reference.

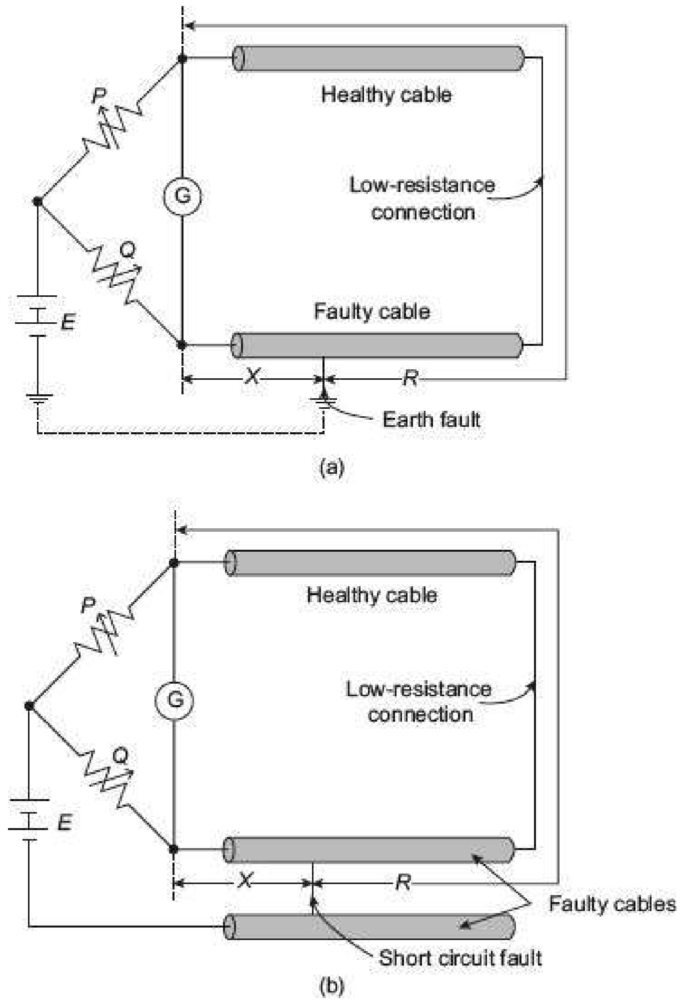


Figure 4.22 *Murray loop test for (a) earth fault, and (b) short-circuit fault localosation in cables*

The loop circuits formed by the cable conductors form a Wheatstone bridge circuit with the two externally controllable resistors *P* and *Q* and the cable resistance *X* and *R* as shown in Figure 4.22. The galvanometer *G* is used for balance detection. The ridge is balanced by adjustment of *P* and *Q* till the galvanometer indicates zero deflection.

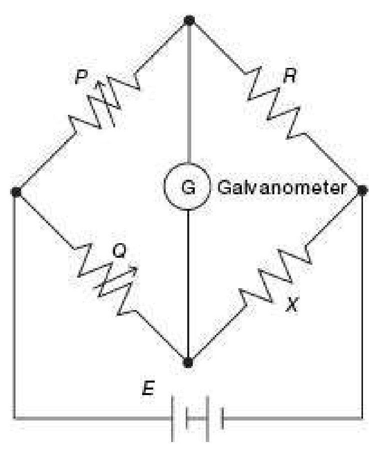


Figure 4.23 *Wheatstone bridge configuration relating to Figure 4.22*

Under balanced condition,

$$\frac{X}{R} = \frac{Q}{P} a$$

or,
$$\frac{X}{R+X} = \frac{Q}{Q+P}$$

$$X = \frac{Q}{Q+P} \times (R+X) \quad (4.25)$$

Here, (R + *X)* is the total loop resistance formed by the good cable and the faulty cable. When the cables have the same cross section and same resistivity, their resistances are proportional to their lengths.

If L_X represents the distance of the fault point from the test end, and *L* is the total length of each cable under test, then we can write

1. The resistance *X* is proportional to the length L_X
2. The resistance (R + X) is proportional to the total length 2L

Equation (4.25) can now be expressed in terms of the lengths as

Thus, position of the fault can easily be located when the total length of the cables are known.

$$L_X = \frac{Q}{P+X} \times 2L \quad (4.26)$$

4.5.2 Varley Loop Test

In this test, the total loop resistance involving the cables is determined experimentally to estimate the fault location, rather than relying upon the information of length of cables and their resistances per unit length. Connections diagrams for Varley loop test to detect ground fault location and short-circuit fault location in low voltage cables is shown in Figure 4.24 (a) and Figure 4.24 (b) respectively.

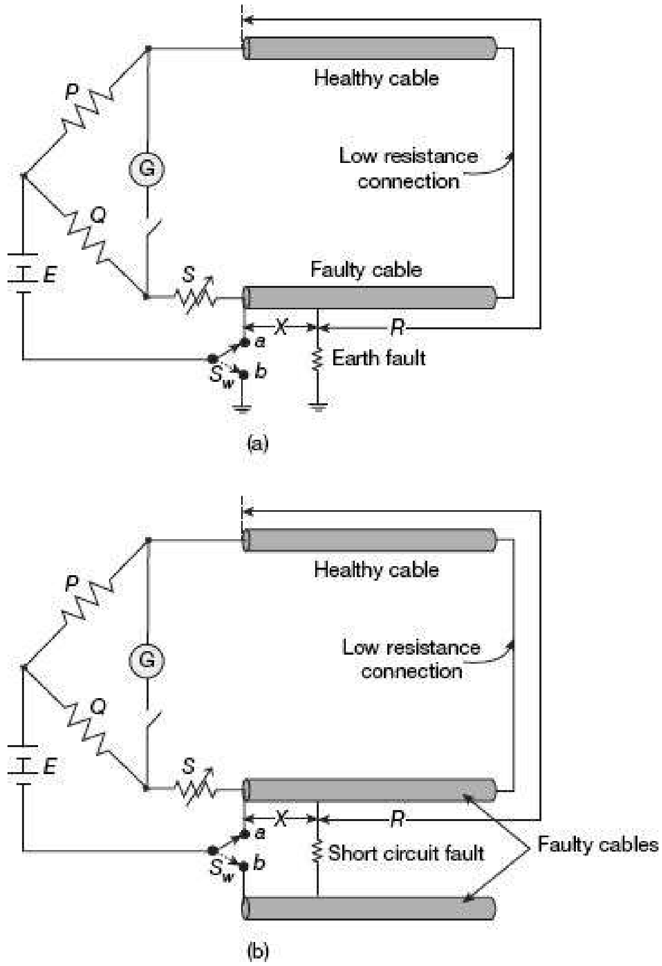


Figure 4.24 *Varley loop test for (a) earth fault, and (b) short-circuit fault localisation in cables*

The single pole double throw switch *Sw* is first connected to terminal 'a' and the resistance *S* is varied to obtain bridge balance.

Let, at this condition, the value of resistance $S = S_1$ when the bridge is balanced. Thus, from Wheatstone bridge principles, at balance condition with the switch at position *a*, we can write

$$\frac{(R+X)}{S_1} = \frac{P}{Q}$$

or,
$$(R+X) = \frac{P}{Q} \times S_1 a \tag{4.27}$$

The total loop resistance (R + *X)* can thus be experimentally determined using (4.27) by reading the values of *P*, *Q* and S_1 under bridge balanced condition with the switch at position *a*.

Now, the switch Sw is changed over to terminal b and the bridge is balanced again by varying S. Let, at this condition, the value of resistance $S = S_2$ when the bridge is balanced. Thus, once again, from Wheatstone bridge principles, at balance condition with the switch at position b, we can write

$$\frac{R}{X+S_2}=\frac{P}{Q}$$

or,

$$\frac{R+X+S_2}{X+S_2}=\frac{P+Q}{Q}$$

$$X=\frac{(R+X)Q-S_2P}{P+Q} \tag{4.28}$$

Thus, *X* can be calculated from (4.28) from known values of P, Q, and S_2 and the value of total loop resistance (R + *X)* obtained from (4.27).

If L_X represents the distance of the fault point from the test end, and *L* is the total length of each cable under test, then we can write:

1. the resistance *X* is proportional to the length L_X
2. the resistance (R + *X)* is proportional to the total length 2L Then we can write

$$\frac{X}{R+X}=\frac{L_X}{2L}$$

$$L_X=\frac{X}{R+X}\times 2L \tag{4.29}$$

Thus, the position of the fault can easily be located when the total length of the cables are known.

Both Murray and Varley loop tests are valid only when the cable cross sections are uniform throughout the loop and also between the healthy and faulty cables. Correction factors need also to be included to take care of temperature variation effects. Too many cable joints within length of the cables may also introduce errors in measurement.

Example 4.8

In a test for fault to earth by Murray loop test, the faulty cable has a length of 5.2 km.
The faulty cable is looped with a sound (healthy) cable of the same length and cross section. Resistances of the ratio arm of the measuring bridge circuit are 100 Ω and 41.2 Ω at balance. Calculate the distance of the fault point from the testing terminal.

Solution If *X* is the resistance of the cable from test end to the fault point, *R* is the resistance of cable loop from the fault point back over to the other end of the test point, and *P* and *Q* are the resistance of the ratio arm, then we can write

$$X=\frac{Q}{Q+P}\times(R+X)$$

If L_X be the distance of the fault point from the test end and *L* be the length of each cable. If resistivity of the cable core material *r* is and each of them has cross-sectional area A, then we can write

$$\therefore \quad X = \rho\frac{L_X}{A} \text{ and } (R+X) = \rho\frac{2L}{A}$$

$$\therefore \quad \rho\frac{L_X}{A} = \frac{Q}{P+Q}\times\rho\frac{2L}{A}$$

$$\text{or,} \quad L_X = \frac{Q}{P+Q}\times 2L$$

$$\text{or,} \quad L_X = \frac{41.2}{41.2+100}\times 2\times 5.2$$

$$\text{or,} \quad L_X = 3.0.3 \text{ km}$$

Thus, the fault has occurred at a distance of 3.03 km from the test end.

Example 4.9

Varley Loop test is being used to locate short circuit fault. The faulty and sound cables are identical with resistances of 0.5 Ω per km. The ratio arms are set at 15 Ω and 40 Ω. Values of the variable resistance connected with the faulty cable are 20 Ω and 10 Ω at the two positions of the selector switch. Determine the length of each cable and fault distance from test end.

Solution Let S_1 be the resistance of the variable resistance at loop resistance measuring position of the selector switch and S_2 be the resistance of the variable resistor at fault location measuring position of the selector switch.

Hence, at the first position of the switch we can write:

$$\frac{(R+X)}{S_1} = \frac{P}{Q}$$

where X is the resistance of the cable from test end to the fault point, R is the resistance of cable loop from the fault point back over to the other end of the test point, and P and Q are the resistance of the ratio arm

Thus, total resistance of the loop is given as

$$(R+X) = \frac{P}{Q}\times S_1 = \frac{15}{40}\times 20 = 7.5\ \Omega$$

Thus, resistance of each cable = 7.5/2 = 3.75 W
length of each cable = 3.75/0.5 = 7.5 km
At the second position of the switch, we have

$$\frac{R}{X+S_2} = \frac{P}{Q}$$

$$\text{or,} \quad \frac{R+X+S_2}{X+S_2} = \frac{P+Q}{Q}$$

$$\text{or,} \quad X = \frac{(R+X)Q - S_2P}{P+Q} = \frac{7.5\times 40 - 10\times 15}{15+40} = 2.73\ \Omega$$

. distance of the fault point from test end = 2.73/0.5 = 5.45 km

EXERCISE

Objective-type Questions

1. In a series-type ohmmeter
   (a) zero marking is on the left-hand side
   (b) zero marking is at the centre
   (c) zero marking is on the right-hand side
   (d) zero marking may be either on left or right-hand side
2. In series type ohmmeters, zero adjustment should be done by
   (a) changing the shunt resistance across the meter movement
   (b) changing the series resistance
   (c) changing the series and the shunt resistance
   (d) changing the battery voltage
3. Screw adjustments are preferred over shunt resistance adjustments for zero calibration in ohmmeters because
   (a) the former method is less costly
   (b) the former method does not disturb the scale calibration
   (c) the former method does not disturb the meter magnetic field
   (d) all of the above
4. The shape of scale in an analog series-type ohmmeter is
   (a) linearly spaced (b) cramped near the start
   (c) cramped near the end (d) directly proportional to the resistance
5. Shunt-type ohmmeters have on their scale
   (a) zero ohm marking on the right corresponding to zero current
   (b) zero ohm marking on the right corresponding to full scale current
   (c) infinite ohm marking on the right corresponding to zero current
   (d) infinite ohm marking on the right corresponding to full scale current
6. Shunt-type ohmmeters have a switch along with the battery to
   (a) disconnect the battery when not in use
   (b) prevent meter from getting damaged when measuring very low resistances
   (c) compensate for thermo-emf effects by reversing battery polarity
   (d) all of the above
7. The shape of scale in an analog shunt-type ohmmeter is
   (a) linearly spaced at lower scales (b) cramped near the start
   (c) linearly spaced at higher scales (d) uniform all throughout the scale
8. High resistances using the voltmeter–ammeter method should be measured with
   (a) voltmeter connected to the source side
   (b) ammeter connected to the source side
   (c) any of the two connections
   (d) readings are to be taken by interchanging ammeter and voltmeter positions
9. Low resistances using the voltmeter–ammeter method should be measured with
   (a) voltmeter connected to the source side
   (b) ammeter connected to the source side
   (c) any of the two connections

(d) readings are to be taken by interchanging ammeter and voltmeter positions

10. Accuracy of the substitution method for measurement of unknown resistance depends on

    (a) accuracy of the ammeter

    (b) accuracy of the standard resistance to which the unknown is compared

    (c) accuracy in taking the readings

    (d) all of the above

11. The null detector used in a Wheatstone bridge is basically a

    (a) sensitive voltmeter (b) sensitive ammeter

    (c) may be any of the above (d) none of (a) or (b)

12. Wheatstone bridge is not preferred for precision measurements because of errors due to

    (a) resistance of connecting leads (b) resistance of contacts

    (c) thermo-electric emf (d) all of the above

13. Error due to thermo-electric emf effects in a Wheatstone bridge can be eliminated by

    (a) taking the readings as quickly as possible

    (b) by avoiding junctions with dissimilar metals

    (c) by using a reversing switch to change battery polarity

    (d) all of the above

14. Low resistances are measured with four terminals to

    (a) eliminate effects of leads

    (b) enable the resistance value to be independent of the nature of contact at the current terminals

    (c) to facilitate connections to current and potential coils of the meters

    (d) all of the above

15. Kelvin's double bridge is called 'double' because

    (a) it has double the accuracy of a Wheatstone bridge

    (b) its maximum scale range is double that of a Wheatstone bridge

    (c) it can measure two unknown resistances simultaneously, i.e., double the capacity of a Wheatstone bridge

    (d) it has two additional ratio arms, i.e., double the number of ratio arms as compared to a Wheatstone bridge

16. Two sets of readings are taken in a Kelvin's double bridge with the battery polarity reversed in order to

    (a) eliminate the error due to contact resistance

    (b) eliminate the error due to thermo-electric effect

    (c) eliminate the error due to change in battery voltage

    (d) all of the above

17. Potentiometers, when used for measurement of unknown resistances, give more accurate results as compared to the voltmeter–ammeter method because

    (a) there is no error due to thermo-electric effect in potentiometers

    (b) the accuracy of voltage measurement is higher in potentiometers

    (c) personnel errors while reading a potentiometer is comparatively less

    (d) all of the above

18. 'Null detection method' is more accurate than 'deflection method' for measurement of unknown resistances because

    (a) the former does not include errors due to nonlinear scale of the meters

    (b) the former does not include errors due to change in battery voltage

(c) the former does to depend on meter sensitivity at balanced condition

(d) all of the above

19. Guard terminals are recommended for high resistance measurements to

    (a) bypass the leakage current

    (b) guard the resistance from effects of stray electro-magnetic fields

    (c) guard the resistance from effects of stray electro-static fields

    (d) none of the above

20. When measuring cable insulation using a dc source, the galvanometer used is initially short circuited to

    (a) discharge the stored charge in the cable

    (b) bypass the high initial charging current

    (c) prevent the galvanometer from getting damaged due to low resistance of the cable

    (d) all of the above

21. The loss of charge method is used for measurement of

    (a) high value capacitances

    (b) dissipation factor of capacitances

    (c) low value resistances

    (d) high value resistances

22. A meggar is used for measurement of

    (a) low value resistances

    (b) medium value resistances

    (c) high value, particularly insulation resistances

    (d) all of the above

23. Controlling torque in a meggar is provided by

    (a) control springs

    (b) balance weights

    (c) control coil

    (d) any one of the above

24. The advantage of Varley loop test over Murray loop test for cable fault localisation is

    (a) the former can be used for localising faults even without knowledge of cable resistance

    (b) the former can be used for localising both earth fault and short circuit faults

    (c) the former can experimentally determine the total loop resistance

    (d) all of the above

25. Possible sources of error in using loop test for cable fault localisation are

    (a) uneven cable resistance/km

    (b) temperature variations

    (c) unknown cable joint resistances

    (d) all of the above

Answers

1. (c)	2. (a)	3. (b)	4. (c)	5. (d)	6. (a)	7. (a)
8. (a)	9. (b)	10. (a)	11. (c)	12. (d)	13. (d)	14. (d)
15. (d)	16. (b)	17. (b)	18. (c)	19. (a)	20. (b)	21. (d)
22. (c)	23. (c)	24. (a)	25. (d)			

Short-answer Questions

1. Describe the operation of a series-type ohmmeter with the help of a schematic diagram. Comment on the scale markings and zero adjustment procedures in such an instrument.
2. Derive an expression for the meter current as a function of the full-scale deflection value in a series type ohmmeter to determine the shape of scale.
3. Describe the operation of a shunt-type ohmmeter with the help of a schematic diagram. Comment on the scale markings and zero adjustment procedures in such an instrument.
4. Derive an expression for the meter current as a function of the full-scale deflection value in a shunt type ohmmeter to determine the shape of scale.
5. List the sources of errors in a Wheatstone bridge that may affect its precision while measuring medium range resistances. Explain how these effects are eliminated/minimised?
6. What are the different problems encountered while measuring low resistances. Explain how a 4-terminal configuration can minimise these errors while measuring low resistances using a voltmeter–ammeter method.
7. Describe how low resistances can be measured with the help of a potentiometer.
8. Describe with suitable schematic diagram, how a high resistance can be effectively measured using Price's guard wire method.
9. Explain the principles of the loss of charge method for measurement of high resistances. Also comment on the compensations required to be made in the calculations to take care of circuit component nonidealities.
10. Draw and explain the operation of a meggar used for high resistance measurement.
11. Describe with suitable schematic diagram, the Murray Loop test for localising earth fault in low voltage cables.
12. Describe with suitable schematic diagram, the Varley loop test for localising earth fault in low voltage cables.

Long-answer Questions

1. (a) Discuss with suitable diagrams, the different ways of zero adjustment in a series-type ohmmeter.

   (b) Design a single range series-type ohmmeter using a PMMC ammeter that has internal resistance of 60 Ω and requires a current of 1.2 mA for full scale deflection. The internal battery has a voltage of 5 V. It is desired to read half scale at a resistance value of 3000 Ω. Calculate (a) the values of shunt resistance and current limiting series resistance, and (b) range of values of the shunt resistance to accommodate battery voltage variation in the range 4.7 to 5.2 V.
2. (a) Describe the operation of a shunt-type ohmmeter with the help of a schematic diagram. Comment son the scale markings and zero adjustment procedures in such an instrument?

   (b) A shunt-type ohmmeter uses a 2 mA basic d'Arsonval movement with an internal resistance of 50 Ω. The battery emf is 3 V. Calculate (a) value of the resistor in series with the battery to adjust the FSD, and (b) at what point (percentage) of full scale will 200 Ω be marked on the scale?
3. (a) Describe in brief, the use of voltmeter–ammeter method for measurement of unknown resistance.

   (b) A voltmeter of 500 Ω resistance and a milliammeter of 0.5 Ω resistance are used to measure two unknown resistances by voltmeter–ammeter method. If the voltmeter reads 50 V and milliammeter reads 50 mA in both the cases, calculate the percentage error in the values of measured resistances if (i) in the first case, the voltmeter is put across the resistance and the milliammeter connected in series with the supply, and (ii) in the second case, the voltmeter is connected in the supply, side and milliammeter connected directly in series with the resistance.
4. (a) Draw the circuit of a Wheatstone bridge for measurement of unknown resistances and derive the condition for balance.

   (b) Four arms of a Wheatstone bridge are as follows: AB = 150 W, BC = 15 W, CD = 6 W, DA = 60 Ω. A galvanometer with internal resistance of 25 Ω is connected between BD, while a battery of 20 V dc is connected between AC. Find the current through the galvanometer. Find the value of the resistance to be put on the arm DA so that the bridge is balanced.
5. (a) Explain the principle of working of a Kelvin's double bridge for measurement of unknown low resistances. Explain how the effects of contact resistance and resistance of leads are eliminated.

(b) A 4-terminal resistor was measured with the help of a Kelvin's double bridge having the following components: Standard resistor = 100.02 |iW, inner ratio arms = 100.022 Ω and 199 W, outer ratio arms = 100.025 Ω and 200.46 W, resistance of the link connecting the standard resistance and the unknown resistance = 300 mW. Calculate value of the unknown resistance.

6. Discuss the difficulties involved for measurement of high resistances. Explain the purpose of guarding a high resistance measurement circuits.

7. (a) Derive an expression for the unknown resistance measured using the loss of charge method.

(b) A cable is tested for insulation resistance by loss of charge method. An electrostatic voltmeter is connected between the cable conductor and earth. This combination is found to form a capacitance of 600 pF between the conductor and earth. It is observed that after charging the cable with 1000 V for sufficiently long time, when the voltage supply is withdrawn, the voltage drops down to 480 V in 1 minute. Calculate the insulation resistance of the cable.

8. (a) Describe with suitable schematic diagrams, the Murray loop test for localization of earth fault and short circuit fault in low voltage cables.

(b) n a test for fault to earth by Murray loop test, the faulty cable has a length of 6.8 km. The faulty cable is looped with a sound (healthy) cable of the same length and cross section. Resistances of the ratio arm of the measuring bridge circuit are 200 Ω and 444 Ω at balance. Calculate the distance of the fault point from the testing terminal.

9. (a) Describe with suitable schematic diagrams, the Varley loop test for localisation of earth fault and short-circuit fault in low voltage cables.

(b) Varley loop test is being used to locate short circuit fault. The faulty and sound cables are identical with resistances of 0.4 Ω per km. The ratio arms are set at 20 Ω and 50 Ω. Values of the variable resistance connected with the faulty cable are 30 Ω and 15 Ω at the two positions of the selector switch. Determine the length of each cable and fault distance from test end.

5 Potentiometers

5.1 INTRODUCTION

A potentiometer is an instrument which is used for measurement of potential difference across a known resistance or between two terminals of a circuit or network of known characteristics. A potentiometer is also used for comparing the emf of two cells. A potentiometer is extensively used in measurements where the precision required is higher than that can be obtained by ordinary deflecting instruments, or where it is required that no current be drawn from the source under test, or where the current must be limited to a small value.

Since a potentiometer measures voltage by comparing it with a standard cell, it can be also used to measure the current simply by measuring the voltage drop produced by the unknown current passing through a known standard resistance. By the potentiometer, power can also be calculated and if the time is also measured, energy can be determined by simply multiplying the power and time of measurement. Thus potentiometer is one of the most fundamental instruments of electrical measurement.

Some important characteristics of potentiometer are the following:

- A potentiometer measures the unknown voltage by comparing it with a known voltage source rather than by the actual deflection of the pointer. This ensures a high degree of accuracy.
- As a potentiometer measures using null or balance condition, hence no power is required for the measurement.
- Determination of voltage using potentiometer is quite independent of the source resistance.

5.2 A BASIC dc POTENTIOMETER

The circuit diagram of a basic dc potentiometer is shown in Figure 5.1.

Operation

First, the switch S is put in the 'operate' position and the galvanometer key K kept open, the battery supplies the working current through the rheostat and the slide wire. The working current through the slide wire may be varied by changing the rheostat setting. The method of measuring the unknown voltage, E_1, depends upon the finding a position for the sliding contact such that the galvanometer shows zero deflection, i.e., indicates null condition, when the galvanometer key K is closed. Zero galvanometer deflection means that the unknown voltage E_1 is equal to the voltage drop E_2, across position a–c of the slide wire. Thus, determination of the values of unknown voltage now becomes a matter of evaluating the voltage drop E_2 along the portion a–c of the slide wire.

When the switch S is placed at ‘ calibrate’ position, a standard or reference cell is connected to the circuit. This reference cell is used to standardize the potentiometer. The slide wire has a uniform cross-section and hence uniform resistance along its entire length. A calibrated scale in cm and fractions of cm, is placed along the slide wire so that the sliding Figure 5.1 *A basic potentiometer circuit* contact can be placed accurately at any desired position along the slide wire. Since the resistance of the slide wire is known accurately, the voltage drop along the slide wire can be controlled by adjusting the values of working current. The process of adjusting the working current so as to match the voltage drop across a portion of sliding wire against a standard reference source is known as ‘ standardisation’.

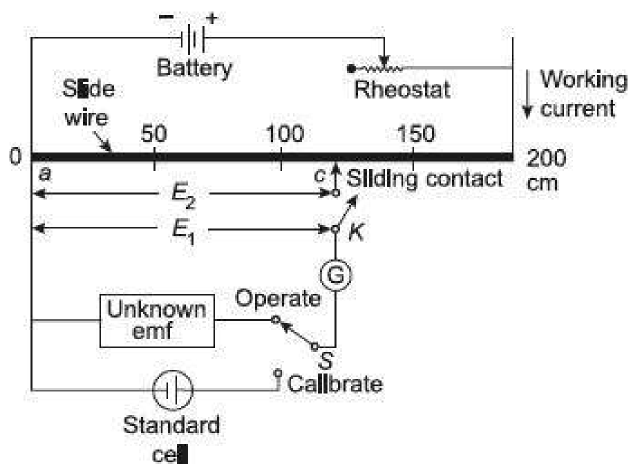


Figure 5.1 *A basic potentiometer circuit*

5.3 CROMPTON’S dc POTENTIOMETER

The general arrangement of a laboratory-type Crompton’s dc potentiometer is shown in Figure 5.2. It consists of a dial switch which has fifteen (or more) steps. Each steep has 10 Ω resistance. So the dial switch has total 150 Ω resistance. The working current of this potentiometer is 10 mA and therefore each step of dial switch corresponds to 0.1 volt. So the range of the dial switch is 1.5 volt.

The dial switch is connected in series with a circular slide wire. The circular slide wire has 10 Ω resistance. So the range of that slide wire is 0.1 volt. The slide wire calibrated with 200 scale divisions and since the total resistance of slide wire corresponds to a voltage drop of 0.1 volt, each division of the slide wire corresponds to $\frac{0.1}{200} = 0.0005$ volt. It is quite comfortable to interpolate readings up to $\frac{1}{5}$ of a scale division and therefore with this Crompton’s potentiometer it is possible to estimate the reading up to 0.0001 volt.

Procedure for Measurement of Unknown emf

- At first, the combination of the dial switch and the slide wire is set to the standard cell voltage. Let the standard sell voltage be 1.0175 volts, then the dial resistor is put in 1.0 volt and the slide wire at 0.0175 volts setting.
- The switch '*S*' is thrown to the calibrate position and the galvanometer switch '*K*' is pressed until the rheostat is adjusted for zero deflection on the galvanometer. The 10 kW protective resistance is kept in the circuit in the initial stages so as to protect the galvanometer from overload.
- After the null deflection on the galvanometer is approached the protective resistance is shorted so as to increase the sensitivity of the galvanometer. Final adjustment is made for the zero deflection with the help of the rheostat. This completes the standardisation process of the potentiometer.
- After completion of the standardisation, the switch '*S*' is thrown to the operate position thereby connecting the unknown emf into the potentiometer circuit. With the protective resistance in the circuit, the potentiometer is balanced by means of the main dial and the slide wire adjustment.
- As soon as the balanced is approached, the protective resistance is shorted and final adjustments are made to obtain true balance.
- After the final true balance is obtained, the value of the unknown emf is read off directly from the setting of the dial switch and the slide wire.
- The standardisation of the potentiometer is checked again by returning the switch '*S*' to the calibrate position. The dial setting is kept exactly the same as in the original standardisation process. If the new reading does not agree with the old one, a second measurement of unknown emf must be made. The standardisation again should be made after the measurement.

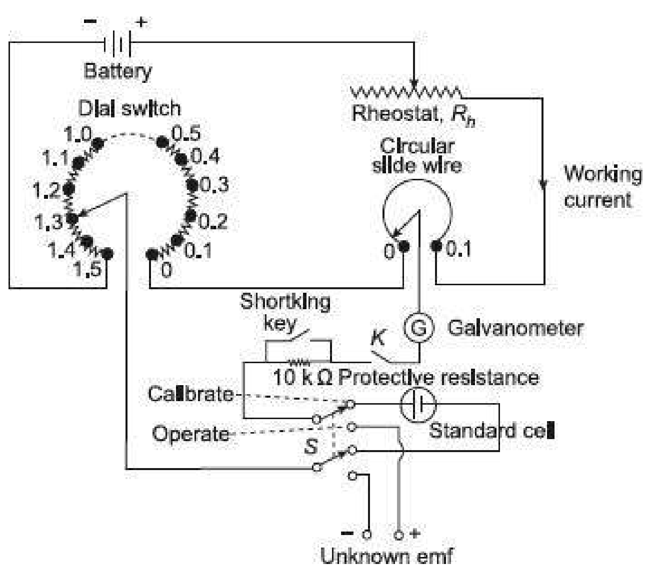


Figure 5.2 *General arrangement of Crompton's dc potentiometer*

A basic slide-wire potentiometer has a working battery

Example 5.1

voltage of 3.0 volts with negligible resistance. The resistance of the slide-wire is 400 Ω and its length is 200 cm. A 200-cm scale is placed along the slide wire. The slide-wire has 1 mm scale divisions and it is possible to read up to 1/5 of a division. The instrument is standardised with 1.018 volt standard cell with with sliding contact at the 101.8 cm mark on scale.

Calculate

(a) Working current

(b) Resistance of series rheostat

(c) Measurement range

(d) Resolution of the instrument

Solution

(a) Working current, I_m

Because the instrument is standardised with an emf of 1.018 volts with sliding contact at 101.8 cm, it is obvious that a length 101.8 cm represents a voltage of 1.018 volts.

$$\text{Resistance of 101.8 cm length of wire} = \frac{1.018}{200} \times 400 = 203.6\ \Omega$$

$$\therefore \quad \text{working current, } I_m = \frac{101.8}{203.6} = 0.005\ \text{A or 5 mA.}$$

(b) Resistance of series rheostat, R_h

Total resistance of battery circuit = Resistance of rheostat (R_h) + Resistance of slide wire.

$\therefore R_h$ = Total resistance – Resistance of slide wire

$$= \frac{3}{0.005} - 400 = 200\ \Omega$$

(c) Measurement range

The measurement range is the total voltage across the slide wire.

$\therefore$ range of voltage = 0.005 ×400 = 2.0 volt.

(d) Resolution of the instrument

A length of 200 cm represents 2.0 volt and therefore 1 mm represents a voltage ofê2 ê 1

$$\left(\frac{2}{200}\right) \times \left(\frac{1}{10}\right) = 1\ \text{mV}$$

Since it is possible to read $\frac{1}{5}$ of I mV, therefore, resolution of the instrument is $\frac{1}{5}$ × 1 = 0.2 mV

A single-range laboratory-type potentiometer has an 18-step dial switch where each step represents 0.1 volt. The

Example 5.2

dial resistors are 10 Ω each. The slide wire of the potentiometer is circular and has 11 turns and a resistance of 1 Ω per turn. The slide wire has 100 divisions and interpolation can be done to one forth of a division. The working battery has a voltage of 6.0 volt. Calculate (a) the measuring range of the potentiometer, (b) the resolution, (c) working current, and (d) setting of the rheostat.

SolutionDial resistor = 10 Weach

Each step = 0.1 v

$$\therefore \quad \text{working current} = \frac{0.1}{10} = 10 \text{ mA}$$

(a) The measuring range of the potentiometer

Total resistance of measuring circuit,

R_m= Resistance of dial + resistance of slide wire

or, R_m = 18 ×10 + 11 = 191

voltage range of the total instrument

= R_m×working current

= 191 ×10 mA

= 1.91 V

(b) The resolution

The slide wire has a resistance of 11 Ω and therefore voltage drop across slide wire

= 11 × 10 mA = 0.11 volt

The slide wire has 11 turns, and therefore voltage drop across each turn

$$= \frac{0.11}{11} = 0.01 \text{ volt}$$

Each turn is divided into 100 divisions and therefore each division represents a voltage drop of $\frac{0.01}{100} = 0.0001$.

Since each turn can be interpolated to of a division,

Resolution of instrument =×0 0001 =0 000025 volt =25 mV

(c) Working current, I_m

As previously mentioned, working current = 1 mA.

(d) Setting of rheostat

Total resistance across battery circuit $\frac{6}{0.01} = 600\ \Omega$

Total resistance of potentiometer circuit is 191 Ω

∴ resistance of series rheostat, R_h = 600 - 191 = 409 W.

5.4 APPLICATIONS OF dc POTENTIOMETERS

Practical uses of dc potentiometers are

- Measurement of current
- Measurement of high voltage
- Measurement of resistance
- Measurement of power
- Calibration of voltmeter
- Calibration of ammeter
- Calibration of wattmeter

5.4.1 Measurement of Current by Potentiometer

The circuit arrangement for measurement of current by a potentiometer is shown in Figure 5.3. The unknown current *I*, whose value is to be measured, is passed through a standard resistor *R* as shown. The standard resistor should be of such a value that voltage drop across it caused by flow of current to be measured, may not exceed the range of the potentiometer. Voltage drop across the standard resistor in volts divided by the value of *R* in ohms gives the value of unknown current in amperes.

$$\text{i.e., unknown current (in Amp) } I = \frac{\text{Voltage drop across } R \text{ (Volt)}}{\text{Value of } R \text{ (ohms)}}$$

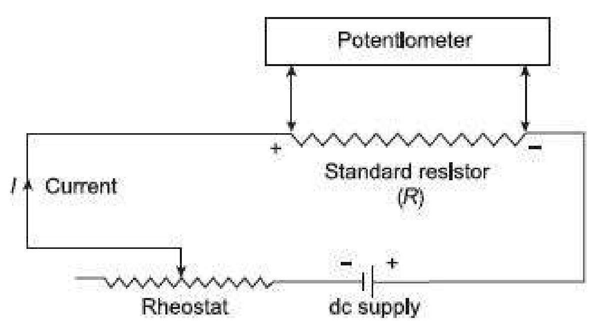


Figure 5.3 *Measurement of current with potentiometer*

Example 5.3 *A simple slide wire potentiometer is used for measurement of current in a circuit. The voltage drop across a standard resistor of 0.1 Ω is balanced at 75 cm. find the magnitude of the current if the standard cell emf of 1.45 volt is balanced at 50 cm.*

Solution For the same working current, if 50 cm corresponds to 1.45 volt. Then 75 cm of the slide wire corresponds to

$$= \frac{1.45}{50} \times 75 = 2.175 \text{ volt}$$

So, across the resistance 0.1 ? the voltage drop is 2.175 volt. Then the value of the current is

$$I = \frac{2.175}{0.1} = 21.75 \text{ A}$$

5.4.2 Measurement of High Voltage by Potentiometer

Special arrangements must be made to measure very high voltage by the potentiometer (say a hundreds of volts) as this high voltage is beyond the range of normal potentiometer. The voltage above the direct range of potentiometer (generally 1.8 volt) can be measured by using a volt–ratio box in conjunction with the potentiometer. The volt–ratio box consists of a simple resistance potential divider with various tapping on the input side. The arrangement is shown in Figure 5.4. Each input terminal is marked with the maximum voltage which can be applied and with the corresponding multiplying factor for the potential scale.

High emf to be measured is applied the suitable input terminal of volt–ratio box and leads to the potentiometer are taken from two tapping points intended for this purpose.

The potential difference across these two points is measured by the potentiometer. If the voltage measured by the potentiometer is v and k be the multiplying factor of the volt–ratio box, then the high voltage to be measured is $V = kv$ volt.

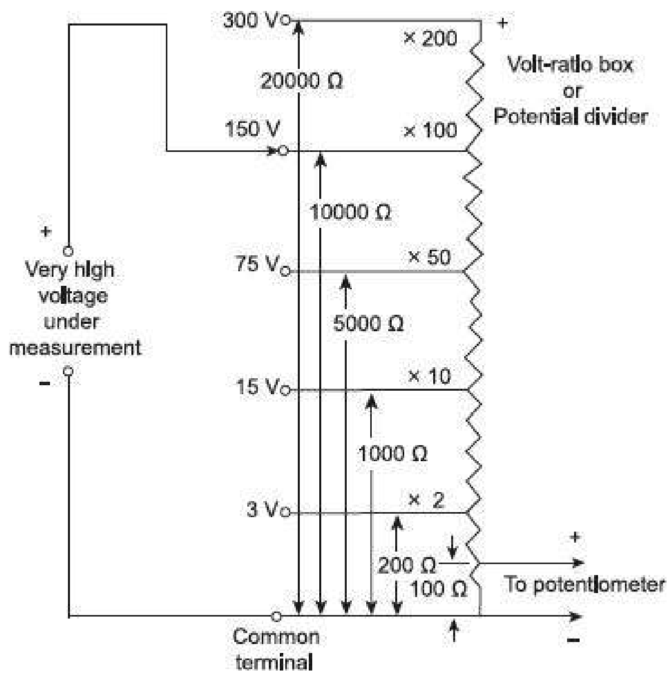


Figure 5.4 *Measurement of high voltage by potentiometer in conjunction with volt–ratio box*

5.4.3 Measurement of Resistance by Potentiometer

The connection diagram for measuring unknown resistance with the help of potentiometer is shown in Figure 5.5. The unknown resistance R, is connected in series with the known standard resistor S. The rheostat connected in the circuit controls the current flowing through the circuit. An ammeter is also connected in the circuit to indicate whether the value of the working current is within the limit of the potentiometer or not. Otherwise, the exact value of the working current need not be known.

When the two–pole double throw switch is put in position 1, the unknown resistance is connected to the potentiometer. Let the reading of the potentiometer in that position is V_R. Then

$$V_R = IR \qquad (5.1)$$

Now the switch is thrown to position 2, this connects the standard resistor S to the potentiometer. If the reading of the potentiometer is that position is V_S then,

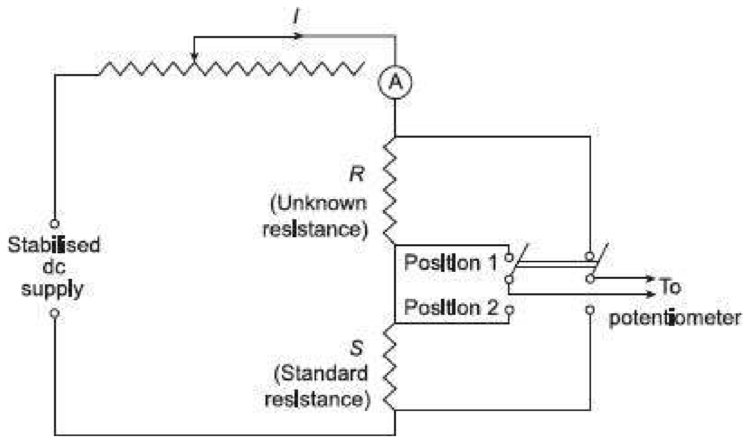


Figure 5.5 *Measurement of resistance by potentiometer*

$$V_S = IS \qquad (5.2)$$

Dividing (5.1) by (5.2), we get

$$\frac{V_R}{V_S} = \frac{IR}{IS}$$

or

$$R = \frac{V_R}{V_S} \times S$$

The value of R can be calculated accurately since the value of the standard resistor S is known. This method of measurement of resistance is used for low value of the resistor.

5.4.4 Measurement of Power by Potentiometer

In measurement of power by potentiometer the measurements are made one across the standard resistor S connected in series with the load and another across the volt–ratio box output terminals. The arrangement is shown in Figure 5.6.

The load current which is exactly equal to the current through the standard resistor S, as it is connected in series with the load, is calculated from the voltage drop across the standard resistor divided by the value of the standard resistor S.

Load current $I = \frac{V_S}{S}$

where V_S = voltage drop across standard resistor S as measured by the potentiometer.

Voltage drop across the load is found by the output terminal of the volt–ratio box. If V_R is the voltage drop across the output terminal of the volt–ratio box and V_L is the voltage drop across load then,

$$V_L = k \times V_R$$

where k is the multiplying factor of the volt–ratio box.

Then the power consumed, $P = V_L I = k \times V_R \times \frac{V_S}{S}$

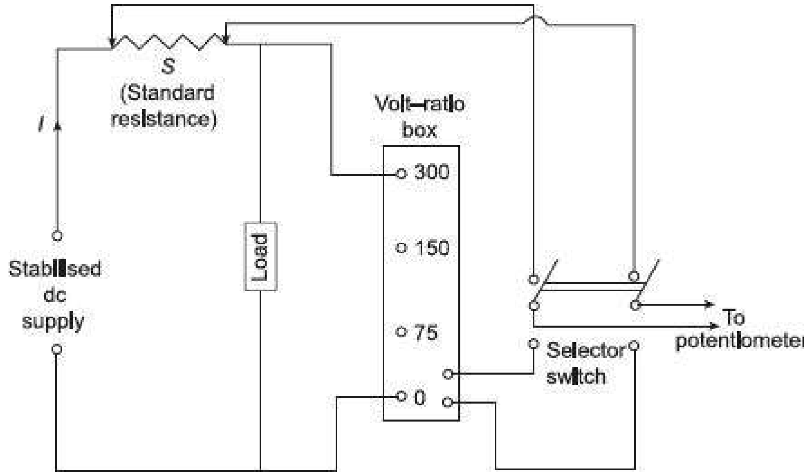


Figure 5.6 *Measurement of power by potentiometer*

5.4.5 Calibration of Voltmeter by Potentiometer

In case of calibration of voltmeter, the main requirement is that a suitable stable dc voltage supply is available, otherwise any change in the supply voltage will cause a change in the calibration process of the voltmeter.

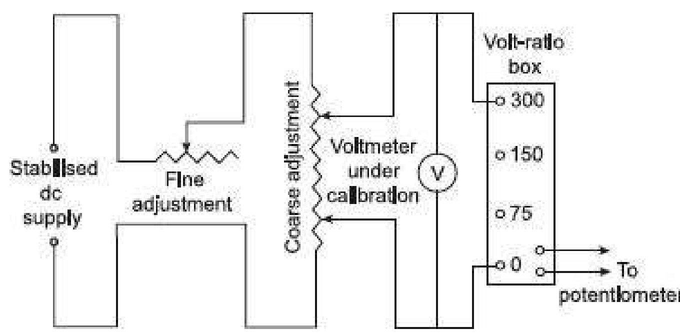


Figure 5.7 *Calibration of voltmeter by potentiometer*

The arrangement for calibrating a voltmeter by potentiometer is shown in Figure 5.7. The potential divider network consists of two rheostats. One for coarse and the other for fine control of calibrating voltage. With the help of these controls, it is possible to adjust the supply voltage so that the pointer coincides exactly with a major division of the voltmeter.

The voltage across the voltmeter is steeped down to a value suitable for the potentiometer with the help of the volt–ratio box. In order to get accurate measurements, it is necessary to measure voltages near the maximum range of the potentiometer, as far as possible.

The potentiometer measures the true value of the voltage. If the reading of the potentiometer does not match with the voltmeter reading, a positive or negative error is indicated. A calibration curve may be drawn with the help of the potentiometer and the voltmeter reading.

5.4.6 Calibration of Ammeter by Potentiometer

Figure 5.8 shows the circuit arrangement for calibration of an ammeter using potentiometer.

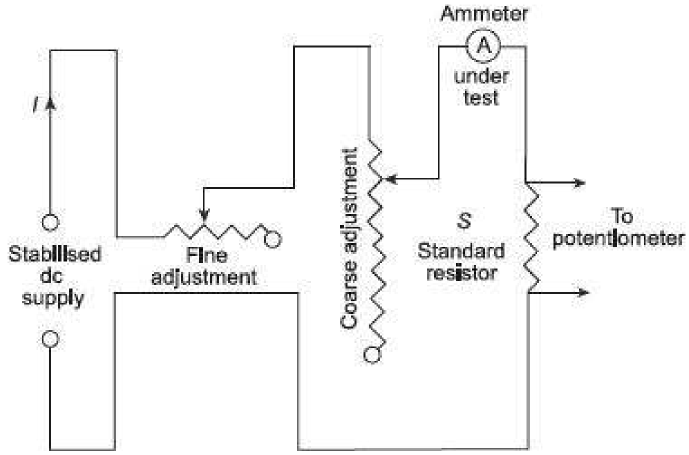


Figure 5.8s *Calibration of ammeter by potentiometer*

A standard resistor *S* of high current carrying capacity is placed in series with the ammeter under test. The voltage drop across *S* measured with the help of the potentiometer and then the current through *S* and hence the ammeter can be computed by dividing the voltage drop by the value of the standard resistor.

Current, $I = \frac{V_S}{S}$ is the voltage drop across the standard resistor *S*.

Now, compare the reading of the ammeter with the current found by calculation. If they do not match, a positive or negative error will be induced. A calibration curve may be drawn between the ammeter reading and the true value of the current as indicated by the potentiometer reading.

As the resistance of the standard resistor *S* is exactly known, the current through *S* is exactly calculated. This method of calibration of ammeter is very accurate.

5.4.7 Calibration of Wattmeter by Potentiometer

In this calibration process, the current coil of the wattmeter is supplied from low voltage supply and potential coil from the normal supply through potential divider. The voltage *V* across the potential coil of the wattmeter under calibration is measured directly by the potentiometer. The current through the current coil is measured by measuring the voltage drop across a standard resistor connected in series with the current coil divided by the value of the standard resistor.

The true power is then *VI*, where *V* is the voltage across the potential coil and *I* is the current through the current coil of the wattmeter. The wattmeter reading may be compared with this value, and a calibration curve may be drawn.

The arrangement for calibrating a wattmeter is shown in Figure 5.9.

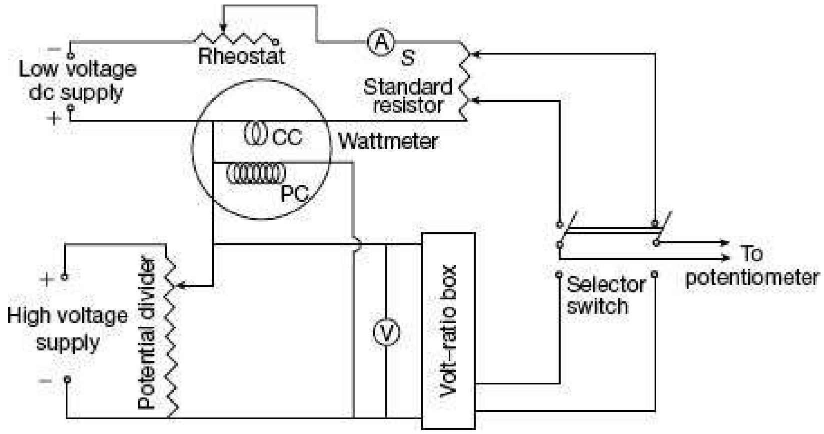


Figure 5.9 *Calibration of wattmeter by dc potentiometer*

Example 5.4

The emf of a standard cell used for standardisation is 1.0186 volt. If the balanced is achieved at a length of 55 cm, determine

(a) The emf of the cell which balances at 70 cm

(b) The current flowing through a standard resistance of 2 Ω if the potential difference across it balances at 60 cm

(c) The voltage of a supply main which is reduced by a volt–ratio box to one hundredth and balance is obtained at 85 cm

(d) The percentage error in a voltmeter reading 1.40 volt when balance is obtained at 80 cm

(e) The percentage error in ammeter reading 0.35 ampere when balance is obtained at 45 cm with the potential difference across a 2.5 Ω resistor in the ammeter circuit

Solution Emf of the standard cell = 1.0186 volts

The voltage drop per cm length of potentiometer wire,

$$v = \frac{1.0186}{55} = 0.01852$$

(a) The emf of a cell balanced at 70 cm,

$= v \cdot l = 0.01852 \times 70 = 1.2964$ volt

(b) The potential difference which is balanced at 60 cm

$= v \cdot l = 0.01852 \times 60 = 1.1112$ volt

Magnitude of the standard resistor, S =2 W

Therefore, current flowing through 2 Ω resistance$V$

$$= \frac{V}{S} = \frac{1.1112}{2} = 0.5556 \text{ A}$$

(c) The potential difference which balances at 85 cm,

$V = v \cdot l = 0.01852 \times 85 = 1.5742$ volt

voltage of supply main = V × ratio of volt–ratio box

1.5742 × 100 = 157.42 volt

(d) The potential difference which balances at 80 cm,

$V = v \cdot l = 0.01852 \times 80 = 1.4816$ volt

Voltmeter reading = 1.40 volt

percentage error in voltmeter reading

$$= \frac{1.4 - 1.4816}{1.4816} \times 100 = -5.507\%$$

(e) The potential difference which balances at 45 cm,

$V = v \cdot l = 0.01852 \times 45 = 0.8334$ volt

Current flowing through 2.5 Ω resistance$V$ 0.8334

$$I = \frac{V}{S} = \frac{0.8334}{2.5} = 0.33336 \text{ A}$$

percentage error in ammeter reading

$$= \frac{0.35 - 0.33336}{0.33336} \times 100 = 4.991\%$$

Example 5.5

The following readings were obtained during the measurement of a low resistance using a potentio meter:
Voltage drop across a 0.1 Ω standard resistance = 1.0437 V
Voltage drop across the low resistance under test = 0.4205 V
Calculate the value of unknown resistance, current and power lost in it.

Solution Given: $S = 0.1$ Ω; $V_S = 1.0437$ V; $V_R = 0.4205$ V

Resistance of unknown resistor, $R = \frac{V_R}{V_S} \times S = \frac{0.4205}{1.0437} \times 0.1 = 0.04\ \Omega$

Current through the resistor, $I = \frac{V_S}{S} = \frac{1.0437}{0.1} = 10.437$ A

Power loss, $PI^2 R = (10.437)^2 \times 0.04 = 4.357$ Wa

Example 5.6

A Crompton's potentiometer consists of a resistance dial having 15 steps of 10 Ω each and a series connected slide wire of 10 Ω which is divided into 100 divisions. If the working current of the potentio meter is 10 mA and each division of slide wire can be read accurately upto 1/5 th of its span, calculate the resolution of the potentiometer in volts.

Solution Total resistance of the potentiometer,

R = Resistance of the dial + Resistance of the slide wire

= 15 × 10 + 10 = 160 W

Working current, I = 10 mA = 0.01 A

Voltage range of the potentiometer = Working current × Total resistance of the potentiometer

= 0.01 × 160 = 1.6

Voltage drop across slide wire = Working current × Slide wire resistance

= 0.01 × 10 = 0.1 V

Since slide wire has 100 divisions, therefore, each division represents $\frac{0.1}{100}$ or 0.001 volt

As each division of slide wire can be read accurately up to $\frac{1}{5}$ potentiometer of its span, therefore, resolution of the potentiometer

$$= \frac{0.001}{5} = 0.0002 \text{ volt}$$

5.5 POTENTIOMETERS

An ac potentiometer is same as dc potentiometer by principle. Only the main difference between the ac and dc potentiometer is that, in case of dc potentiometer, only the magnitude of the unknown emf is compared with the standard cell emf, but in ac potentiometer, the magnitude as well as phase angle of the unknown voltage is compared to achieve balance.

This condition of ac potentiometer needs modification of the potentiometer as constructed for dc operation.

The following points need to be considered for the satisfactory operation of the ac potentiometer:

1. To avoid error in reading, the slide wire and the resistance coil of an ac potentiometer should be non-inductive.
2. The reading is affected by stray or external magnetic field, so in the time of measurement they must be eliminated or measured and corresponding correction factor should be introduced.
3. The sources of ac supply should be free from harmonics, because in presence of harmonics the balance may not be achieved.
4. The ac source should be as sinusoidal as possible.
5. The potentiometer circuit should be supplied from the same source as the voltage or current being measured.

5.6 CLASSIFICATION OF AC POTENTIOMETERS

There are two general types of ac potentiometers:

1. Polar Potentiometer

As the name indicates, in these potentiometers, the unknown emf is measured in polar form, i.e., in terms of its magnitude and relative phase. The magnitude is indicated by one scale and the phase with respect to some reference axis is indicated by another scale. There is provision for reading phase angles up to 360°.

The voltage is read in the form $V - \theta$.

Example: Drysdale polar potentiometer

2. Coordinate Potentiometer

Here, the unknown emf is measured in Cartesian form. Two components along and perpendicular to some standard axis are measured and indicated directly by two different scales known as in phase (V_1) and quadrature (V_2) scales (Figure 5.10). Provision is made in this instrument to read both positive and negative values of voltages so that all angles up to 360° are covered.

$$\text{Voltage } V = \sqrt{(V_1)^2 + (V_2)^2}\,;\ \theta = \tan^{-1}\left(\frac{V_2}{V_1}\right)$$

Example: Gall–Tinsley and Campbell–Larsen type potentiometer

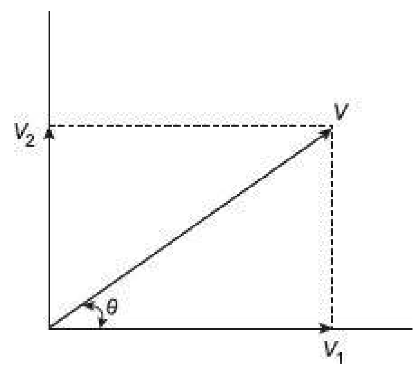


Figure 5.10 *Polar and coordinate representation of unknown emf*

5.6.1 Drysdale Polar Potentiometer

The different components of a Drysdale polar potentiometer is shown in Figure 5.11.

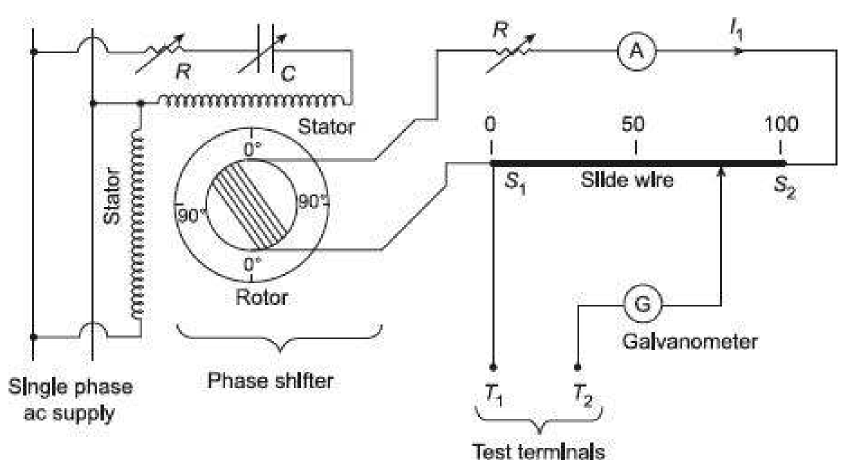


Figure 5.11 *Drysdale polar potentiometer*

The slide wire S_1–S_2 is supplied from a phase shifting circuit for ac measurement. The phase shifting circuit is so arranged that the magnitude of the voltage supplied by it remains constant while its phase can be varied through 360°. Consequently, slide wire current can be maintained constant in magnitude but varied in phase.

The phase shifting circuit consists of two stator coils connected in parallel supplied from the same source; their currents are made to differ by 90° by using very accurate phase shifting technique. The two windings produce rotating flux which induces a secondary emf in the rotor winding which is of constant magnitude but the phase of which can be varied by rotating the rotor in any position. The phase of the rotor emf is read from the circular dial attached in the potentiometer.

Before the ac measurement, the potentiometer is first calibrated by using dc supply for slide wire and standard cell for test terminals T_1 and T_2. The unknown alternating voltage to be measured is applied across test terminals and the balance is achieved by varying the slide wire contact and the position of the rotor. The ammeter connected in the slide wire circuit gives the magnitude of the unknown emf and the circular dial in the rotor circuit gives the phase angle of it.

5.6.2 Gall Coordinate Potentiometer

The Gall coordinate potentiometer consists of two separate potentiometer circuit in a single case. One of them is called the '*in-phase*' potentiometer and the other one is called the *quadrature* potentiometer. The slide-wire circuits of these two potentiometers are supplied with two currents having a phase difference of 90°. The value of the unknown voltage is obtained by balancing the voltages of *in-phase* and *quadrature* potentiometers slide wire simultaneously. If the measured values of *in-phase* and *quadrature* potentiometer slide-wires are V_1 and V_2 respectively then the magnitude of the unknown voltage is $V=\sqrt{V_1^2+V_2^2}$ $V = V_1^2 + V_2^2$ and the phase angle of the unknown voltage is given by *q* = $\tan^{-1}\frac{V_2}{V_1}$.

Figure 5.12 shows the schematic diagram of a Gall coordinate-type potentiometer. *W–X* and *Y–Z* are the sliding contacts of the *in-phase* and *quadrature* potentiometer respectively. *R* and *R′* are two rheostats to control the two slide-wire currents. The *in-phase* potentiometer slide-wire is supplied from a single-phase supply and the *quadrature* potentiometer slide-wire is supplied from a phase-splitting device to create a phase difference of 90° between the two slide-wire currents. T_1 and T_2 are two step-down transformers having an output voltage of 6 volts. These transformers also isolate the potentiometer from the high-voltage supply. *R* and *C* are the variable resistance and capacitance for phase-splitting purpose. *VG* is a vibration galvanometer which is tuned to the supply frequency and *K* is the galvanometer key. *A* is a dynamometer ammeter which is used to display the current in both the slide-wires so that they can be maintained at a standard value of 50 mA. SW_1 and SW_2 are two *sign- changing* switches which may be necessary to reverse the direction of unknown emf applied to the slide wires. SW_3 is a selector switch and it is used to apply the unknown voltage to the potentiometer.

Operation Before using the potentiometer for ac measurements, the current in the *in-*

phase potentiometer slide wire is first standardised using a standard dc cell of known value. The vibration galvanometer *VG* is replaced by a D'Arsonval galvanometer. Now the *in-phase* slide wire current is adjusted to the standard value of 50 mA by varying the rheostat *R*. This setting is left unchanged for ac calibration; the dc supply is replaced by ac and the D'Arsonval galvanometer by the vibration galvanometer.

The magnitude of the current in the quadrature potentiometer slide wire must be equal to the in- phase potentiometer slide wire current and the two currents should be exactly in quadrature. The switch SW_3 is placed to *test position* (as shown in Figure 5.12) so that the emf induced in the secondary winding of mutual inductance *M* is impressed across the *in-phase* potentiometer wire through the vibration galvanometer. Since the induced emf in the secondary of mutual inductance *M* will be equal to $2pfMi$ volt in magnitude; where *f* is the supply frequency and will lag 90° behind the current in the quadrature slide-wire *i*, so the value of emf calculated from the relation $e' = 2pf\,Mi$ for a current $i = 50$ mA is set on *in-phase* potentiometer slide wire and *R*' are adjusted till exact balance point is obtained. At balance position, the current in the potentiometer wires will be exactly equal to 50 mA in magnitude and exactly in quadrature with each other. The polarity difference between the two circuits is corrected by changing switches SW_1 and SW_2.

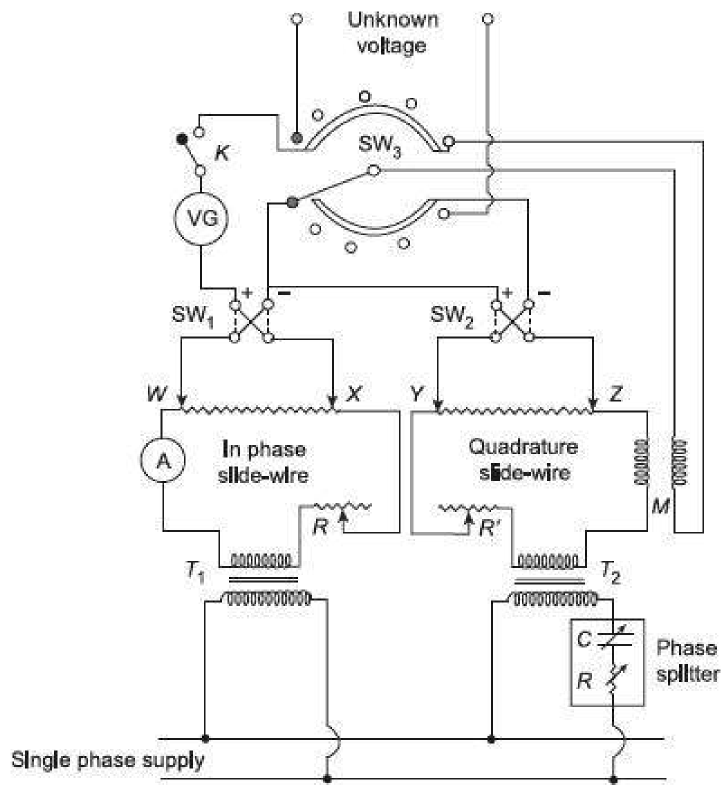


Figure 5.12 *Gall coordinate potentiometer*

Lastly, the unknown voltage is applied to the potentiometer by means of the switch SW_3 and balance is obtained on both the potentiometer slide-wire by adjusting the slide-wire setting. The reading of slide-wire *WX* gives the in-phase component (V_1) and slide wire *YZ* gives quadrature component (V_2) of the unknown voltage.

ADVANTAGES AND DISADVANTAGES OF ac

5.7 POTENTIOMETERS

Advantages

1. An ac potentiometer is a very versatile instrument. By using shunt and volt–ratio box, it can measure wide range of voltage, current and resistances.
2. As it is able to measure phase as well as magnitude of two signals, it is used to measure power, inductance and phase angle of a coil, etc.
3. The principle of ac potentiometer is also incorporated in certain special application like Arnold circuit for the measurement of CT (Current Transformer) errors.

Disadvantages

1. A small difference in reading of the dynamometer instrument either in dc or ac calibration brings on error in the alternating current to be set at standard value.
2. The normal value of the mutual inductance *M* is affected due to the introduction of mutual inductances of various potentiometer parts and so a slight difference is observed in the magnitude of the current of quadrature wire with compared to that in the in–phase potentiometer wire.
3. Inaccuracy in the measured value of frequency will also result in the quadrature potentiometer wire current to differ from that of in–phase potentiometer wire.
4. The presence of mutual inductances in the various parts of the potentiometer and the inter capacitance, the potential gradient of the wires is affected.
5. Since the standardisation is done on the basis of rms value and balance is obtained dependent upon the fundamental frequency only, therefore, the presence of harmonics in the input signal introduces operating problem and the vibration galvanometer tuned to the fundamental frequency may not show full null position at all.

5.8 APPLICATIONS OF ac POTENTIOMETER

The major applications of the ac potentiometers are

1. Measurement of self-inductance
2. Calibration of voltmeter
3. Calibration of ammeter
4. Calibration of wattmeter

5.8.1 Measurement of Self–inductance

The circuit diagram for measurement of self inductance of a coil by ac potentiometer is shown in Figure 5.13(a). A standard non-inductive resistor is connected in series with the coil under test and two potential differences V_1 and V_2 are measured in magnitude and phase by the potentiometer.

The vector diagram is shown in Figure 5.13(b). Refer to this figure.

Voltage drop across standard resistor $R_S, V2 = IR_S$

where, I= current flowing through the circuit, and

R_S= resistance of the standard non-inductive resistor

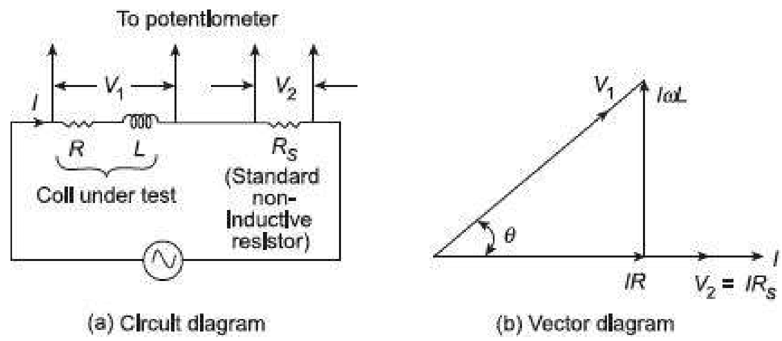


Figure 5.13 *Measurement of self-inductance by ac potentiometer*

or, $$I = \frac{V_2}{R_S}$$

Voltage drop across inductive coil = V_1

Phase angle between voltage across and current through the coil = θ

Voltage drop due to resistance of coil, $IR = V_1 \cos\theta$

$$\therefore \text{ resistance of the coil, } R = \frac{V_1 \cos\theta}{I} = \frac{V_1 \cos\theta}{\frac{V_2}{R_S}} = \frac{R_S V_1 \cos\theta}{V_2}$$

Voltage drop due to inductance of coil, $I\omega L = V_1 \sin\theta$

$$\therefore \text{ inductance of the coil, } L = \frac{V_1 \sin\theta}{I\omega} = \frac{V_1 \sin\theta}{\omega\left(\frac{V_2}{R_S}\right)} = \frac{R_S V_1 \sin\theta}{V_2 \omega}$$

5.8.2 Calibration of Ammeter

The method of calibration of an ac ammeter is similar to dc potentiometer method for dc ammeter (refer to Section 5.4.6) i.e., ac ammeter under calibration is connected in series with a non inductive variable resistance for varying the current, and a non-inductive standard resistor and voltage drop across standard resistor is measured on ac potentiometer. However, the standardising of the ac potentiometer involves the use of a suitable transfer instrument.

5.8.3 Calibration of Voltmeter

The method of calibration of an ac voltmeter by using ac potentiometer is similar to that adopted for calibration of dc voltmeter by using dc potentiometer (refer Section 5.4.5).

5.8.4 Calibration of Wattmeter

The circuit diagram for calibration of wattmeter by ac potentiometer is shown in Figure 5.14. The calibration process is same that adopted in case of calibration of wattmeter by dc potentiometer (refer Section 5.4.7).

The current coil of the wattmeter is supplied through a stepdown transformer and the potential coil from the secondary of a variable transformer whose primary is supplied from the rotor of a phase shifting transformer.

The voltage V across the potential coil of the wattmeter and the current I through the current coil of wattmeter are measured by the potentiometer, introducing a volt–ratio box and a standard resistor as shown in Figure 5.14.

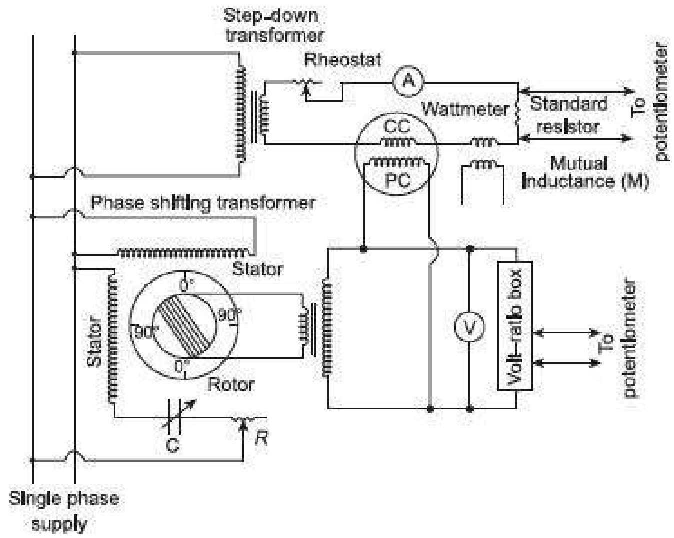


Figure 5.14 *Calibration of wattmeter by ac potentiometer*

The power factor cos q is varied by rotating the phase shift rotor, the phase angle between voltage and current, F being given by the reading on the dial of the phase shifter. The power is then VI cos F and the wattmeter reading may be compared with this reading. A calibration curve may be drawn if necessary. A small mutual inductance M is included to ensure accuracy of measurement of zero power factor.

Example 5.7

The following readings were obtained during measurement of inductance of a coil on an ac potentiometer:Voltage drop across 0.1 Ω standard resistor connected in series with the coil= 0.613 -12°6'Voltage across the test coil through a 100:1 volt–ratio box = 0.781 -50°48'Frequency 50 Hz. Determine the value of the inductance of the coil.

Solution

Current through the coil $= \bar{I} = \dfrac{0.613 \angle 12°6'}{0.1} = 6.13 \angle 12°6'$ A

Voltage across the coil $\bar{V} = 100 \times 0.781 \angle 50°48' = 78.1 \angle 50°48'$ V

∴impedance of the coil $\bar{Z} = \dfrac{\bar{V}}{\bar{I}} = \dfrac{78.1 \angle 50°48'}{6.13 \angle 12°6'} = 12.74 \angle 38°42'\ \Omega\alpha$

Resistance of the coil $R = 12.74 \cos 38°42' = 9.94\ \Omega$

Reactance of the coil $X = 12.74 \sin 38°42' = 7.96\ \Omega$

$$\therefore \text{inductance of the coil } L = \frac{X}{2\pi f} = \frac{7.96}{2 \times \pi \times 50} = 0.0253 \text{ H}$$

The power factor cos q is varied by rotating the phase shift rotor, the phase angle between voltage and current, F being given by the reading on the dial of the phase shifter. The power is then VI cos F and the wattmeter reading may be compared with this reading.

A calibration curve may be drawn if necessary. A small mutual inductance *M* is included to ensure accuracy of measurement of zero power factor.

Example 5.7

The following results were obtained for determination of impedance of a coil by using a coordinate type potentiometer: Voltage across the 1.0 Ωresistor in series with the coil = +0.2404 V in phase dial and 0.0935 V on quadrature dial Voltage across 10:1 potential divider used with the coil = +0.3409 V on in phase dial and +0.2343 V on quadrature dial Calculate the resistance and reactance of the coil.

Solution

Current through the coil

$$\overline{I} = \frac{(+0.2404 - j0.0935)}{1.0} = (+0.2404 - j0.0935)\ \text{A}$$

Voltage across the coil

$$\overline{V} = 10(0.3409 + j0.2343) = (3.409 + j2.343)\ \text{V}$$

∴ impedance of the coil

$$\overline{Z} = \frac{\overline{V}}{\overline{I}} = \frac{3.409 + j2.343}{0.2404 - j0.0935} = \frac{4.136\angle 34.5^\circ}{0.258\angle -21.2^\circ} = 16.03\angle 55.7^\circ$$

$$= (9.03 + j13.24)\ \Omega$$

∴ resistance of the coil $R = 9.03\ \Omega$

Reactance of the coil $L = 13.24\ \Omega$

Example 5.9

The following results were obtained during the measurement of power by a polar potentiometer:Voltage across 0.2 Ωstandard resistor in series with the load = 1.52 -35°Voltage across 200:1 potential divider across the line = 1.43 -53°Calculate the current, voltage, power and power factor of the load.

Solution

(a) Current through the load, $\overline{I} = \frac{1.52\angle 35^\circ}{0.2} = 7.6\angle 35^\circ\ \text{A}$

Magnitude of the current *I*= 7.6 A

(b) Voltage across the load *V*=200 ×(1.43 <53°)=286 <53°V

Magnitude of the voltage *V*= 286 V

(c) Phase angle of the load = 53° <35° = 18°

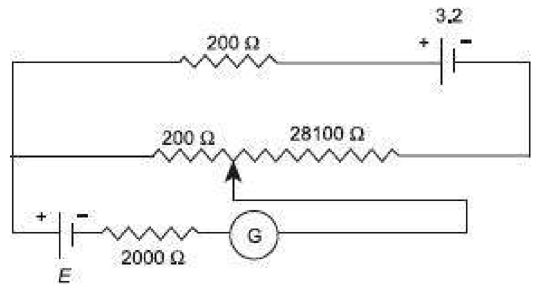
∴power factor of the load cos *φ* = cos 18° = 0.951 (lagging)

(d) Power consumed by the load, *P* = *I*cos *φ*=286 ×× 0.951

EXERCISE

Objective-type Questions

1. The transfer instrument which is used for standardisation of a polar-type ac potentiometer is
   (a) an electrostatic instrument
   (b) a dynamometer instrument
   (c) a moving coil instrument
   (d) a thermal instrument
2. A dc potentiometer is designed to measure up to about 2 volts with a slide wire of 800 mm. A standard cell of emf 1.18 volt obtains balance at 600 mm. a test cell is seen to obtained balance at 680 mm. The emf of the test cell is
   (a) 1.50 volts
   (b) 1.00 volts
   (c) 1.34 volts
   (d) 1.70 volts
3. For measuring an ac voltage by an ac potentiometer, it is desirable that the supply for the potentiometer is taken from
   (a) a battery
   (b) the same source as the unknown voltage
   (c) a source other than the source of unknown voltage
   (d) any of the above
4. The calibration of a voltmeter can be carried out by using
   (a) an ammeter
   (b) a function generator
   (c) a frequency meter
   (d) a potentiometer
5. A slide wire potentiometer has 10 wires of 1 m each. With the help of a standard voltage source of 1.018 volt it is standardise by keeping the jockey at 101.8 cm. If the resistance of the potentiometer wire is 1000 Ω then the value of the working current is
   (a) 1 mA
   (b) 0.5 mA
   (c) 0.1 A
   (d) 10 mA
6. In the potentiometer circuit, the value of the unknown voltage *E* under balance condition will be
   (a) 3 V
   (b) 200 mV
   (c) 2.8 V
   (d) 3.2 V



7. The potentiometer is standardised for making it
   (a) precise
   (b) accurate
   (c) accurate and precise
   (d) accurate and direct reading
8. Consider the following statements.A dc potentiometer is the best means available for the measurement of dc voltage because
   1. The precision in measurement is independent of the type of detector used
   2. It is based on null balance technique
   3. It is possible to standardize before a measurement is undertaken
   4. It is possible to measure dc voltages ranging in value from mV to hundreds of volts Of these statements,
   (a) 2 and 3 are correct
   (b) 1 and 4 are correct
   (c) 2 and 4 are correct
   (d) 3 and 4 are correct
9. In a dc potentiometer measurements, a second reading is often taken after reversing the polarities of dc supply and the unknown voltage, and the average of the two readings is taken. This is with a view to eliminate the effects of
   (a) ripples in the dc supply
   (b) stray magnetic field
   (c) stray thermal emfs
   (d) erroneous standardisation

Answers

1. (d)	2. (c)	3. (b)	4. (d)	5. (d)	6. (b)	7. (d)
8. (c)	9. (c)					

Short-answer Questions

1. Explain why a potentiometer does not load the voltage source whose voltage is being measured.
2. Describe the procedure of standardisation of a dc potentiometer.
3. What is a *volt ratio* box? Explain its principle with a suitable block diagram.
4. Explain the reasons why dc potentiometers cannot be used for ac measurement directly.
5. Explain the procedure for measurement of self-reactance of a coil with the help of ac potentiometer.
6. What is the difference between a slide-wire potentiometer and direct reading potentiometer?
7. How can a dc potentiometer be used for calibration of voltmeter?
8. Explain with suitable diagram how a dc potentiometer can be used for calibration of an ammeter.
9. Explain with a suitable diagram how a dc potentiometer can be used for calibration of wattmeter.
10. What are the different forms of ac potentiometers and bring out the differences between them.

Long-answer Questions

1. (a) Explain the working principle of a Crompton dc potentiometer with a suitable diagram.

   (b) The emf of a standard cell is measured with a potentiometer which gives a reading of 1.01892 volts. When a 1 MΩ resistor is connected across the standard cell terminals, the potentiometer reading drops to 1.01874 volts. Calculate the internal resistance of the cell.

   [Ans: 176.6 Ω]

2. (a) Name the different types of dc potentiometers and explain one of them.

   (b) A slidewire potentiometer is used to measure the voltage between the two points of a certain dc circuit. The potentiometer reading is 1.0 volt. Across the same two points when a 10000 Ω/V voltmeter is connected, the reading on the voltmeter is 0.5 volt of its 5–volt range. Calculate the input resistance between two points.

   [Ans: 50000 Ω]

3. (a) Write down the procedure of standardisation of a dc potentiometer. How can it be used for calibration of ammeters and voltmeters?

   (b) A slidewire potentiometer has a battery of 4 volts and negligible internal resistance. The resistance of slide wire is 100 Ω and its length is 200 cm. A standard cell of 1.018 volts is used for standardising the potentiometer and the rheostat is adjusted so that balance is obtained when the sliding contact is at 101.8 cm.

   (i) Find the working current of the slidewire and the rheostat setting. [Ans: 20 mA, 100 Ω]

   (ii) If the slidewire has divisions marked in mm and each division can be interpolated to one fifth, calculate the resolution of the potentiometer. [Ans: 0.2 mV]

4. (a) What are the problems associated with ac potentiometers? Describe the working of any one ac potentiometer.

   (b) Power is being measured with an ac potentiometer. The voltage across a 0.1 Ω standard resistance connected in series with the load is $(0.35 - j0.10)$ volt. The voltage across 300:1 potential divider connected to the supply is $(0.8 + j0.15)$ volt. Determine the power consumed by the load and the power factor.

   [Ans: 801 W, 0.8945]

5. (a) Describe the construction and working of an ac coordinate–type potentiometer.

   (b) Measurements for the determination of the impedance of a coil are made on a coordinate type potentiometer. The result are: Voltage across 1 Ω standard resistance in series with the coil = +0.952 V on inphase dial and –0.340 V on quadrature dial; voltage across 10:1 potential divider connected to the terminals of the coil = +1.35 V on inphase dial and +1.28 V on quadrature dial.

   Calculate the resistance and reactance of the coil.

   [Ans: R = 8.32 Ω, × = 16.41 Ω]

6. Describe briefly the applications of ac potentiometers.

7. Write short notes on the following (any three):

   (a) Simple dc potentiometer and its uses

   (b) Calibration of low range ammeter

   (c) Measurement of high voltage by dc potentiometer

   (d) Polar potentiometer

   (e) Coordinate–type potentiometer

   (f) Comparison between ac and dc potentiometer

6 AC Bridges

6.1 INTRODUCTION

Alternating current bridges are most popular, convenient and accurate instruments for measurement of unknown inductance, capacitance and some other related quantities. In its simplest form, ac bridges can be thought of to be derived from the conventional dc Wheatstone bridge. An ac bridge, in its basic form, consists of four arms, an alternating power supply, and a balance detector.

6.2 SOURCES AND DETECTORS IN ac BRIDGES

For measurements at low frequencies, bridge power supply can be obtained from the power line itself. Higher frequency requirements for power supplies are normally met by electronic oscillators. Electronic oscillators have highly stable, accurate yet adjustable frequencies. Their output waveforms are very close to sinusoidal and output power level sufficient for most bridge measurements.

When working at a single frequency, a tuned detector is preferred, since it gives maximum sensitivity at the selected frequency and discrimination against harmonic frequencies. *Vibration galvanometers* are most commonly used as tuned detectors in the power frequency and low audio-frequency ranges. Though vibration galvanometers can be designed to work as detectors over the frequency range of 5 Hz to 1000 Hz, they have highest sensitivity when operated for frequencies below 200 Hz.

Head phones or *audio amplifiers* are popularly used as balance detectors in ac bridges at frequencies of 250 Hz and above, up to 3 to 4 kHz.

Transistor amplifier with *frequency tuning* facilities can be very effectively used as balance detectors with ac bridges. With proper tuning, these can be used to operate at a selective band of frequencies with high sensitivity. Such detectors can be designed to operate over a frequency range of 10 Hz to 100 kHz.

6.3 GENERAL BALANCE EQUATION FOR FOUR-ARM BRIDGE

An ac bridge in its general form is shown in Figure 6.1, with the four arms being represented by four unspecified impedances Z_1, Z_2, Z_3 and Z_4.

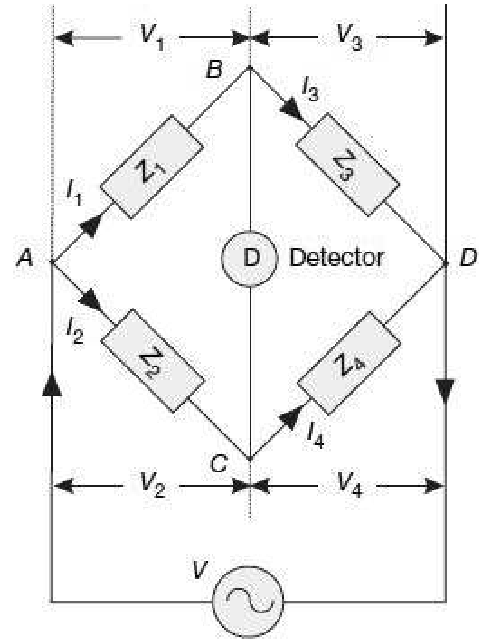


Figure 6.1 *General 4-arm bridge confi guration*

Balance in the bridge is secured by adjusting one or more of the bridge arms. Balance is indicated by zero response of the detector. At balance, no current flows through the detector, i.e., there is no potential difference across the detector, or in other words, the potentials at points *B* and *C* are the same. This will be achieved if the voltage drop from *A* to *B* equals the voltage drop from *A* to *C*, both in magnitude and phase.

Thus, we can write in terms of complex quantities:

$$\bar{V}_1 = \bar{V}_2$$

or, $$\bar{I}_1\bar{Z}_1 = \bar{I}_2\bar{Z}_2 \quad (6.1)$$

Also at balance, since no current flows through the detector,

$$\bar{I}_1 = \bar{I}_3 = \frac{\bar{V}}{\bar{Z}_1 + \bar{Z}_3} \quad (6.2)$$

and $$\bar{I}_2 = \bar{I}_4 = \frac{\bar{V}}{\bar{Z}_2 + \bar{Z}_4} \quad (6.3)$$

Combining Eqs (6.2) and (6.3) into Eq. (6.1), we have

$$\frac{\bar{V}}{\bar{Z}_1 + \bar{Z}_3}\bar{Z}_1 = \frac{\bar{V}}{\bar{Z}_2 + \bar{Z}_4}\bar{Z}_2$$

or, $$\bar{Z}_1\bar{Z}_2 + \bar{Z}_1\bar{Z}_4 = \bar{Z}_2\bar{Z}_1 + \bar{Z}_2\bar{Z}_3$$

or, $$\bar{Z}_1\bar{Z}_4 = \bar{Z}_2\bar{Z}_3 \quad (6.4)$$

or, $$\frac{\bar{Z}_1}{\bar{Z}_3} = \frac{\bar{Z}_2}{\bar{Z}_4} \quad (6.5)$$

When using admittances in place of impedances, Eq. (6.4) can be re-oriented as

$$\bar{Y}_1\bar{Y}_4 = \bar{Y}_2\bar{Y}_3 \quad (6.6)$$

Equations (6.4) and (6.6) represent the basic balance equations of an ac bridge. Whereas (6.4) is convenient for use in bridge configurations having series elements, (6.6) is more useful when bridge configurations have parallel elements.

Equation (6.4) indicates that under balanced condition, the product of impedances of one pair of *opposite* arms must be equal to the product of impedances of the other pair of *opposite* arms, with the impedances expressed as complex numbers. This will mean, both magnitude and phase angles of the complex numbers must be taken into account.

Re-writing the expressions in polar form, impedances can be expressed as $\overline{Z} = Z\angle\theta$ where *Z* represents the magnitude and *θ* represents the phase angle of the complex impedance.

If similar forms are written for all impedances and substituted in (6.4), we obtain:

$$Z_1\angle\theta_1 \times Z_4\angle\theta_4 = Z_2\angle\theta_2 \times Z_3\angle\theta_3$$

Thus, for balance we have,

$$Z_1Z_4\angle(\theta_1+\theta_4) = Z_2Z_3\angle(\theta_2+\theta_3) \quad (6.7)$$

Equation (6.7) shows that two requirements must be met for satisfying balance condition in a bridge.

The first condition is that the magnitude of the impedances must meet the relationship;
$$Z_1Z_4 = Z_2Z_3 \quad (6.8)$$

The second condition is that the phase angles of the impedances must meet the relationship; $\angle(\theta_1+\theta_4) = \angle(\theta_2+\theta_3)$ (6.9)

Example 6.1

In the AC bridge circuit shown in Figure 6.1, the supply voltage is 20 V at 500 Hz. Arm AB is 0.25 mμ pure capacitance; arm BD is 400 Ω pure resistance and arm AC has a 120 Ω resistance in parallel with a 0.15 mμ capacitor. Find resistance and inductance or capacitance of the arm CD considering it as a series circuit.

Solution Impedance of the arm *AB* is

$$Z_1 = \frac{1}{2\pi fC_1} = \frac{1}{2\pi \times 500 \times 0.25 \times 10^{-6}} = 1273\ \Omega$$

Since it is purely capacitive, in complex notation, $\overline{Z}_1 = 1273\angle -90°\ \Omega$

Impedance of arm *BD* is $Z_3 = 400\ \Omega$

Since it is purely resistive, in complex notation, $\overline{Z}_3 = 400\angle 0°\ \Omega$

Impedance of arm AC containing 120 Ω resistance in parallel with a 0.15 μF capacitor is

$$\bar{Z}_2 = \frac{R_2}{1 + j2\pi f C_2 R_2} = \frac{120}{1 + j(2\pi \times 500 \times 0.15 \times 10^{-6} \times 120)}$$
$$= 119.8\angle -3.2^\circ\ \Omega$$

For balance, $\bar{Z}_1\bar{Z}_4 = \bar{Z}_2\bar{Z}_3$

$\therefore$ impedance of arm *CD* required for balance is $\bar{Z}_4 = \dfrac{\bar{Z}_2\bar{Z}_3}{\bar{Z}_1}$

or, $$\bar{Z}_4 = \frac{119.88 \times 400}{1273}\angle(-3.2^\circ + 0^\circ + 90^\circ) = 37.65\angle 86.8^\circ$$

The positive angle of impedance indicates that the branch consists of a series combination of resistance and inductance.

Resistance of the unknown branch $R_4 = 37.65 \times \cos(86.8^\circ) = 2.1\ \Omega$

Inductive reactance of the unknown branch

$$X_4 = 37.65 \times \sin(86.8^\circ) = 37.59\ \Omega$$

Inductance of the unknown branch $L_4 = \dfrac{37.59}{2\pi \times 500} H = 11.97$ mH

6.4 MEASUREMENT OF SELF-INDUCTANCE

6.4.1 Maxwell's Inductance Bridge

This bridge is used to measure the value of an unknown inductance by comparing it with a variable standard self-inductance. The bridge configuration and phasor diagram under balanced condition are shown in Figure 6.2.

The unknown inductor L_1 of resistance R_1 in the branch *AB* is compared with the standard known inductor L_2 of resistance R_2 on arm *AC*. The inductor L_2 is of the same order as the unknown inductor L_1. The resistances R_1, R_2, etc., include, of course the resistances of contacts and leads in various arms. Branch *BD* and *CD* contain known no-inductive resistors R_3 and R_4 respectively.

The bridge is balanced by varying L_2 and one of the resistors R_3 or R_4. Alternatively, R_3 and R_4 can be kept constant, and the resistance of one of the other two arms can be varied by connecting an additional resistor.

Under balanced condition, no current flows through the detector. Under such condition, currents in the arms *AB* and *BD* are equal (I_1). Similarly, currents in the arms *AC* and *CD* are equal (I_2). Under balanced condition, since nodes *B* and *D* are at the same potential, voltage drops across arm *BD* and *CD* are equal ($V_3 = V_4$); similarly, voltage drop across arms *AB* and *AC* are equal ($V_1 = V_2$).

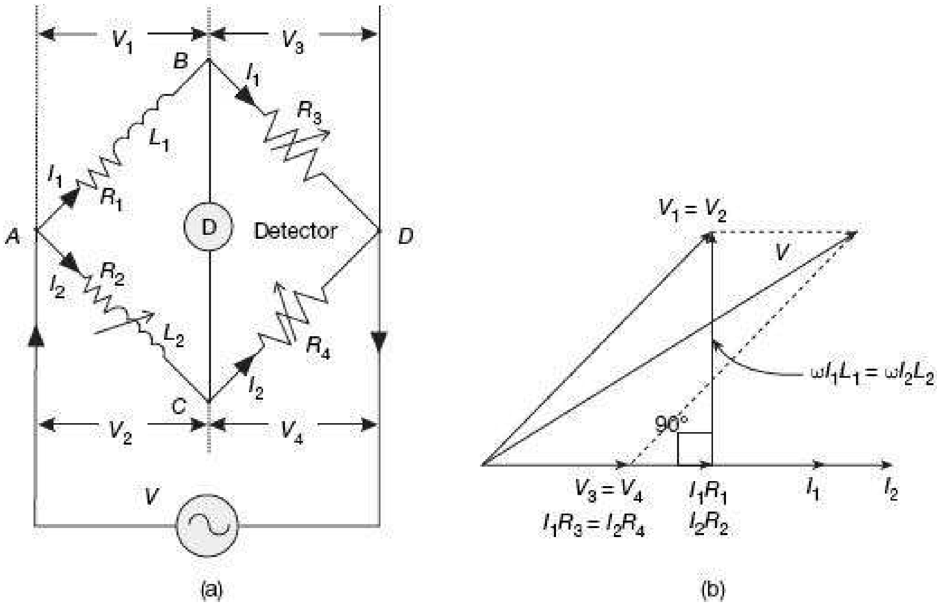


Figure 6.2 *Maxwell's inductance bridge under balanced condition: (a) Configuration (b) Phasor diagram*

As shown in the phasor diagram of Figure 6.2 (b), V_3 and V_4 being equal, they are overlapping. Arms *BD* and *CD* being purely resistive, currents through these arms will be in the same phase with the voltage drops across these two respective branches. Thus, currents I_1 and I_2 will be collinear with the phasors V_3 and V_4. The same current I_1 flows through branch *AB* as well, thus the voltage drop I_1R_1 remains in the same phase as I_1. Voltage drop wI_1L_1 in the inductor L_1 will be 90° out of phase with I_1R_1 as shown in Figure 6.2(b). Phasor summation of these two voltage drops I_1R_1 and wI_1L_1 will give the voltage drop V_1 across the arm *AB*. At balance condition, since voltage across the two branches *AB* and *AC* are equal, thus the two voltage drops V_1 and V_2 are equal and are in the same phase. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2 + j\omega L_2}{R_4}$$

or, $$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

Equating real and imaginary parts, we have

$$R_1R_4 = R_2R_3$$

or, $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

and also, $$j\omega L_1R_4 = j\omega L_2R_3$$

or, $$\frac{L_1}{L_2} = \frac{R_3}{R_4}$$

Thus, $$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L_2}$$

Unknown quantities can hence be calculated as

$$L_1 = L_2 \times \frac{R_3}{R_4} \text{ and } R_1 = R_2 \times \frac{R_3}{R_4} \qquad (6.10)$$

Care must be taken that the inductors L_1 and L_2 must be placed at a distance from each other to avoid effects of mutual inductance.

The final expression (6.10) shows that values of L1 and R1 do not depend on the supply frequency. Thus, this bridge confi guration is immune to frequency variations and even harmonic distortions in the power supply.

6.4.2 Maxwell's Inductance–Capacitance Bridge

In this bridge, the unknown inductance is measured by comparison with a standard variable capacitance. It is much easier to obtain standard values of variable capacitors with acceptable degree of accuracy. This is however, not the case with finding accurate and stable standard value variable inductor as is required in the basic Maxwell's bridge described in Section 6.4.1.

Configuration of a Maxwell's inductance–capacitance bridge and the associated phasor diagram at balanced state are shown in Figure 6.3.

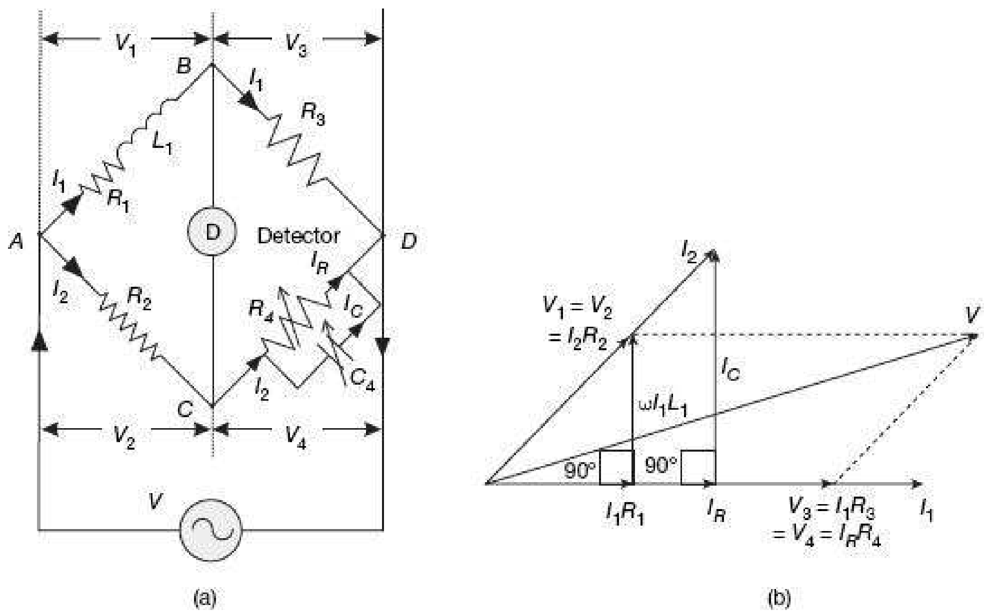


Figure 6.3 Maxwell's inductance–capacitance bridge under balanced condition: (a) Confi guration (b) Phasor diagram

The unknown inductor L_1 of effective resistance R_1 in the branch *AB* is compared with the standard known variable capacitor C_4 on arm *CD*. The other resistances R_2, R_3, and R_4 are known as non– inductive resistors.

The bridge is preferably balanced by varying C_4 and R_4, giving independent adjustment settings.

Under balanced condition, no current flows through the detector. Under such condition, currents in the arms *AB* and *BD* are equal (I_1). Similarly, currents in the arms *AC* and *CD* are equal (I_2). Under balanced condition, since nodes *B* and *D* are at the same potential, voltage drops across arm *BD* and *CD* are equal ($V_3 = V_4$); similarly, voltage drops across arms *AB* and *AC* are equal ($V_1 = V_2$).

As shown in the phasor diagram of Figure 6.3 (b), V_3 and V_4 being equal, they are overlapping both in magnitude and phase. The arm *BD* being purely resistive, current I_1

through this arm will be in the same phase with the voltage drop V_3 across it. Similarly, the voltage drop V_4 across the arm *CD*, current I_R through the resistance R_4 in the same branch, and the resulting resistive voltage drop $I_R R_4$ are all in the same phase [horizontal line in Figure 6.3(b)]. The resistive current I_R when added with the quadrature capacitive current I_C, results in the main current I_2 flowing in the arm *CD*. This current I_2 while flowing through the resistance R_2 in the arm *AC*, produces a voltage drop $V_2 = I_2 R_2$, that is in same phase as I_2. Under balanced condition, voltage drops across arms *AB* and *AC* are equal, i.e., $V_1 = V_2$. This voltage drop across the arm *AB* is actually the phasor summation of voltage drop $I_1 R_1$ across the resistance R_1 and the quadrature voltage drop $wI_1 L_1$ across the unknown inductor L_1. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance,
$$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2}{\left(\dfrac{R_4}{1 + j\omega C_4 R_4}\right)}$$

or,
$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_2 R_3 R_4$$

Equating real and imaginary parts, we have

$$R_1 R_4 = R_2 R_3$$

or,
$$R_1 = R_2 \times \frac{R_3}{R_4}$$

and also,
$$j\omega L_1 R_4 = j\omega C_4 R_2 R_3 R_4$$

or,
$$L_1 = C_4 R_2 R_3$$

Thus, the unknown quantities are

$$L_1 = C_4 R_2 R_3 \text{ and } R_1 = R_2 \times \frac{R_3}{R_4} \tag{6.11}$$

Once again, the final expression (6.11) shows that values of L_1 and R_1 do not depend on the supply frequency. Thus, this bridge confi guration is immune to frequency variations and even harmonic distortions in the power supply.

It is interesting to note that both in the Maxwell's Inductance Bridge and Inductance-Capacitance Bridge, the unknown Inductor L_1 was always associated with a resistance R_1. This series resistance has been included to represent losses that take place in an inductor coil. An ideal inductor will be lossless irrespective of the amount of current flowing through it. However, any real inductor will have some non-zero resistance associated with it due to resistance of the metal wire used to form the inductor winding. This series resistance causes heat generation due to power loss. In such cases, the Quality Factor or the *Q*-Factor of such a lossy inductor is used to indicate how closely the real inductor comes to behave as an ideal inductor. The *Q*-factor of an inductor is defined as the ratio of its inductive reactance to its resistance at a given frequency. *Q*-factor is a measure of the efficiency of the inductor. The higher the value of *Q*-factor, the closer it approaches the behavior of an ideal, loss less inductor. An ideal inductor would have an infinite *Q* at all frequencies.

The Q-factor of an inductor is given by the formula $Q = \frac{\omega L}{R}$, where R is its internal resistance R (series resistance) and wL is its inductive reactance at the frequency ω.

Q-factor of an inductor can be increased by either increasing its inductance value (by using a good ferromagnetic core) or by reducing its winding resistance (by using good quality conductor material, in special cases may be super conductors as well).

In the Maxwell's Inductance-Capacitance Bridge, Q-factor of the inductor under measurement can be found at balance condition to be $Q = \frac{\omega L_1}{R_1}$ or,

$$Q = \frac{\omega C_4 R_2 R_3}{R_2 \times \frac{R_3}{R_4}} = \omega C_4 R_4 \qquad (6.12)$$

The above relation (6.12) for the inductor Q factor indicate that this bridge is not suitable for measurement of inductor values with high Q factors, since in that case, the required value of R_4 for achieving balance becomes impractically high.

Advantages of Maxwell's Bridge

1. The balance equations (6.11) are independent of each other, thus the two variables C_4 and R_4 can be varied independently.
2. Final balance equations are independent of frequency.
3. The unknown quantities can be denoted by simple expressions involving known quantities.
4. Balance equation is independent of losses associated with the inductor.
5. A wide range of inductance at power and audio frequencies can be measured.

Disadvantages of Maxwell's Bridge

1. The bridge, for its operation, requires a standard variable capacitor, which can be very expensive if high accuracies are asked for. In such a case, fixed value capacitors are used and balance is achieved by varying R_4 and R_2.
2. This bridge is limited to measurement of low Q inductors ($1 < Q < 10$).
3. Maxwell's bridge is also unsuited for coils with very low value of Q (e.g., $Q < 1$). Such low Q inductors can be found in inductive resistors and RF coils. Maxwell's bridge finds difficult and laborious to obtain balance while measuring such low Q inductors.

6.4.3 Hay's Bridge

Hay's bridge is a modification of Maxwell's bridge. This method of measurement is particularly suited for high Q inductors.

Configuration of Hay's bridge and the associated phasor diagram under balanced state are shown in Figure 6.4.

The unknown inductor L_1 of effective resistance R_1 in the branch AB is compared with the standard known variable capacitor C_4 on arm *CD*. This bridge uses a resistance R_4 in series with the standard capacitor C_4 (unlike in Maxwell's bride where R_4 was in parallel with C_4). The other resistances R_2 and R_3 are known no-inductive resistors.

The bridge is balanced by varying C_4 and R_4.

Under balanced condition, since no current flows through the detector, nodes *B* and *D* are at the same potential, voltage drops across arm *BD* and *CD* are equal ($V_3 = V_4$); similarly, voltage drops across arms *AB* and *AC* are equal ($V_1 = V_2$).

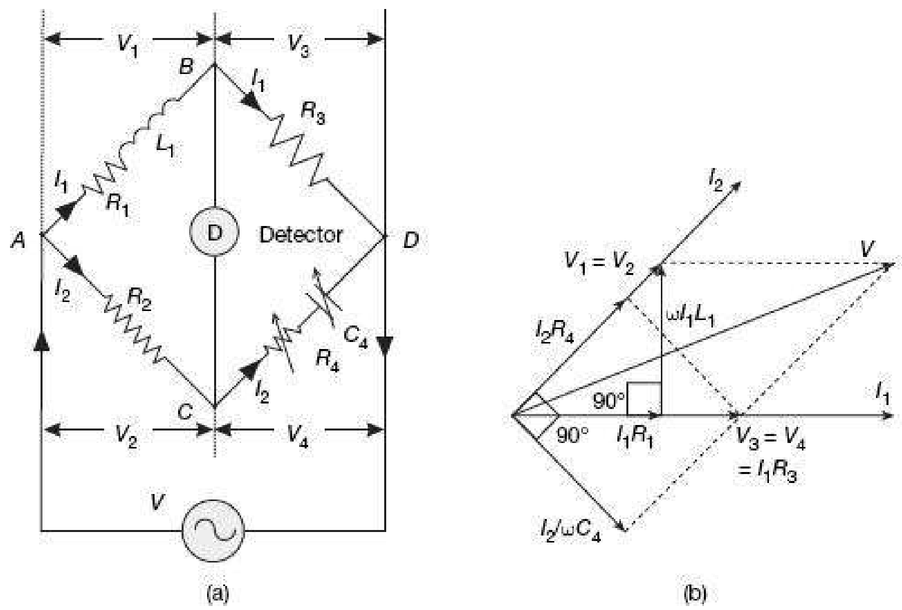


Figure 6.4 *Hay's bridge under balanced condition: (a) Confi guration, (b) Phasor diagram*

As shown in the phasor diagram of Figure 6.4 (b), V_3 and V_4 being equal, they are overlapping both in magnitude and phase and are draw on along the horizontal axis. The arm *BD* being purely resistive, current I_1 through this arm will be in the same phase with the voltage drop $V_3 = I_1R_3$ across it. The same current I_1, while passing through the resistance R_1 in the arm *AB*, produces a voltage drop I_1R_1 that is once again, in the same phase as *I* 1. Total voltage drop *V* 1 across the arm *AB* is obtained by adding the two quadrature phasors *I* 1*R* 1 and *wI* 1*L* 1 representing resistive and inductive voltage drops in the same branch *AB*. Since under balance condition, voltage drops across arms *AB* and *AC* are equal, i.e., (*V* 1 = *V* 2), the two voltages *V* 1 and *V* 2 are overlapping both in magnitude and phase. The branch *AC* being purely resistive, the branch current *I* 2 and branch voltage *V* 2 will be in the same phase as shown in the phasor diagram of Figure 6.4 (b). The same current *I* 2 fl ows through the arm *CD* and produces a voltage drop *I* 2*R* 4 across the resistance *R* 4. This resistive voltage drop *I* 2*R* 4, obviously is in the same phase as *I* 2. The capacitive voltage drop *I* 2/*wC* 4 in the capacitance *C* 4 present in the same arm *AC* will however, lag the current *I* 2 by 90°. Phasor summation of these two series voltage drops across *R* 4 and *C* 4 will give the total voltage drop *V* 4 across the arm *CD*. Finally, phasor summation of *V* 1 and *V* 3 (or *V* 2 and *V* 4) results in the supply voltage *V*.

At balance, $$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2}{\left(R_4 - \frac{j}{\omega C_4}\right)}$$

or, $$R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2 R_3$$

Equating real and imaginary parts, we have

Equating real and imaginary parts, we have

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \tag{6.13}$$

and $$\omega L_1 R_4 = \frac{R_1}{\omega C_4} \tag{6.14}$$

Solving Eqs (6.13) and (6.14) we have the unknown quantities as

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2} \tag{6.15}$$

and

$$R_1 = \frac{R_2 R_3 R_4 \omega^2 C_4^2}{1 + \omega^2 R_4^2 C_4^2} \tag{6.16}$$

Q factor of the inductor in this case can be calculated at balance condition as

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4} \tag{6.17}$$

Hay's bridge is more suitable for measurement of unknown inductors having Q factor more than 10. In those cases, bridge balance can be attained by varying R_2 only, without losing much accuracy.

From (6.15) and (6.17), the unknown inductance value can be written as

$$L_1 = \frac{R_2 R_3 C_4}{1 + (1/Q)^2} \tag{6.18}$$

For inductors with Q > 10, the quantity $(1/Q)^2$ will be less than 1/100, and thus can be neglected from the denominator of (6.18). In such a case, the inductor value can be simplifi ed to L1 = R2R3C4, which essentially is the same as obtained in Maxwell's bridge.

6.4.4 Anderson's Bridge

This method is a modification of Maxwell's inductance–capacitance bridge, in which value of the unknown inductor is expressed in terms of a standard known capacitor. This method is applicable for precise measurement of inductances over a wide range of values.

Figure 6.5 shows Anderson's bridge configuration and corresponding phasor diagram under balanced condition.

The unknown inductor L_1 of effective resistance R_1 in the branch *AB* is compared with the standard known capacitor *C* on arm *ED*. The bridge is balanced by varying *r*.

Under balanced condition, since no current flows through the detector, nodes *B* and *E* are at the same potential.

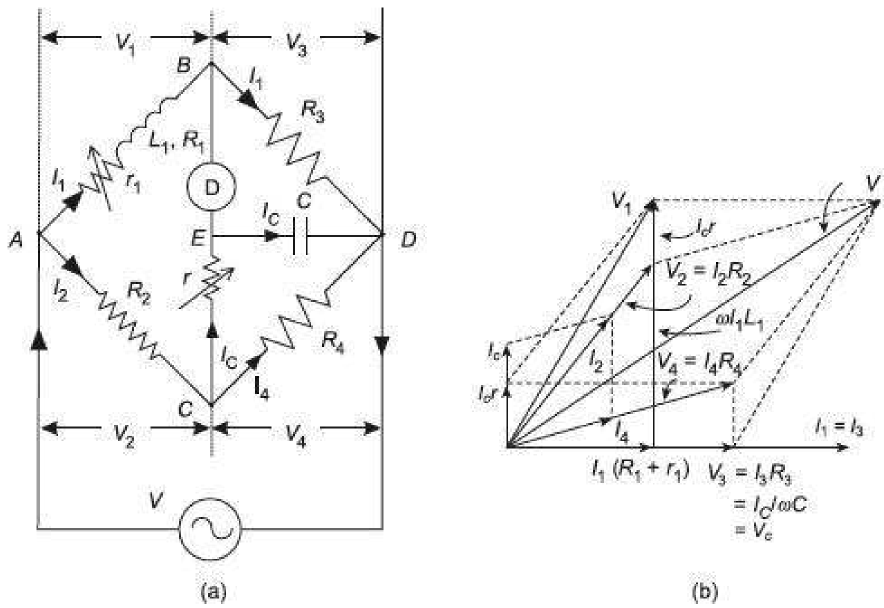


Figure 6.5 *Anderson's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

As shown in the phasor diagram of Figure 6.5 (b), I_1 and $V_3 = I_1R_3$ are in the same phase along the horizontal axis. Since under balance condition, voltage drops across arms *BD* and *ED* are equal, $V_3 = I_1R_3 = I_C/wC$ and all the three phasors are in the same phase. The same current I_1, when flowing through the arm *AB* produces a voltage drop $I_1(R_1 + r_1)$ which is once again, in phase with I_1. Since under balanced condition, no current flows through the detector, the same current I_C flows through the resistance *r* in arm *CE* and then through the capacitor *C* in the arm *ED*. Phasor summation of the voltage drops I_Cr in arm the *CE* and I_C/wC in the arm *ED* will be equal to the voltage drop V_4 across the arm *CD*. V_4 being the voltage drop in the resistance R_4 on the arm *CD*, the current I_4 and V_4 will be in the same phase. As can be seen from the Anderson's bridge circuit, and also plotted in the phasor diagram, phasor summation of the currents I_4 in the arm *CD* and the current I_C in the arm *CE* will give rise to the current I_2 in the arm *AC*. This current I_2, while passing through the resistance R_2 will give rise to a voltage drop $V_2 = I_2R_2$ across the arm *AC* that is in phase with the current I_2. Since, under balance, potentials at nodes *B* and *E* are the same, voltage drops between nodes *A* -*B* and between *A* -*C* -*E* will be equal. Thus, phasor summation of the voltage drop $V_2 = I_2R_2$ in the arm *AC* $I_C\,r$ in arm the *CE* will build up to the voltage V_1 across the arm *AB*. The voltage V_1 can also be obtained by adding the resistive voltage drop $I_1(R_1 + r_1)$ with the quadrature inductive voltage drop wI_1L_1 in the arm *AB*. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $I_2 = I_C + I_4$

and, $$V_{BD} = V_{ED}, \text{ or } I_1R_3 = I_C \times \frac{1}{j\omega C}$$

$\therefore$ $$I_C = j\omega I_1R_3C \tag{6.19}$$

The other balance equations are:

$$V_{AB} = V_{AC} + V_{CE}, \text{ or } I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + I_Cr \tag{6.20}$$

and, $V_{CD} = V_{CE} + V_{ED}$, or $I_C\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_C)R_4$ (6.21)

Putting the value of I_C from Eq. (6.19) in Eq. (6.20), we have:*Ir (+R +jwL) =IR +jwI RCr*

$$I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + j\omega I_1R_3Cr$$

or, $$I_1(r_1 + R_1 + j\omega L_1 - j\omega R_3Cr) = I_2R_2 \quad (6.22)$$

Then, putting the value of I_C from Eq. (6.19) in Eq. (6.21), we have:Ê1 ˆ

$$j\omega I_1R_3C\left(r + \frac{1}{j\omega C}\right) = (I_2 - j\omega I_1R_3C)R_4$$

or, $$I_1(j\omega R_3Cr + R_3 + j\omega R_3CR_4) = I_2R_4 \quad (6.23)$$

From Eqs (6.22) and (6.23), we obtain:

From Eqs (6.22) and (6.23), we obtain:

$$I_1(r_1 + R_1 + j\omega L_1 - j\omega R_3Cr) = I_1(j\omega R_3Cr + R_3 + j\omega R_3CR_4)\frac{R_2}{R_4}$$

Equating real and imaginary parts, we get

$$R_1 = \frac{R_2R_3}{R_4} - r_1 \quad (6.24)$$

and, $$L_1 = C\frac{R_3}{R_4}\left[r(R_2 + R_4) + R_2R_4\right] \quad (6.25)$$

The advantage of Anderson’s bridge over Maxwell’s bride is that in this case a fixed value capacitor is used thereby greatly reducing the cost. This however, is at the expense of connection complexities and balance equations becoming tedious.

6.4.5 Owen’s Bridge

This bridge is used for measurement of unknown inductance in terms of known value capacitance.

Figure 6.6 shows the Owen’s bridge configuration and corresponding phasor diagram under balanced condition.

The unknown inductor L_1 of effective resistance R_1 in the branch *AB* is compared with the standard known capacitor C_2 on arm *AC*. The bridge is balanced by varying R_2 and C_2 independently.

Under balanced condition, since no current flows through the detector, nodes *B* and *C* are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.5 (b), I_1, $V_3 = I_1R_3$ and $V_4 = I_2/wC_4$ are all in the same phase along the horizontal axis. The resistive voltage drop I_1R_1 in the arm *AB* is also in the same phase

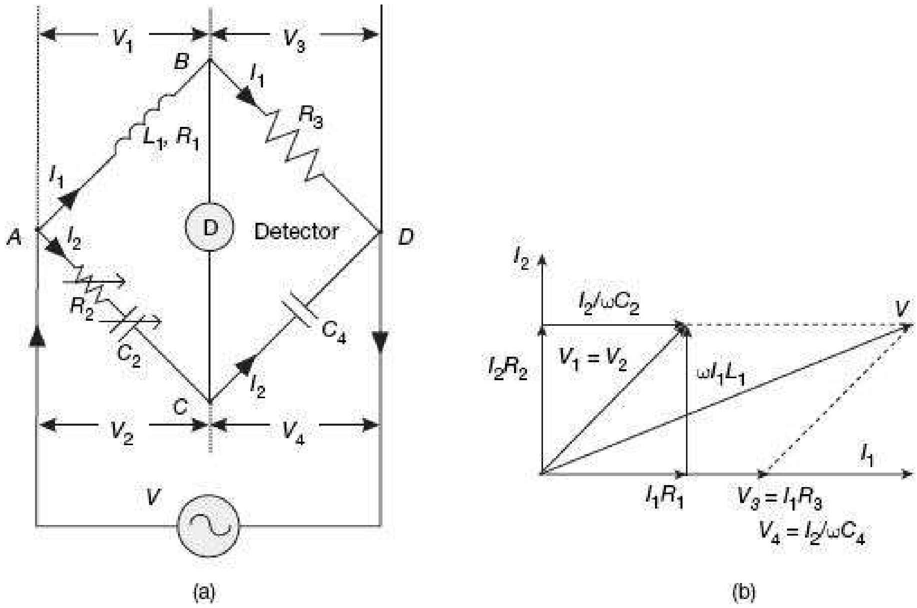


Figure 6.6 *Owen's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

with I_1. The inductive voltage drop wI_1L_1 when added in quadrature with the resistive voltage drop I_1R_1 gives the total voltage drop V_1 across the arm *AB*. Under balance condition, voltage drops across arms *AB* and *AC* being equal, the voltages V_1 and V_2 coincide with each other as shown in the phasor diagram of Figure 6.6 (b). The voltage V_2 is once again summation of two mutually quadrature voltage drops I_2R_2 (resistive) and I_2/wC_2 (capacitive) in the arm *AC*. It is to be noted here that the current I_2 leads the voltage V_4 by 90° due to presence of the capacitor C_4. This makes I_2 and hence I_2R_2 to be vertical, as shown in the phasor diagram. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

$$\text{At balance,}\quad \frac{(R_1 + j\omega L_1)}{R_3} = \frac{\left(R_2 + \dfrac{1}{j\omega C_2}\right)}{\dfrac{1}{j\omega C_4}}$$

Simplifying and separating real and imaginary parts, the unknown quantities can be found out as

$$R_1 = R_3 \frac{C_4}{C_2} \tag{6.26}$$

and

$$L_1 = R_2 R_3 C_4 \tag{6.27}$$

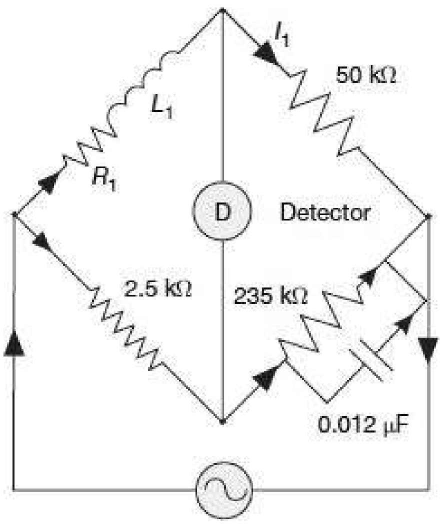
It is thus possible to have two independent variables C_2 and R_2 for obtaining balance in Owen's bridge. The balance equations are also quite simple. This however, does come with additional cost for the variable capacitor

Example 6.2

A Maxwell's inductance–capacitance bridge is used to measure a unknown inductive impedance. The bridge constants at bridge balance are: Pure resistance arms = 2.5 *kΩ and 50 kΩ. In between these two resistors, the third arm has a capacitor of value 0.012 μF in series with a resistor of value 235 kΩ. Find the series equivalent of the unknown impedance.*

Solution Referring to the diagram of a Maxwell's inductance–capacitance bridge:

Using the balance equation,



$$L_1 = C_4 R_2 R_3 = 0.012 \times 10^{-6} \times 2.5 \times 10^3 \times 50 \times 10^3 = 1.5 \text{ H}$$

$$\text{and } R_1 = R_2 \times \frac{R_3}{R_4} = 2.5 \times 10^3 \times \frac{50 \times 10^3}{235 \times 10^3} = 0.53 \text{ k}\Omega$$

Example 6.3 *The four arms of a bridge are connected as follows:*

Arm AB: A choke coil L_1 with an equivalent series resistance r_1

Arm BC: A noninductive resistance R_3

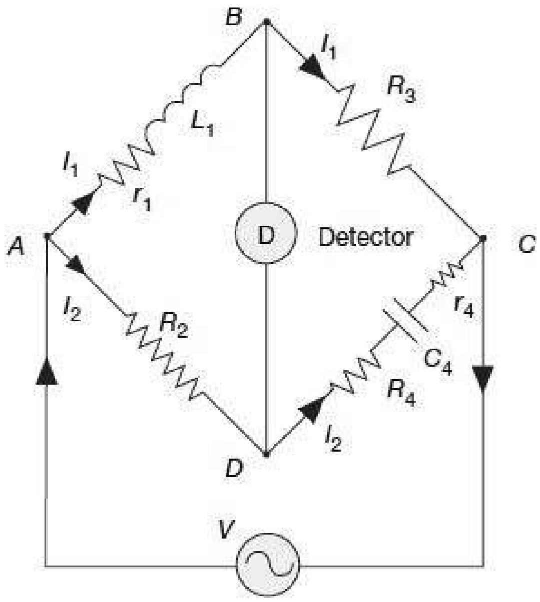
Arm CD: A mica capacitor C_4 in series a noninductive resistance R_4

Arm DA: A noninductive resistance R_2

When the bridge is supplied from a source of 450 Hz is given between terminals A and C and the detector is connected between nodes B and D, balance is obtained the following conditions: R_2 = 2400 Ω, R_3 = 600 Ω, C_4 = 0.3 μF and R_4 = 55.4 Ω. Series resistance of the capacitor is 0.5 Ω. Calculate the resistance and inductance of the choke coil.

Solution The bridge configuration is shown below:

Given that at balance,



$R_2 = 2400\ \Omega$, $R_3 = 600\ \Omega$, $C_4 = 0.3\ \mu F$, $R_4 = 55.4\ \Omega$ and $r_4 = 0.5\ \Omega$.

At balance, $\dfrac{r_1 + j\omega L_1}{R_3} = \dfrac{R_2}{r_4 + R_4 - \dfrac{j}{\omega C_4}}$

or,

$$r_1 r_4 + r_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 + j\omega L_1 r_4 - \frac{jr_1}{\omega C_4} = R_2 R_3$$

Equating real and imaginary parts, we have

$$r_1 r_4 + r_1 R_4 \frac{L_1}{C_4} = R_2 R_3 \qquad \text{(i)}$$

and $\omega L_1 R_4 + \omega L_1 r_4 = \dfrac{r_1}{\omega C_4}$ (ii)

Solving (i) and (ii), we have the unknown quantities as

$$L_1 = \frac{R_2 R_3 C_4}{1+\omega(R_4+r_4)^2 C_4^2} = \frac{2400\times 600\times 0.3\times 10^{-6}}{1+(2\pi\times 450\times 55.9\times 0.3\times 10^{-6})^2} = 0.43\ \text{H}$$

and

$$r_1 = \frac{R_2 R_3 (R_4+r_4)\omega^2 C_4^2}{1+\omega(R_4+r_4)^2 C_4^2} = \frac{2400\times 600\times 55.9\times(2\pi\times 450)^2\times(0.3\times 10^{-6})^2}{1+(2\pi\times 450\times 55.9\times 0.3\times 10^{-6})^2} = 57.8\ \Omega$$

6.5 MEASUREMENT OF CAPACITANCE

Bridges are used to make precise measurements of unknown capacitances and associated losses in terms of some known external capacitances and resistances. An ideal capacitor is formed by placing a piece of dielectric material between two conducting plates or electrodes. In practical cases, this dielectric material will have some power losses in it due to dielectric's conduction electrons and also due to dipole relaxation phenomena. Thus, whereas an ideal capacitor will not have any losses, a real capacitor will have some losses associated with its operation. The potential energy across a capacitor is thus dissipated in all real capacitors as heat loss inside its dielectric material. This loss is equivalently represented by a series resistance, called the equivalent series resistance (ESR). In a good capacitor, the ESR is very small, whereas in a poor capacitor the ESR is large. *C* real *C* ideal *ESR*

A real, lossy capacitor can thus be equivalently represented by an ideal loss less capacitor in series *CR* with its equivalent series resistance (ESR) shown Figure 6.7 *Equivalent series resistance (ESR)* in Figure 6.7.

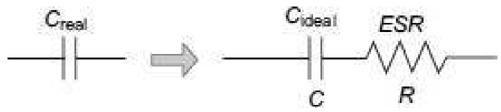


Figure 6.7 *Equivalent series resistance (ESR)*

The quantifying parameters often used to describe performance of a capacitor are ESR, its dissipation factor (DF), Quality Factor (*Q*-factor) and Loss Tangent (tan *d*). Not only that these parameters describe operation of the capacitor in radio frequency (RF) applications, but ESR and DF are also particularly important for capacitors operating in power supplies where a large dissipation factor will result in large amount of power being wasted in the capacitor. Capacitors with high values of ESR will need to dissipate large amount of heat. Proper circuit design needs to be practiced so as to take care of such possibilities of heat generation.

Dissipation factor due to the non-ideal capacitor is defined as the ratio of the resistive power loss in the ESR to the reactive power oscillating in the capacitor, or

$$DF = \frac{i^2 R}{i^2 X_C} = \frac{R}{1/\omega C} = \omega CR$$

When representing the electrical circuit parameters as phasors, a capacitor's dissipation factor is equal to the tangent of the angle between the capacitor's impedance phasor and the negative reactive axis, as shown in the *impedance triangle diagram* of Figure 6.8 This gives rise to the parameter known as the loss tangent *d* where Figure. 6.8 *Impedance*

$$\tan\delta = \frac{R}{X_C} = \frac{R}{1/\omega C} = \omega CR$$

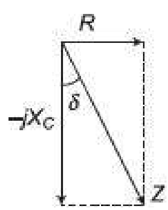


Figure. 6.8 *Impedance triangle diagram*

Loss tangent of a real capacitor can also be defined in the *voltage triangle diagram* of Figure 6.9 as the ratio of voltage drop across the *ESR* to the voltage drop across the capacitor only, i.e. tangent of the angle between the capacitor voltage only and the total voltage drop across the combination of capacitor and *ESR*.

$$\tan\delta = \frac{V_R}{V_C} = \frac{iR}{iX_C} = \frac{R}{1/\omega C} = \omega CR$$

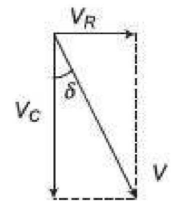


Figure. 6.9 *Voltage triangle diagram*

Though the expressions for dissipation factor (*DF*) and loss tangent (tan δ) are the same, normally the dissipation factor is used at lower frequencies, whereas the loss tangent is more applicable for high frequency applications. A good capacitor will normally have low values of dissipation factor (*DF*) and loss tangent (tan δ).

In addition to *ESR*, *DF* and loss tangent, the other parameter used to quantify performance of a real capacitor is its Quality Factor or *Q*-Factor. Essentially for a capacitor it is the ratio of the energy stored to that dissipated per cycle.

$$Q = \frac{i^2 X_C}{i^2 R} = \frac{X_C}{R}$$

It can thus be deduced that the *Q* can be expressed as the ratio of the capacitive reactance to the *ESR* at the frequency of interest.

$$Q = \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{1}{DF} = \frac{1}{\tan\delta}$$

A high quality capacitor (high *Q*-factor) will thus have low values of dissipation factor (*DF*) and loss tangent (tan δ), i.e. less losses.

The most commonly used bridges for capacitance measurement are De Sauty's bridge and Schering Bridge.

6.5.1 De Sauty's Bridge

This is the simplest method of finding out the value of a unknown capacitor in terms of a known standard capacitor. Configuration and phasor diagram of a De Sauty's bridge under balanced condition is shown in Figure 6.10.

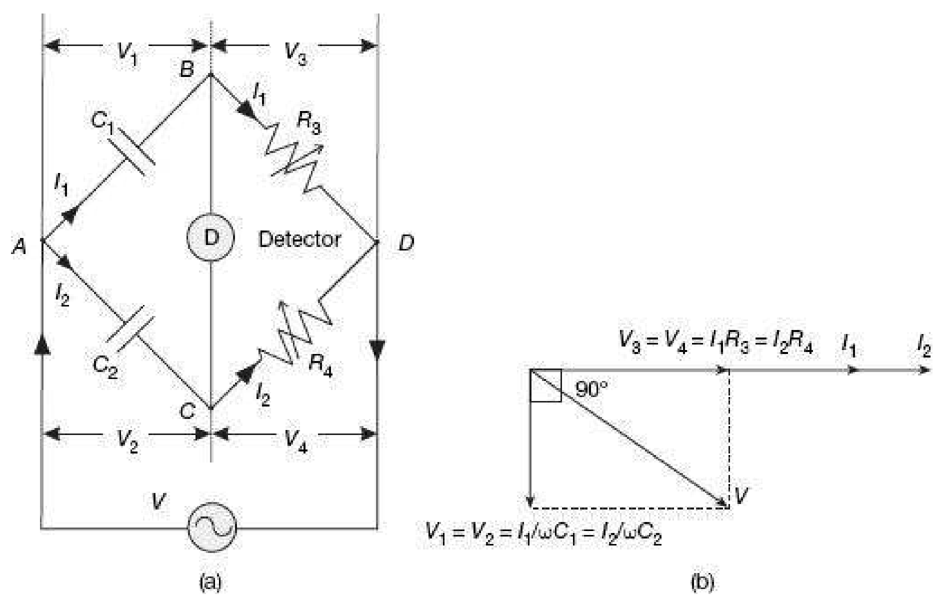


Figure 6.10 *De Sauty's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

The unknown capacitor C_1 in the branch AB is compared with the standard known standard capacitor C_2 on arm AC. The bridge can be balanced by varying either of the non-inductive resistors R_3 or R_4.

Under balanced condition, since no current flows through the detector, nodes B and C are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.7 (b), $V_3 = I_1R_3$ and $V_4 = I_2R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm BD and I_2 in the arm CD are also in the same phase with I_1R_3 and I_2R_4 along the horizontal line. Capacitive voltage drop $V_1 = I_1/wC_1$ in the arm AB lags behind I_1 by 90°. Similarly, the other capacitive voltage drop $V_2 = I_2/wC_2$ in the arm AC lags behind I_2 by 90°. Under balanced condition, these two voltage drops V_1 and V_2 being equal in magnitude and phase, they overlap each other along the vertical axis as shown in Figure 6.7 (b). Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage V.

$$\text{At balance, } \frac{\left(\frac{1}{j\omega C_1}\right)}{R_3} = \frac{\left(\frac{1}{j\omega C_2}\right)}{R_4}$$

$$\text{or, } \quad C_1 = C_2 \frac{R_4}{R_3} \tag{6.28}$$

The advantage of De Sauty's bridge is its simplicity. However, this advantage may be nullifi ed by impurities creeping in the measurement if the capacitors are not free from dielectric losses. This method is thus best suited for loss-less air capacitors.

In order to make measurement in capacitors having inherent dielectric losses, the *modified De Sauty's bridge* as suggested by Grover, can be used. This bridge is also called the series resistance-capacitance bridge. Configuration of such a bridge and its corresponding phasor diagram under balanced condition is shown in Figure 6.11.

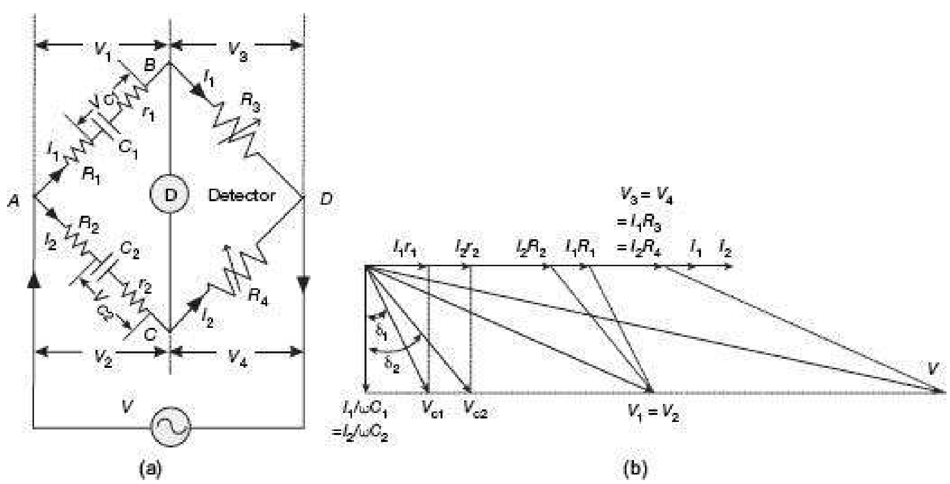


Figure 6.11 *Modified De Sauty's bridge under balanced condition: (a) Configuration, and (b) Phasor diagram*

The unknown capacitor C_1 with internal resistance r_1 representing losses in the branch AB is compared with the standard known standard capacitor C_2 along with its internal resistance r_2 on arm AC. Resistors R_1 and R_2 are connected externally in series with C_1

and C_2 respectively. The bridge can be balanced by varying either of the non-inductive resistors R_3 or R_4.

Under balanced condition, since no current flows through the detector, nodes B and C are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.11 (b), $V_3 = I_1R_3$ and $V_4 = I_2R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm BD and I_2 in the arm CD are also in the same phase with I_1R_3 and I_2R_4 along the horizontal line. The other resistive drops, namely, I_1R_1 in the arm AB and I_2R_2 in the arm AC are also along the same horizontal line. Finally, resistive drops inside the capacitors, namely, I_1r_1 and I_2r_2 are once again, in the same phase, along the horizontal line. Capacitive voltage drops I_1/wC_1 lags behind I_1r_1 by 90°. Similarly, the other capacitive voltage drop I_2/wC_2 lags behind I_2r_2 by 90°. Phasor summation of the resistive drop I_1r_1 and the quadrature capacitive drop I_1/wC_1 produces the total voltage drop V_{C1} across the series combination of capacitor C_1 and its internal resistance r_1. Similarly, phasor summation of the resistive drop I_2r_2 and the quadrature capacitive drop I_2/wC_2 produces the total voltage drop V_{C2} across the series combination of capacitor C_2 and its internal resistance r_2. d_1 and d_2 represent loss angles for capacitors C_1 and C_2 respectively. Phasor summation of I_1R_1 and V_{C1} gives the total voltage drop V_1 across the branch AB. Similarly, phasor summation of I_2R_2 and V_{C2} gives the total voltage drop V_2 across the branch AC. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage V.

At balance, $$\frac{\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right)}{R_3} = \frac{\left(R_2 + r_2 + \frac{1}{j\omega C_2}\right)}{R_4}$$

or, $$R_1R_4 + r_1R_4 - \frac{jR_4}{\omega C_1} = R_2R_3 + r_2R_3 - \frac{jR_3}{\omega C_2}$$

Equating real and imaginary parts, we have

$$\frac{C_1}{C_2} = \frac{(R_2 + r_2)}{(R_1 + r_1)} = \frac{R_4}{R_3} \tag{6.29}$$

$$\therefore \quad C_1 = C_2 \frac{R_4}{R_3} \tag{6.30}$$

The modified De Sauty's bridge can also be used to estimate dissipation factor for the unknown capacitor as described below:

Dissipation factor for the capacitors are defined as

$$D_1 = \tan\delta_1 = \frac{I_1r_1}{\frac{I_1}{\omega C_1}} = \omega C_1 r_1 \text{ and } D_2 = \tan\delta_2 = \frac{I_2r_2}{\frac{I_2}{\omega C_2}} = \omega C_2 r_2 \tag{6.31}$$

From Eq. (6.29), we have

$$\frac{C_1}{C_2} = \frac{(R_2 + r_2)}{(R_1 + r_1)}$$

or,
$$C_2 r_2 - C_1 r_1 = C_1 R_1 - C_2 R_2$$

or,
$$\omega C_2 r_2 - \omega C_1 r_1 = \omega C_1 R_1 - C_2 R_2$$

Using Eq. (6.31), we getor, *D* -*D* =*w* (*CR* -*CR*)

or,
$$D_2 - D_1 = \omega(C_1 R_1 - C_2 R_2)$$

Substituting the value of C_1 from Eq. (6.30), we have

$$D_2 - D_1 = \omega C_2 \left(\frac{R_1 R_4}{R_3} - R_2 \right) \quad (6.32)$$

Thus, dissipation factor for one capacitor can be estimated if dissipation factor of the other capacitor is known.

6.5.2 Schering Bridge

Schering bridges are most popularly used these days in industries for measurement of capacitance, dissipation factor, and loss angles. Figure 6.12 illustrates the confi guration of a Schering bridge and corresponding phasor diagram under balanced condition.

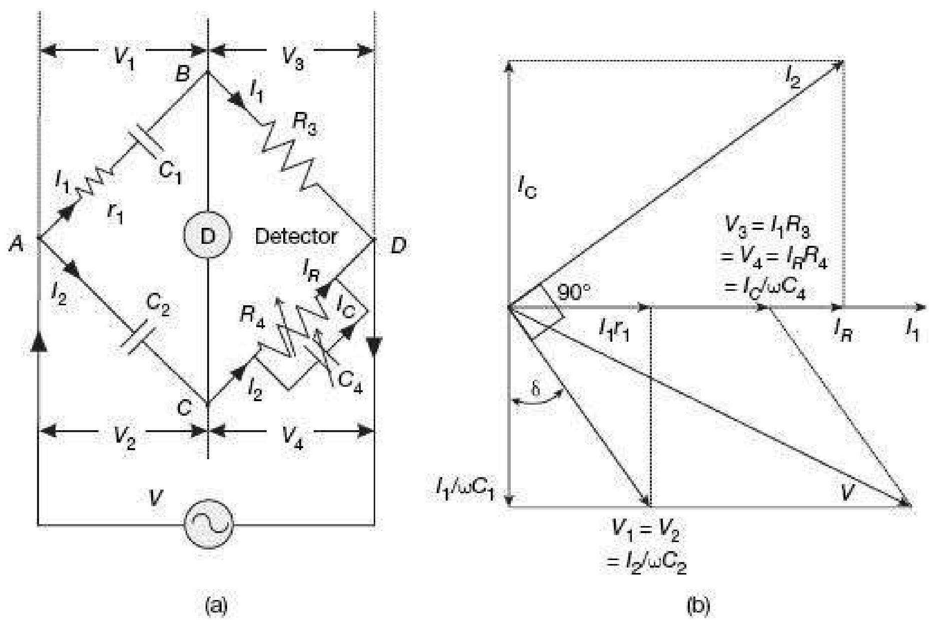


Figure 6.12 *Schering bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

The unknown capacitor C_1 along with its internal resistance r_1 (representing loss) placed on the arm *AB* is compared with the standard loss-less capacitor C_2 placed on the arm *AC*. This capacitor C_2 is either an air or a gas capacitor to make it loss free. R_3 is a non-inductive resistance placed on arm *BD*. The bridge is balanced by varying the capacitor C_4 and the non-inductive resistor R_4 parallel with C_4, placed on arm *CD*.

Under balanced condition, since no current flows through the detector, nodes *B* and *C* are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.12 (b), $V_3 = I_1 R_3$ and $V_4 = I_R R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm *BD* and I_R flowing through R_4 are also in the same phase with I_1R_3 and I_RR_4 along the horizontal line. The other resistive drop namely, I_1R_1 in the arm *AB* is also along the same horizontal line. The resistive current I_R through R_4 and the quadrature capacitive current I_C through C_4 will add up to the total current I_2 in the branch *CD* (and also in *A* C under balanced condition). Across the arm *AB*, the resistive drop I_1r_1 and the quadrature capacitive drop I_1/wC_1 will add up to the total voltage drop V_1 across the arm. At balance, voltage drop V_1 across arm *AB* will be same as the voltage drop $V_2 = I_2/wC_2$ across the arm *AC*. It can be confirmed from the phasor diagram in Figure 6.12(b) that the current I_2 has quadrature phase relationship with the capacitive voltage drop I_2/wC_2 in the arm *AC*. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $$\frac{\left(r_1+\frac{1}{j\omega C_1}\right)}{R_3}=\frac{\left(\frac{1}{j\omega C_2}\right)}{\left(\frac{R_4}{1+j\omega C_4R_4}\right)}$$

or, $$R_4\left(r_1+\frac{1}{j\omega C_1}\right)=\left(\frac{R_3}{j\omega C_2}\right)(1+j\omega C_4R_4)$$

or, $$R_4r_1-\frac{jR_4}{\omega C_1}=\frac{R_3R_4C_4}{C_2}-\frac{jR_3}{\omega C_2}$$

Equating real and imaginary parts, we have the unknown quantities:

$$r_1=\frac{R_3C_4}{C_2} \qquad (6.33)$$

and

$$C_1=C_2\frac{R_4}{R_3} \qquad (6.34)$$

Dissipation Factor

$$D_1=\tan\,\delta_1=\frac{I_1r_1}{\frac{I_1}{\omega C_1}}=\omega C_1r_1=\omega\times C_2\frac{R_4}{R_3}\times\frac{R_3C_4}{C_2}=\omega R_4C_4 \qquad (6.35)$$

Thus, using Schering bridge, dissipation factor can be obtained in terms of the bridge parameters at balance condition.

6.6 MEASUREMENT OF FREQUENCY

6.6.1 Wien's Bridge

Wien's bridge is primarily used for determination of an unknown frequency. However, it can be used for various other applications including capacitance measurement, in harmonic distortion analysers, where it is used as notch filter, and also in audio and HF oscillators.

Configuration of a Wien's bridge for determination of unknown frequency and corresponding phasor diagram under balanced condition is shown in Figure 6.13.

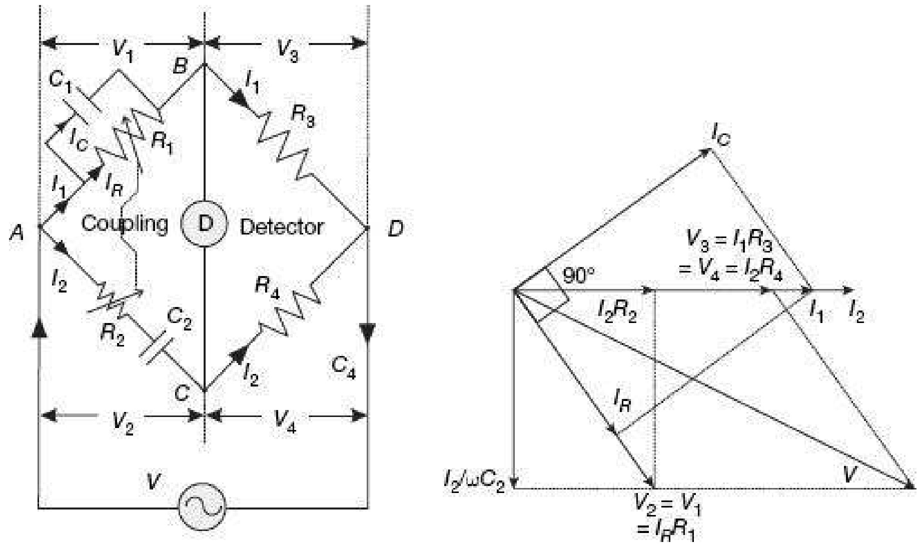


Figure 6.13 *Wien's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

Under balanced condition, since no current fl ows through the detector, nodes B and C are at the same potential, i.e., $V_1=V_2$ and $V_3=V_4$.

As shown in the phasor diagram of Figure 6.13 (b), $V_3 = I_1R_3$ and $V_4 = I_2R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm BD and I_2 flowing through R4 are also in the same phase with I_1R_3 and I_2R_4 along the horizontal line. The other resistive drop, namely, I_2R_2 in the arm *AC* is also along the same horizontal line. The resistive voltage drop I_RR_2 across R_2 and the quadrature capacitive drop I_2/wC_2 across C_2 will add up to the total voltage drop V_2 in the arm *AC*. Under balanced condition, voltage drops across arms *AB* and *AC* are equal, thus $V_1 = V_2$ both in magnitude and phase. The voltage V_1 will be in the same phase as the voltage drop I_RR_1 across the resistance R_1 in the same arm *AB*. The resistive current I_R will thus be in the same phase as the voltage $V_1 = I_RR_1$. Phasor addition of the resistive current I_R and the quadrature capacitive current I_C, which flows through the parallel R_1C_1 branch, will add up to the total current I_1 in the arm *AB*. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $$\frac{\left(\dfrac{R_1}{1+j\omega C_1R_1}\right)}{R_3} = \frac{\left(R_2 - \dfrac{j}{\omega C_2}\right)}{R_4}$$

or, $$\frac{R_1R_4}{1+j\omega C_1R_1} = \frac{\omega C_2R_2R_3 - jR_3}{\omega C_2}$$

or, $$\omega C_2R_1R_4 = \omega C_2R_2R_3 - jR_3 + j\omega^2C_1C_2R_1R_2R_3 + \omega C_1R_1R_3$$

or, $$\omega(C_2R_1R_4) = \omega(C_2R_2R_3 + C_1R_1R_3) - j(R_3 - \omega^2C_1C_2R_1R_2R_3)$$

Equating real and imaginary parts, we get

$$C_2R_1R_4 = C_2R_2R_3 + C_1R_1R_3$$

$$\text{or,} \quad \frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} \tag{6.36}$$

and

$$\omega^2 C_1 C_2 R_1 R_2 R_3 = R_3$$

$$\text{or,} \quad \omega = \sqrt{\frac{1}{C_1 C_2 R_1 R_2}}$$

$$\text{or,} \quad f = \frac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}} \tag{6.37}$$

In most bridges, the parameters are so chosen that,

$$R_1 = R_2 = R \text{ and } C_1 = C_2 = C$$

Then, from Eq. (6.37), we get

$$f = \frac{1}{2\pi RC} \tag{6.38}$$

Sliders for the resistors R_1 and R_2 are mechanically coupled to satisfy the criteria $R_1 = R_2$.

Wien's bridge is frequency sensitive. Thus, unless the supply voltage is purely sinusoidal, achieving balance may be troublesome, since harmonics may disturb balance condition. Use of filters with the null detector in such cases may solve the problem.

Example 6.4 *The four arms of a bridge are connected as follows:*

Arm AB: A capacitor C_1 with an equivalent series resistance r_1

Arm BC: A noninductive resistance R_3

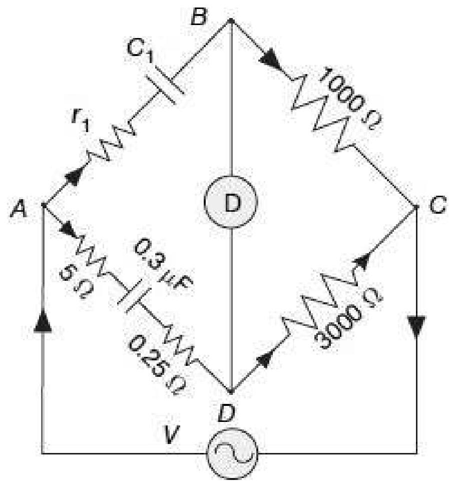
Arm CD: A noninductive resistance R_4

Arm DA: A capacitor C_2 with an equivalent series resistance r_2 in series with a resistance R_2

A supply of 500 Hz is given between terminals A and C and the detector is connected between nodes B and D. At balance, $R_2 = 5\ \Omega$, $R_3 = 1000\ \Omega$, $R_4 = 3000\ \Omega$, $C_2 = 0.3\ \mu F$ and $r_2 = 0.25\ \Omega$. Calculate the values of C_1 and r_1, and also dissipation factor of the capacitor.

Solution The configuration can be shown as

$$\text{At balance,} \quad \frac{r_1 + \frac{1}{j\omega C_1}}{1000} = \frac{5.25 + \frac{1}{j2\pi \times 500 \times 0.3 \times 10^{-6}}}{3000}$$



or, $$3r_1 + \frac{3}{j\omega C_1} = 5.25 + \frac{1}{j2\pi \times 500 \times 0.3 \times 10^{-6}}$$

Equating real and imaginary terms, we get

$$r_1 = \frac{5.25}{3} = 1.75\ \Omega$$

and, $$\frac{3}{j2\pi \times 500 \times C_1} = \frac{1}{j2\pi \times 500 \times 0.3 \times 10^{-6}}$$

or, $$C_1 = 3 \times 0.3 \times 10^{-6} = 0.9\ \mu\text{F}$$

Example 6.5 *The four arms of a bridge supplied from a sinusoidal source are configured as follows:*

Arm AB: A resistance of 100 Ω in parallel with a capacitance of 0.5 μF

Arm BC: A 200 Ω noninductive resistance

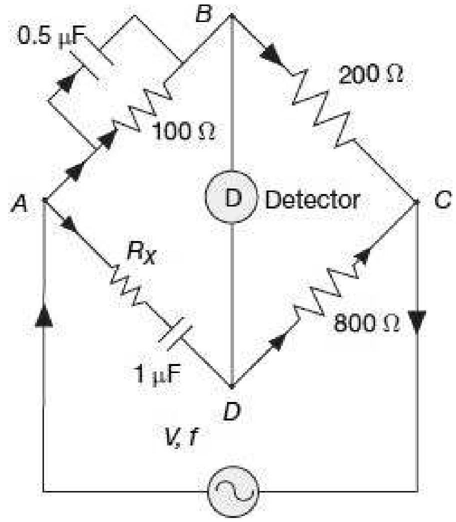
Arm CD: A 800 Ω noninductive resistance

Arm DA: A resistance R_x in series with a 1 μF capacitance

Determine the value of R_x and the frequency at which the bridge will balance.

Supply is given between terminals A and C and the detector is connected between nodes B and D.

Solution The configuration can be shown as



The configuration shows that it is a Wien's bridge. Thus, following Eq. (6.36), the balance equation can be written as

$$\frac{R_4}{R_3} = \frac{R_x}{R_1} + \frac{C_1}{C_2}$$

Thus,
$$R_x = R_1 \times \left(\frac{R_4}{R_3} - \frac{C_1}{C_2} \right) = 100 \times \left(\frac{800}{200} - \frac{0.5 \times 10^{-6}}{1 \times 10^{-6}} \right) = 350\ \Omega$$

The frequency at which bridge is balanced is given by Eq. (6.37):

$$f = \frac{1}{2\pi\sqrt{0.5 \times 10^{-6} \times 1 \times 10^{-6} \times 100 \times 350}} = 1203\ \text{Hz}$$

6.7 WAGNER EARTHING DEVICE

A serious problem encountered in sensitive ac bridge circuits is that due to stray capacitances. Stray capacitances may be formed in an ac bridge between various junction points within the bridge configuration and nearest ground (earthed) object. These stray capacitors affect bridge balance in severe ways since these capacitors carry leakage current when the bridge is operated with ac, especially at high frequencies. Formation of such stray capacitors in a simple ac bridge circuit is schematically shown in Figure 6.14.

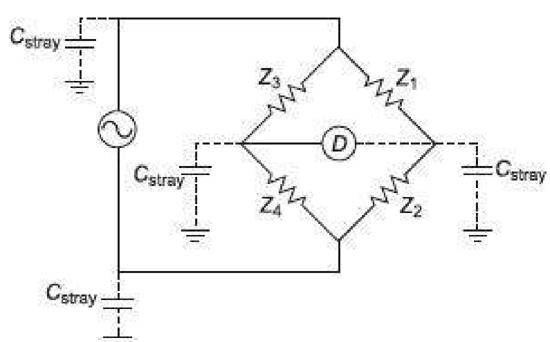


Figure 6.14 *Formation of stray capacitors in an ac bridge*

One possible way of reducing this effect is to keep the detector at ground potential, so there will be no ac voltage between it and the ground, and thus no current through Figure 6.14

Formation of stray capacitors in an ac bridge the stray capacitances can leak out. However, directly connecting the null detector to ground is not an option, since it would create a direct current path for other stray currents. Instead, a special voltage-divider circuit, called a *Wagner ground* or *Wagner earth*, may be used to maintain the null detector at ground potential without having to make a direct connection between the detector and ground.

The Wagner earth circuit is nothing more than a voltage divider as shown in Figure 6.15. There are two additional (auxiliary) arms Z_A and Z_B in the bridge configuration with a ground connection at their junction *E*. The switch *S* is used to connect one end of the detector alternately to the ground point *e* and the bridge connection point *d*. The two impedances Z_A and Z_B must be made of such components (*R*, *L*, or *C*) so that they are capable of forming a balanced bridge with the existing bride arm pairs Z_1–Z_2 or Z_3–Z_4. Stray capacitances formed between bridge junctions and the earthing point *E* are shown as C_1, C_2, C_3 and C_4.

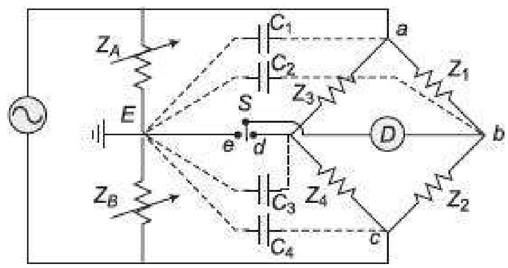


Figure 6.15 *Wagner earthing device*

The bridge is first balanced with the arms Z_1–Z_2 and Z_3–Z_4 with the switch at the position *d*. The switch is next thrown to position *e* and balance is once again attained between arms Z_1–Z_2 and Z_5–Z_6. The process is repeated till both bridge configurations become balanced. At this point, potential at the points *b*, *d*, *e* are the same and are all at earth potential. Thus, the Wagner earthing divider forces the null detector to be at ground potential, without a direct connection between the detector and ground. Under these conditions, no current can flow through the stray capacitors C_2 and C_3 since their terminals are both at earth potential. The other two stray capacitors C_1 and C_4 become part of (shunt to) the Wagner arms Z_A and Z_B, and thus get eliminated from the original bridge network.

The Wagner earthing method gives satisfactory results from the point of view of eliminating stray capacitance charging effects in ac bridges, but the entire balancing process is time consuming at times.

EXERCISE

Objective-type Questions

1. A bridge circuit works at a frequency of 2 kHz. Which of the following can be used as null detector in such a bridge?
   (a) Vibration galvanometers and tunable amplifiers
   (b) Headphones and tunable amplifiers

(c) Vibration galvanometers and headphones

(d) All of the above

2. Under balanced condition of a bridge for measuring unknown impedance, if the detector is suddenly taken out

(a) measured value of the impedance will be lower

(b) measured value of the impedance will be higher

(c) measured value of the impedance will not change

(d) the impedance can not be measured

3. Harmonic distortions in power supply does not affect the performance of Maxwell's bridge since

(a) filters are used to remove harmonics

(b) final expression for unknown inductance contain only fundamental frequency

(c) mechanical resonance frequency of null detectors are beyond the range of harmonic frequencies

(d) final expression for unknown inductance is independent of frequency

4. Maxwell's bridge can be used for measurement of inductance with

(a) high Q factors

(b) very low Q factors

(c) medium Q factors

(d) wide range of Q factor variations

5. The advantage of Hay's bridge over Maxwell's inductance–capacitance bridge is that

(a) its final balance equations are independent of frequency

(b) it reduces cost by not making capacitor or inductor as the variable parameters

(c) it can be used measuring low Q inductors

(d) it can be used measuring high Q inductors

6. The advantage of Anderson's bridge over Maxwell's bridge is that

(a) its final balance equations are independent of inductor losses

(b) it reduces cost by not making capacitor or inductor as the variable parameters

(c) number of bridge components required are less

(d) attaining balance condition is easier and less time consuming

7. The main advantage of Owen's bridge for measurement of unknown inductance is that

(a) it has two independent elements R and C for achieving balance

(b) it can be used for measurement of very high Q coils

(c) it is very inexpensive

(d) it can be used for measurement of unknown capacitance as well

8. DeSauty's bridge is used for measurement of

(a) high Q inductances

(b) low Q inductances

(c) loss less capacitors

(d) capacitors with dielectric losses

9. Schering bridge can be used for measurement of

(a) capacitance and dissipation factor

(b) dissipation factor only

(c) inductance with inherent loss

(d) capacitor but not dissipation factor

10. Frequency can be measured using
    (a) Anderson's bridge
    (b) Maxwell's bridge
    (c) De Sauty's bridge
    (d) Wien's bridge

Answers

1. (b)	2. (c)	3. (d)	4. (c)	5. (d)	6. (b)	7. (a)
8. (c)	9. (a)	10. (d)				

Short-answer Questions

1. Derive the general equations for balance in ac bridges. Show that both magnitude and phase conditions need to be satisfied for balancing an ac bridge.
2. Derive the expression for balance in Maxwell's inductance bridge. Draw the phasor diagram under balanced condition.
3. Show that the final balance expressions are independent of supply frequency in a Maxwell's bridge. What is the advantage in having balance equations independent of frequency?
4. Discuss the advantages and disadvantages of Maxwell's bridge for measurement of unknown inductance.
5. Explain why Maxwell's inductance–capacitance bridge is suitable for measurement of inductors having quality factor in the range 1 to 10.
6. Explain with the help of phasor diagram, how unknown inductance can be measured using Owen's bridge.
7. Explain how Wien's bridge can be used for measurement of unknown frequencies. Derive the expression for frequency in terms of bridge parameters.

Long-answer Questions

1. (a) Explain with the help of a phasor diagram, how unknown inductance can be measured using Maxwell's inductance–capacitance bridge.
   (b) The following data relate to a basic ac bridge:

$$\overline{Z}_1 = 50\ \Omega\angle 80^\circ \quad \overline{Z}_2 = 125\ \Omega \quad \overline{Z}_3 = 200\ \Omega\angle 30^\circ \quad \overline{Z}_4 = \text{unknown}$$

   Determine the unknown arm parameters. [10 + 5]

2. Describe the working of Hay's bridge for measurement of inductance. Derive the equations for balance and draw the phasor diagram under balanced condition. Explain how this bridge is suitable for measurement of high *Q* chokes?
3. Derive equations for balance for an Anderson's bridge. Draw its phasor diagram under balance. What are its advantages and disadvantages?
4. Describe how unknown capacitors can be measured using De Sauty's bridge. What are the limitations of this bridge and how they can be overcome by using a modified De Sauty's bridge? Draw relevant phasor diagrams
5. Describe the working of a Schering bridge for measurement of capacitance and dissipation factor. Derive relevant equations and draw phasor diagram under balanced condition.
6. In an Anderson's bridge for measurement of inductance, the arm *AB* consists of an unknown impedance with *L* and *R*, the arm *BC* contains a variable resistor, fixed resistances of 500 Ω each in arms *CD* and *DA*, a known variable resistance in the arm *DE*, and a capacitor of fixed capacitance 2 μF in the arm *CE*. The ac supply of 200 Hz is connected across *A* and *C*, and the detector is connected between *B* and *E*. If balance is obtained with a resistance of 300 Ω in the arm *DE* and a resistance of 600 Ω in the arm *BC*, calculate values of unknown impedance *L* and *R*. Derive the relevant equations for balance and draw the phasor diagram.
7. The four arms of a Maxwell's inductance–capacitance bridge at balance are Arm *AB* : A choke coil L1 with an equivalent series resistance R_1 Arm *BC* : A non-inductive resistance of 800 ΩArm *CD* : A mica capacitor of 0.3 μF in parallel with a noninductive resistance of 800 ΩArm *DA* : A non-inductive resistance 800 ΩSupply is given between terminals *A* and *C* and the detector is connected between nodes *B* and *D*. Derive the equations for balance

of the bridge and hence determine values of L_1 and R_1. Draw the phasor diagram of the bridge under balanced condition.

8. The four arms of a Hay's bridge used for measurement of unknown inductance is configured as follows: Arm *AB* : A choke coil of unknown impedance Arm *BC* : A non-inductive resistance of 1200 ΩArm *CD* : A non-inductive resistance of 900 Ω in series with a standard capacitor of 0.4 μF Arm *DA* : A noninductive resistance 18000 ΩIf a supply of 300 V at 50 Hz is given between terminals *A* and *C* and the detector is connected between nodes *B* and *D*, determine the inductance and inherent resistance of the unknown choke coil. Derive the conditions for balance and draw the phasor diagram under balanced condition.

9. A capacitor busing forms the arm *AB* of a Schering bridge and a standard capacitor of 400 μF capacitance and negligible loss, form the arm *AD*. Arm *BC* consists of a non-inductive resistance of 200 Ω. When the detector connected between nodes *B* and *D* shows no deflection, the arm *CD* has a resistance of 82.4 Ωin parallel with a capacitance of 0.124 μF. The supply frequency is 50 Hz. Calculate the capacitance and dielectric loss angle of the capacitor. Derive the equations for balance and draw the relevant phasor diagram at balanced state.

10. (a) An ac bridge is configured as follows:

    Arm *AB* : A resistance of 600 Ω in parallel with a capacitance of 0.3 μF

    Arm *BC* : An unknown non-inductive resistance

    Arm *CD* : A noninductive resistance of 1000 Ω

    Arm *DA* : A resistance of 400 Ω in series with a capacitance of 0.1 μF

    If a supply is given between terminals *A* and *C* and the detector is connected between nodes *B* and *D*, find the resistance required in the arm *BC* and also the supply frequency for the bridge to be balanced.

    (b) Explain how Wien's bridge can be used for measurement of unknown frequency. Draw the phasor diagram under balanced condition and derive the expression for balance.

7 Power Measurement

7.1 INTRODUCTION

Measurement of electric power is as essential in industry as in commercial or even domestic applications. Prior estimation and subsequent measurements of instantaneous and peak power demands of any installation are mandatory for design, operation and maintenance of the electric power supply network feeding it. Whereas an under-estimation of power demand may lead to blowing out of power supply side accessories, on the other hand, over-estimation can end up with over-design and additional cost of installation. Knowledge about accurate estimation, calculation and measurement of electric power is thus of primary concern for designers of new installations. In this chapter, the most popular power measurement methods and instruments in dc and ac circuits are illustrated.

7.2 POWER MEASUREMENT IN dc CIRCUITS

Electric power (P) consumed by a load (R) supplied from a dc power supply (V_S) is the product of the voltage across the load (V_R) and the current flowing through the load (I_R):

$$P = V_R \times I_R \tag{7.1}$$

Thus, power measurement in a dc circuit can be carried out using a voltmeter (V) and an ammeter (A) using any one of the arrangements shown in Figure 7.1.

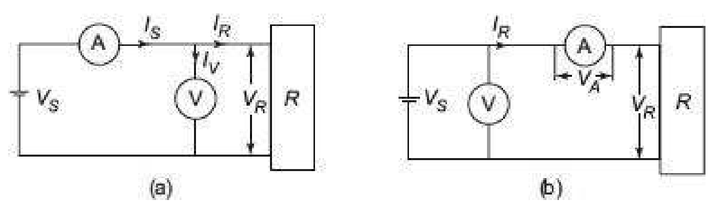


Figure 7.1 *Two arrangements for power measurement in dc circuits*

One thing should be kept in mind while using any of the two measuring arrangements shown in Figure 7.1; that both the voltmeter and the ammeter requires power for their own operations. In the arrangement of Figure 7.1(a), the voltmeter is connected between the load and the ammeter. The ammeter thus, in this case measures the current flowing into the voltmeter, in addition to the current flowing into the load.

$$\text{Current through the voltmeter} = I_V = V_R/R_V \tag{7.2}$$

where, R_V is the internal resistance of the voltmeter.

$$\begin{aligned}\text{Power consumed by the load} &= V_R \times I_R = V_R \times (I_S - I_V)\\ &= V_R \times I_S - V_R \times I_V\\ &= V_R \times I_S - V_R^2/R_V \qquad (7.3)\\ &= \text{Power indicated by instruments} - \text{Power loss in voltmeter}\end{aligned}$$

Thus, Power indicated = Power consumed + Power loss in voltmeter

In the arrangement of Figure 7.1(b), the voltmeter measures the voltage drop across the ammeter in addition to that dropping across the load.

$$\text{Voltage drop across ammeter} = V_A = I_R \times R_A \tag{7.4}$$

where, R_A is the internal resistance of the ammeter.

$$\begin{aligned}\text{Power consumed by the load} &= V_R \times I_R = (V_S - V_A) \times I_R \\ &= V_S \times I_R - V_A \times I_R \\ &= V_S \times I_R - I_R^2 \times R_A \qquad (7.5)\\ &= \text{power indicated by instruments} - \text{Power loss in ammeter}\end{aligned}$$

Thus, Power indicated = Power consumed + Power loss in Ammeter

Thus, both arrangements indicate the additional power absorbed by the instruments in addition to indicating the true power consumed by the load only. The corresponding measurement errors are generally referred to as insertion errors.

Ideally, in theory, if we consider voltmeters to have infi nite internal impedance and ammeters to have zero internal impedance, then from (7.3) and (7.5) one can observe that the power consumed by the respective instruments go down to zero. Thus, in ideal cases, both the two arrangements can give correct indication of the power consumed by the load. Under practical conditions, the value of power loss in instruments is quite small, if not totally zero, as compared with the load power, and therefore, the error introduced on this account is small.

Example 7.1

Two incandescent lamps with 80 Ω and 120 Ω resistances are connected in series with a 200 V dc source. Find the errors in measurement of power in the 80 Ω lamp using a voltmeter with internal resistance of 100 kΩ and an ammeter with internal resistance of 0.1 mΩ, when (a) the voltmeter is connected nearer to the lamp than the ammeter, and (b) when the ammeter is connected nearer to the lamp than the voltmeter

Solution Assuming both the instruments to be ideal, i.e., the voltmeter with infinite internal impedance and ammeter with zero internal impedance, the current through the series circuit should be

= 200/(80 + 120) = 1 A

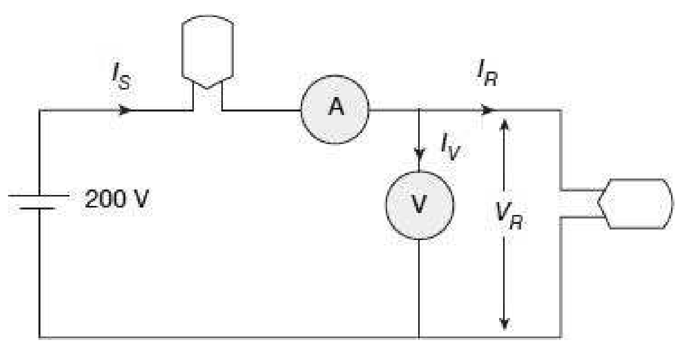


Figure 7.2 *Actual connections for Example 7.1(a)*

Hence, true power consumed by the 80 Ω lamp would have been

$= 1^2 \text{ X } 80 = 80\ \Omega$

However, considering the internal resistance of the ammeter and voltmeter, the equivalent circuit will look like Figure 7.3.

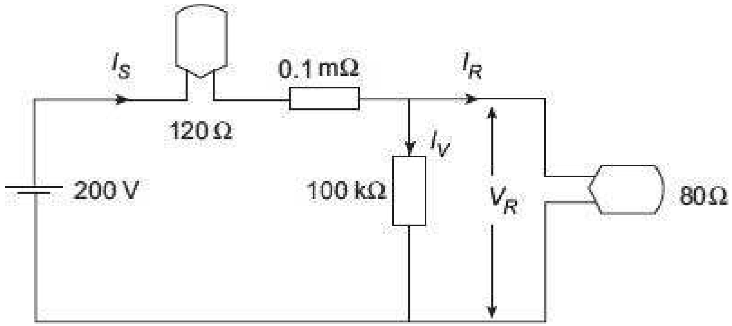


Figure 7.3 *Equivalent circuit for Example 7.1(a)*

Supply current (ammeter reading)

I_S = Supply voltage/Equivalent resistance of the circuit

$$= \frac{\text{Supply voltage}}{(\text{Series of Lamp 1 and ammeter}) + (\text{Parallel of Lamp 2 and voltmeter})}$$

$$= 200\Big/\left((120+0.1\times10^{-3})+{(100\times10^3\times80)}\Big/{(100\times10^3+80)}\right)$$

$= 1.0003$ A

Actual current through the 80 Ω lamp is

$$I_R = 1.0003\times\frac{100\times10^3}{100\times10^3+80}\text{ A}$$

$= 0.9995$ A

Voltage across the 80 Ω lamp (voltmeter reading) is

$V_R = I_R \times 80$

$= 79.962$ V

Thus, actual power consumed by the 80 Ω lamp is

$V_R \times I_R = 79.962 \times 0.9995 = 79.922$ W

Power consumption as indicated by the two meters

= Voltmeter reading × Ammeter reading

$= 79.962 \times 1.0003 = 79.986$ W

(b) In this case, the actual circuit and its equivalent will look like Figure 7.4.

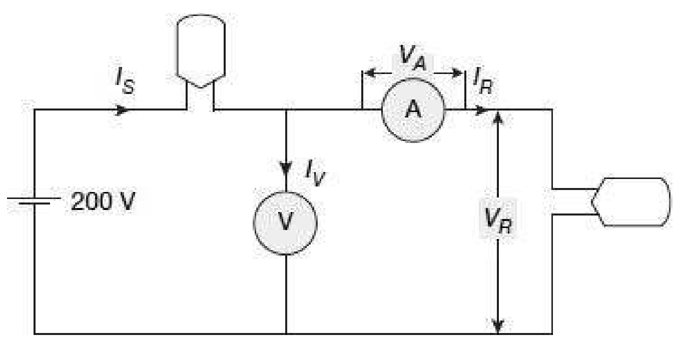


Figure 7.4 *Actual connection for Example 7.1(b)*

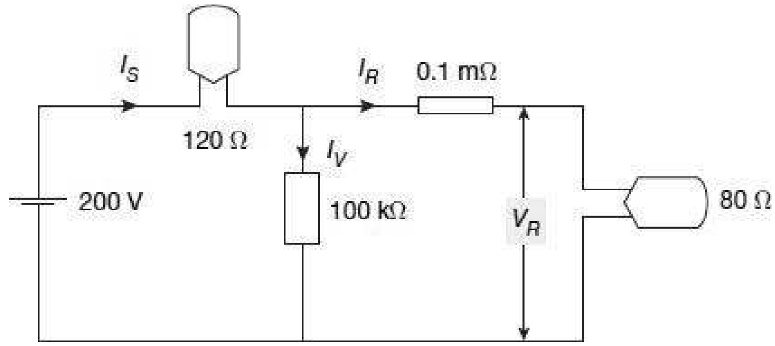


Figure 7.5 *Equivalent circuit for Example 7.1(a)*

Supply current

$$I_S = \text{Supply voltage/Equivalent resistance of the circuit}$$

$$= \frac{\text{Supply voltage}}{\text{Series of Lamp 1 and [Parallel of voltmeter and (Series of ammeter and Lamp 2)]}}$$

$$= 200\bigg/\left(120 + \frac{[100\times10^3\times(80+0.1\times10^{-3})]}{100\times10^3+(80+0.1\times10^{-3})}\right)$$

$$= 1.003 \text{ A}$$

Current through the 80 Ω lamp (ammeter reading) is

$$I_R = 1.0003\times\frac{100\times10^3}{100\times10^3+(80+0.1\times10^{-3})}$$

$$= 0.9995 \text{ A}$$

Voltage across the 80 Ω lamp

$$= I_R \times 80 = 79.96 \text{ V}$$

Voltmeter reading

$$= I_V \times 100 \times 10^3$$

$$= (I_S - I_R) \times 100 \times 10^3 = 80 \text{ V}$$

Thus, actual power consumed by the 80 Ω lamp is

$$V_R \times I_R = 79.96 \times 0.9995 = 79.92 \text{ W}$$

Power consumption as indicated by the two meters

$$= \text{Voltmeter reading} \times \text{Ammeter reading}$$

$$= 80 \times 0.9995 = 79.96 \text{ W}$$

Thus, we can have the following analysis:

Case	*Power Consumption by 80 W lamp (W)*			*% Error from ideal*
	Ideal Power	*Actual Power*	*Meter Indication*	
a	80	79.922	79.986	0.0175
b	80	79.92	79.96	0.05

Power in dc circuits can also be measured by wattmeter. Wattmeter can give direct indication of power and there is no need to multiply two readings as in the case when ammeter and voltmeter is used.

The type of wattmeter most commonly used for such power measurement is the *dynamometer*. It is built by (1) two fixed coils, connected in series and positioned coaxially with space between them, and (2) a moving coil, placed between the fixed coils and fitted with a pointer. Such a construction for a dynamometer-type wattmeter is shown in Figure 7.6.

It can be shown that the torque produced in the dynamometer is proportional to the product of the current flowing through the fixed coils times that through the moving coil.

The fixed coils, generally referred to as current coils, carry the load current while the moving coil, generally referred to as voltage coil, carries a current that is proportional, via the multiplier resistor R_V, to the voltage across the load resistor *R*. As a consequence, the deflection of the moving coil is proportional to the power consumed by the load.

A typical connection of such a wattmeter for power measurement in dc circuit is shown in Figure 7.7.

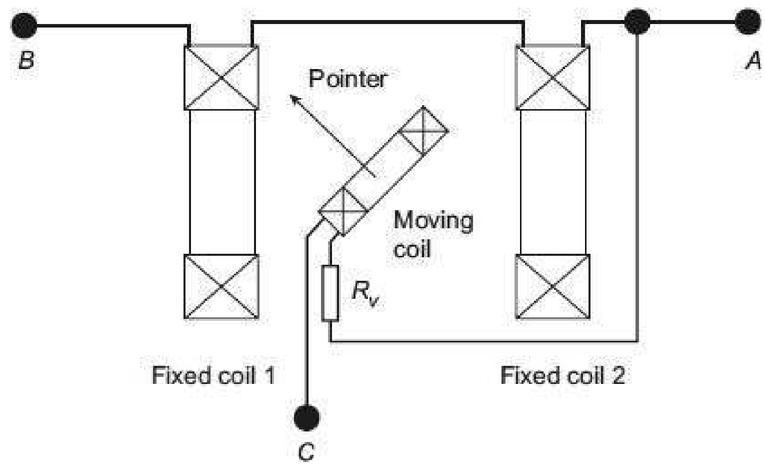


Figure 7.6 *Basic construcion of dynamometer-type wattmeter*

In such a connection of the wattmeter, the insertion error, as in the pervious case with ammeter and voltmeter, still exists. Relative β positioning of the current coil and the voltage coil with respect to load, introduce similar V_S errors in measurement of actual power. In particular, by connecting the voltage coil between *A* and *C* (Figure 7.7), the current coils carry the surplus current flowing through the voltage coil. On the other hand, by connecting the moving coil between *B* and *C*, this current error can be avoided, but now the voltage coil measures the surplus voltage drop across the current coils.

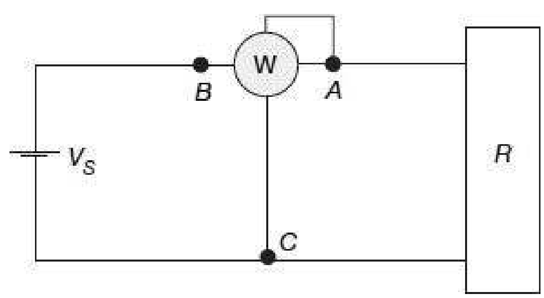


Figure 7.7 *Connection of dynamometer-type wattmeter for power measurement in dc circuit*

7.3 POWER MEASUREMENT IN ac CIRCUITS

In alternating current circuits, the instantaneous power varies continuously as the voltage and current varies while going through a cycle. In such a case, the power at any instant is given by

$$p(t) = v(t) \times i(t) \tag{7.6}$$

where, $p(t)$, $v(t)$, and $i(t)$ are values of instantaneous power, voltage, and current respectively.

Thus, if both voltage and current can be assumed to be sinusoidal, with the current lagging the voltage by phase-angle φ, then

$$v(t) = V_m \sin \omega t$$

and

$$i(t) = I_m \sin (\omega t - \varphi)$$

where, V_m and I_m are peak values of voltage and current respectively, and *w* is the angular frequency.

The instantaneous power *p*is therefore given by

$$p(t) = V_m I_m \sin \omega t \sin (\omega t - \varphi) \tag{7.7}$$

or,

$$p(t) = \frac{V_m I_m}{2}[\cos\varphi - \cos(2\omega t - \varphi)]$$

Average value of power over a complete cycle in such a case will be

$$\begin{aligned}
P &= \frac{1}{2T}\int_0^{2T} p(t)\,dt = \frac{1}{2T}\int_0^{2T} \frac{V_m I_m}{2}[\cos\varphi - \cos(2\omega t - \varphi)]\,dt \\
&= \frac{V_m I_m}{2T}\int_0^{2T}\left[\cos\varphi - \cos\left(\frac{4\pi}{T}t - \varphi\right)\right]dt \\
&= \frac{V_m I_m}{2T}\left[\cos\varphi\, t\Big|_0^T - \frac{T}{4\pi}\sin\left(\frac{4\pi}{T}t - \varphi\right)\Big|_0^T\right] \\
&= \frac{V_m I_m}{4T}[\cos\varphi T - 0] \\
&= \frac{V_m I_m}{2}\cos\varphi \\
&= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi \\
&= V\,I\cos\varphi
\end{aligned} \tag{7.8}$$

where, *V*and *I* are rms values of voltage and current respectively and cos *j* is power factor of the load.

Involvement of the power-factor term cos *j* in the expression for power in ac circuit indicates that ac power cannot be measured simply by connecting a pair of ammeter and voltmeter. A wattmeter, with in-built facility for taking in to account the power factor, can only be used for measurement of power in ac circuits.

Figure 7.8 plots the waveforms of instantaneous power $p(t)$, voltage $v(t)$, and current $i(t)$.

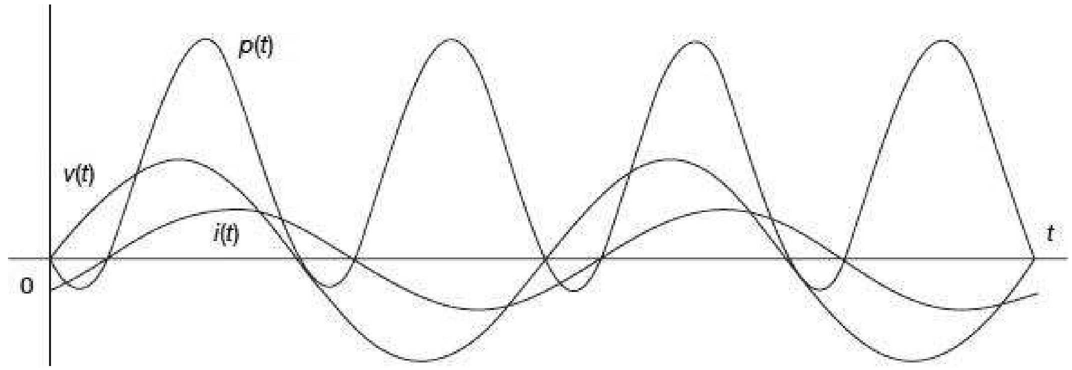


Figure 7.8 *Plot of the waveforms of instantaneous power voltage and current in ac circuit*

Readers may find interesting to note in Figure 7.8 that though voltage and current waveforms have zero average value over a complete cycle, the instantaneous power has offset above zero having non-zero average value.

7.4 ELECTRODYNAMOMETER-TYPE WATTMETER

An electrodynamometer-type wattmeter is similar in design and construction with the analog electrodynamometer-type ammeter and voltmeter described in Chapter 2.

7.4.1 Construction of Electrodynamometer-type Wattmeter

Schematic diagram displaying the basic constructional features of a electrodynamometer-type wattmeter is shown in Figure 7.9.

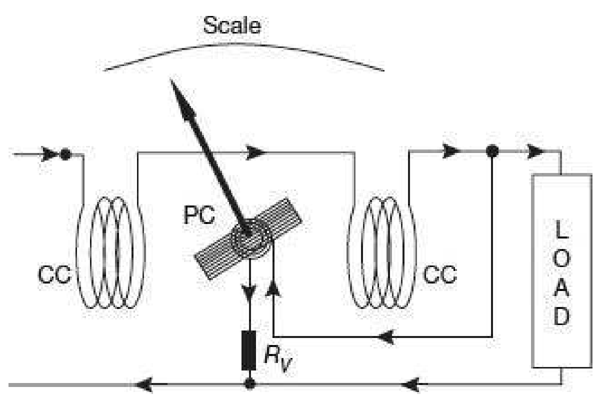


Figure 7.9 *Schematic of electrodynamometer-type wattmeter*

Internal view of such an arrangement is shown in the photograph of Figure 7.10.

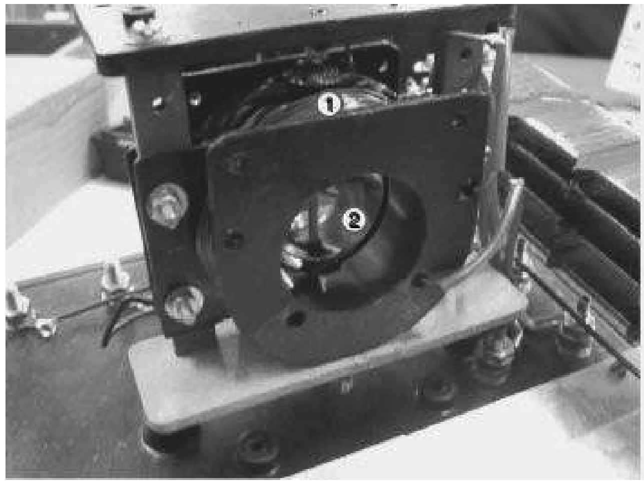


Figure 7.10 *nternal photograph of electrodynamometer-type wattmeter:(1) Fixed (current) coil (2) Moving (potential) coil*

1. Fixed Coil System

Such an instrument has two coils connected in different ways to the same circuit of which power is to be measured. The *fixed coils* or the *field coils* are connected in series with the load so as to carry the same current as the load. The fixed coils are hence, termed as the *Current Coils (CC)* of the wattmeter. The main magnetic field is produced by these fixed coils. This coil is divided in two sections so as to provide more uniform magnetic field near the centre and to allow placement of the instrument moving shaft.

Fixed coils are usually wound with thick wires for carrying the main load current through them. Windings of the fixed coil is normally made of stranded conductors running together but, insulated from each other. All the strands are brought out to an external commutating terminator so that a number of current ranges of the instrument may be obtained by grouping them all in series, all in parallel, or in a series–parallel combination. Such stranding of the fixed coils also reduces Eddy-current loss in the conductors. Still higher current or voltage ranges, however, can be accommodated only through the use of instrument transformers.

Fixed coils are mounted rigidly with the coil supporting structures to prevent any small movement whatsoever and resulting field distortions. Mounting supports are made of ceramic, and not metal, so as not to disturb the magnetic field distribution.

2. Moving Coil System

The **moving coil** that is connected across the load carries a current proportional to the voltage. Since the moving coil carries a current proportional to the voltage, it is called the *voltage coil* or the *pressure coil* or simply *PC* of the wattmeter. The moving coil is entirely embraced by the pair of fixed coils. A high value **non-inductive resistance** is connected in series with the voltage coil to restrict the current through it to a small value, and also to ensure that voltage coil current remains as far as possible in phase with the load voltage.

The moving coil, made of fine wires, is wound either as a self-sustaining air-cored coil, or else wound on a nonmetallic former. A metallic former, otherwise would induce Eddy-currents in them under influence of the alternating field.

3. Movement and Restoring System

The moving, or voltage coil along with the pointer is mounted on an aluminum spindle in case jewel bearings are used to support the spindle. For higher sensitivity requirements, the moving coil may be suspended from a torsion head by a metallic suspension which serves as a lead to the coil. In other constructions, the coil may be suspended by a silk fibre together with a spiral spring which gives the required torsion. The phosphor-bronze springs are also used to lead current into and out of the moving coil. In any case, the torsion head with suspension, or the spring, also serves the purpose of providing the restoring torque to bring the pointer back to its initial position once measurement is over.

The moving, or voltage coil current must be limited to much low values keeping in mind the design requirements of the movement system. Current is lead to and out of the moving coil through two spiral springs. Current value in the moving coil is thus to be limited to values that can be safely carried by the springs without appreciable heating being caused.

4. Damping System

Damping in such instruments may be provided by small aluminum vanes attached at the bottom of the spindle. These vanes are made to move inside enclosed air chambers, thereby creating the damping torque. In other cases, the moving coil itself can be stitched on a thin sheet of mica, which acts as the damping vane while movements. Eddy-current damping, however, cannot be used with these instruments. This is due to the fact that any metallic element to be used for Eddy-current damping will interfere and distort the otherwise weak operating magnetic field. Moreover, introduction of any external permanent magnet for the purpose of Eddy-current damping will severely hamper the operating magnetic field.

5. Shielding System

The operating field produced by the fixed coils, is comparatively lower in electrodynamometer-type instruments as compared to other type of instruments. In some cases, even the earth's magnetic field can pollute the measurement readings. It is thus essential to shield the electrodynamometer-type instruments from effects of external magnetic fields. Enclosures of such instruments are thus made of alloys with high permeability to restrict penetration of external stray magnetic fields into the instrument.

7.4.2 Operation of Electrodynamometer-type Wattmeter

The schematic operational circuit of an electrodynamometer-type wattmeter being used for measurement of power in a circuit is shown in Figure 7.11.

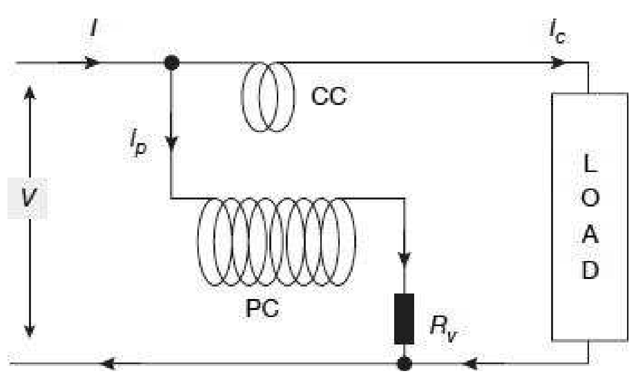


Figure 7.11 *Operational circuit of electrodynamometer-type wattmeter*

V = voltage to be measured (rms)

I = current to be measured (rms)

i_P = voltage (pressure) coil instantaneous current

i_C = current coil instantaneous current

R_V = external resistance connected with pressure coil

R_P = resistance of pressure coil circuit (PC resistance + R_V)

M = mutual inductance between current coil and pressure coil

θ = angle of deflection of the moving system

ω = angular frequency of supply in radians per second

φ = phase-angle lag of current I with respect to voltage V

As described in Chapter 2, the instantaneous torque of the electrodynamometer wattmeter shown in Figure 7.11 is given by

$$T_i = i_P i_C \frac{dM}{d\theta} \tag{7.9}$$

Instantaneous value of voltage across the pressure-coil circuit is

$$v_P = \sqrt{2} \times V \sin \omega t$$

If the pressure coil resistance can be assumed to be very high, the whole pressure coil can be assumed to be behaving like a resistance only. The current i_P in the pressure coil thus, can be assumed to in phase with the voltage v_P, and its instantaneous value is

$$i_P = {v_P}/{R_P} = \sqrt{2} \times {V}/{R_P} \sin \omega t = \sqrt{2} \times I_P \sin \omega t$$

where $I_P = V/R_P$ is the rms value of current in pressure coil.

Assuming that the pressure-coil resistance is sufficiently high to prevent branching out of any portion of the supply current towards the pressure coil, the current coil current can be written as

$$i_C = \sqrt{2} \times I \sin(\omega t - \varphi)$$

Thus, instantaneous torque from (7.9) can be written as

$$T_i = \sqrt{2} \times I_P \sin\omega t \times \sqrt{2} \times I \sin(\omega t - \varphi)\frac{dM}{d\theta}$$

$$= 2I_P I \sin\omega t \sin(\omega t - \varphi)\frac{dM}{d\theta}$$

$$= I_P I\{\cos\varphi - \cos(2\omega t - \varphi)\}\frac{dM}{d\theta} \quad (7.10)$$

Presence of the term containing 2*w t*, indicates the instantaneous torque as shown in (7.10) varies at twice the frequency of voltage and current.

Average deflecting torque over a complete cycle is

$$T_d = \frac{1}{T}\int_0^T T_i \, d\omega t = \frac{1}{2\pi}\int_0^{2\pi} I_P I\{\cos\varphi - \cos(2\omega t - \varphi)\}\frac{dM}{d\theta}\, d\omega t$$

$$= \frac{I_P I}{2\pi}\left[\omega t \cos\varphi\right]_0^{2\pi} \frac{dM}{d\theta}$$

$$= I_P I \cos\varphi \frac{dM}{d\theta} \quad (7.11)$$

$$= \frac{V}{R_P} I \cos\varphi \frac{dM}{d\theta}$$

$$= \frac{VI\cos\varphi}{R_P}\frac{dM}{d\theta} \quad (7.12)$$

With a spring constant K, the controlling torque provided by the spring for a final steady-state deflection of θ is given by

$$T_C = K\theta$$

Under steady-state condition, the average deflecting torque will be balanced by the controlling torque provided by the spring. Thus, at balanced condition $T_C = T_d$

$$T_C = T_d$$

$$K\theta = \frac{VI\cos\varphi}{R_P}\frac{dM}{d\theta}$$

$$\theta = \frac{VI\cos\varphi}{KR_P}\frac{dM}{d\theta}$$

$$\theta = \left(K_1 \frac{dM}{d\theta}\right) P \quad (7.13)$$

where, P is the power to be measured and $K_1 = 1/KR_P$ is a constant.

Steady-state deflection θ is thus found to be an indication of the power P to be measured.

7.4.3 Shape of scale in Electrodynamometer-type Wattmeter

Steady-state deflection θ can be made proportional to the power P to be measured, i.e., the deflection will vary linearly with variation in power if the rate of change of mutual inductance is constant over the range of deflection. In other words, the scale of measurement will be uniform if the mutual inductance between the fixed and moving coils varies linearly with angle of deflection. Such a variation in mutual inductance can be achieved by careful design of the instrument. Figure 7.12 shows the expected nature of variation of mutual inductance between fixed and moving coils with respect to angle of deflection.

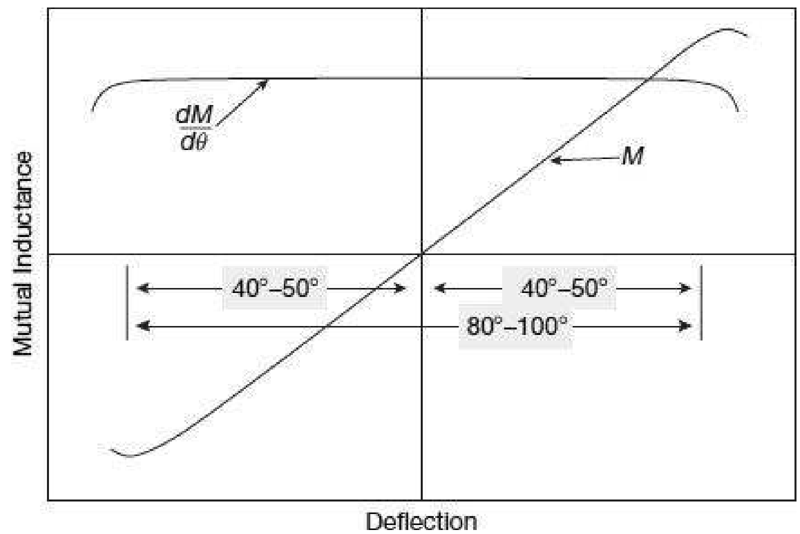


Figure 7.12 *Variation of mutual inductance with deflection*

By a suitable design, the mutual inductance between fixed and moving coils can be made to vary linearly with deflection angle over a range of 40° to 50° on either side of zero mutual inductance position, as shown in Figure 7.12. If the position of zero mutual inductance can be kept at the mid-scale, then the scale can be graduated to be uniform over 80° to 100°, which covers almost entire range of the scale.

7.4.4 Errors in Electrodynamometer-type Wattmeter

1. Error due to Pressure-Coil Inductance

It was assumed during the discussions so far that the pressure coil circuit is purely resistive. In reality, however, the pressure coil will have certain inductance along with resistance. This will introduce errors in measurement unless necessary compensations are taken care of. To have an estimate of such error, let us consider the following:

V = voltage applied to the pressure coil circuit (rms)

I = current in the current coil circuit (rms)

I_P = current in the voltage (pressure) coil circuit (rms)

r_P = resistance of pressure coil only

L = inductance of pressure coil

R_V = external resistance connected with pressure coil

R_P = resistance of pressure coil circuit (PC resistance + R_V)

Z_P = impedance of pressure coil circuit

M = mutual inductance between current coil and pressure coil

ω = angular frequency of supply in radian per second

φ = phase-angle lag of current I with respect to voltage V

Due to inherent inductance of the pressure coil circuit, the current and voltage in the pressure coil will no longer be in phase, rather the current through the pressure coil will lag the voltage across it by a certain angle given by

$$\alpha = \tan^{-1}\left(\frac{\omega L}{R_P}\right) = \tan^{-1}\left(\frac{\omega L}{r_P + R_V}\right)$$

As can be seen from Figure 7.13, current through the pressure coil lags voltage across it by a phase-angle which is less than that between the current coil current and the pressure coil voltage.

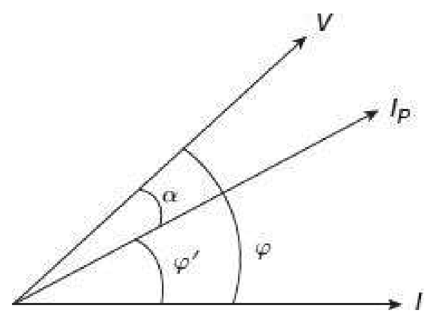


Figure 7.13 *Wattmeter phasor diagram with pressure coil inductance*

In such a case, phase-angle difference between the *with pressure coil inductance*pressure coil current and current coil current is

$$\varphi' = \varphi - \alpha$$

Following from (7.11), the wattmeter deflection will be

$$\theta' = \frac{I_P I}{K}\cos\varphi' \cdot \frac{dM}{d\theta}$$

$$\theta' = \frac{V}{Z_P K} I \cos(\varphi - \alpha) \cdot \frac{dM}{d\theta}$$

Relating to $R_P = Z_P \cos\alpha$ in the pressure coil circuit, the wattmeter deflection can be re-written as *VI dM*

$$\theta' = \frac{VI}{R_P K}\cos\alpha \cdot \cos(\varphi - \alpha) \cdot \frac{dM}{d\theta} \tag{7.14}$$

In the absence of inductance, $Z_P = R_P$ and $\alpha = 0$; wattmeter in that case will read true power, given by,

$$\theta = \frac{VI}{R_P K}\cos\varphi \frac{dM}{d\theta} \tag{7.15}$$

Taking the ratio of true power indication to actual wattmeter reading, we get

$$\frac{\text{True power indication}}{\text{Actual wattmeter reading}} = \frac{\theta}{\theta'} = \frac{\frac{VI}{R_p K}\cos\varphi \frac{dM}{d\theta}}{\frac{VI}{R_p K}\cos\alpha \cdot \cos(\varphi-\alpha)\cdot \frac{dM}{d\theta}} = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi-\alpha)}$$

Thus, the correction factor can be identified as

$$CF = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi-\alpha)}$$

True power indication can thus be obtained from the actual wattmeter reading using the correction factor CF as

True power indication = CF X Actual wattmeter reading

Thus, for lagging power factor loads,

$$\text{True power indication} = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi-\alpha)} \times \text{ Actual wattmeter readings}$$

The above relations, along with Figure 7.13 indicate that under lagging power factor loads, unless special precautions are taken, actual wattmeter reading will tend to display higher values as compared to true power consumed.

For leading power factor loads, however, the wattmeter phasor diagram will be as shown in Figure 7.14.

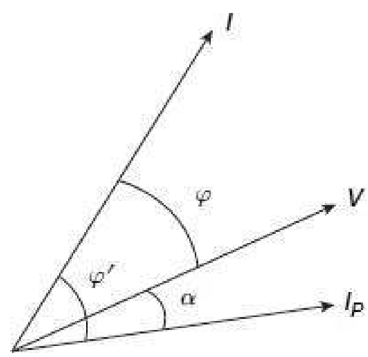


Figure 7.14 *Wattmeter phasor diagram with pressure coil inductance during leading load*

For leading power factor loads,

$$\text{True power indication} = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi+\alpha)} \times \text{ Actual wattmeter readings}$$

The above relations, along with Figure 7.13 indicate that under leading power factor loads, unless special precautions are taken, actual wattmeter reading will tend to display higher values as compared to true power consumed.

2. Compensation for Pressure Coil Inductance

A wattmeter can be compensated for pressure coil inductance by connecting a preset value of capacitance across a certain portion of the external resistance connected in series with the pressure coil, as shown in Figure 7.15.

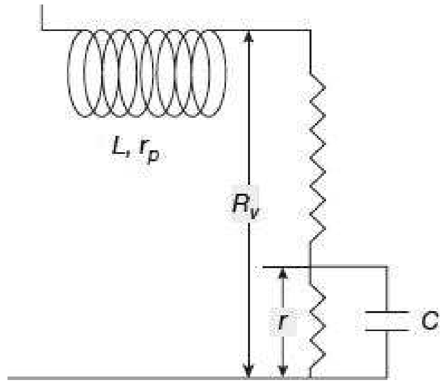


Figure 7.15 *Compensation for pressure coil inductance*

The total impedance of the circuit in such a case can be written as

$$Z_P = (r_p + R_V - r) + j\omega L + \frac{r - j\omega Cr^2}{1 + \omega^2 C^2 r^2}$$

To make the entire circuit behave as purely resistive, if we can design the circuit parameters in such a case that for power frequencies

$$w^2 C^2 r^2 << 1$$

Then we can re-write the total impedance of the pressure coil as

$$Z_P = (r_p + R_V - r) + jwL + r - jwCr^2 = r_P + R_V + jw(L - Cr^2)$$

If by proper design, we can make $L = Cr^2$

Then, impedance = $r_p + R_V = R_P$

Thus error introduced due pressure coil inductance can be substantially eliminated.

3. Error due to Pressure Coil Capacitance

The voltage, or pressure coil circuit may have inherent capacitance in addition to inductance. This capacitance effect is mainly due to inter-turn capacitance of the winding and external series resistance. The effect of stray capacitance of the pressure coil is opposite to that due to inductance. Therefore, the wattmeter reads low on lagging power factors and high on leading power factors of the load. Actual reading of the wattmeter, thus, once again needs to be corrected by the corresponding correction factors to obtain the true reading. The effect of capacitance (as well as inductance) varies with variable frequency of the supply.

4. Error due to Connection

There are two alternate methods of connection of wattmeter to the circuit for measurement of power. These are shown in Figure 7.16. In either of these connection modes, errors are introduced in measurement due power losses in pressure coil and current coil.

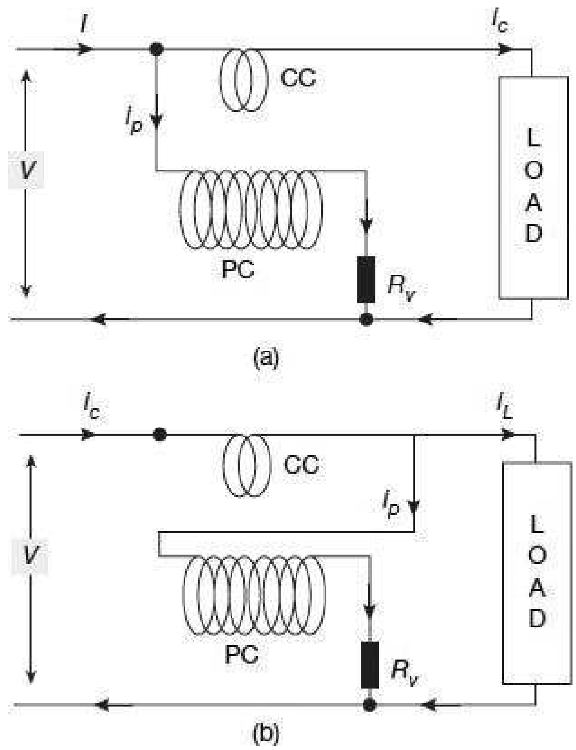


Figure 7.16 *Wattmeter connections*

In the connection of Figure 7.16(a), the pressure coil is connected across the supply, thus pressure coil measures the voltage across the load, plus the voltage drop across the current coil. Wattmeter reading in this case will thus include power loss in current coil as well, along with power consumed by the load.

Wattmeter reading = Power consumed by load + Power loss in CC

In the connection of Figure 7.16(b), the current coil is connected to the supply side; therefore, it carries load current plus the pressure coil current. Hence, wattmeter reading in this case includes, along with power consumed by the load, power loss in the pressure coil as well.

Wattmeter reading = Power consumed by load + Power loss in PC

In the case when load current is small, power loss in the current coil is small and hence the connection of Figure 7.16(a) will introduce comparatively less error in measurement.

On the other hand, when load current is large, current branching through the pressure coil is relatively small and error in measurement will be less if connection of Figure 7.16(b) is used.

Errors due to branching out of current through the pressure coil can be minimised by the use of compensating coil as schematically shown in Figure 7.17.

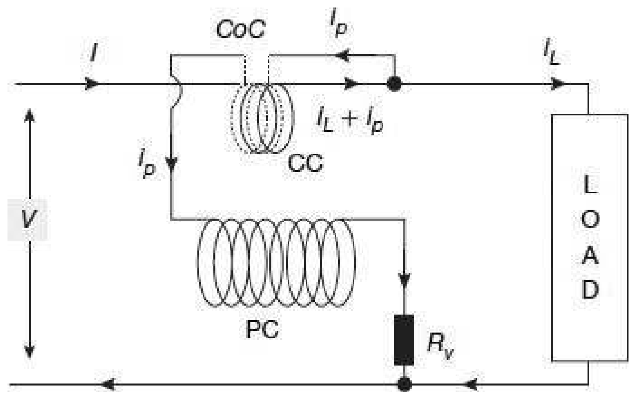


Figure 7.17 *Schematic connection diagram of compensated wattmeter*

In the compensated connection, the current coil consists of two windings, each winding having the same number of turns. The two windings are made as far as possible identical and coincident. One of the two windings (CC) is made of heavy wire that carries the load current plus the current for the pressure coil. The other winding (compensating coil–CoC) which is connected in series with the pressure coil, uses thin wire and carries only the current to the pressure coil. This current in the compensating coil is, however, in a direction opposite to the current in main current coil, creating a flux that opposes the main flux. The resultant magnetic field is thus due to the current coil only, effects of pressure coil current on the current coil flux mutually nullifying each other. Thus, error due to pressure coil current flowing in the current coil in cancelled out and the wattmeter indicates correct power.

5. Eddy-current Errors

Unless adequate precautions are adopted, Eddy-currents may be induced in metallic parts of the instrument and even within the thickness of the conductors by alternating magnetic field of the current coil. These Eddy-currents produce spurious magnetic fields of their own and distort the magnitude and phase of the main current coil magnetic field, thereby introducing error in measurement of power.

Error caused by Eddy-currents is not easy to estimate, and may become objectionable if metal parts are not carefully avoided from near the current coil. In fact, solid metal in coil supports and structural part should be kept to a minimum as far as practicable. Any metal that is used is kept away and is selected to have high resistivity so as to reduce Eddy-currents induced in it. Stranded conductors are recommended for the current coil to restrict generation of Eddy-current within the thickness of the conductor.

6. Stray Magnetic Field Errors

The operating field in electrodynamometer-type instruments being weak, special care must be taken to protect these instruments from external magnetic fields. Hence, these instruments should be shielded against effects of stray magnetic fields. Laminated iron shields are used in portable laboratory instruments, while steel casings are provided as shields in switchboard mounted wattmeter. Precision wattmeters, however, are not provided with metals shields, for that will introduce errors due to Eddy-current, and also some dc error due to permanent magnetization of the metal shield under influence of external magnetic field. Such wattmeters are manufactured to have *astatic* system as shown in Figure

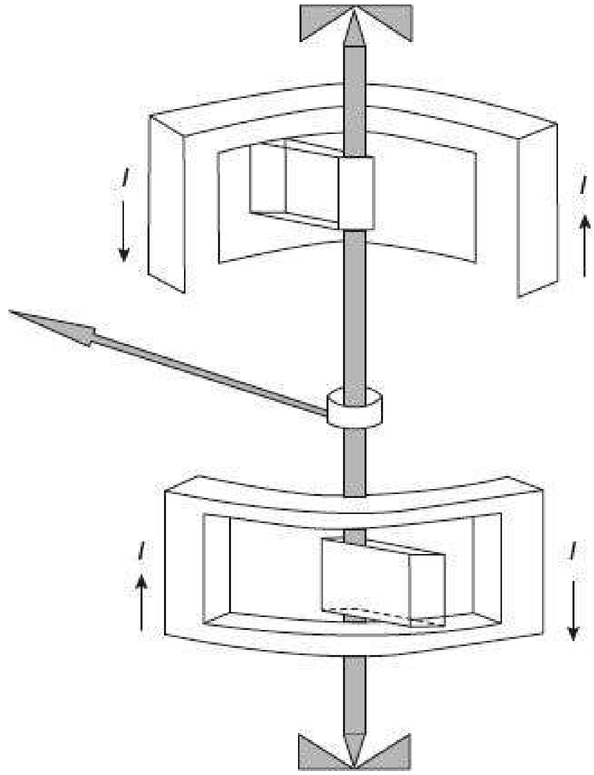


Figure 7.18 *Astatic systems for electrodynamometer wattmeter*

Astatic electrodynamometer instruments are constructed with two similar sets of fixed and moving coils mounted on the same shaft. The pair of fixed coils is so connected that their magnetic fields are in opposition. Similarly, the pair of moving coils is also connected to produce magnetic fields in opposite directions. This makes the deflecting torque acting on the two moving coils to be in the same direction. Deflection of the pointer is thus due to additive action of the two moving coils. However, since the two fields in the two pairs of fixed and moving coils are in opposition, any external uniform field will affect the two sets of pairs differently. The external field will reduce the field in one coil and will enhance the field in the other coil by identical amount. Therefore, the deflecting torque produced by one coil is increased and that by the other coil is reduced by an] equal amount. This makes the net torque on account of the external magnetic field to zero.

7. Error Caused by Vibration of the Moving System

The instantaneous torque on the moving system varies cyclically at twice the frequency of the voltage and current (7.10). If any part of the moving system, such as the spring or the pointer has natural frequency close to that of torque pulsation, then accidental resonance may take place. In such a case, the moving system may vibrate with considerable amplitude. These vibrations may pose problems while noting the pointer position on the scale. These errors due to vibrations may be avoided by designing the moving elements to have natural frequencies much further away from twice the frequency of the supply voltage.

8. Temperature Errors

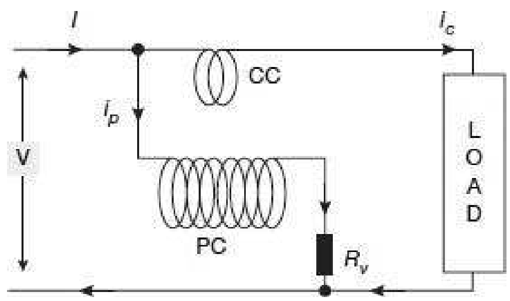
Temperature changes may affect accuracy of wattmeter by altering the coil resistances. Temperature my change due to change in room temperature or even due to heating effects in conductors with flow of current. Change in temperature also affects the spring stiffness, thereby introducing error in the deflection process. High-precision instruments are fitted with temperature compensating resistors that tend to neutralise the effects of temperature variation.

Example 7.2

An electrodynamometer-type wattmeter has a current coil with a resistance of 0.1 Ω and a pressure coil with resistance of 6.5 kΩ. Calculate the percentage errors while the meter is connected as (i) current coil to the load side, and (ii) pressure coil to the load side. The load is specified as (a) 12 A at 250 V with unity power factor, and (b) 12 A at 25 V with 0.4 lagging power factor.

Solution

(a) Load specified as 12 A at 250 V with unity power factor



(i) Current coil (CC) on load side

True power = $VI \cos j$

= 250 X 12 X 1

= 3000 Ω

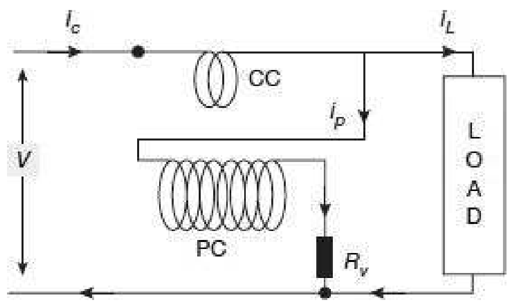
Power lost in CC = I^2 X r_C (where r_C is the resistance of CC)

= 12^2 X 0.1

= 14.4 Ω

The wattmeter will thus read total power = 3000 + 14.4 = 3014.4 Ω

Hence, error in measurement $= \frac{14.4}{3000} \times 100\% = 0.48\%$



(ii) Pressure coil (CC) on load side

True power = $VI \cos j$

= 250 X 12 X 1

= 3000 Ω

Power lost in PC = V^2/R_P (where R_P is the resistance of PC)

= $250^2/6500$

= 9.6 W

The wattmeter will thus read total power = 3000 + 9.6 = 3009.6 Ω

Hence, error in measurement $= \frac{9.6}{3000} \times 100\% = 0.32\%$

(b) Load specified as 12 A at 250 V with 0.4 power factor

(i) Current coil (CC) on load side

True power = $VI \cos j$

= 250 X 12 X 0.4

= 1200 Ω

Power lost in CC = I^2 X r_C (where r_C is the resistance of CC)

= 12^2 X 0.1

= 14.4 W

The wattmeter will thus read total power = 1200 + 14.4 = 1214.4 Ω

Hence, error in measurement $= \frac{14.4}{1200} \times 100\% = 1.2\%$

(ii) Pressure coil (CC) on load side

True power = 1200 Ω

Power lost in PC = V^2/R_P (where R_P is the resistance of PC)

= $250^2/6500$

= 9.6 W

The wattmeter will thus read total power = 1200 + 9.6 = 1209.6 Ω

Hence, error in measurement $= \frac{9.6}{1200} \times 100\% = 0.8\%$

An electrodynamometer-type wattmeter is used for power

Example 7.3 *measurement of a load at 100 V and 9 A at a power factor of 0.1 lagging. The pressure coil circuit has a resistance of 3000 Ω and inductance of 30 mH. Calculate the percentage error in wattmeter reading when the pressure coil is connected (a) on the load side, and (b) on the supply side. The current coil has a resistance of 0.1 Ω and negligible inductance. Assume 50 Hz supply frequency.*

Solution

(a) Load specified as 9 A at 100 V with 0.1 power factor:

True power = $VI \cos \varphi$

= 100 X 9 X 0.1

= 90 W

Phase-angle $\varphi = \cos^{-1}(0.1) = 1.471$ rad

Given, resistance of pressure coil circuit = 3000 Ω

and reactance of pressure coil circuit = $2p \times 50 \times 30 \times 10^{-3} = 9.42\ \Omega$

∴ phase-angle of the pressure coil circuit $\alpha = \tan^{-1}\dfrac{9.42}{3000} = 0.00313$ rad

(i) Pressure coil connected on the load side

Following the expression (7.15) for actual power indication of the wattmeter in the presence of pressure coil inductance, the actual wattmeter reading is given by

$$\text{Actual wattmeter reading} = \frac{\text{True power indication}}{\cos\varphi} \times \cos\alpha \cdot \cos(\varphi - \alpha)$$

$$\text{Actual wattmeter reading} = \frac{90}{0.1} \times \cos(0.00313) \cdot \cos(1.471 - 0.00313) = 92.47 \text{ W}$$

$$\text{Power loss in wattmeter} = V^2/R_P$$

$= 100^2/3000$

$= 3.33$ W

Total power indication of wattmeter, taking power loss in PC into account as well is

$= 92.47 + 3.33$

$= 95.8$ W

$$\therefore \text{error in measurement} = \frac{95.8 - 90}{90} \times 100\% = 6.44\%$$

(ii) Current coil connected on the load side

True power remains = 90 W

The CC gets in series with the load, and power loss in CC gets included in the total power.

Total power = $90 + I^2 \times r_C = 90 + 9^2 \times 0.1 = 98.1$ W

Load impedance $Z = V/I = 100/9 = 11.1\ \Omega$

Load resistance $R_L = Z$ X cos φ = 11.1 X 0.1 = 1.11 Ω

Load reactance $X_L = Z$ X sin φ = 11.1 X 0.995 = 11.05 Ω

Total load resistance including CC resistance

R_L¢ = 1.11 + 0.1 = 11.21 Ω

Since CC has no inductance, total load reactance including CC reactance = X_L¢ = 11.05 + 0 = 11.05 Ω

Total load power factor including CC

$$\cos\varphi' = \cos(\tan^{-1}(11.05/1.21)) = 1.462 \text{ rad}$$

$$\text{Actual wattmeter reading} = \frac{\text{Total power indication}}{\cos\varphi'} \times \cos\alpha \cdot \cos(\varphi' - \alpha)$$

$$\text{Actual wattmeter reading} = \frac{98.1}{0.109} \times \cos(0.00313) \cdot \cos(1.462 - 0.00313) = 100.52 \text{ W}$$

$$\therefore \text{ error in measurement} = \frac{100.52 - 90}{90} \times 100\% = 11.69\%$$

Larger error in case (ii) as compared to case (i) confi rms the fact that low power factor power measurements should not have the CC of wattmeter connected to the load side.

7.5 INDUCTION-TYPE WATTMETER

Induction-type wattmeters work in similar principles as for induction type ammeters and voltmeters previously discussed in Chapter 2. Induction-type wattmeters, however, following the very basic principles of mutual induction, can only be used for measurement of ac power, in contrast to electrodynamometer type wattmeters that can be used for power measurements in both ac and dc circuits. Induction type wattmeters, in contradiction to electrodynamometer-type wattmeters, can be used only with circuits having relatively steady values of frequency and voltage.

7.5.1 Construction of Induction-type Wattmeter

Schematic diagram displaying the basic constructional features of an induction-type wattmeter is shown in Figure 7.19.

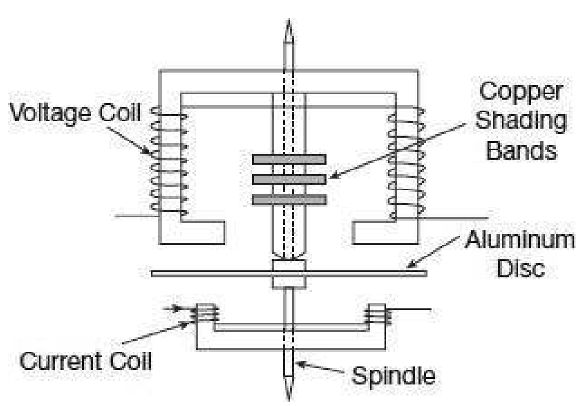


Figure 7.19 *Constructional details of induction-type wattmeter*

Induction-type wattmeters have two laminated iron-core electromagnets. One of the electromagnets is excited by the load current, and the other by a current proportional to the voltage of the circuit in which the power is to be measured. The upper magnet in Figure 7.19, which is connected across the voltage to be measured, is named as the *shunt* magnet, whereas the other electromagnet connected in series with the load to carry load current is called the *series* magnet. A thin aluminum disc, mounted in the space between the two magnets is acted upon by a combined effect of fluxes coming out of these two electromagnets. In ac circuits, interaction of these changing fluxes will induce Eddy-current within the aluminum disc.

The two voltage coils, connected in series, are wound in such a way that both of them send flux through the central limb. Copper shading bands fitted on the central limb of the shunt magnet makes the flux coming out of the magnet lag behind the applied voltage by 90°.

The series magnet houses two small current coils in series. These are wound in a way that the fluxes they create within the core of the magnet are in the same direction.

7.5.2 Operation of Induction-type Wattmeter

A simplified phasor diagram for describing the theory of operation of induction-type wattmeter is shown in Figure 7.20.

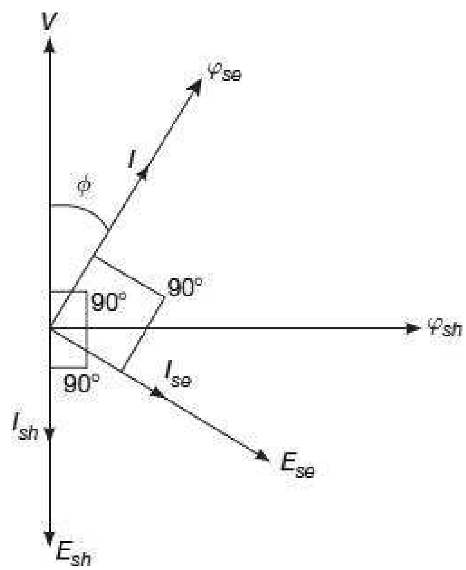


Figure 7.20 *Phasor diagram for induction-type wattmeter*

V = voltage to be measured

I = current to be measured

φ = phase-angle lag of current I with respect to voltage V

φ_{sh} = flux of the shunt magnet

φ_{se} = flux of the series magnet

E_{sh} = eddy emf induced in the disc by the flux jsh of shunt magnet

I_{sh} = eddy-current fl owing in the disc due to and in phase with Esh (neglecting inductance of the Eddy-current path)

E_{se} = eddy emf induced in the disc by the fl ux jse of series magnet

I_{se} = eddy-current fl owing in the disc due to and in phase with Ese (neglecting inductance of the Eddy-current path)

Z = impedance of the Eddy-current path

ω = angular frequency of supply in radians per second

T_d = average torque acting on the disc

The shunt magnet flux φ_{sh} is made to lag behind the applied voltage *(V)* by 90°. This is achieved by the use of copper shading rings. On the other hand, the series magnet flux φ_{se} is in the same phase as the load current *(I)* through it.

Continuing from theories of induction-type instruments in Chapter 2, the instantaneous torque acting on the aluminum disc is proportional to ($\varphi_{sh} \cdot i_{se}$ - $\varphi_{se} \cdot i_{sh}$).

Let, instantaneous value the applied voltage is

$$v = V_m \sin w\, t$$

Then, the instantaneous current is given by

$$i = I_m \sin (w\, t - j)$$

The shunt magnet flux generated is

$$\varphi_{sh} = k' \int v \cdot dt = -k' \frac{V_m}{\omega} \cos \omega t \tag{7.16}$$

where *k*¢ is a constant and the minus (-) sign indicating the fact the flux φ_{sh} lags behind the voltage by 90°.

The series magnet flux generated is

$$\varphi_{se} = kI_m \sin(\omega t - \phi) \tag{7.17}$$

where *k* is another constant.

The eddy emf induced in the disc due to the shunt magnet flux is

$$E_{sh} = -\frac{d\varphi_{sh}}{dt} = -k'V_m \sin\omega t$$

The resultant eddy-current fl owing in the disc is

$$I_{sh} = -\frac{k'V_m}{Z}\sin(\omega t - \alpha) \tag{7.18}$$

where $\alpha$is the phase-angle of the eddy path impedance (Z).

Similarly, the eddy emf induced in the disc due to the series magnet flux is

$$E_{se} = -\frac{d\varphi_{se}}{dt} = -kI_m\omega\cos(\omega t - \phi)\,a$$

The resultant Eddy-current flowing in the disc is

$$I_{se} = -\frac{kI_m}{Z}\omega\cos(\omega t - \phi - \alpha) \tag{7.19}$$

*Z*The instantaneous deflecting torque (*T*) acting on the disc can now be calculated as

$$T = \frac{kk'}{Z}V_m I_m[\cos\omega t\cos(\omega t - \phi - \alpha) + \sin(\omega t - \phi)\sin(\omega t - \alpha)] \tag{7.20}$$

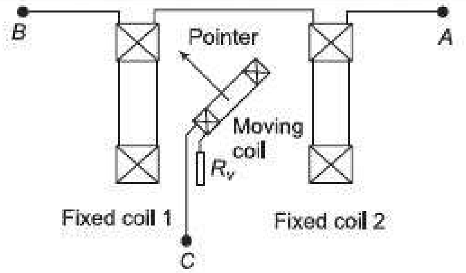
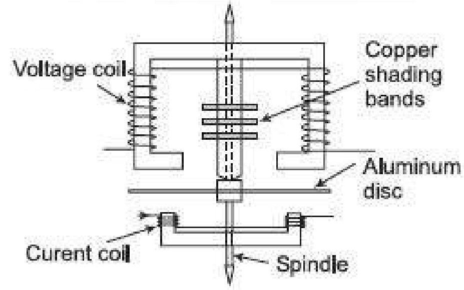
The average torque acting on the disc is thus

$$\begin{aligned} T_d &= \frac{1}{2\pi}\int_0^{2\pi}\frac{kk'}{Z}V_m I_m[\cos\omega t\cos(\omega t - \phi - \alpha) + \sin(\omega t - \phi)\sin(\omega t - \alpha)]\,dt \\ &= \frac{kk'}{Z}V_m I_m\frac{1}{2}[\cos(\phi + \alpha) + \cos(\phi - \alpha)] \\ &= \frac{kk'}{Z}V_m I_m\cos\alpha\cos\phi \\ &= \left(\frac{2kk'}{Z}\cos\alpha\right)VI\cos\phi \end{aligned} \tag{7.21}$$

where, *V* and *I* are rms values of voltage and current. Average torque on the instrument is thus found to be proportional to the power in the circuit.

7.5.3 Differences Between Dynamometer Wattmeter and Induction-type Wattmeters

Though both electrodynamometer and induction type wattmeters can be used for measurement of power, following are the differences between the two.

Dynamometer Type Watteter	Induction Type Wattmeter
	
Schematic of electrodynamometer type wattmeter	Schematic of induction type wattmeter
Current coil split in two parts, but a single pressure coil	Both current and pressure coils split in two parts each, placed on each of two arms of the two magnets
Pressure coil is the moving coil	None of the coils are moving, rather there is an aluminum disc placed between the two electromagnets, that moves
Pointer is attached with the moving (pressure) coil	Pointer is attached with the aluminum disc
Can be used for measurement of power both in AC as well as DC circuits	Can only be used for measurement of power in AC circuits
Fluid friction damping is used	Eddy current damping is used
Both the coils are air-cored	Both the coils are mounted on laminated iron core
Can be used in circuits even with fluctuating frequency and voltage	Can be used only with circuits having relatively steady values of frequency and voltage

7.6 POWER MEASUREMENT IN POLYPHASE SYSTEMS

Blondel's Theorem

The theorem states that 'in an *n*-phase network, the total power can be obtained by taking summation of the *n* wattmeters so connected that current elements of the wattmeters are each in one of the *n* lines and the corresponding voltage element is connected between that line and a common point'.

Consider the case of measuring power usingthree wattmeters in a 3-phase, 3-wire system as shown

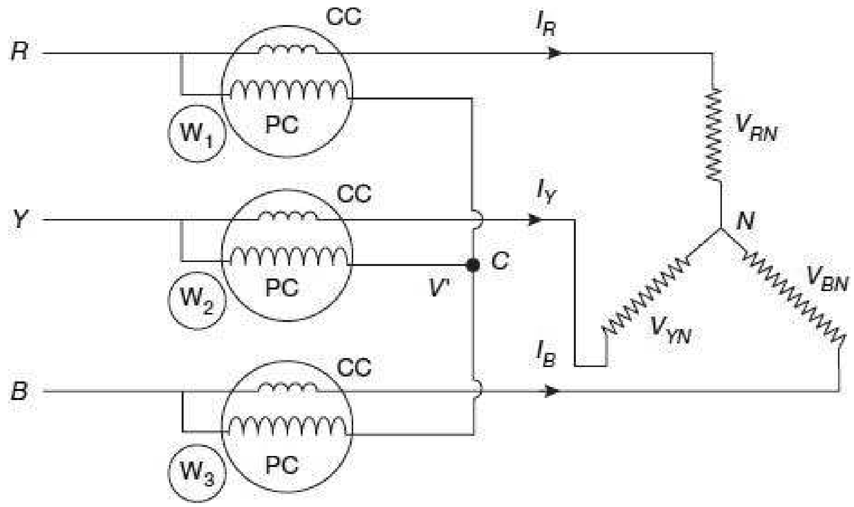


Figure 7.21 *Power measurement in a 3-phase 3-wire system*

Current coils of the three wattmeters, W_1, W_2 and W_3 are connected to the three lines *R*, *Y*, and *B*. Potential coils of the three wattmeters are connected to the common point *C*. The potential at the point *C* may be different from the neutral point *(N)* potential of load.

Power consumed by the load

$$P = V_{RN} \times I_R + V_{YN} \times I_Y + V_{BN} \times I_B \qquad (7.22)$$

Reading of wattmeter W_1, $P_1 = V_{RC}$ X I_R

Reading of wattmeter W_2, $P_2 = V_{YC}$ X I_Y

Reading of wattmeter W_3, $P_3 = V_{BC}$ X I_B

Now, if the voltage difference between the nodes *C* and *N* is taken as $V_{CN} = V_C - V_N$, then we can have

$$V_{RN} = V_R - V_N = V_R - V_C + V_C - V_N = V_{RC} + V_{CN}$$

$$V_{YN} = V_Y - V_N = V_Y - V_C + V_C - V_N = V_{YC} + V_{CN}$$

$$V_{BN} = V_B - V_N = V_B - V_C + V_C - V_N = V_{BC} + V_{CN}$$

Sum of the three wattmeter readings can now be combined as

$$\mathrm{P}_1 + P_2 + P_3 = (V_{RN} - V_{CN}) \text{ X } I_R + (V_{YN} - V_{CN}) \text{ X } I_Y + (V_{BN} - V_{CN}) \text{ X } I_{\mathrm{B}}$$

$$= V_{RN} \text{ X } I_R + V_{YN} \text{ X } I_Y + V_{BN} \text{ X } I_B - V_{CN} (I_R + I_Y + I_B)$$

Applying Kirchhoff's current law at node *N*, $(I_R + I_Y + I_B) = 0$

Thus, sum of wattmeter readings,

$$P_1 + P_2 + P_3 = V_{RN} \times I_R + V_{YN} \times I_Y + V_{BN} \times I_B \qquad (7.23)$$

Comparing with Eq. (7.22), it is observed that sum of the three individual wattmeter readings indicate the total power consumed by the load.

7.7 POWER MEASUREMENT IN THREE-PHASE SYSTEMS

[redacted]

7.7.1 Three-Wattmeter Method

1. Three-Phase Three-Wire Systems

Measurement of power in a 3-phase 3-wire system using three wattmeters has already been described in Section 7.6.1.

2. Three-Phase Four-Wire Systems

Wattmeter connections for measurement of power in a 3-phase 4-wire system are shown in Figure 7.22.

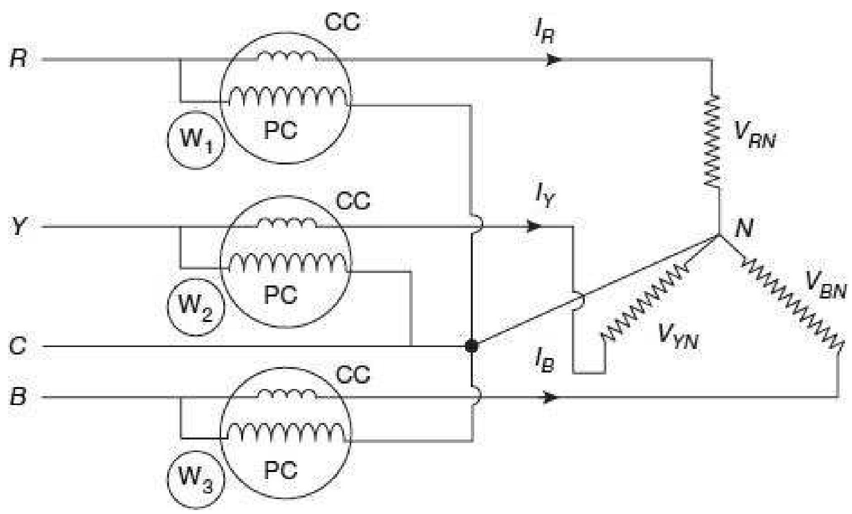


Figure 7.22 *Power measurement in 3-phase 4-wire system*

In this case, the common point of the three pressure coils coincides with the neutral *N* of the system. Voltage across each potential coil is thus, effectively the per-phase voltages of the corresponding phases. Current through current coils of the three wattmeters are nothing but the phase currents of the corresponding phases.

Sum of the three wattmeter readings in such a case will be

$$P_1 + P_2 + P_3 = V_{RN} \text{ X } I_R + V_{YN} \text{ X } I_Y + V_{BN} \text{ X } I_B$$

This is exactly the same as the power consumed by the load.

Hence, summation of the three wattmeter readings display the total power consumed by the load.

7.7.2 Two-Wattmeter Method

This is the most common method of measuring three-phase power. It is particularly useful when the load is unbalanced.

1. Star-Connected System

The connections for measurement of power in the case of a star-connected three-phase load are shown

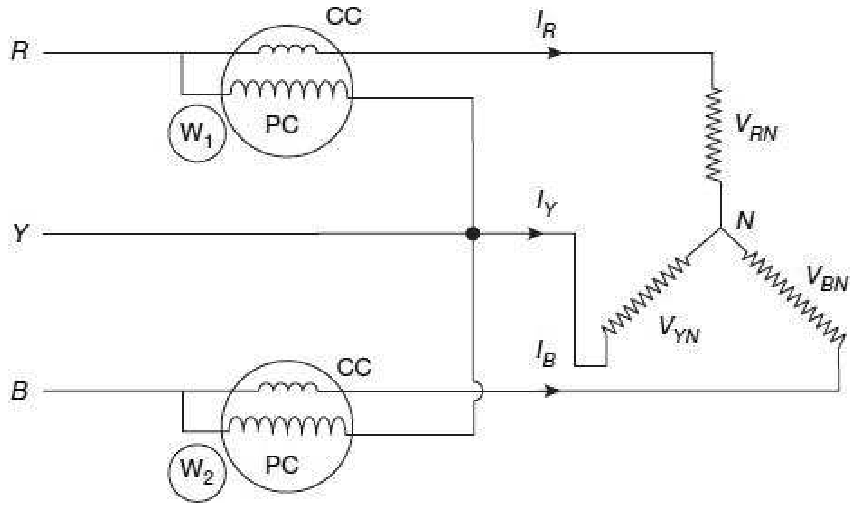


Figure 7.23 *Two-wattmeter method for star-connected load*

The current coils of the wattmeters are connected in lines R and B, and their voltage coils are connected between lines R and Y, and B and Y respectively.

Power consumed by the load

$$P = V_{RN} \times I_R + V_{YN} \times I_Y + V_{BN} \times I_B \tag{7.24}$$

Reading of wattmeter W_1, $P_1 = V_{RY}$ X $I_R = (V_{RN} - V_{YN})$ X I_R

Reading of wattmeter W_2, $P_2 = V_{BY}$ X $I_B = (V_{BN} - V_{YN})$ X I_B

Summation of the two wattmeter readings:

$$= P_1 + P_2 = (V_{RN} - V_{YN}) \text{ X } I_R + (V_{BN} - V_{YN}) \text{ X } I_B$$

$$= V_{RN} \times I_R + V_{BN} \times I_B - V_{YN} \times (I_R + I_B) \tag{7.25}$$

From Kirchhoff's law, summation of currents at node N must be zero, i.e.,

$$I_R + I_Y + I_B = 0$$

or $$I_R + I_B = -I_Y$$

Thus, from Eq. (7.25), we can re-write,

$$P_1 + P_2 = V_{RN} \times I_R + V_{YN} \times I_Y + V_{BN} \times I_B \tag{7.26}$$

It can thus, be concluded that sum of the two wattmeter readings is equal to the total power consumed by the load. This is irrespective of fact whether the load is balanced or not.

2. Delta-Connected System

Two wattmeters can also be used for measurement of total power in a three-phase delta-connected system. The connections in the case of a delta-connected three-phase load are shown in Figure 7.24.

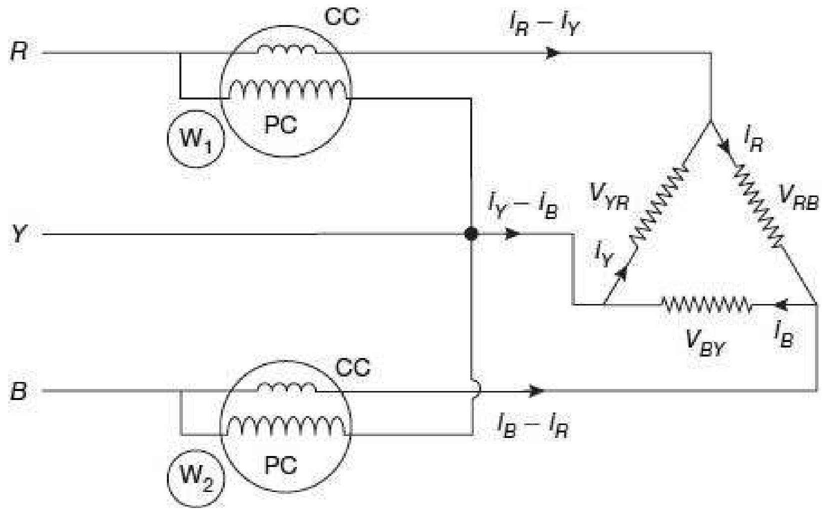


Figure 7.24 *Two-wattmeter method for delta-connected load*

The current coils of the wattmeters are connected in lines *R* and *B*, and their voltage coils are connected between lines *R* and *Y*, and *B* and *Y* respectively.

Power consumed by the load

$$P = V_{RB} \times i_R + V_{YR} \times i_Y + V_{BY} \times i_B \tag{7.27}$$

Reading of wattmeter W_1, $P_1 = -V_{YR}$ X $(i_R - i_Y)$

Reading of wattmeter W_2, $P_2 = V_{BY}$ X $(i_B - i_R)$

Summation of the two-wattmeter readings:

$$= P_1 + P_2 = -V_{YR} \text{ X } (i_R - i_Y) + V_{BY} \text{ X } (i_B - i_R)$$

$$= V_{YR} \times i_Y + V_{BY} \times i_B - i_R \times (V_{YR} + V_{BY}) \tag{7.28}$$

From Kirchhoff's voltage law, summation of voltage drops across a closed loop is zero, i.e.,

$$V_{YR} + V_{BY} + V_{RB} = 0$$

or $$V_{YR} + V_{BY} = -V_{RB}$$

Thus, from Eq. (7.28) we can re-write,

$$P_1 + P_2 = V_{YR} \times i_Y + V_{BY} \times i_B + V_{RB} \times i_R \tag{7.29}$$

Therefore, sum of the two wattmeter readings is equal to the total power consumed by the load. This is once again, irrespective of fact whether the load is balanced or not.

3. Effect of Power Factor on Wattmeter Readings

Phasor diagram for the star-connected load of Figure 7.23 with a balanced load is shown in Figure 7.25.

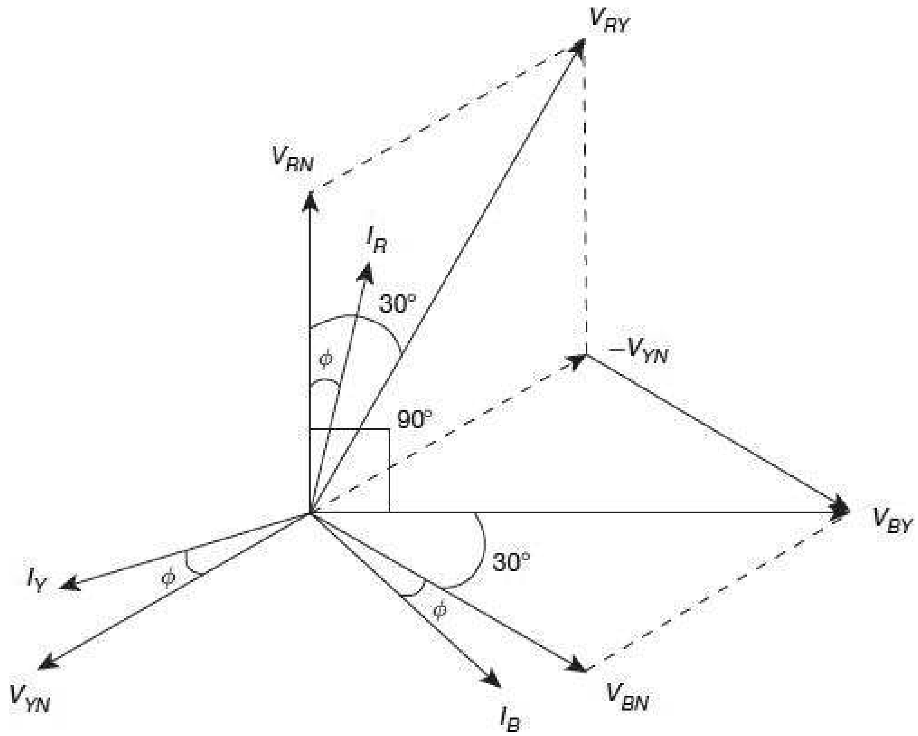


Figure 7.25 *Phasor diagram for balanced three-phase star-connected load*

Let, V_{RN}, V_{BN}, and V_{YN} are phase voltages and I_R, I_B, and I_Y are phase currents for the balanced three phase star-connected system under study.

For a balanced system, phase voltages, $V_{RN} = V_{BN} = V_{YN} = V$ (say)

And, phase currents, $I_R = I_B = I_Y = I$ (say)

For a star-connected system,

Line voltages $V_{RY} = V_{YB} = V_{BR} = \sqrt{3}$ V

Line currents $I_R = I_B = I_Y = I$

Power factor = cos ϕ, where ϕ is the angle by which each of the phase currents lag the corresponding phase voltages.

Current through the CC of wattmeter W_1 is I_R and voltage across its potential coil is V_{RY}. The current I_R leads the voltage by V_{RY} an angle (30° − ϕ), as shown in Figure 7.25.

∴ reading of wattmeter W_1 is,

$$P_1 = V_{RY} \times I_R \cos(30° - \phi) = \sqrt{3}\, VI \cos(30° - \phi) \quad (7.30)$$

Current through the CC of wattmeter W_2 is I_B and voltage across its potential coil is V_{BY}. The current I_B lags the voltage by V_{BY} an angle (30° + ϕ), as shown in Figure 7.25.

∴ reading of wattmeter W_2 is,

$$P_2 = V_{BY} \times I_B \cos(30° + \phi) = \sqrt{3}\, VI \cos(30° + \phi) \quad (7.31)$$

Sum of these two-wattmeter readings:

$$P_1 + P_1 = \sqrt{3}\, VI \cos(30° - \phi) + \sqrt{3}\, VI \cos(30° + \phi) = 3\, VI \cos\phi \quad (7.32)$$

This is the total power consumed by the load, adding together the three individual phases.

Thus, at any power factor, the total power consumed by the load will be, in any case, summation of the two wattmeter readings.

There is way to find out value of the load power factor, if unknown, by a few steps of manipulation.

Using Eq. (7.30) and Eq. (7.31), difference of the two wattmeter readings:

$$P_1 - P_1 = \sqrt{3}\,VI\cos(30° - \phi) - \sqrt{3}\,VI\cos(30° + \varphi) = \sqrt{3}\,VI\sin\phi \qquad (7.33)$$

Taking the ratio Eq. (7.33) to Eq. (7.32), one can have,

or, $$\phi = \tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right) \qquad (7.34)$$

Then, power factor, $\cos\phi = \cos\left[\tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right)\right]$

4. Unity Power Factor

With unity power factor, $\cos\phi = 1$, $\phi = 0$

Total power $P = 3\,VI\cos\phi = 3\,VI\cos 0° = 3\,VI$

Reading of wattmeter W_1 is,

$$P_1 = \sqrt{3}\,VI\cos(30° - \phi) = \sqrt{3}\,VI\cos 30° = \frac{3}{2}VI$$

Reading of wattmeter W_2 is,

$$P_1 = \sqrt{3}\,VI\cos(30° + \phi) = \sqrt{3}\,VI\cos 30° = \frac{3}{2}VI$$

Thus, summation of the two wattmeter readings $= P_1 + P_2 = \frac{3}{2}VI + \frac{3}{2}VI = 3\,VI$, which is same as the total power.

Thus, at unity power factor, readings of the two wattmeters are equal; each wattmeter reads half the total power.

5. 0.5 Power Factor

With power factor, $\cos\phi = 0.5$, $\phi = 60°$

Total power $P = 3\,VI\cos\phi = 3\,VI\cos 60° = \frac{3}{2}VI$

$$P_1 = \sqrt{3}\,VI\cos(30° - \phi) = \sqrt{3}\,VI\cos(30° - 60°) = \frac{3}{2}VI$$

$$P_2 = \sqrt{3}\,VI\cos(30° + \phi) = \sqrt{3}\,VI\cos(30° + 60°) = 0$$

Thus, summation of the two wattmeter readings $= P_1 + P_2 = \frac{3}{2}VI + 0 = \frac{3}{2}VI$, which is same as the 22 total power.

Therefore, at 0.5 power factor, one of the wattmeters reads zero, and the other reads total power.

6. Zero Power Factor

With power factor, $\cos\phi = 0$, $\phi = 90°$

Total power $P = 3VI\cos\phi = 3VI\cos 90° = 0$

$$P_1 = \sqrt{3}\,VI\cos(30° - \phi) = \sqrt{3}\,VI\cos(30° - 90°) = \sqrt{3}\,VI\cos(-60°) = \frac{\sqrt{3}}{2}VI$$

$$P_2 = \sqrt{3}\,VI\cos(30° + \phi) = \sqrt{3}\,VI\cos(30° + 90°) = \sqrt{3}\,VI\cos(120°) = -\frac{\sqrt{3}}{2}VI$$

Thus, summation of two-wattmeter readings $= P_1 + P_2 = \frac{\sqrt{3}}{2}VI - \frac{\sqrt{3}}{2}VI = 0$, which is same as the total power.

Therefore, at zero power factor, readings of the two wattmeters are equal but of opposite sign.

It should be noted from the above discussions that, at power factors below 0.5, one of the wattmeters would tend to give negative readings. Figure 7.26 plots the nature of variations in the two wattmeter readings with changing power factor. However, in many cases, meter scales are not marked on the negative side, and hence negative readings cannot be recorded. In such a case, either the current or the pressure coil needs to be reversed; such that the meter reads along positive scale. This positive reading, however, should be taken with a negative sign for calculation of total power.

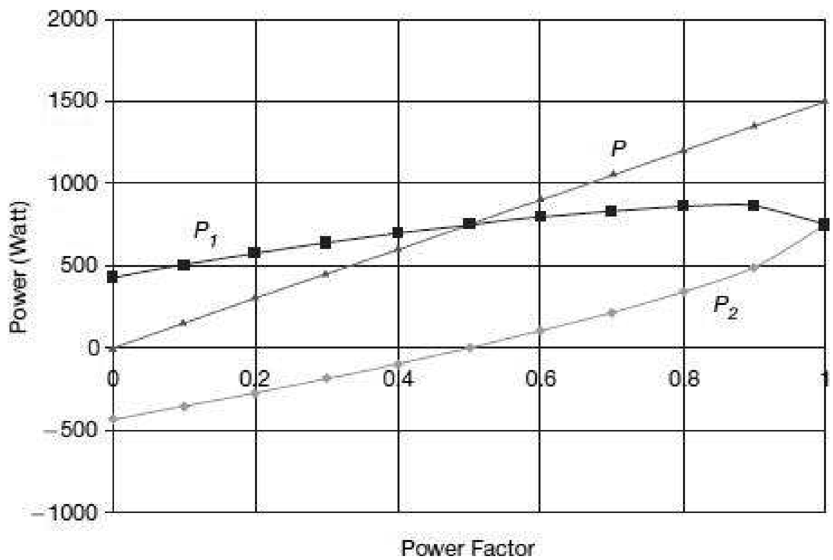


Figure 7.26 *Variation of two wattmeter readings (P_1 and P_2) in comparison to the total power (P) with respect to changing power factor; load is assumed to be 5 A at 100 V*

Example 7.4

Two wattmeters are connected to measure the power consumed by a 3-phase balanced load. One of the wattmeters read 1500 Ω and the other 700 Ω. Find power factor of the load, when (a) both the readings are positive, and (b) when the reading of the second wattmeter is obtained after reversing its current coil connection.

Solution

(a) Given, $P_1 = 1500$ W and $P_2 = 700$ W

According to (7.34), power factor angle is given by

$$\phi = \tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right)$$

or, $$\phi = \tan^{-1}\left(\sqrt{3}\frac{1500 - 700}{1500 + 700}\right) = 32.2°$$

$\therefore$ power factor = cos ϕ = cos 32.2° = 0.846

(b) Given, $P_1 = 1500$ W and $P_2 = -700$ W

$\therefore$ $$\phi = \tan^{-1}\left(\sqrt{3}\frac{1500 - (-700)}{1500 + (-700)}\right) = 78.1°$$

$\therefore$ power factor = cos ϕ = cos 78.1° = 0.21

Example 7.5

Two wattmeters are connected to measure the power consumed by a 3-phase load with power factor 0.4. Total power consumed by the load, as indicated by the two wattmeters is 30 kW. Find the individual wattmeter readings.

Solution If P_1 and P_2 are the two individual wattmeter readings then according to the problem, $P_1 + P_2 = 30$ kW

Given, power factor, cos $\phi = 0.4$

$\therefore$ power factor angle $\phi = \cos^{-1} 0.4 = 66.4°$

According to Eq. (7.3),

$$\phi = \tan^{-1}\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right)$$

or, $$\left(\sqrt{3}\frac{P_1 - P_2}{P_1 + P_2}\right) = \tan\phi = 2.29$$

Solving (i) and (ii), we get $P_1 = 34.85$ kW and $P_2 = -4.85$ kW

7.8 REACTIVE POWER MEASUREMENTS

If *V*and *I* are the rms values of the voltage and current in a single-phase circuit and *Φ* is the phase-angle difference between them, then the active power in the circuit is, *VI*cos *Φ*. The active power is obtained by multiplying the voltage by the component of current which is in same phase with the voltage (i.e., *I*cos *Φ*). The component of current which is 90° out of phase with the voltage (i.e., *I*sin *Φ*) is called the reactive component of current and the product *VI* sin *Φ* is called the reactive power. Measurement of reactive power, along with active power is sometimes important, since the phase-angle *Φ* of the circuit can be obtained from the ratio (reactive power)/(active power).

$$\frac{\text{Reactive power}}{\text{Active power}} = \frac{VI\sin\phi}{VI\cos\phi} = \tan\phi$$

The same wattmeter that is used for measurement of active power can also be used for measurement of reactive power with slight modifications in the connections. Observing the fact that sin $\Phi = \cos(90° - \Phi)$, the wattmeter may be so connected that the current coil carries the load current and the voltage coil should have a voltage that is 90° out of phase with the actual voltage of the circuit. Under these circumstances, the wattmeter will read

$VI \cos(90^{\circ}\ \Phi)$, or $VI \sin \Phi$

For single-phase measurements, the pressure coil circuit may be made to behave as large inductive by inclusion of external inductors in series with it. This will cause the pressure coil current to lag behind the voltage by 90°, and thus the wattmeter will read

$$VI \cos(90^{\circ} - \Phi) = VI \sin \Phi = \text{Reactive power}$$

For measurement of reactive power in a circuit with a balanced 3-phase load, a single wattmeter may be used with suitable connections as shown in Figure 7.27, the corresponding phasor diagram is drawn in Figure 7.28.

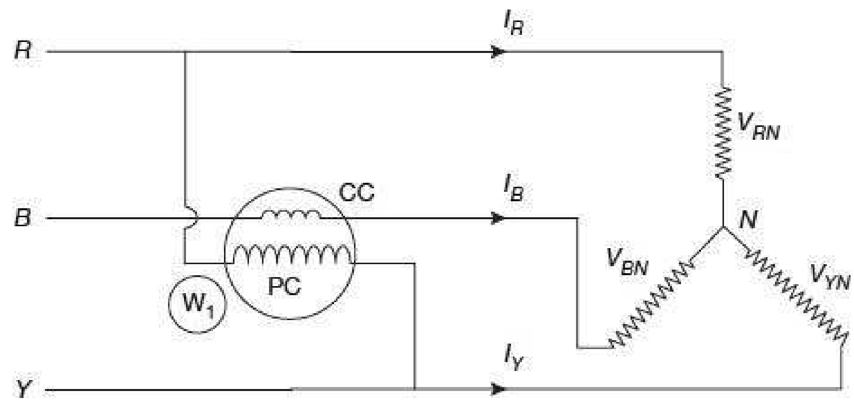


Figure 7.27 *Connection diagram for reactive power measurement*

The phasor I_B represents the current flowing through the CC of the wattmeter. The voltage applied across the voltage coil is the voltage difference between the lines *R* and *Y*, i.e., the difference between the phasors V_{RN} and V_{YN}, i.e., the phasor V_{RY}. The phase-angle between V_{RY} and $-V_{YN}$ is 30°, for the balanced system, and that between $-V_{YN}$ and V_{BN} is 60°. Thus, the total phase-angle difference between V_{RY} and V_{BN} is 90°, and the angle between V_{RY} and I_B is $(90° + \Phi)$. The wattmeter will read

$$V_{RY} \times I_B \times \cos(90^{\circ} + \phi) = \sqrt{3}\ VI \cos(90^{\circ} + \phi) = -\sqrt{3}\ VI \sin\phi = -W_R$$

where, *V* is the per phase rms voltage and *I* is the line current of the system.

The total reactive power of the circuit is thus:

$$3\ VI \sin\phi = -\sqrt{3}\ W_R$$

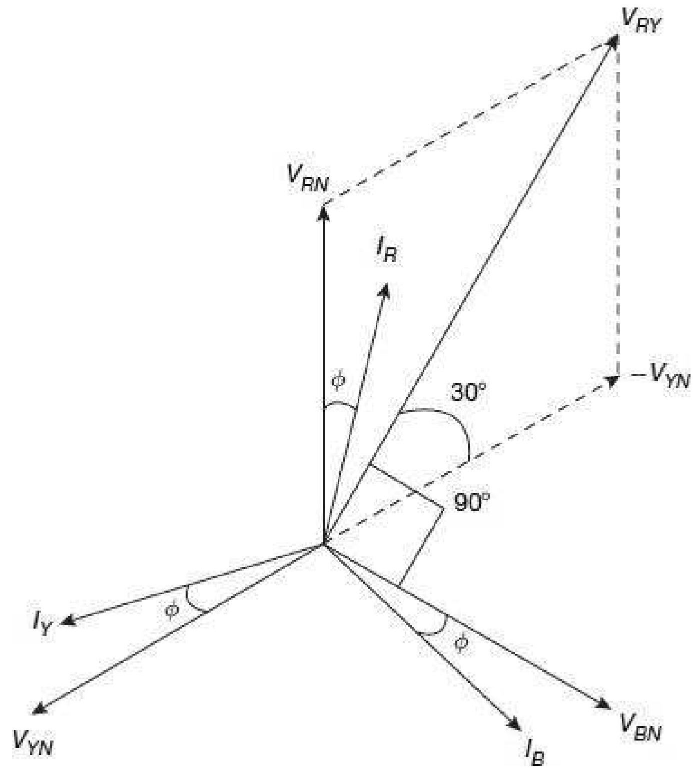


Figure 7.28 *Phasor diagram for reactive power measurement*

7.9 POWER MEASUREMENT WITH INSTRUMENT TRANSFORMERS

Power measurements in high-voltage, high-current circuits can be effectively done by the use of Potential Transformers (PT) and Current Transformers (CT) in conjunction with conventional wattmeters. By using a number of current transformers of different ratio for the current coil, and a number of potential transformers with different turns ratio for the voltage coil, the same wattmeter can be utilised for power measurements over a wide range.

Primary winding of the CT is connected in series with the load and the secondary winding is connected with the wattmeter current coil, often with an ammeter in between.

Primary winding of the PT is connected across the main power supply lines, and its secondary winding is connected in parallel with the wattmeter voltage coil.

The connections of a wattmeter when so used are shown in Figure 7.29, an ammeter and a voltmeter is also connected in the circuit.

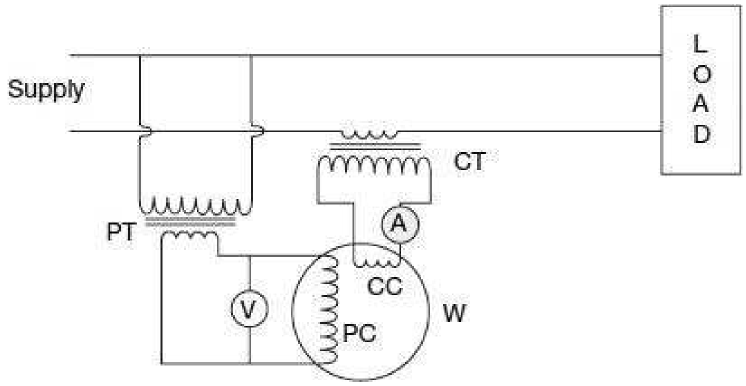


Figure 7.29 *Power measurement using instrument transformers*

PT and CT are, however, unless carefully designed and compensated, have inherent ratio and phase-angle errors. Ratio errors are more severe for ammeters and voltmeters, where phase-angle errors have more pronounced effects on power measurements. Unless properly compensated for, these can lead to substantial errors in power measurements.

Error introduced in power measurement due to ratio errors in PT and CT can be explained with the help of phasor diagrams of current and voltage in load and also in wattmeter coils, as shown in Figure 7.30. Figure 7.30(a) refers to a load with lagging power factor, and Figure 7.30(b) to a load with leading power factor.

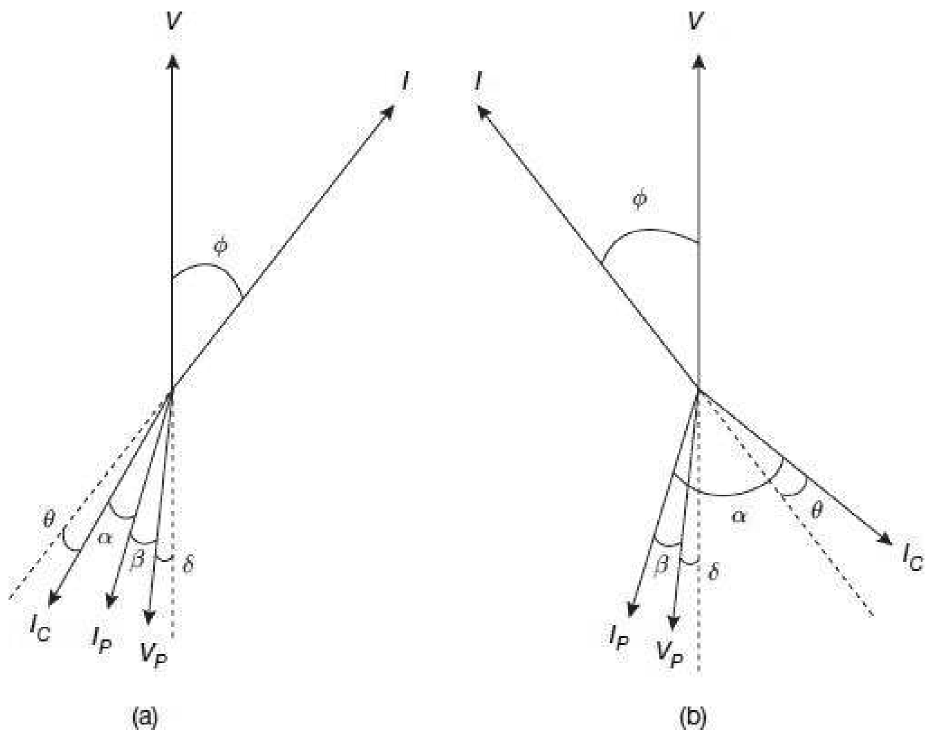


Figure 7.30 *Phasor diagram showing phase-angle error during power measurement using instrument transformers for (a) lagging load, and (b) leading load*

V = voltage across the load

I = load current

Φ = load phase-angle between V and I

V_P = voltage across secondary wining of PT Voltage across PC of Wattmeter

I_P = current through the PC

β = phase-angle lag between current I_P and voltage V_P due to inductance of the PC

I_C = secondary current of CT Current through CC of wattmeter

α = phase-angle difference the currents in CC and PC of wattmeter

δ = phase-angle error of PT, i.e., the deviation from secondary being exactly 180° out of phase with Respect to primary

θ = phase-angle error of CT, i.e., the deviation from secondary being exactly 180° out of phase with respect to primary

The Phasor shown with dotted lines are images of *V* and *I* with 180° phase shift.

1. Lagging Power Factor

In general, phase-angle (θ) of CT is positive, whereas phase-angle of PT (δ) may be positive or negative.

Thus, phase-angle of the load is $\Phi = \theta + \alpha + \beta \pm \delta$

2. Leading Power Factor

Phase-angle of the load is $\Phi = \alpha - \theta - \beta \pm \delta$

3. Correction Factors

For the time being, neglecting the ratio error, the correction factor that need to be incorporated for lagging power factor load is

$$K = \frac{\cos\phi}{\cos\beta \cdot \cos\alpha} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi - \theta - \beta \pm \delta)}$$

For leading power factor, the correction factor is

$$K = \frac{\cos\phi}{\cos\beta \cdot \cos\alpha} = \frac{\cos\phi}{\cos\beta \cdot \cos(\phi + \theta + \beta \pm \delta)}$$

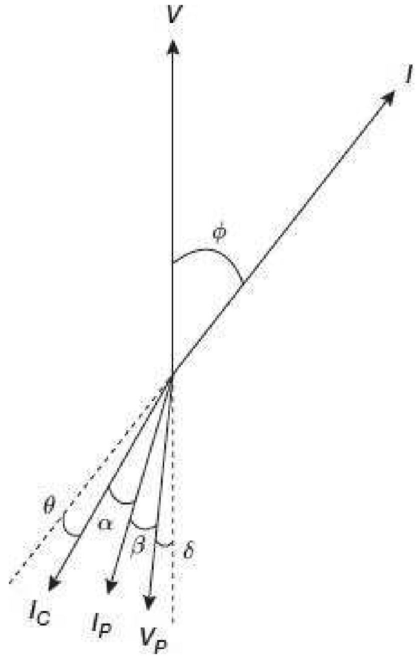
Considering ratio errors for both PT and CT, the corresponding correction factors for ratio errors also need to be incorporated.

General expression for the power to be measured can now be written as

Power = K x Actual PT ratio x Actual CT ratio x Wattmeter reading

Example 7.6

A 110 V, 5 A wattmeter indicates 450 Ω, when used along with a PT with nominal ratio 110:1 and a CT with nominal ratio 25:1 respectively. Resistance and inductance of the wattmeter pressure coil circuit is 250 Ω and 7 mH *respectively. The ratio errors and phase-angles of PT and CT are +0.6%, -60', and -0.3% and +75' respectively. What is the true value of power being measured? The load phase-angle is 35° lagging, and the supply frequency is 50 Hz.*



Solution Resistance of pressure coil circuit = 250 Ω

Reactance of pressure coil circuit

$= 2p \times 50 \times 70 \times 10^{-3} = 2.2\ \Omega$

∴ phase-angle of pressure coil

$$\beta = \tan^{-1}\frac{2.2}{250} \approx 30'$$

Given,

Phase-angle of CT = $\theta = +75'$

Phase-angle of PT = $\delta = -60'$

Phase-angle of load = $\Phi = 35^\circ$

∴ phase-angle between wattmeter pressure coil current I_P and current coil current I_C is

$$\alpha = \Phi - \theta - \beta - \delta = 35^\circ - 75' - 60' - 30'$$

or, $\alpha = 32^\circ 15'$

Correction factor for phase-angle error,

$$K = \frac{\cos\phi}{\cos\beta \cdot \cos\alpha} = \frac{\cos 35^\circ}{\cos 30' \times \cos 32^\circ 15'} = 1.495$$

For PT or CT, the percentage ratio error is given by

$$\%E = \frac{Kn - Ka}{Ka} \times 100$$

where *Kn* = Nominal ratio and *Ka* = Actual ratio

Thus, actual ratio $Ka = \frac{Kn \times 100}{100 + \%E}$

Thus, for CT, actual ratio $= \frac{25 \times 100}{100 - 0.3} = 25.08$

And, for PT, actual ratio $= \frac{110 \times 100}{100 + 0.6} = 109.34$

Hence, true power of load $P = 1.495 \text{ X } 25.08 \text{ X } 109.34 \text{ X } 450 \text{ X } 10^{-3}$

$= 1844.8$ kW

EXERCISE

Objective-type Questions

1. Power consumed by a simple dc circuit can be measured with the help of a single ammeter if
   (a) the load resistance is known
   (b) the supply voltage is known
   (c) both of (a) and (b) known
   (d) cannot be measured
2. Power indicated while measuring power in a dc circuit using an ammeter and a voltmeter, when the voltmeter is connected to the load side, is
   (a) true power consumed by the load
   (b) power consumed by the load plus power lost in ammeter
   (c) power consumed by the load plus power lost in voltmeter
   (d) power consumed by load plus power lost in both ammeter and voltmeter
3. In ammeter–voltmeter method for measurement of power in dc circuits, true power will be obtained with ammeter connected to the load side when
   (a) voltmeter has zero internal impedance
   (b) ammeter has zero internal impedance
   (c) ammeter has infinite internal impedance
   (d) voltmeter has infinite internal impedance
4. Electrodynamometer-type wattmeters have a construction where
   (a) current coil is fixed
   (b) voltage coil is fixed
   (c) both voltage and current coils are movable
   (d) both voltage and current coils are fixed
5. In electrodynamometer-type wattmeters, current coil is made of two sections
   (a) to reduce power loss
   (b) to produce uniform magnetic field
   (c) to prevent Eddy-current loss
   (d) to reduce errors due to stray magnetic field
6. In electrodynamometer-type wattmeters, current coils carrying heavy currents are made of stranded wire
   (a) to reduce iron loss

(b) to reduce Eddy-current loss in conductor

(c) to reduce hysteresis loss

(d) all of the above

7. In electrodynamometer-type wattmeters, a high value noninductive resistance is connected in series with the pressure coil

    (a) to prevent overheating of the spring leading current in the pressure coil

    (b) to restrict the current in the pressure coil

    (c) to improve power factor of the pressure coil

    (d) all of the above

8. In electrodynamometer-type wattmeters, both current coil and pressure coil are preferably air-cored

    (a) to reduce effects of stray magnetic field

    (b) to reduce Eddy-current losses under AC operation

    (c) to increase the deflecting torque

    (d) all of the above

9. In electrodynamometer-type wattmeters, pressure coil inductance produce error which is

    (a) constant irrespective of load power factor

    (b) higher at low power factors of load

    (c) lower at low power factors of load

    (d) same at lagging and leading power factors of load

10. When measuring power in a circuit with low current, the wattmeter current coil should be connected

    (a) to the load side

    (b) to the source side

    (c) anywhere, either load side or source side, does not matter

    (d) in series with the load along with CT for current amplification

11. When measuring power in a circuit with high current

    (a) current coil should be connected to the load side

    (b) pressure coil should be connected to the load side

    (c) pressure coil should be connected to the source side

    (d) the placement of pressure and current coils are immaterial

12. In induction-type wattmeters,

    (a) voltage coil is the moving coil

    (b) current coil is the moving coil

    (c) both are moving

    (d) none are moving

13. Three wattmeters are used to measure power in a 3-phase balanced star connected load.

    (a) The reading will be higher when the neutral point is used as the common point for wattmeter pressure coils.

    (b) The reading will be lower when the neutral point is used as the common point for wattmeter pressure coils.

    (c) Wattmeter reading will remain unchanged irrespective whether the neutral is used for measurement or not.

    (d) The three-wattmeter method cannot be used in systems with 4 wires.

14. Two wattmeters can be used to measure power in a

    (a) three-phase four-wire balanced load

    (b) three-phase four-wire unbalanced load

(c) three-phase three-wire unbalanced load

(d) all of the above

15. In 2-wattmeter method for measurement of power in a star-connected 3 phase load, magnitude of the two wattmeter readings will be equal

(a) at zero power factor

(b) at unity power factor

(c) at 0.5 power factor

(d) readings of the two wattmeters will never be equal

Answers

1. (a)	2. (c)	3. (b)	4. (a)	5. (b)	6. (b)	7. (d)
8. (b)	9. (a)	10. (a)	11. (b)	12. (d)	3. (c)	14. (d)
15. (b)						

Short-answer Questions

1. Draw and label the different internal parts of an electrodynamometer-type wattmeter. What are the special constructional features of the fixed and the moving coils in such an instrument?
2. Discuss about the special shielding requirements and measures taken for shielding internal parts of an electrodynamometer-type wattmeter against external stray magnetic fields.
3. Draw and explain the operation of a compensated wattmeter used to reduce errors due to connection of pressure coil nearer to the load side as pertinent to electrodynamometer-type wattmeters.
4. Draw and explain the constructional features of an induction-type wattmeter used for measurement of power.
5. Draw the schematic and derive how three-phase power consumed by a delta-connected load can be measured by two wattmeters.
6. Explain how power factor of an unknown three-phase load can be estimated by using two wattmeters.
7. Two wattmeters are connected to measure the power consumed by a 3-phase load with a power factor of 0.35. Total power consumed by the load, as indicated by the two wattmeters, is 70 kW. Find the individual wattmeter readings.
8. With the help of suitable diagrams and sketches, explain how a single wattmeter can be used for measurement of reactive power in a 3-phase balanced system.
9. Derive an expression for the correction factor necessary to be incorporated in wattmeter readings to rectify phase-angle error in instrument transformers while used for measurement of power.

Long-answer Questions

1. Describe the constructional details of an electrodynamometer-type wattmeter. Comment upon the shape of scale when spring control is used.
2. Derive the expression for torque when an electrodynamometer-type wattmeter is used for measurement of power in ac circuits. Why should the pressure-coil circuit be made highly resistive?
3. List the different sources of error in electrodynamometer-type wattmeters. Briefly discuss the error due to pressure coil inductance. How is this error compensated?
4. (a) Discuss the special features of damping systems in electrodynamometer-type wattmeters.

   (b) An electrodynamometer-type wattmeter is used for power measurement of a load at 200 V and 9 A at a power factor of 0.3 lagging. The pressure-coil circuit has a resistance of 4000 and inductance of 40 mH. Calculate the percentage error in the wattmeter reading when the pressure coil is connected (a) on the load side, and (b) on the supply side. The current coil has a resistance of 0.2 and negligible inductance. Assume 50 Hz supply frequency.
5. A 250 V, 10 A electrodynamometer wattmeter has resistance of current coil and potential coil of 0.5 and 12.5 k respectively. Find the percentage error due to the two possible connections: (a) pressure coil on load side, and (b) current coil on load side with unity power factor loads at 250 V with currents (i) 4A, and (ii) 8A. Neglect error due to pressure coil inductance.

6. Discuss in brief the constructional details of an induction-type wattmeter. Show how the deflecting torque in such in instrument can be made proportional to the power in ac circuits.

7. (a) Draw the schematic and derive how three-phase power consumed by a star-connected load can be measured by two wattmeters.

   (b) Two wattmeters are connected to measure the power consumed by a 3-phase balanced load. One of the wattmeters reads 1500 Ω and the other, 700 Ω. Calculate power and power factor of the load, when (a) both the readings are positive, and (b) when the reading of the second wattmeter is obtained after reversing its current coil connection.

8. A 110 V, 5 A range wattmeter is used along with instrument transformers to measure power consumed by a load at 6 KV taking 100 A at 0.5 lagging power factor. The instrument transformers have the following specifications:

   PT: Nominal ratio = 80:1; Ratio error = +1.5%; Phase error = –2°

   CT: Nominal ratio = 20:1; Ratio error = –1.0%; Phase error = +1°

   Assuming wattmeter reads correctly, find the error in the indicated power due to instrument-transformer errors.

8 Measurement of Energy

8.1 INTRODUCTION

Energy is the total power consumed over a time interval, that is Energy = Power × Time. Generally, the process of measurement of energy is same as that for measurement of power except for the fact that the instrument used should not merely measure power or rate of consumption of energy, but must also take into account the time interval during which the power is being supplied.

The unit of energy can be expressed in terms of Joule or Watt-second or Watt-hour as per convenience. A larger unit that is most commonly used is kilowatt-hour (kWh), which is defined as the energy consumed when power is delivered at an average rate of 1 kilowatt for one hour. In commercial metering, this amount of 1 kilowatt-hour (kWh) energy is specified as 1 unit of energy.

Energy meters used for measurement of energy have moving systems that revolve continuously, unlike in indicating instruments where it deflects only through a fraction of a revolution. In energy meters, the speed of revolution is proportional to the power consumed. Thus, total number of revolutions made by the meter moving system over a given interval of time is proportional to the energy consumed. In this context, a term called meter constant, defined as the number of revolutions made per kWh, is used. Value of the meter constant is usually marked on the meter enclosure.

8.2 SINGLE-PHASE INDUCTION-TYPE ENERGY METER

Induction-type instruments are most commonly used as energy meters for measurement of energy in domestic and industrial ac circuits. Induction-type meters have lower friction and higher torque/weight ratio; they are inexpensive, yet reasonably accurate and can retain their accuracy over considerable range of loads and temperature.

8.2.1 Basic Theory of Induction-type Meters

In all induction-type instruments, two time-varying fluxes are created in the windings provided on the static part of the instrument. These fluxes are made to link with a metal disc or drum and produce emf therein. These emfs in turn, circulate eddy current on the body of the metal disc. Interaction of these fluxes and eddy currents produce torques that make the disc or drum to rotate. Schematic diagrams representing front and top views of such an instrument is shown in Figure 8.1.

A thin aluminum disc free to rotate about its central axis is fitted with a spindle and placed below the two poles φ_1 and φ_2. Fluxes φ_1 and φ_2 coming out of the two electromagnets φ_1 and φ_2 link with the aluminum disc placed below. These fluxes are alternating in nature, and hence they induce emfs in the aluminum disc. These induced emfs will in turn produce eddy currents i_1 and i_2 on the disc, as shown in Figure 8.1. There are two sets of fluxes φ_1 and φ_2, and two sets of currents i_1 and i_2. Current i_1 interacts with flux φ_2 to produce a force F_1 and hence a torque *Td1* on the disc. Similarly, current i_2 interacts with flux φ_1 to produce a force F_2 and hence a torque T_{d2} on the disc. Total torque is resultant of the torques T_{d1} and T_{d2}.

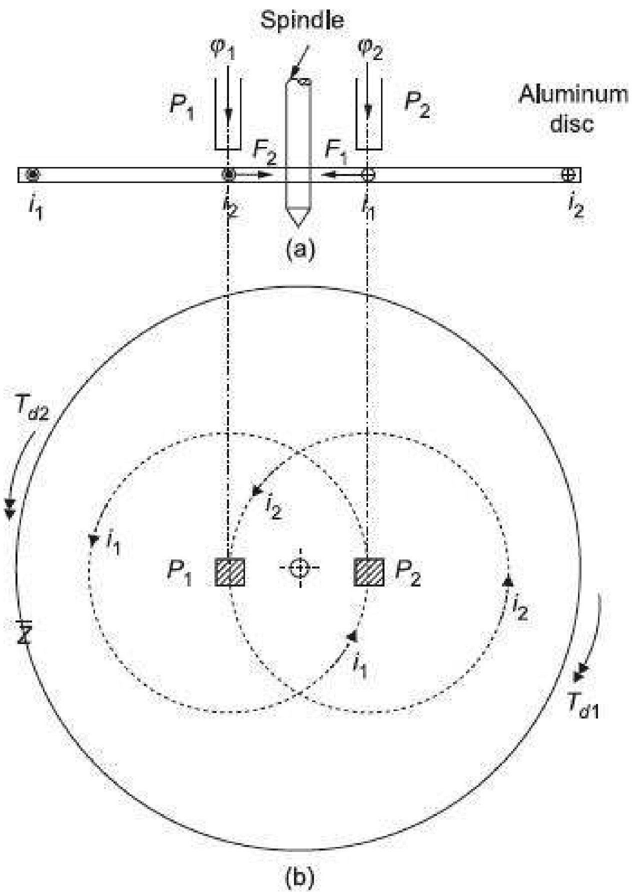


Figure 8.1 *Working principle of induction-type instrument: (a) Front view (b) Top view*

Let φ_1 and φ_2 are the instantaneous values of two fluxes having a phase difference of α between them. Therefore, we can write

$$\varphi_1 = \varphi_1 = \varphi_{1m} \sin \omega t$$

and $$\varphi_2 = \varphi_{2m} \sin(\omega t - \alpha)$$

where, φ_{1m} and φ_{2m} are peak values of fluxes φ_1 and φ_2 respectively.

The flux φ_1 will produce an alternating emf is the disc, given by

$$e_1 = -\frac{d\varphi_1}{dt} = -\frac{d}{dt}(\varphi_{1m} \sin \omega t) = -\varphi_{1m} \omega \cos \omega t$$

Similarly, the alternating emf produced in the disc due to the flux φ_2 is given by

$$e_2 = -\varphi_{2m} \omega \cos(\omega t - \alpha)$$

If, $\overline{Z}$ is considered to the impedance of the aluminum disc with power factor β then eddy current induced in the disc due to the emf e_1 can be expressed as

$$i_1 = \frac{e_1}{\bar{\bar{Z}}} = -\frac{\varphi_{1m}\omega\cos(\omega t - \beta)}{Z}$$

Similarly, eddy current induced in the disc due to the emf e_2 is given by

$$i_2 = \frac{e_2}{\bar{\bar{Z}}} = -\frac{\varphi_{2m}\omega\cos(\omega t - \alpha - \beta)}{Z}$$

Instantaneous torque developed in proportional to the product of instantaneous current and instantaneous flux are those that interact with each other to produce the torque in question.

∴ instantaneous torque T_{d1} produced due to interaction of the current i_1 and flux φ_2 is given by

$$T_{d1} \propto \varphi_2 i_1$$

Similarly, instantaneous torque T_{d2} produced due to interaction of the current i_2 and flux φ_1 is given by

$$T_{d2} \propto \varphi_1 i_2$$

Total deflecting torque can thus be calculated as

$$T_d \propto T_{d1} - T_{d2} \propto \varphi_2 i_1 - \varphi_1 i_2$$

$$T_d \propto \begin{bmatrix} \{\varphi_{2m}\sin(\omega t - \alpha)\} \times \left\{-\dfrac{\varphi_{1m}\omega\cos(\omega t - \beta)}{Z}\right\} \\ -\{\varphi_{1m}\sin\omega t\} \times \left\{-\dfrac{\varphi_{2m}\omega\cos(\omega t - \alpha - \beta)}{Z}\right\} \end{bmatrix}$$

$$T_d \propto \frac{\varphi_{1m}\varphi_{2m}\omega}{Z}[\sin\omega t\cos(\omega t - \alpha - \beta) - \sin(\omega t - \alpha)\cos(\omega t - \beta)]$$

$$T_d \propto \frac{\varphi_{1m}\varphi_{2m}\omega}{Z}\cdot\frac{1}{2}\begin{bmatrix} \sin(\omega t + \omega t - \alpha - \beta) + \sin(\omega t - \omega t + \alpha + \beta) \\ -\sin(\omega t - \alpha + \omega t - \beta) - \sin(\omega t - \alpha - \omega t + \beta) \end{bmatrix}$$

$$T_d \propto \frac{\varphi_{1m}\varphi_{2m}\omega}{Z}\cdot\frac{1}{2}\begin{bmatrix} \sin(2\omega t - \alpha - \beta) + \sin(\alpha + \beta) \\ -\sin(2\omega t - \alpha - \beta) - \sin(\beta - \alpha) \end{bmatrix}$$

$$T_d \propto \frac{\varphi_{1m}\varphi_{2m}\omega}{Z}\cdot\frac{1}{2}[\sin(\alpha + \beta) - \sin(\beta - \alpha)]$$

$$T_d \propto \frac{\varphi_{1m}\varphi_{2m}\omega}{Z}\cdot\frac{1}{2}[\sin\alpha\cos\beta + \cos\alpha\sin\beta - \sin\beta\cos\alpha + \cos\beta\sin\alpha]$$

$$T_d \propto \frac{\varphi_{1m}\varphi_{2m}\omega}{Z}\cdot\sin\alpha\cos\beta \qquad (8.1)$$

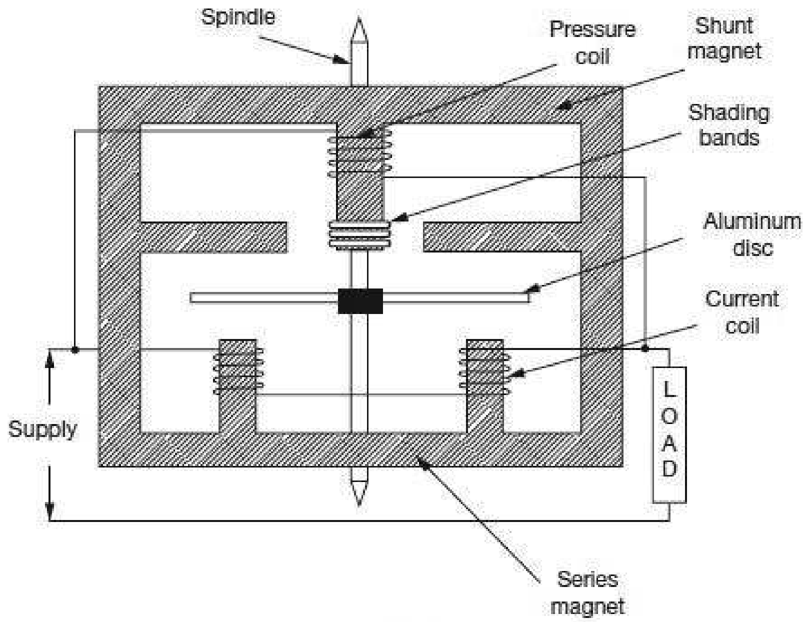
The following two observations can be made from Eq. (8.1):

1. The torque is directly proportional to the power factor of the aluminum disc (cos *β*). Thus, to increase the deflecting torque, the path of eddy current in the disc must be as resistive as possible, so that value of cos *β* is as high as possible.
2. The torque is directly proportional to sin ***α*****.** Therefore, to have large deflecting

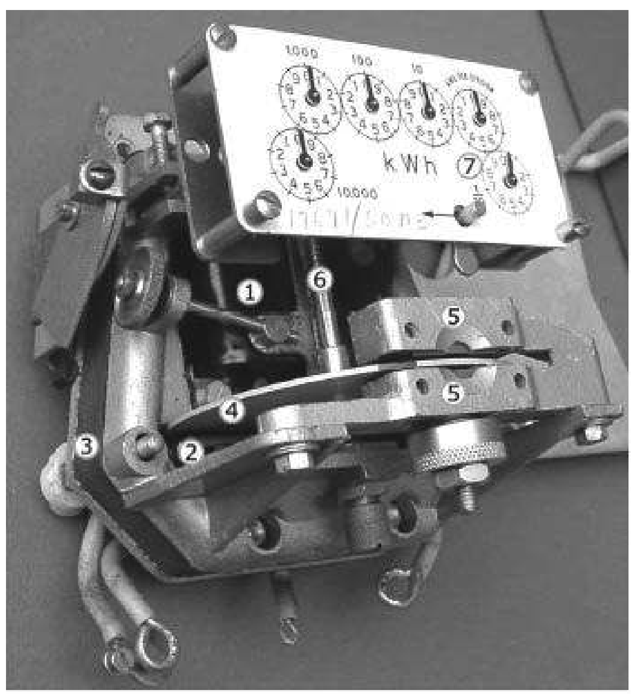
torque, the angle α between the two fluxes should preferably be as nearly as possible close to 90°.

8.2.2 Constructional Details of Induction-Type Energy Meter

Constructional details of an induction-type single-phase energy meter are schematically shown in Figure 8.2(a). The photograph of such an arrangement is shown in Figure 8.2(b).



(a)



(b)

Figure 8.2 *(a) Constructional details of induction-type single-phase energy meter*
(b) Photograph of induction-type single-phase energy meter
(Courtesy, Creative Commons Attribution Share Alike 2.5)

1. **Volatage coil**–many turns of fine wire encased in plastic, connected in parallel with load.

2. **Current coil**–few turns of thick wire, connected in series with load

3. **Stator**–concentrates and confines magnetic field.
4. **Aluminum rotor disc**.
5. **Rotor brake magnets**
6. **Spindle with worm gear**.
7. **Display dials**

A single phase energy meter has four essential parts:

(i) Operating system

(ii) Moving system

(iii) Braking system

(iv) Registering system

1. Operating System

The operating system consists of two electromagnets. The cores of these electromagnets are made of silicon steel laminations. The coils of one of these electromagnets (series magnet) are connected in series with the load, and is called the current coil. The other electromagnet (shunt magnet) is wound with a coil that is connected across the supply, called the pressure coil. The pressure coil, thus, carries a current that is proportional to supply voltage.

Shading bands made of copper are provided on the central limb of the shunt magnet. Shading bands, as will be described later, are used to bring the flux i — Bearing produced by a shunt magnet exactly in quadrature Pivot with the applied voltage.

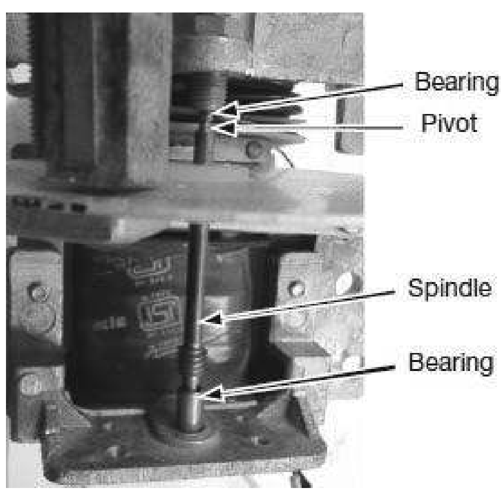


Figure 8.3 *Pivot and jewel bearing*

2. Moving System

The moving system consists of a light aluminum disc mounted on a spindle. The disc is placed in the space between the series and shunt magnets. The disc is so positioned that it intersects the flux produced by both the magnets. The deflecting torque on the disc is produced by interaction between these fluxes and the eddy current they induce in the disc. In energy meters, there is no control spring as such, so that there is continuous rotation of the disc.

The spindle is supported by a steel pivot supported by jewel bearings at the two ends, as shown in Figure 8.3. A unique design for suspension of the rotating disc is used in 'floating-shaft' energy meters. In such a construction, the rotating shaft has one small piece of permanent magnet at each end. The upper magnet is attracted by a magnet placed in the upper bearing, whereas the lower magnet is attracted by another magnet placed in the lower bearing. The moving system thus floats without touching either of the bearing surfaces. In this way, friction while movement of the disc is drastically reduced.

3. Braking System

The braking system consists of a braking device which is usually a permanent magnet positioned near the edge of the aluminum disc. The arrangement is shown in Figure 8.4.

The emf induced in the aluminum disc due to relative motion between the rotating disc and the fixed permanent magnet (brake magnet) induces eddy current in the disc. This eddy current, while interacting with the brake magnet flux, produces a retarding or braking torque. This braking torque is proportional to speed of the rotating disc. When the braking torque becomes equal to the operating torque, the disc rotates at a steady speed.

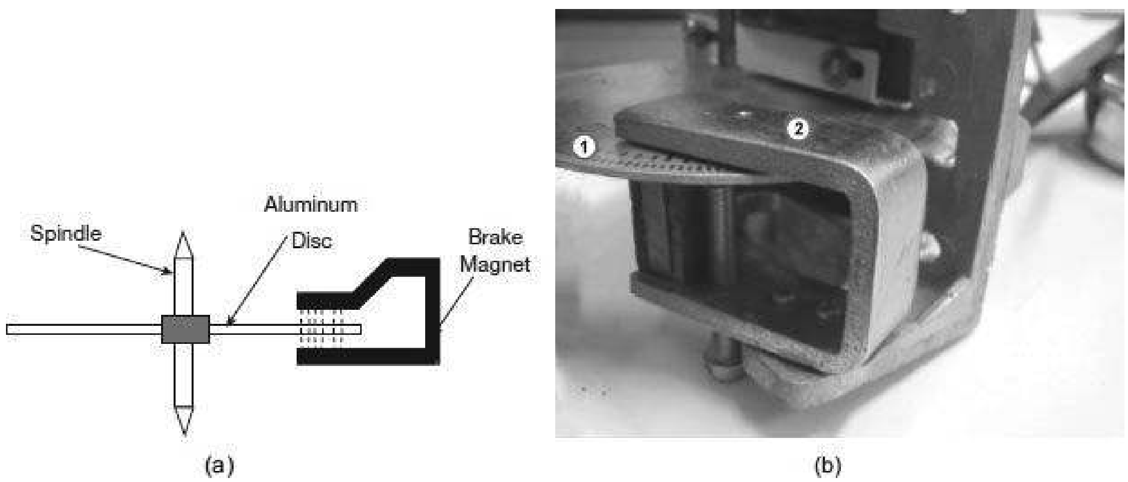


Figure 8.4 *Brake magnet to provide eddy current braking in induction-type single-phase energy meter: (a) Schematic diagram (b) Actual picture (1) Aluminum rotating disc (2) Brake magnet*

The position of the permanent magnet with respect to the rotating disc is adjustable. Therefore, braking torque can be adjusted by shifting the permanent magnet to different radial positions with respect to the disc.

It is pertinent to mention here that the series magnet also acts as a braking magnet, since it opposes the main torque producing flux generated by the shunt magnet.

4. Registering System

The function of a registering or counting system is to continuously record a numerical value that is proportional to the number of revolutions made by the rotating system. By suitable combination of a train of reduction gears, rotation of the main aluminum disc can be transmitted to different pointers to register meter readings on different dials. Finally, the kWh reading can be obtained by multiplying the number of revolutions as pointed out by the dials with the meter constant. The photograph of such a dial-type registering system is shown in Figure 8.5.

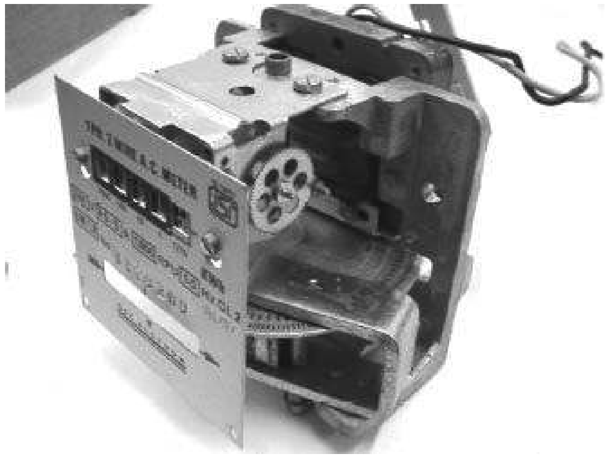


Figure 8.5 *Photograph of dial-type single phase energy meter*

8.2.3 Operation of Induction-Type Energy Meter

As per construction, the pressure coil winding is made highly inductive by providing a large number of turns. The air gaps in a shunt magnet circuit are also made small to reduce the reluctance of shunt flux paths. Thus, as supply voltage is applied across the pressure coil, the current I_P through the pressure coil is proportional to the supply voltage and lags behind it by an angle that is only a few degrees less than 90°. Ideally, this angle of lag should have been 90° but for the small unavoidable resistance present in the winding itself and the associated iron losses in the magnetic circuit.

Figure 8.6 shows the path of different fluxes while the meter is under operation. The corresponding phasor diagram is shown in Figure 8.7.

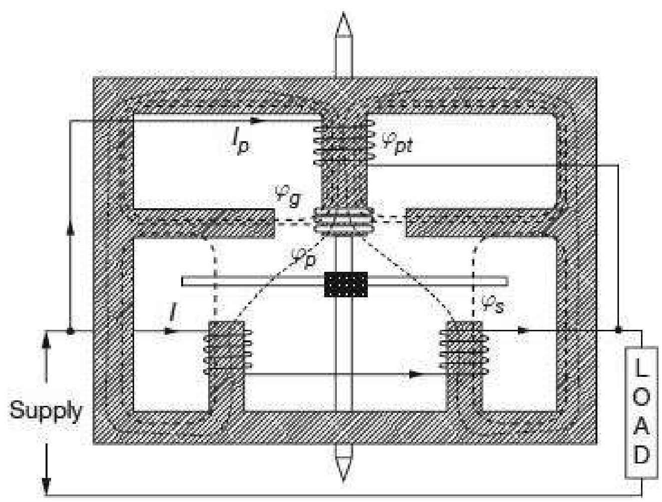


Figure 8.6 *Flux paths in induction-type single-phase energy meter*

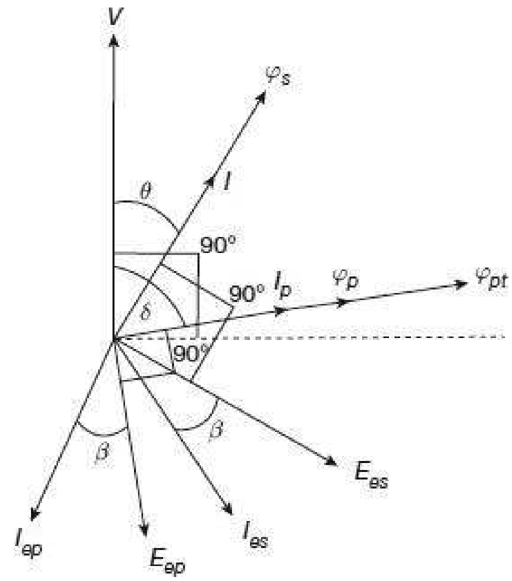


Figure 8.7 *Phasor diagram of single-phase induction-type energy meter*

Let,

V = supply voltage
I = load current
θ = phase angle of load
β = phase angle of aluminum disc
α = phase angle between shunt magnet and series magnet fluxes
δ = phase angle between supply voltage and pressure coil flux

The current I_P produces a flux φ_{pt} that is in same phase as I_P. This flux is made to divide itself in two parts, φ_g and φ_p. The major portion of total pressure coil flux, i.e., φ_g passes through the side gaps as shown in Figure 8.6, as reluctance of these paths are low due very small air gaps. Remaining portion of the flux, i.e., φ_p passes through the disc and is responsible for production of the driving torque. Due to larger reluctance of the path, this flux φ_p is relatively weaker.

The flux φ_p is proportional to the current I_P and is in the same phase, as shown in the phasor diagram of Figure 8.7. The flux φ_p is thus proportional to the supply voltage V and lags it by an angle δ which is only a few degrees less than 90°. The flux φ_p being alternating in nature, induces and eddy emf E_{ep} in the disc, which in turn produces eddy current I. Depending on the impedance angle β of the aluminum disc, eddy current I will lag behind the eddy emf E_{ep} by an angle β.

The load current I flows through the series magnet current coil and produces a flux φ_s. This flux is proportional to the load current I and is in phase with it. This flux, in the same way, induces and eddy emf E_{es} in the disc, which in turn produces eddy current I_{es}. The eddy current I_{es} lags behind the eddy emf E_{es} by the same angle β.

Now, the eddy current I_{es} interacts with flux φ_p to produce a torque and the eddy current I_{ep} interacts with flux φ_s to produce another torque. These two torques are in opposite direction as shown in Figure 8.1, and the resultant torque is the difference of these two.

Following (8.1), the resultant deflecting torque on the disc due to combined action of two fluxes φ_p and φ_s is given as

$$T_d \propto \frac{\varphi_p \varphi_s \omega}{Z} \cdot \sin\alpha \cos\beta \quad (8.2)$$

where, Z is the impedance of the aluminum disc and ω is the angular frequency of supply voltage.

The driving torque can be re-written following the phasor diagram in Figure 8.7 as

$$T_d = K_1 \frac{\varphi_p \varphi_s \omega}{Z} \cdot \sin(\delta - \theta)\cos\beta \text{, where } K_1 \text{ is a constant}$$

Since we have, $j_p \propto V$ and $\varphi_s \propto$ I,

$$\therefore \text{ driving torque } T_d = K_2 VI \frac{\omega}{7} .\sin(\delta - \theta)\cos\beta \quad (8.3)$$

$$\text{If } \omega, Z \text{ and } \beta \text{ are constants, then } T_d = K_3 VI \sin(\delta - \theta) \quad (8.4)$$

If N is the speed of rotation of the disc, then braking torque $T_b = K_4 N$

At steady running condition of the disc, the driving torque must equal the braking torque,

$$\therefore \quad K_4 N = K_3 VI \sin(\delta - \theta)$$

$$\text{or,} \quad N = KVI \sin(\delta - \theta) \quad (8.5)$$

If we can make $\delta = 90°$

$$\text{Then speed of disc is } N = KVI \sin(90° - \theta) = KVI \cos\theta \quad (8.6)$$

$$\text{Thus speed } N = K \times Power$$

Thus, in order that the speed of rotation can be made to be proportionate to the power consumed, the angle difference δ between the supply voltage V and the pressure coil flux φ_p must be made 90°

Total number of revolutions within a time interval dt is

$$= \int N dt = \int KVI \sin(\delta - \theta) dt \quad (8.7)$$

If, $\varepsilon = 90°$, total number of revolutions

$$= K \int VI \cos\theta dt$$

$$= K \int (\text{power}) \times dt$$

$$= K \times \text{Energy}$$

Thus, total number of revolutions is proportional to the energy consumed.

8.3 ERRORS IN INDUCTION-TYPE ENERGY METERS AND THEIR COMPENSATION

8.3.1 Phase-angle Error

It is clear from (8.7) that the meter will indicate true energy only if the phase angle between the pressure coil flux φ_p and the supply voltage V is 90°. This requires that the pressure coil winding should be designed as highly inductive and its resistance and iron losses should be made minimum. But, even then the phase angle is not exactly 90°, rather a few degrees less than 90°. Suitable adjustments can be implemented such that the shunt magnet flux linking with the disc can be made to lag the supply voltage by an angle exactly equal to 90°.

1. Shading Coil with Adjustable Resistance

Figure 8.8 shows the arrangement where an additional coil (shading coil) with adjustable resistance is placed on the central limb of the shunt magnet close to the disc. Main flux created by the shunt magnet induces an emf in this shading coil. This emf creates its own flux. These two fluxes result in a modified flux to pass through the air gap to link the disc and thus produce the driving torque. With proper adjustment of the shading coil resistance, the resultant flux can be made to lag the supply voltage exactly by an angle of 90°.

Operation of the shading coil can be explained with the help of the phasor diagram shown in Figure 8.9. The pressure coil, when excited from the supply voltage V, carries a current I_P and produces an mmf AT_{pt} which in turn produces the total flux φ_{pt}. The flux φ_{pt} lags the supply voltage V by an angle φ which is slightly less than 90°. The current I_P produces a flux φ_{pt} that is in same phase as I_P. The flux φ_{pt} gets divided in two parts, φ and φ_p. The portion of flux φ passes through the side gaps as shown in Figure 8.8, and remaining portion of the flux, i.e., φ_p passes through the disc and also the shading coil. Due to linkage with the time varying flux, an emf E_{sc} is induced in the shading coil that lags behind its originating flux φ_p by 90° (i.e. E_{SC} is 180°) lagging behind the supply voltage V. This emf circulates and eddy current I_{sc} through the shading coil itself. I_{sc} lags behind the emf E_{sc} by an angle λ that depends on the impedance of the shading coil. The shading coil current I_{sc} produces an mmf AT_{sc} which is in phase with I_{sc}. The flux φ_p passing through to the disc will thus be due to the resultant mmf AT_p which is summation of the original mmf AT_{pt} and the mmf AT_{sc} due to the shading coil. This flux φ_p will be in phase with the mmf AT_p. As seen in the phasor diagram of Figure 8.9, the flux φ_p can be made to lag the supply voltage V by exactly 90° if the mmf AT_p or in other words, the shading coil phase angle λ can be adjusted properly. The shading coil phase angle can easily be adjusted by varying the external resistance connected to the shading coil.

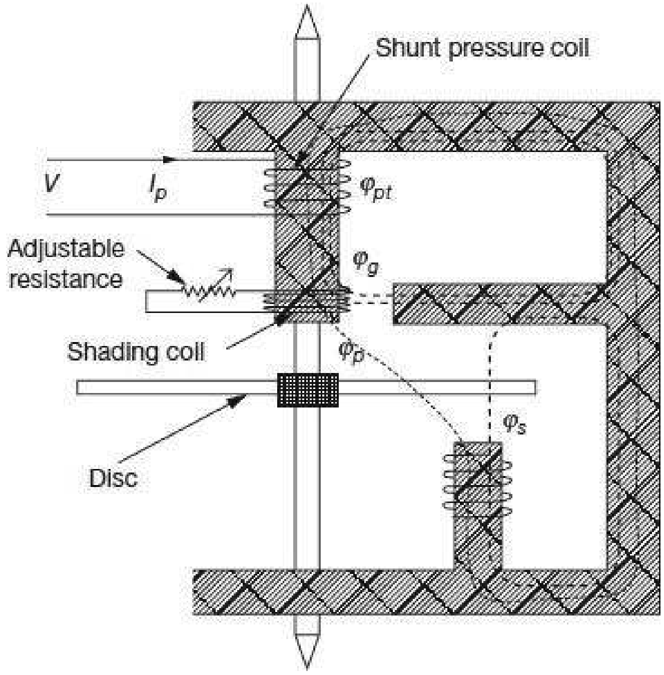


Figure 8.8 *Shading coil for lag adjustment V*

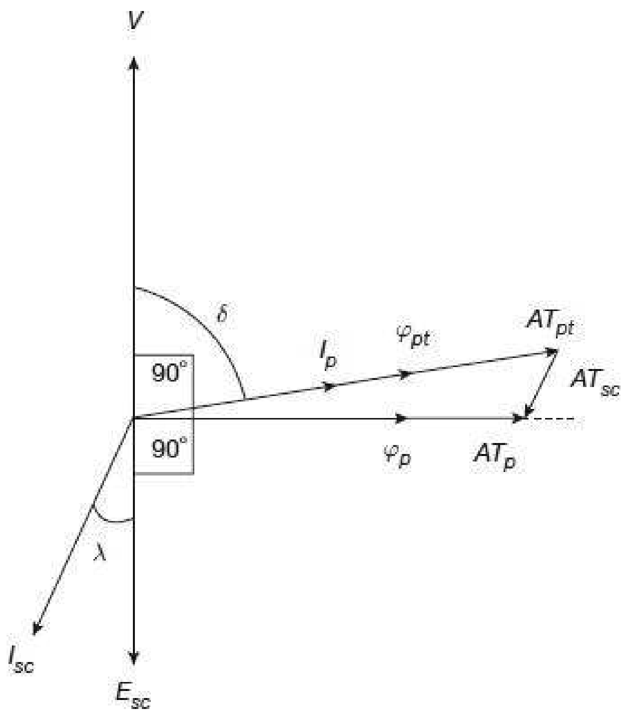


Figure 8.9 *Phasor diagram showing operation of shading coil for lag adjustment*

2. Copper Shading Bands

A similar result of lag adjustment can be obtained by the use of copper shading bands placed on the central limb of the shunt magnet. Such an arrangement is shown in Figure 8.10. Following the same arguments, the resultant flux φ_p crossing over to the disc can be made to lag the supply voltage V by exactly 90° by proper adjustment of the mmf produced by the copper shading bands. Adjustments in this case can be done by moving the shading bands along the axis of the limb. As the bands are moved upwards along the limb, they embrace more flux. This results in increased values of induced emf, increased values of induced eddy current and hence increased values of the mmf produced by the bands. Similarly, as the bands are moved downwards, mmf produced by the bands is reduced. This changes the phase angle difference between φ_p and φ_{pt}, as can be observed from the phasor diagram of Figure 8.9. Thus, careful adjustments of the copper shading bands position can make the phase difference between the supply voltage V and resultant shunt magnet flux φ_p to be exactly 90°.

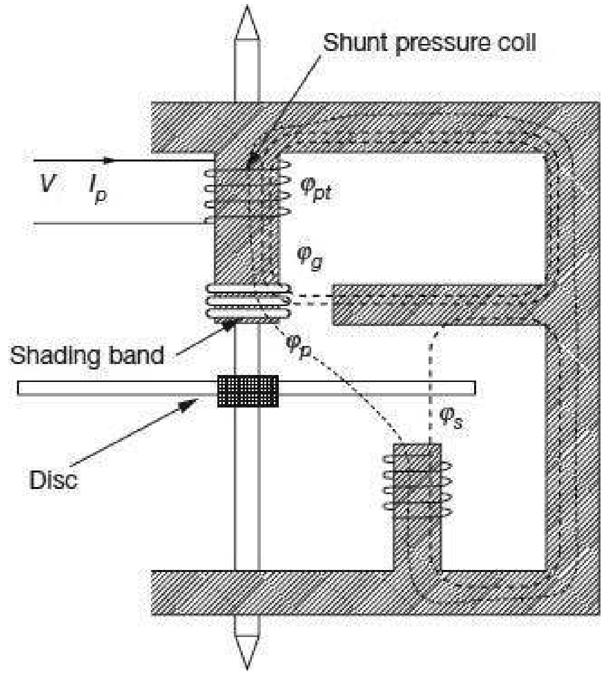


Figure 8.10 *Copper shading bands for lag adjustment*

8.3.2 Error due to Friction at Light Loads

Friction in bearings can pose serious errors in measurement of energy in the form of that it will impede proper movement of the rotating disc. This problem is particularly objectionable at low loads, when the driving torque itself is very low; therefore, unwanted friction torque can even stop the disc from rotating. To avoid this, it is necessary to provide an additional torque that is essentially independent of the load, to be applied in the direction of the driving torque, i.e., opposite to the frictional torque to compensate for the frictional retarding torque. This is achieved by means of a small vane or shading loop placed in the air gap between the central limb of the shunt magnet and the aluminum disc, and slightly off-centre from the central limb, as shown in Figure 8.11. Interactions between fluxes which are linked and not linked by the shading or compensating vane and the currents they induce in the disc result in a small driving torque that can compensate for the frictional retarding torque. The value of this small additional torque can be adjusted by lateral movement of the vane in and out of its position in the air gap.

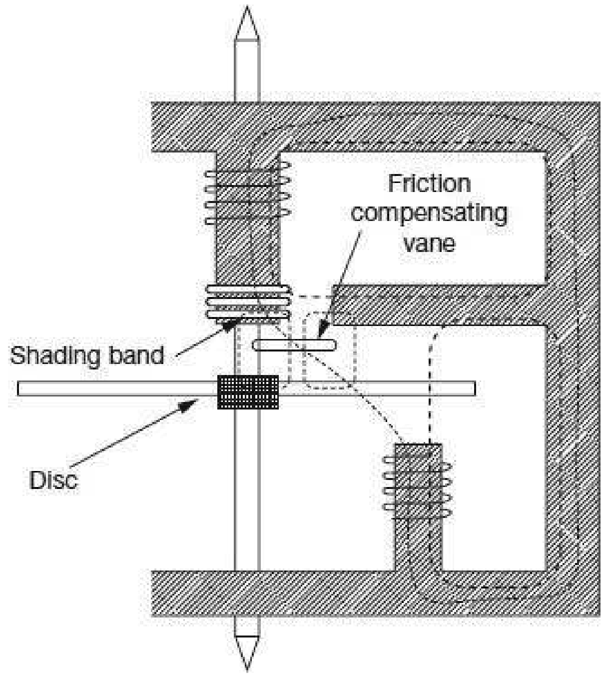


Figure 8.11 *Shading vane for friction compensation*

8.3.3 Creeping Error

In some meters, a slow but continuous rotation of the disc can be observed even when there is no current flowing through the current coil, and only pressure coil is energised. This is called **creeping**. The primary reason for creeping is due to over-compensation for friction. Though the main driving torque is absent at no-load, the additional torque provided by the friction compensating vane will make the disc continue to rotate. Other causes of creeping may be excessive voltage across the potential coil resulting in production of excessive torque by the friction compensating device, or vibrations, and stray magnetic fields.

Creeping can be avoided by drilling two holes on the aluminum disc placed on diametrically opposite locations. Drilling such holes will distort the eddy current paths along the disc and the disc will tend to stop with the holes coming underneath the shunt magnet poles. The disc can thus creep only till a maximum of half the rotation till one of the holes comes below the shunt magnet pole. This effect is however, too insignificant to hamper disc movement during normal running operations under load.

Creeping can also be avoided by attaching a tiny piece of iron to the edge of the disc. The brake magnet in such a case can lock the iron piece to itself and prevent creeping of the disc. Once again, this action is too insignificant to hamper disc movement during normal running operations under load. The arrangement is schematically shown in Figure 8.12.

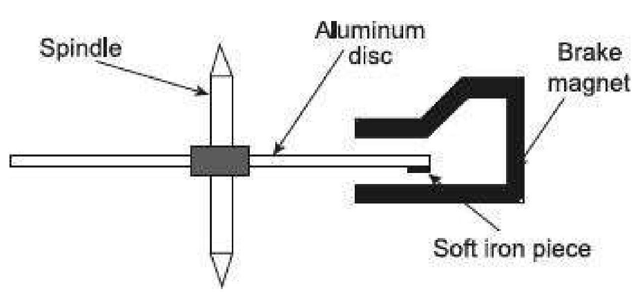


Figure 8.12 *Soft iron piece at the end of the disc to prevent creeping*

8.3.4 Error due to Change in Temperature

Errors introduced by variation of temperature in induction-type energy meters are usually small since the various effects tend to neutralise each other. An increase in temperature increases the pressure coil resistance, thereby reducing pressure coil current and reducing pressure coil flux. This will tend to reduce the driving torque. But the flux of the brake magnet also reduces due to increase in temperature, thereby reducing the braking torque. Again, an increase in temperature increases the resistance to eddy current path in the disc, which reduces both driving torque and braking torque. The various effects thus tend to neutralise each other.

The effects of increasing temperature, however, in general cause the meter to rotate faster and hence record higher values. Temperature effects thus need to be compensated for by using temperature shunts in the brake magnet.

8.3.5 Error due to Overload

At a constant voltage, the deflecting torque becomes simply proportional to the series magnet flux and hence proportional to the load current. This is due to the fact that from Eq. (8.2), at constant voltage as the shunt magnet flux φ_p is constant, the driving torque $T_d \propto \varphi_s \propto I$.

On the other hand, as the disc rotates continuously in the field of the series magnet, an emf is induced dynamically in the disc due to its linkage with the series magnet flux φ_s. This emf induces eddy currents in the disc that interact with the series magnet flux to create a retarding or braking torque that opposes motion of the disc. This self braking torque is proportional to the square of the series magnet flux or is proportional to the square of the load current; i.e., $T_b \propto \varphi^2_s \propto I^2$.

At higher loads, thus the braking torque overpowers the deflecting torque and the meter tends to rotate at slower speed, and consequently reads lower than actual.

To avoid such errors, and to minimise the self-braking action, the full load speed of the disc is set at lower values. The current coil flux φ_s is made smaller as compared to the pressure coil flux φ_p. Thus, the dynamically induced emf that causes the braking torque is restricted as compared to the driving torque. Magnetic shunts are also sometimes used with series magnets to compensate for overload errors at high current values.

8.3.6 Error due to Voltage Variations

Voltage variations can cause errors in induction-type energy meters mainly due to two reasons:

1. At too high voltages, the relationship between the supply voltage V and the shunt magnet flux φ_p no longer remain linear due to saturation of iron parts, and
2. For sudden fluctuations in supply voltage, the shunt magnet flux φ_p produces a dynamically induced emf in the disc which in turn results in a self-braking torque and the disc rotation is hampered.

Compensation for voltage variation is provided by using a suitable magnetic shunt that diverts a major portion of the flux through the disc when the voltage rises, thereby increasing the driving torque to overcome the self-braking torque. Such compensation can be achieved by increasing the reluctance of the side limbs of the shunt magnet. This is done by providing holes in the side limbs as shown in Figure 8.13.

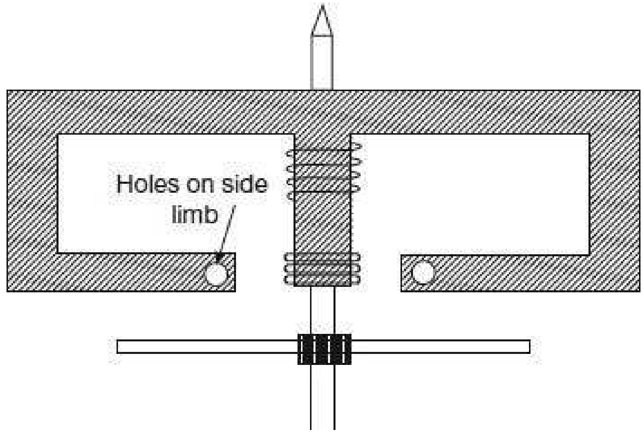


Figure 8.13 *Holes provided on side limbs to compensate for errors due to sudden voltage variations*

8.4 TESTING OF ENERGY METERS

Energy meters are tested at the following conditions:

1. At 5% of rated current at unity power factor
2. At 100% or 125% of rated current with unity power factor
3. At one intermediate load with unity power factor
4. At rated current and 0.5 lagging power factor
5. *Creep test* With pressure coil supplied with 110% of rated voltage and current coil open circuited, the meter disc should not rotate by more than one revolution, i.e., it should not creep
6. *Starting test* At 0.5% of rated current and full rated voltage, the meter disc should start rotating

8.4.1 Phantom Loading

When the current rating of the meter under test is high, a test with actual loading arrangements would involve considerable wastage of energy and also it is difficult to arrange for such large loads under laboratory test conditions. In such cases, to avoid this, ‘phantom’ or ‘fictitious’ loading arrangements are done for testing of energy meters.

Phantom loading consists of supplying the shunt magnet pressure coil circuit from a rated voltage source. The series magnet current coil is supplied from a separate low voltage supply source. It is possible to circulate rated current through the current coil circuit with the low voltage source since impedance of this circuit is very low. The energy indicated by the meter under phantom loading condition is the same as the energy indication as would have been with a real load. With this arrangement, the total energy consumed for the test is comparatively smaller. The total energy required for the test is that due to the small pressure coil current at rated voltage and small current coil voltage at rated current.

Example 8.1

The meter constant of a 220 V, 5 A energy meter is 2000 revolutions per kWh. The meter is tested at half load at rated voltage and unity power factor. The meter is found to make 34 revolutions in 116 s. Determine the meter error at half load.

Solution Actual energy consumed at half load during 166 s:

$$= VI\cos\phi \times t \times 10^{-3} = 220 \times 2.5 \times \frac{116}{60 \times 60} \times 10^{-3} = 17.72 \times 10^{-3} \text{ kWh}$$

Energy as recorded by the meter

$$= \frac{\text{Number of revolutions made}}{\text{Meter constant (rev/kWh)}} = \frac{34}{2000} = 17 \times 10^{-3} \text{ kWh}$$

$$\therefore \quad \text{Error} = \frac{17 - 17.72}{17.72} \times 100\% = -4.06\% \text{ (meter runs slower)}$$

Example 8.2

A 230 V, 5 A energy meter on full load unity power factor test makes 60 revolutions in 360 seconds. If the designed speed of the disc is 520 revolutions per kWh, find the percentage error.

Solution Actual energy consumed at full load during 360 s:

$$= VI\cos\phi \times t \times 10^{-3} = 230 \times 5 \times \frac{360}{60 \times 60} \times 10^{-3} = 115 \times 10^{-3} \text{ kWh}$$

Energy as recorded by the meter

$$= \frac{\text{Number of revolutions made}}{\text{Meter constant (rev/kWh)}} = \frac{60}{520} = 115.385 \times 10^{-3} \text{ kWh}$$

$$\therefore \quad \text{Error} = \frac{115.385 - 115}{115} \times 100\% = 0.34\% \text{ (meter runs faster)}$$

Example 8.3

An energy meter is designed to have 80 revolutions of the disc per unit of energy consumed. Calculate the number of revolutions made by the disc when measuring the energy consumed by a load carrying 30 A at 230 V and 0.6 power factor. Find the percentage error if the meter actually makes 330 revolutions.

Solution Actual energy consumed by the load in one hour:

$$= VI\cos\phi \times t \times 10^{-3} = 230 \times 30 \times 0.6 \times 1 \times 10^{-3} = 4.14 \text{ kWh}$$

The meter makes 80 revolutions per unit of energy consumed, i.e., per kWh.

Thus number of revolutions made by the meter to record 4.14 kWh is

$$= 4.14 \times 80 = 331.2$$

In case the meter makes 330 revolutions, then error is given as

$$\text{Error} = \frac{330 - 331.2}{331.2} \times 100\% = -0.36\% \text{ (meter runs slower)}$$

Example 8.4

A 230 V, single-phase watt hour meter records a constant load of 10 A for 4 hours at unity power factor. If the meter disc makes 2760 revolutions during this period, what is the meter constant in terms of revolutions per unit? Calculate the load power factor if the number of revolutions made by the meter is 1104 when recording 5 A at 230 V for 6 hours.

Solution Actual energy consumed by the load in 4 hours:

$$= VI \cos\phi \times t \times 10^{-3} = 230 \times 10 \times 1 \times 4 \times 10^{-3} = 9.2 \text{ kWh}$$

$$\therefore \text{ Meter constant} = \frac{\text{Number of revolutions made}}{\text{kWh consumed}} = \frac{2760}{9.2} = 300 \text{ rev/kWh}$$

With this value of meter constant, with 1104 revolutions, the meter records an energy = 1104/300 3.68 kWh

Thus, energy consumed

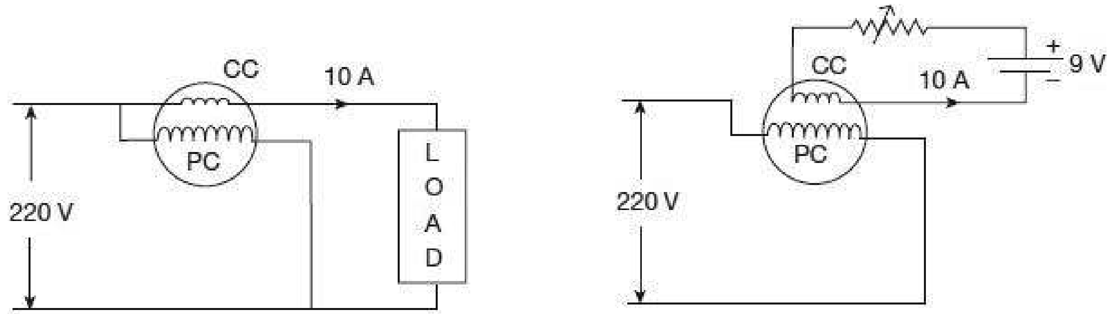
$$= VI \cos\phi \times t \times 10^{-3} = 230 \times 5 \times \cos\phi \times 6 \times 10^{-3} = 3.68 \text{ kWh}$$

Hence, power factor of load is $\cos\phi = 0.533$

Example 8.5

A 220 V, 10 A dc energy meter is tested for its name plate ratings. Resistance of the pressure coil circuit is 8000 Ω and that of current coil itself is 0.12 Ω. Calculate the energy consumed when testing for a period of 1 hour with
(a) Direct loading arrangement
(b) Phantom loading with the current coil circuit excited by a separate 9 V battery

Solution Test arrangements with direct and phantom loading arrangements are schematically shown in the figure.



(a) (b)

(a) With direct loading

Power consumed in the pressure coil circuit $= \frac{(220)^2}{8000} = 6.05$ W

Power consumed in the current coil (series) circuit = 220 × 10 = 2200 W

∴ total power consumed with direct measurement = 2206.05 W

∴ total energy consumed during 1 hour with direct measurement

$$= 2206.05 \times 1 \times 10^{-3} = 2.20605 \text{ kWh}$$

(b) With phantom loading

Power consumed in the pressure coil circuit $= \frac{(220)^2}{8000} = 6.05$ W

Power consumed in the current coil (series) circuit = 9 × 10 = 90 W

∴ total power consumed with direct measurement = 96.05 W

∴ total energy consumed during 1 hour with direct measurement

$$= 96.05 \times 1 \times 10^{-3} = 0.09605 \text{ kWh}$$

Thus, energy consumed is considerably less in phantom loading as compared to direct loading for energy meter testing.

EXERCISE

EXERCISE

Objective-type Questions

1. Energy meters do not have a control spring to
   (a) avoid unnecessary friction losses
   (b) enable continuous rotation of the disc
   (c) avoid damping during movement
   (d) all of the above
2. In induction-type energy meters, the speed of rotation of the disc is proportional to the
   (a) energy consumption
   (b) power consumption
   (c) derivative of power consumption
   (d) none of the above
3. The advantages of induction-type energy meters are
   (a) low torque/weight ratio
   (b) low friction
   (c) high and sustained accuracy
   (d) all of the above
4. Induction-type energy meters have aluminum disc as the rotating part so that
   (a) flux can pass through the rotating part
   (b) eddy current can be induced in the rotating part

(c) creeping error can be avoided

(d) all of the above

5. In induction-type energy meters

(a) pressure coil is the moving part

(b) current coil is the moving part

(c) both current and pressure coils are moving

(d) both current and pressure coils are stationary

6. In induction-type energy meters, high driving torque can be obtained by

(a) making the disc purely resistive

(b) making the phase difference between the two operating fluxes as large as possible

(c) making the disc impedance as low as possible

(d) all of the above

7. Braking torque provided by the permanent magnet in an induction-type energy meter is proportional to

(a) speed of the rotating disc

(b) square of the flux of the permanent magnet

(c) distance of the permanent magnet with respect to centre of the disc

(d) all of the above

8. Braking torque provided by the permanent magnet in an induction-type energy meter can be changed by

(a) providing a metal shunt and shifting its position

(b) moving the position of the permanent magnet with respect to the disc

(c) both (a) and (b)

(d) none of the above

9. In single-phase induction-type energy meters, maximum torque is produced when the shunt magnet flux

(a) leads the supply voltage by 90°

(b) lags the supply voltage by 90°

(c) lags the supply voltage by 45°

(d) is in phase with the supply voltage

10. In single-phase induction-type energy meters, lag adjustments are done by

(a) permanent magnet placed on the edge of the disc

(b) holes provided on the side limbs of the pressure coil

(c) copper shading bands placed on the central limb of the pressure coil

(d) metal shunts placed on the series magnets

11. In single-phase induction-type energy meters, lag adjustments can be done by

(a) shifting the copper shading band along the axis of the central limb

(b) varying the external resistance connected to the shading coil placed on the central limb

(c) either of (a) or (b) as the case may be

(d) none of the above

12. In single-phase induction-type energy meters, friction compensation can be done by

(a) placing shading bands in the gap between central limb and the disc

(b) drilling diametrically opposite holes on the disc

(c) providing holes on the side limbs

(d) all of the above

13. Creeping in a single-phase energy meter may be due to
    (a) vibration
    (b) overcompensation of friction
    (c) over voltages
    (d) all of the above
14. Creeping in a single-phase energy meter can be avoided by
    (a) using good quality bearings
    (b) increasing strength of the brake magnet
    (c) placing small soft iron piece on edge of the rotating disc
    (d) all of the above
15. Increase in operating temperature in an induction-type energy meter will
    (a) reduce pressure coil flux
    (b) reduce braking torque
    (c) reduce driving torque
    (d) all of the above
16. Overload errors in induction-type energy meters can be reduced by
    (a) designing the meter to run at lower rated speeds
    (b) designing the current coil flux to have lower rated values as compared to pressure coil flux
    (c) providing magnetic shunts along with series magnets that saturate at higher loads
    (d) all of the above
17. Over voltages may hamper rotation of the disc in induction-type energy meters since
    (a) the pressure coil flux no longer remains in quadrature with the current coil flux
    (b) dynamically induced emf in the disc from the pressure coil flux produces a self-braking torque
    (c) effect of the brake magnet is enhanced
    (d) all of the above
18. If an induction-type energy meter runs fast, it can be slowed down by
    (a) moving up the copper shading bands placed on the central limb
    (b) adjusting the magnetic shunt placed on the series magnets
    (c) moving the permanent brake magnet away from centre of the disc
    (d) bringing the permanent brake magnet closer to centre of the disc
19. Phantom loading for testing of energy meters is used
    (a) for meters having low current ratings
    (b) to isolate current and potential circuits
    (c) to test meters having a large current rating for which loads may not be available in the laboratory
    (d) all of the above
20. In single-phase induction-type energy meters, direction of rotation of the disc can be reversed by
    (a) reversing supply terminals
    (b) reversing load terminals
    (c) opening the meter and reversing either the potential coil terminals or the current coil terminals
    (d) opening the meter and reversing both the potential coil terminals and the current coil terminals

Answers

1. (b)	2. (b)	3. (a)	4. (b)	5. (d)	6. (a)	7. (d)
8. (c)	9. (b)	10. (c)	11. (c)	12. (d)	13. (b)	14. (c)
15. (d)	16. (d)	17. (b)	18. (d)	19. (c)	20. (c)	

Short-answer Questions

1. Draw a schematic diagram showing construction details of an induction-type energy meter and label its different parts. Comment on the different materials used for the different internal components.
2. Why braking torque is necessary in induction type energy meters? Draw and explain how braking arrangement is done such instruments.
3. Why is it necessary to have lag adjustment devices in induction type energy meters? Draw and explain in brief, operation of such arrangements in a single-phase energy meter.
4. What is the effect of friction in induction-type energy meters? How is it overcome in practice?
5. What is creeping error in an energy meter? What are its possible causes? How can it be compensated in an induction type energy meter?
6. Explain the effects of over-load in induction type energy meters. How can this effect be avoided?
7. Why can sudden voltage variations cause errors in induction type energy meter readings? Discuss how these errors can be minimized.
8. List the tests normally carried out on single phase energy meters? Why phantom loading arrangement is done for testing high capacity energy meters?

Long-answer Questions

1. Derive an expression for the driving torque in a single phase induction type meter. Show that the driving torque is maximum when the phase angle between the two fluxes is 900 and the rotating disc is purely non-inductive.
2. Draw the schematic diagram of the internal operating parts of a single phase induction type energy meter. Comment of the materials used and operation of the different internal parts.
3. Draw and describe the relevant phasor diagram and derive how the number of revolutions in a single phase induction type energy meter is proportional to the energy consumed.
4. (a) Explain the sources of error in a single phase induction type energy meter.

   (b) A 220 V, 5 A energy meter on full load unity power factor test makes 60 revolutions in 360 seconds. If the designed speed of the disc is 550 revolutions per kWh, find the percentage error.
5. (a) What is phase angle error in an induction type energy meter? Explain how this error is reduced in a single phase induction type energy meter.

   (b) An energy meter is designed to have 60 revolutions of the disc per unit of energy consumed. Calculate the number of revolutions made by the disc when measuring the energy consumed by a load carrying 20 A at 230 V and 0.4 power factor. Find the percentage error if the meter actually makes 110 revolutions.
6. (a) What is the effect of friction in induction-type energy meters? How is it overcome? What is creeping error? How is it overcome?

   (b) A 230 V single-phase watthour meter records a constant load of 5 A for 6 hours at unity power factor. If the meter disc makes 2760 revolutions during this period, what is the meter constant in terms of revolutions per unit? Calculate the load power factor if the number of revolutions made by the meter is 1712 when recording 4 A at 230 V for 5 hours.
7. (a) What are the tests normally carried out on single-phase energy meters? Why is phantom loading arrangement is done for testing high capacity energy meters?

   (b) A 230 V, 5 A dc energy meter is tested for its name plate ratings. Resistance of the pressure coil circuit is 6000 Ω and that of current coil itself is 0.15 Ω. Calculate the energy consumed when testing for a period of 2 hours with

      (i) Direct loading arrangement

(ii) Phantom loading with the current coil circuit excited by a separate 6 V battery