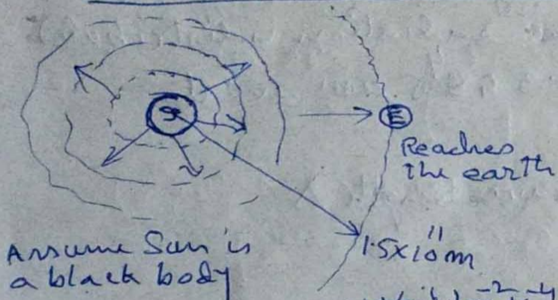
# Solar Radiation Constant & Solar Intensity




Sun emits energy at huge rate

$= 3.95\times10^{26}$ J/s

→ E.M. Radiations

As per Stefen Boltzman law
Total emissive power from Sun

$P = e\sigma A T^4$

e – emissivity
σ – Stefen Boltzman const.
A – area of the sun
T – Temp in Kelvin
Radius of Sun = $7.0\times10^{8}$
Surface temp. of sun $= 5800^\circ K$

$$P = (1)\cdot\underbrace{(5.67\times10^{-8})}_{W m^{-2} K^{-4}}\left[4\pi(7.0\times10^{8})^2\right](5800)^4$$

$= 3.95\times10^{26}$ W

Intensity on the surface of the earth atmosphere

$$= I = \frac{P}{A} = \frac{3.95\times10^{26}\,W}{4\pi(1.5\times10^{11} m)^2} = 1397\ W/m^2$$

→ assuming no absorption

1397 W/m² is call the Solar Constant.

$32' = 0.53^\circ$


Total energy incident on the equivalent two dimensional disk of radius r

$= I\times\pi r^2$ watt.

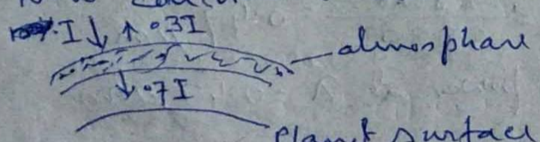
Area of the earth surface

$= 4\pi r^2$ (Earth radius $= 6.5\times10^{6}$ m)

So the average intensity on the earth surface

$$I = \frac{1397\times\pi r^2}{4\pi r^2} = 349.25\ W/m^2$$

Now consider the absorption effect.




Actual average intensity on the earth surface

$= 0.7\times349.25\ W/m^2$

~~244 W/m²~~

$= 244.475\ W/m^2$

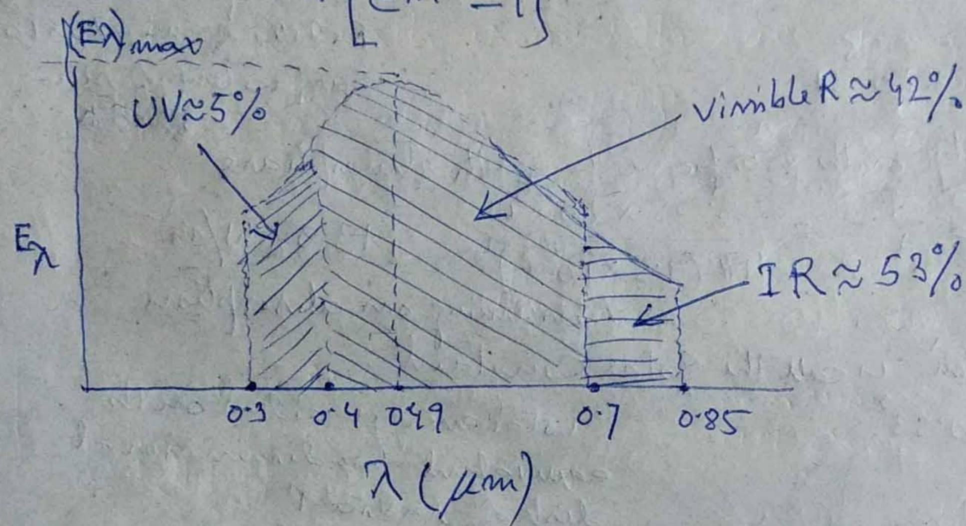
## ② Distribution of Solar Radiation

Solar radiation covers entire range of visible region and some part of UV & IR. As the solar radiation is distributed non-uniformly over the range of 0.3-4 μm and its distribution is given by Plank

Emissive Energy at wave length $\lambda$

$$E_\lambda = f(\lambda, T)$$

$$E_\lambda = \frac{C_1}{\lambda^5\left[e^{\frac{C_2}{\lambda T}} - 1\right]} \qquad C_1 \,\&\, C_2 \text{ are constants}$$




$E_\lambda$ = monochromatic or spectral emissive power

$\Rightarrow W\, m^{-2}\, \mu m^{-1}$

Weins Displacement Law

$$\lambda T = 2898 = \text{constant} \quad [\text{only for black body}]$$

Sun surface temperature

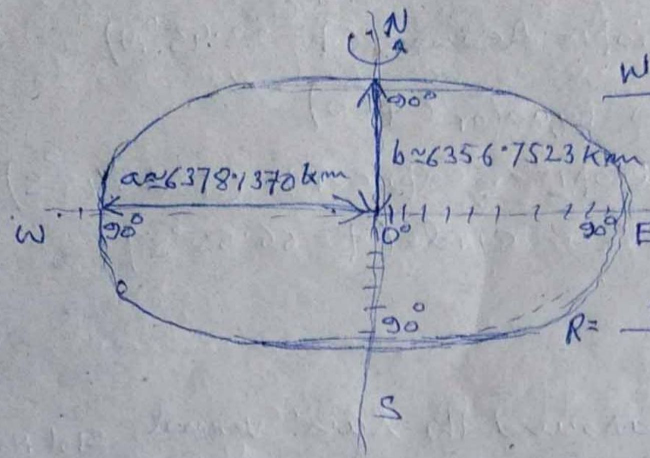
$$T_s = 5880^\circ K \quad \therefore \lambda = \frac{2898}{5880} = 0.49\ \mu m$$

$(E_\lambda)_{max} \Rightarrow$ maximum emissive power at $\lambda = 0.49\ \mu m$.

# Solar Geometry



Wikipedia

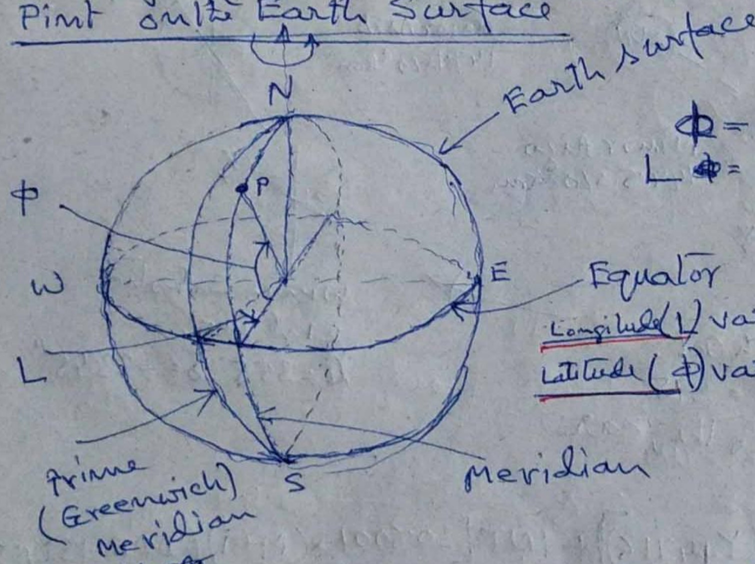



$$R = \frac{2a+b}{3} = 6371\cdot0088\text{ km}$$

Earth is a obligate ~~sphere~~ sphere having flattend at the poles and bulged around the equator.

For ~~graph~~ solar power calculation it is consider as sphere having ~~radius~~ dia 12,800 km (approx.)

Point on the Earth Surface




Φ = Latitude of point P

L ~~Φ~~ = Longitude of point P

Longitude (L) varies from 0°E to 90°W

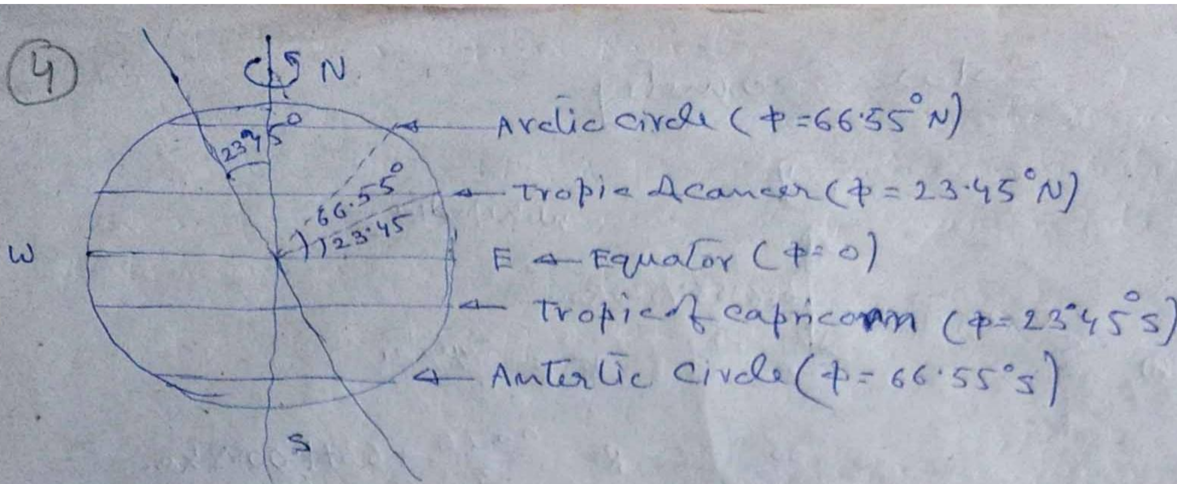
Latitude (Φ) varies from 90°N to 90°S

(Φ) ~~Longitude~~ ~~Latitude~~ Latitude of a point (P) is angle between a ~~p~~ radius drawn from the point to the center of the earth and a radius drawn from the center of the earth to equator

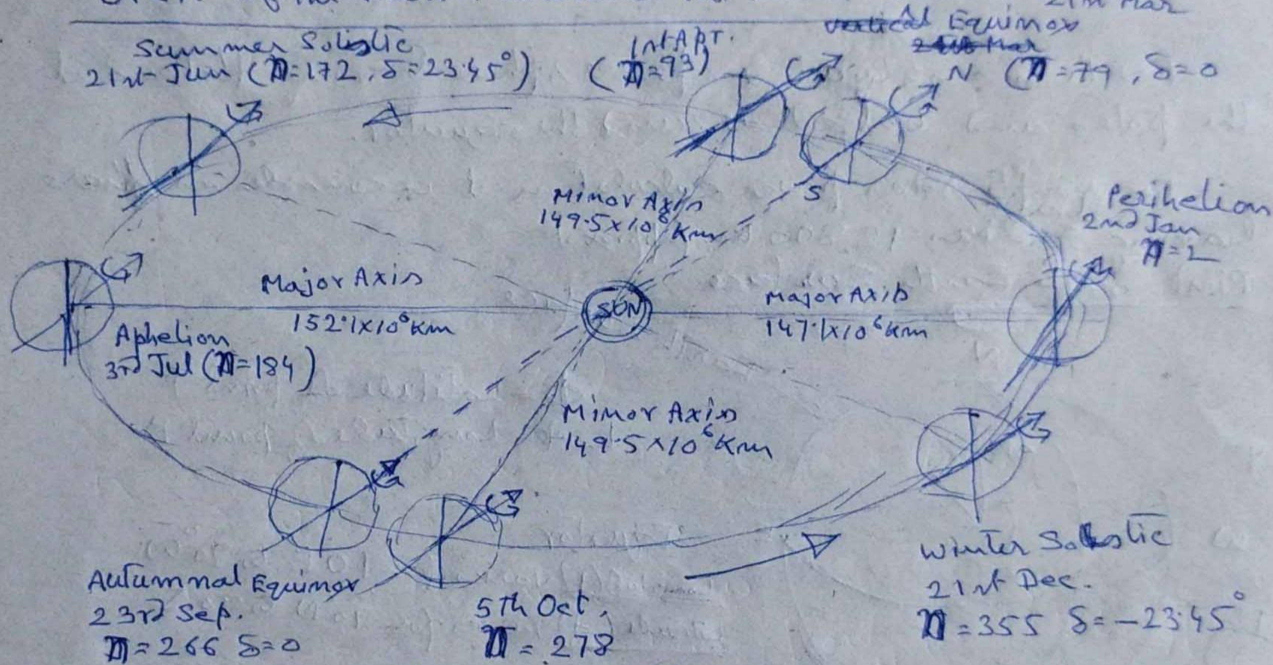
(L) ~~Latitude~~ ~~Longitude~~ Longitude of a point (P) is the angle between the Greenwich (or prime) meridian and the meridian passes through the point.

Meridian is the circumference that passes through N & S pole.

Declination angle (δ) — ~~Angle~~ Angle between equitorial plane and a line joining to the center of the Sun and Earth is called the declination angle




Orbit of the Earth around the Sun




$n \to$ no. of day of the year

**Julian Day**

$$JD = INT[365.25 \times (Y + 4716)] + INT[30.6001 \times (M+1) + D + B - 1524.5]$$

Solar Declination angle $\delta = 23.45 \sin\left[\frac{360}{365}(284 + n)\right]$ deg.

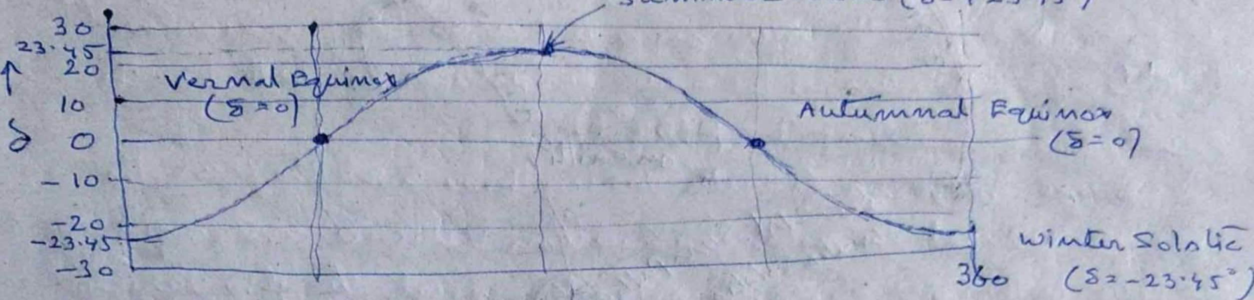
$n \to$ day of the year counted from 1st January.

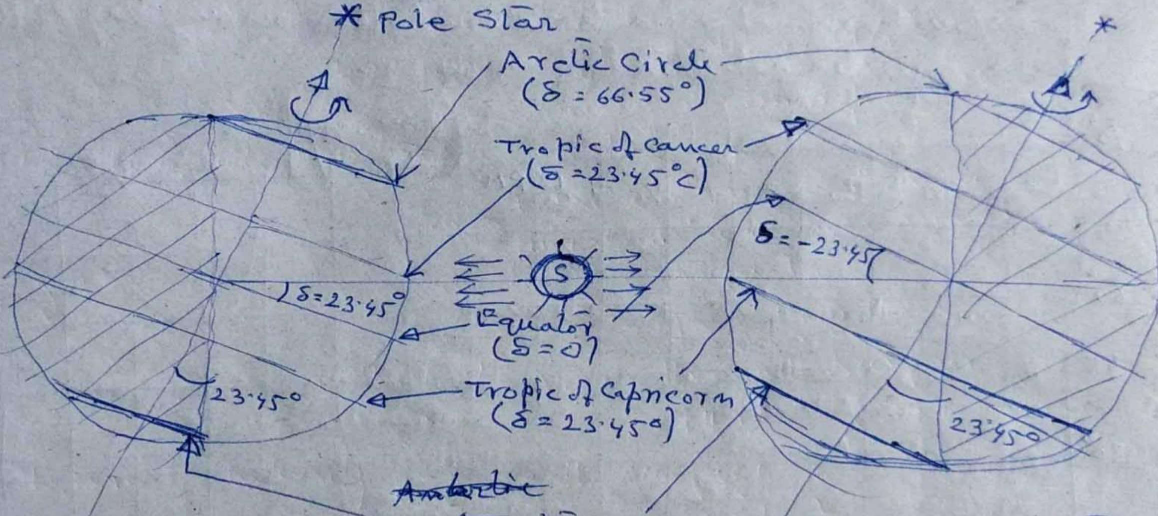
Value of $n$ for any day "D" of a month

| Month | $n$ for the day of the Month, D | Month | $n$ for the day of the Month, D |
|---|---|---|---|
| Jan | D | Jul | 181 + D |
| Feb | 31 + D | Aug | 212 + D |
| Mar | 59 + D | Sep | 243 + D |
| Apr | 90 + D | Oct. | 273 + D |
| May | 120 + D | Nov | 304 + D |
| Jun | 151 + D | Dec. | 334 + D |

Solar declination angle (δ) varies continuously in a sinusoidal fashion over the year due to the earth motion along the path of nearly circular orbit around the sun.

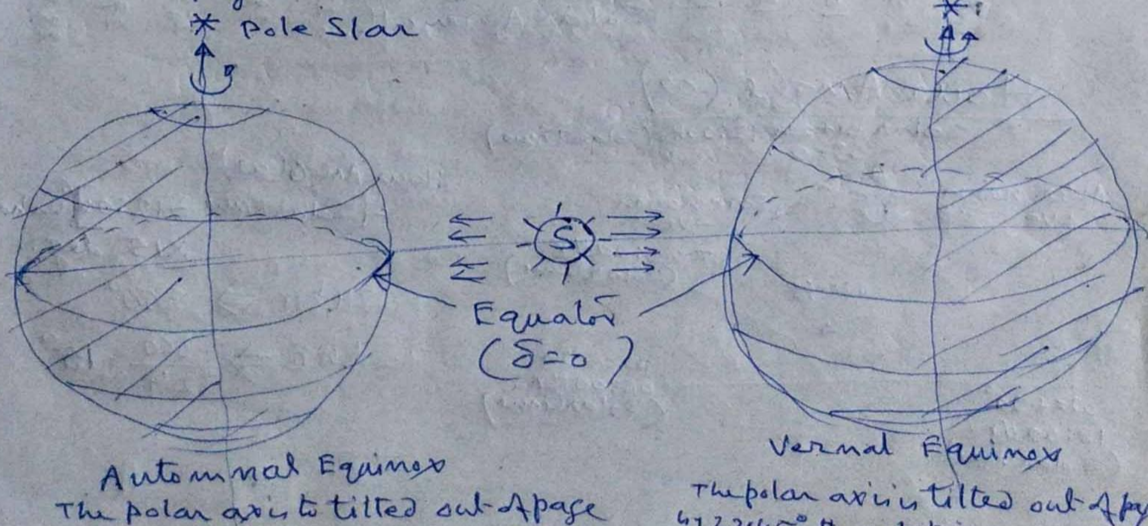
Solar declination angle (δ) varies from $-23.45^\circ$ to $+23.45^\circ$.




Summer Solstic
Earth is moving out the page

Winter Solstic
Earth is moving into the page.

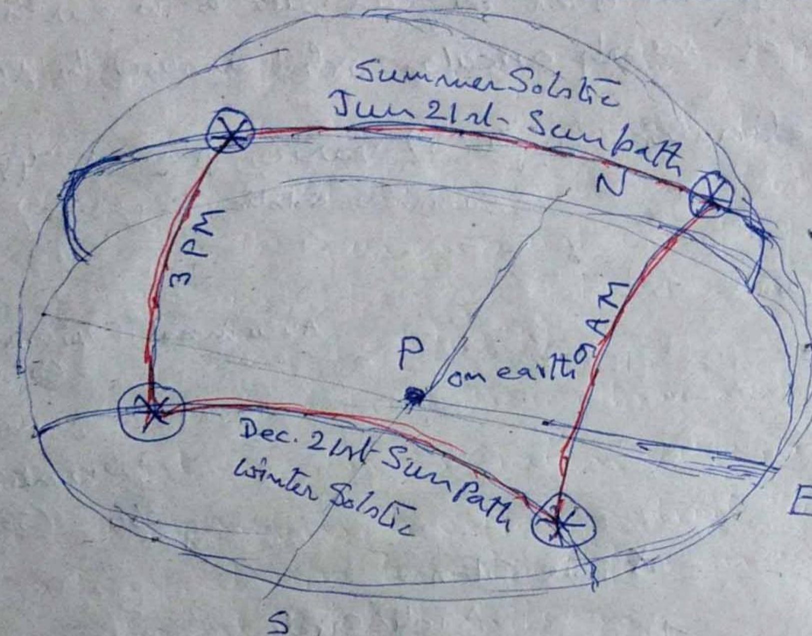


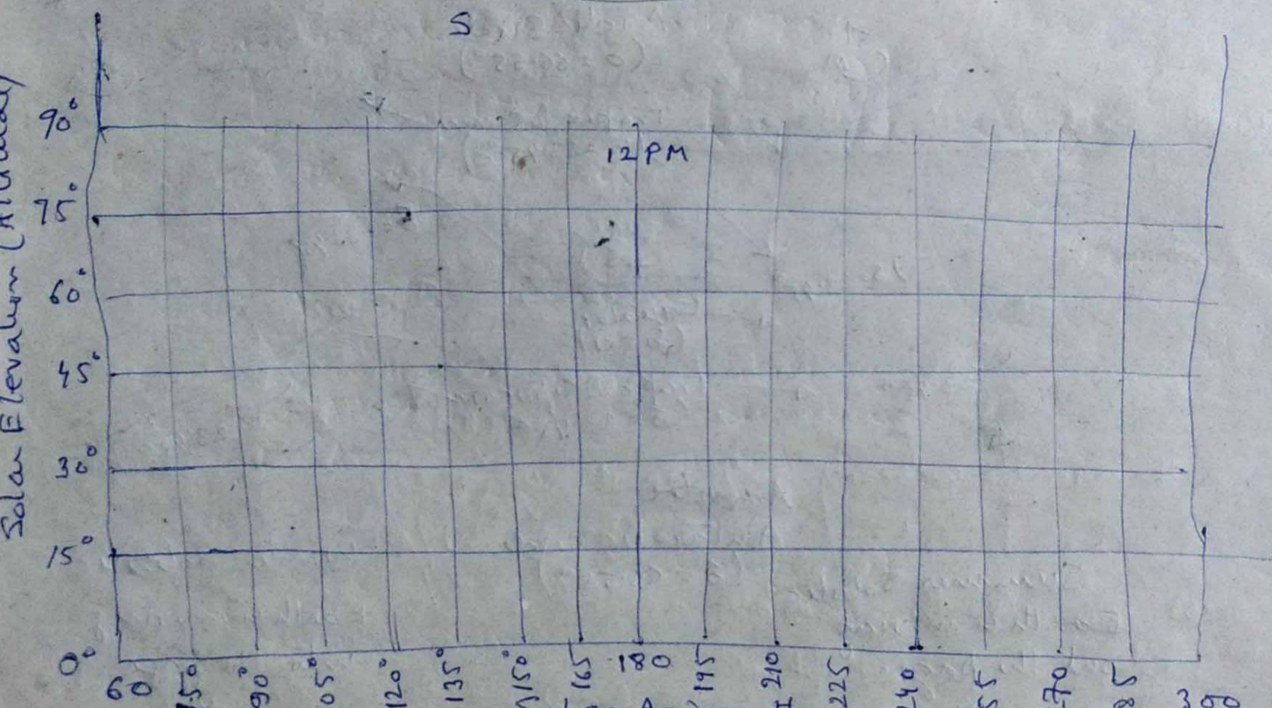

Automnal Equinox
The polar axis is tilted out of page by 23.45°, orbit is moving the Earth out of page.

Vernal Equinox
The polar axis is tilted out of page by 23.45°, the orbit is moving the earth into the page.

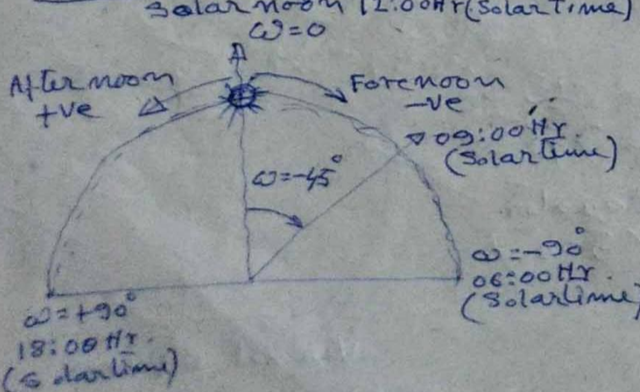
(6)

# Solar window for a particular position




## Hour Angle (ω)

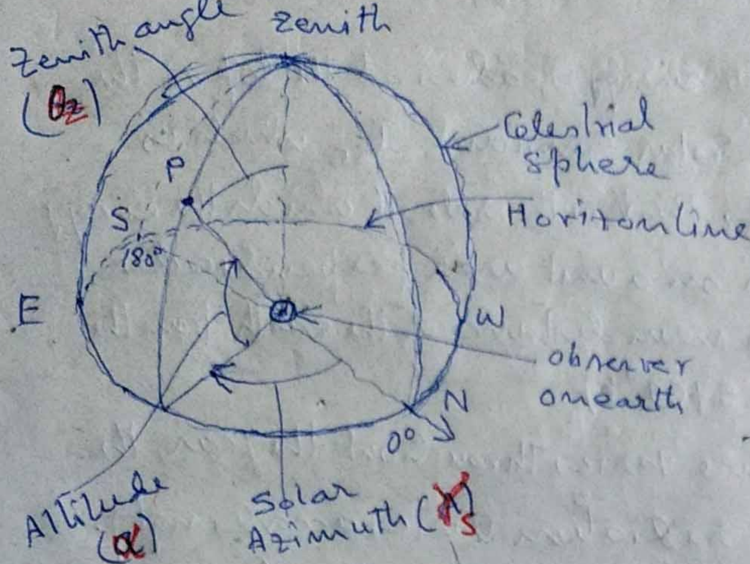


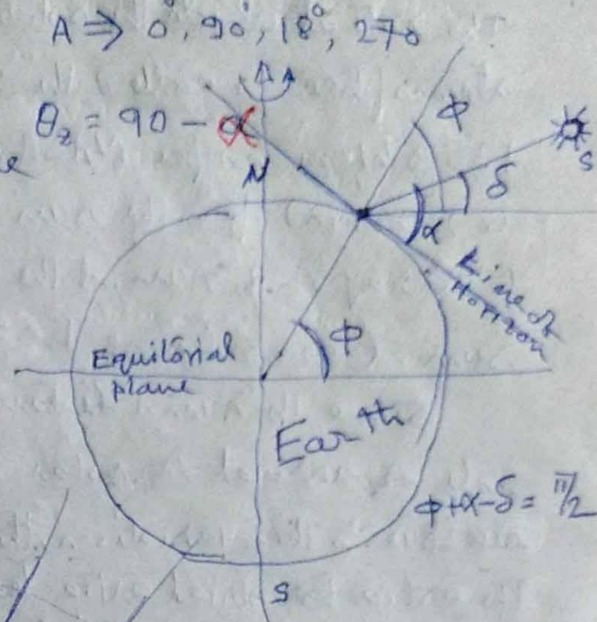

Hour Angle (ω)
= [Solar Time − 12:00] (in hours) × 15 deg.

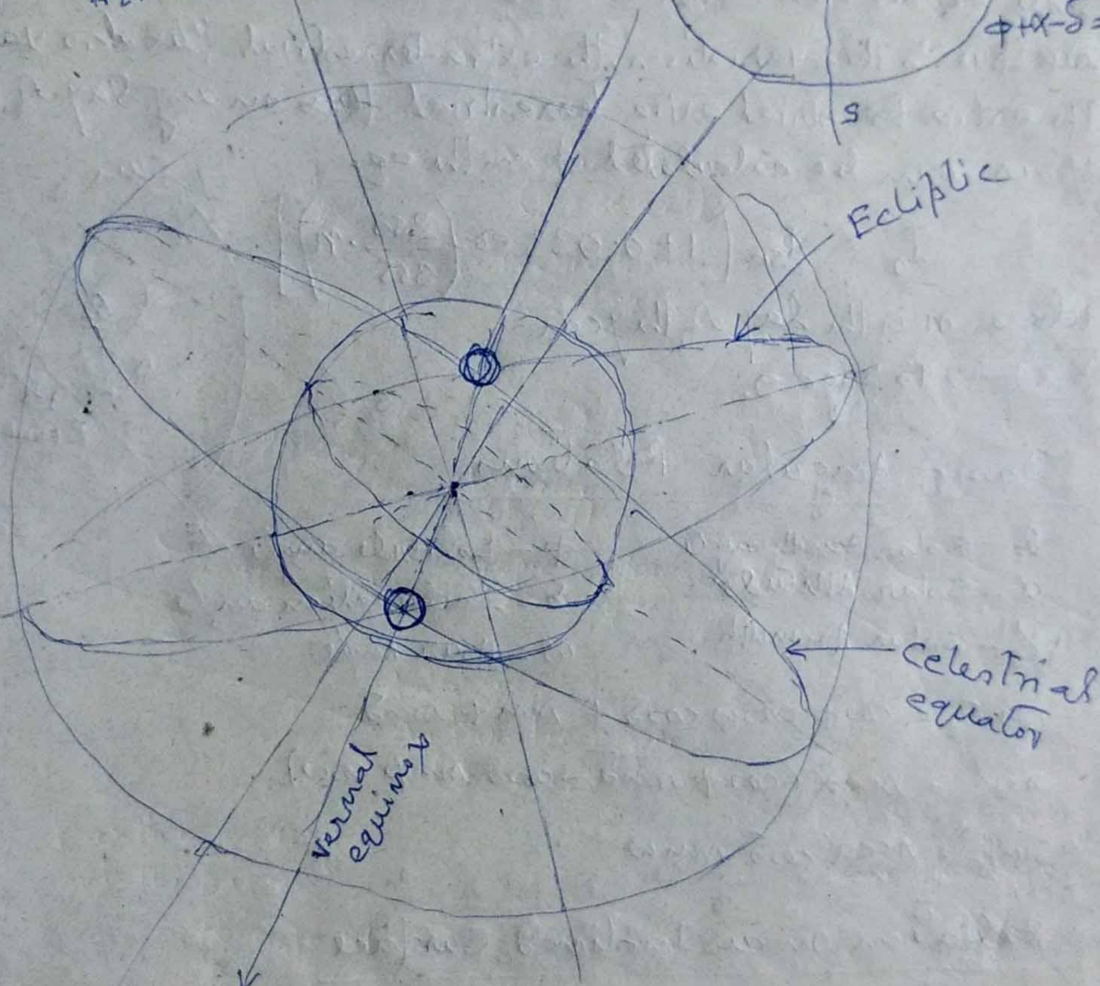
$24\,hr \Rightarrow 360°$

$1\,hr \Rightarrow \frac{360}{24} = 15°$

# Position of Object in the Celestial Sphere



## Extra-Terrestrial Solar Radiation and Terrestrial Solar Radiation

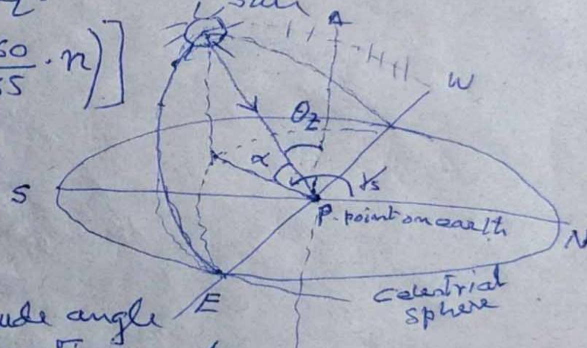
The rate at which the solar energy reaches at the top of the atmosphere is called the Solar constant $I_{sc}$ which is 1353 $W/m^2$ as per NASA. This is the rate at which the energy is received from the sun on a unit area perpendicular to the ray of the sun, at the mean distance of the earth from the sun. (considering $T = 5762°K$)

Since the sun's distance varies throughout the year, the rate of arrival of solar radiation varies accordingly. Due to this variation, the extra-terrestrial flux also varies. The ~~extra-terrestrial~~ extra-Terrestrial flux on any day of the year can be calculated from the eq.

$$I_0 = I_{sc}\left[1 + 0.033\cos\left(\frac{360}{365}\cdot n\right)\right]$$

where $n$ is the day of the year

(***) → To page 9.




## Basic Angular Relations

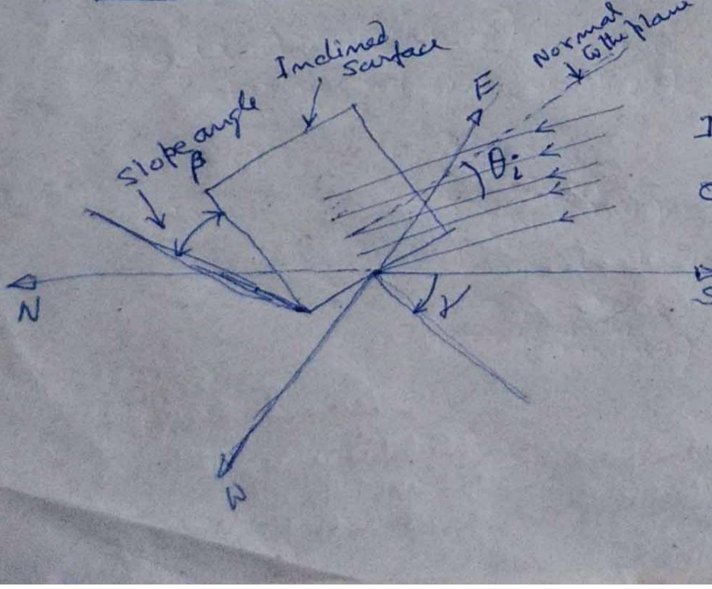
$\theta_z$ – Solar zenith angle
$\alpha$ – Solar Altitude
$\gamma_s$ – Solar Azimuth
$\phi$ – Latitude angle
$\delta$ – Declination angle
$\omega$ – Hour Angle

$$\cos\theta_z = \cos\phi\cos\omega\cos\delta + \sin\phi\sin\delta$$

$$\cos\gamma_s = \sec\alpha(\cos\phi\sin\delta - \cos\delta\sin\phi\cos\omega)$$

$$\sin\gamma_s = \sec\alpha\cos\delta\sin\omega$$

## Radiation on an Inclined Surface




Slope angle – $\beta$
Surface Azimuth Angle – $\gamma$
Incident Angle – $\theta_i$

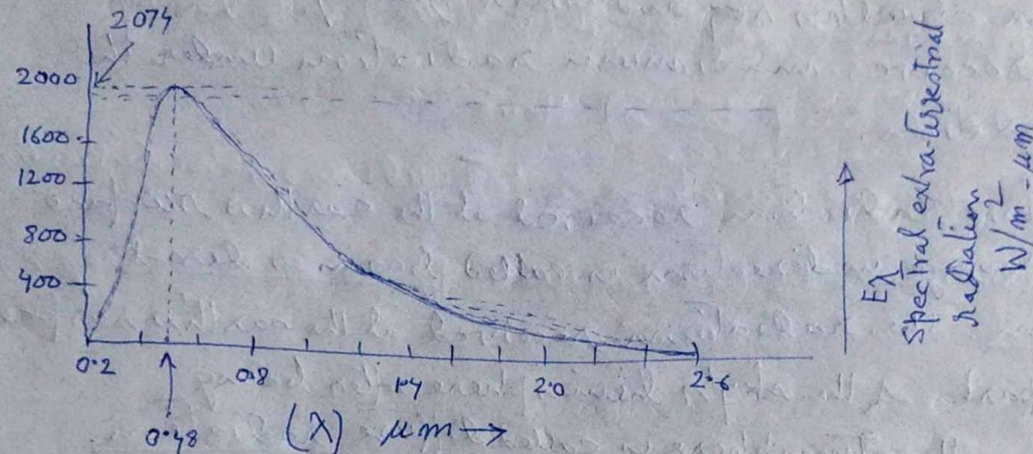
$$\cos\theta_i = \sin\phi(\sin\delta\cos\beta + \cos\delta\cos\gamma\cos\omega\sin\beta) + \cos\phi(\cos\delta\cos\omega\cos\beta - \sin\delta\cos\gamma\sin\beta) + \cos\delta\sin\gamma\sin\omega\sin\beta.$$

To page- 11. (**)

From (***) → From page 8. 

The earth is closest to the sun in the summer and furthest away in the winter.

Measurements of the spectral distribution of extra-terrestrial solar radiation are made and recorded by M.P. Thekaekara. The spectral distribution of extra-terrestrial solar radiation is shown here.




The curve show that the spectral first increases sharply with wave ~~length~~ length and passes through the maximum value of 2074 W/m²-μm at a wavelength of 0·48 μm and then decreases asymptotically to zero. 99% of the sun's radiation is obtained upto a wavelength of 4 μm.

The solar energy that arrive at the earth's atmosphere, consists of about 8% UV radiation ($\lambda$ less than 0·39 μm), 46% visible light ($\lambda$ from 0·39 to 0·78 μm), 46% infra-red radiation ($\lambda > 0{\cdot}78\mu m$).

When the solar radiation passes through the earth's ~~surface~~ atmosphere, it gets subjected to absorption and scattering mechanisms before reaching the earth's surface. that is because of presence of ozone and water vapour in the atmosphere and to a ~~lower extent~~ lessar extent of the other gases, e.g., $CO_2$, $NO_2$, $O_2$, $CH_4$ and CO etc. and particulate matter, absorb the solar radiation. This leads to an increase in the internal energy of the atmosphere. ~~assist~~ on the other hand, all gaseous molecules as well as particulate matter in the atmosphere assist scattering the solar radiation. Some scattered radiation goes back into space and some reaches the earth's surface.

Moreover, the earth's atmosphere may be of without clouds or with clouds. In the former case, the sky is cloudless everywhere, while in the later, the sky is partly or fully covered by clouds. For both type of atmosphere the absorption and scattering mechanisms, as stated earlier, occur. It is fact that less attenuation takes place in a cloudless sky and consequently earth's surface receives maximum radiation under the said condition.

Solar radiation received at the earth's surface without change in direction is called beam or direct radiation. The radiation received at the earth's surface from all parts of the sky's hemisphere after being scattered in the atmosphere is called diffuse radiation. The sum of the beam and diffuse radiation is referred to as total or global radiation. This is the total solar radiation received at any point on the earth's surface. In general sense, it is also referred to as the insolation at that point

From page 8 (**)



For different conditions the expression for $\theta_i$ can be simplified as follows:

For vertical surface, $\beta = 90°$, $\cos\beta = 0$, $\sin\beta = 1$

$$\cos\theta_i = \sin\phi\cos\delta\cos\gamma\cos\omega - \cos\phi\sin\delta\cos\gamma + \cos\delta\sin\gamma\sin\omega$$

For Horizontal surface, $\beta = 0$, $\omega = 0$, $\cos\beta = 1$, $\sin\beta = 0$, $\theta_i = \theta_z$

$$\cos\theta_i = \cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$$
$$= \sin\phi\sin\delta + \cos\phi\cos\delta$$
$$= \cos(\phi - \delta) = \cos(\pi/2 - \alpha) = \sin\alpha$$
$$[\phi + \alpha - \delta = \pi/2]$$
$$\therefore \cos\theta_i = \cos\theta_z = \sin\alpha$$

For surface facing south $\gamma = 0$, $\sin\gamma = 0$, $\cos\gamma = 1$

$$\cos\theta_i = \sin\phi(\sin\delta\cos\beta + \cos\delta\cos\omega\sin\beta) + \cos\phi(\cos\delta\cos\omega\cos\beta - \sin\delta\sin\beta)$$
$$= \sin\delta(\sin\phi\cos\beta - \cos\phi\sin\beta) + \cos\delta\cos\omega(\sin\phi\sin\beta + \cos\phi\cos\beta)$$
$$= \sin\delta\sin(\phi - \beta) + \cos\delta\cos\omega\cos(\phi - \beta)$$

For vertical surface facing south, $\gamma = 0$, $\beta = 90°$

$$\cos\theta_i = \sin\delta\sin(-90 + \phi) + \cos\delta\cos\omega\cos(-90 + \phi)$$
$$= \sin\phi\cos\delta\cos\omega - \cos\phi\sin\delta$$

Sunrise or Sunset Hour Angle ($\omega_s$) and Day Length

The hour angle corresponding to sunrise or sunset at horizontal surface is called sunrise or sunset hour angle ($\omega_s$). At the time of sunrise or sunset the zenith angle $\theta_z$ is 90°, so for the horizontal surface,

$$\cos\theta_i = \cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega_s = 0$$
$$\therefore \cos\omega_s = -\frac{\sin\phi\sin\delta}{\cos\phi\cos\delta} = -\tan\phi\tan\delta$$
$$\therefore \omega_s = \cos^{-1}[-\tan\phi\tan\delta]$$

$\omega_s$ may be '+' or '−'. The positive value corresponds to sunset and negative value corresponds to sunrise.

The angle between sunrise and sunset i.e. the day angle is

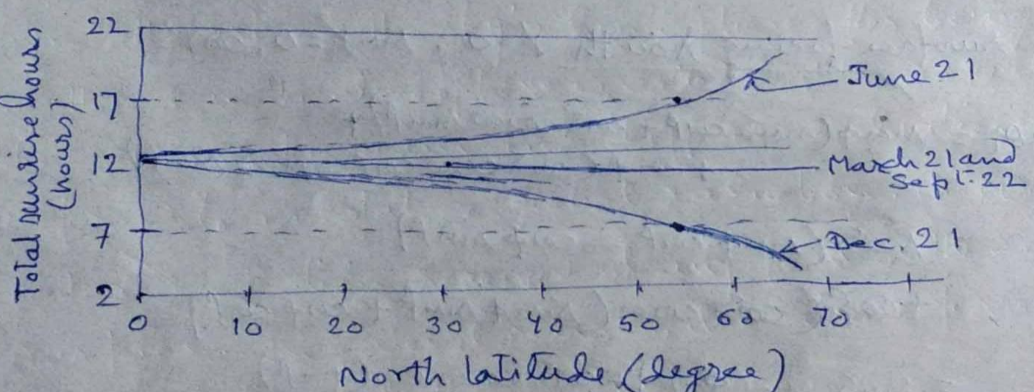
$$2\omega_s = 2\cos^{-1}(-\tan\phi \tan\delta)$$

Since $15^\circ$ of the hour angle is equivalent to 1 hour, the corresponding day length (in hour) is given by

$$t_d = \frac{2}{15}\cos^{-1}(-\tan\phi \tan\delta) \text{ Hour}$$

so the day length is a function of latitude and solar declination.

The variation of $t_d$ with latitude ($\phi$) for different day ($n$) of the year is shown below:




The hour angle at sunrise as seen by the observer on an inclined surface facing south (i.e. $\gamma = 0$) will also be

$$\omega_s = \cos^{-1}(\tan\phi \tan\delta)$$

if the day under consideration lies between Sept. 22 and Mar. 21, and the location is in the northern hemisphere.

However, if the day under consideration lies between Mar. 21 and Sept. 22, the hour angle at sunrise or sunset would be smaller in magnitude and would be obtained from

$$\cos\theta_i = \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)$$

by putting $\theta_i = 90^\circ$, $\omega = \omega_{st}$