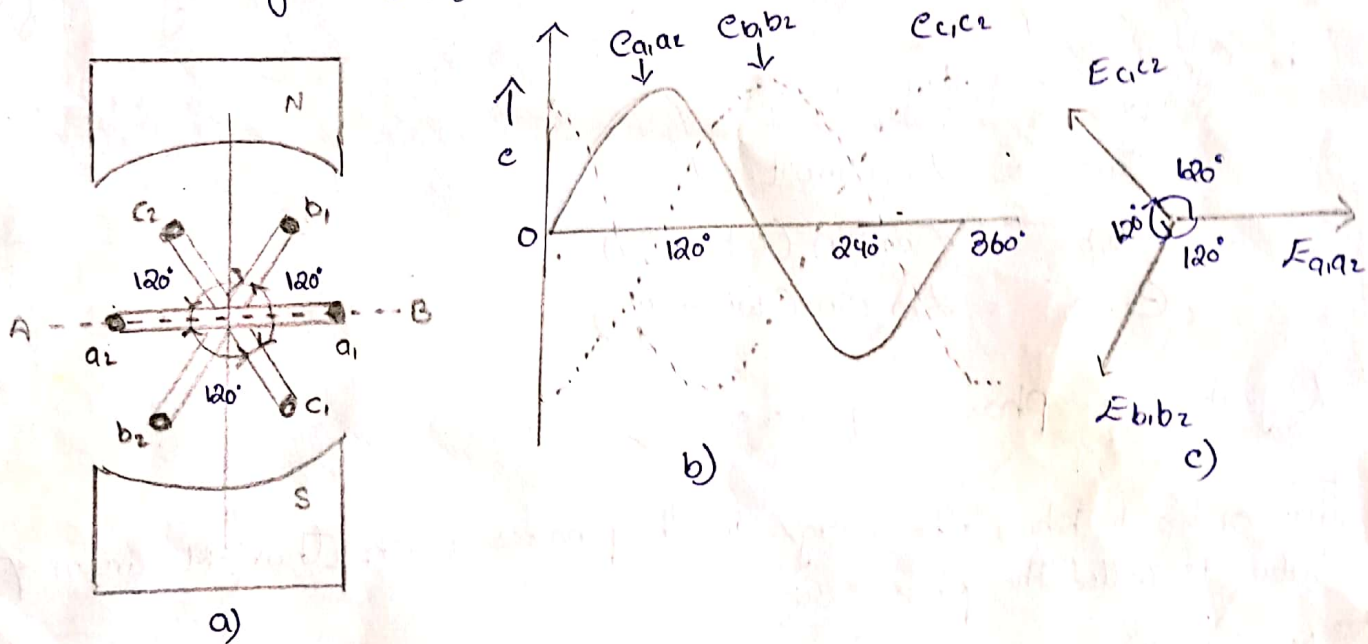


Module 2

b) Three Phase Circuits :-

Generation of Three Phase Voltages :-

- These are three equal voltages of same frequency but displaced from one another by 120° Electrical
- These voltages are produced by 3 generators which has 3 identical winding or phases displaced by 120° Electrical apart.
- When these winding are rotated in magnetic field EMF is induced in each winding / phase
- These EMF are of the same magnitude and frequency but are displaced from one another by 120° Electrical.



- Consider 3 electrical coils a_1, a_2, b_1, b_2 and c_1, c_2 wound on some axis but displaced from each other by 120° Electrical.
- Let the 3 coils be rotated in an anticlockwise direction in a bipolar magnetic field with an angular velocity of radians/sec
- Here a_1, b_1 and c_1 are starting terminals
 a_2, b_2 and c_2 are ending terminals

- When the coil $a_1 a_2$ is in the position AB the magnitude and direction of the E.M.F induced in various coil is

so
necessarily
sites and at
So that the

a) E.M.F induced in coil $a_1 a_2$ is zero and is increasing in \oplus ve direction
This is indicated by $e_{a_1 a_2}$ wave

b) The coil $b_1 b_2$ is 120° electrically behind coil $a_1 a_2$.
The E.M.F induced in the coil is \ominus ve and approaching maximum negative value.
This is indicated by $e_{b_1 b_2}$ wave

c) The coil $c_1 c_2$ is 240° electrically behind coil $a_1 a_2$ or
coil $c_1 c_2$ is 120° electrically behind coil $b_1 b_2$.
The E.M.F induced in this coil is \oplus ve and is decreasing.
This is indicated by $e_{c_1 c_2}$

- The Rms value of 3 phase voltages are shown vectorially in fig 'c'

The Eqn for 3 voltages are

$$e_{a_1 a_2} = E_m \sin \omega t$$

$$e_{b_1 b_2} = E_m \sin (\omega t - \frac{2\pi}{3})$$

$$e_{c_1 c_2} = E_m \sin (\omega t - \frac{4\pi}{3})$$

Definition of Phase Sequence:-

The order in which the voltage in the phases reach their maximum positive value is called the phase sequence

Naming the Phases :-

3 ϕ may be numbered as 1, 2, 3
alphabets as a, b, c
colours as R, Y, B

R Y B is considered as positive
R B Y is considered as negative

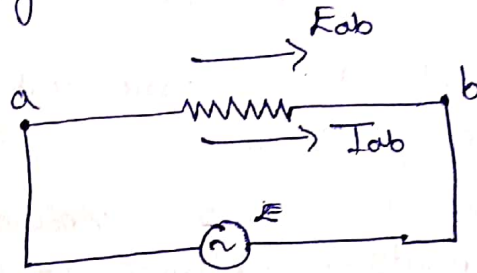
Direction is shown

Subscript Notation :-

Necessarily we employ some systematic notation for the solⁿ of ac circuits and system containing a number of E.M.F.s acting and currents flowing so that the process of solution is simplified and less prone to error.

- It is normally preferred to employ double-subscript notation while dealing with ac electrical circuits.
- In this system the order in which the subscripts are written indicates the direction in which EMF acts or current flows.

• For example if EMF is expressed as E_{ab}



• this indicates that EMF acts from a to b

• if EMF is expressed as E_{ba}

• this indicates that EMF acts from b to a (EMF is acting in opposite direction)
i.e. $E_{ab} = -E_{ba}$

Similarly I_{ab} indicates current flows in the direction from a to b but I_{ba} indicates current flows in the direction from b to a

i.e. $I_{ba} = -I_{ab}$

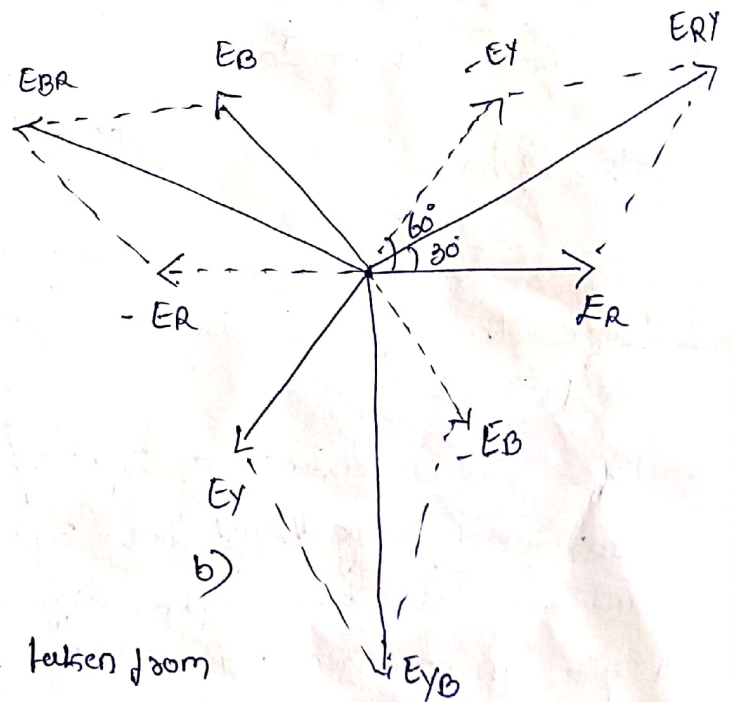
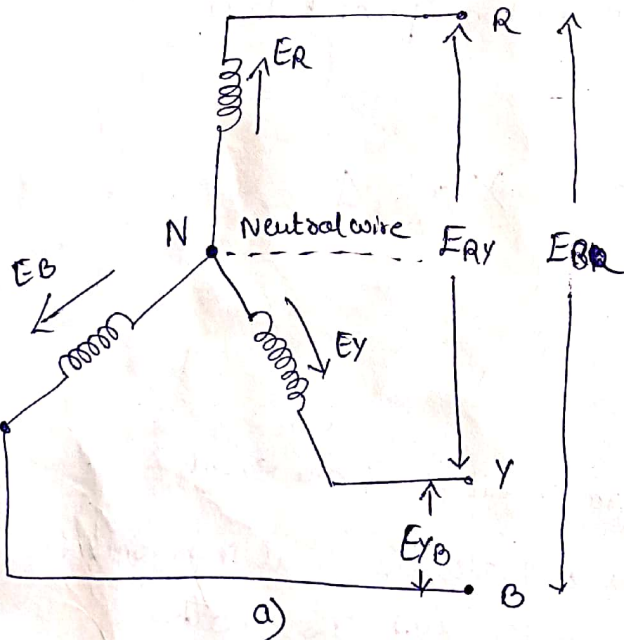
* Three-phase balanced supply and load Y and Δ

- When a balanced generating supply, where the 3 phase voltage are equal phase difference is 120°
- Where the impedances of 3 phase / three circuit load are equal then current flowing through these 3 phases will also be equal in magnitude and will also have a phase difference of 120° with one another. Such an arrangement is called a balanced load.

* Relationship b/w line & phase Quantities and Expression for balanced 3 phase star connection.

1400.
Figure b) F
Star connection with
one wire in open

- Three phase star connection is obtained by joining similar terminals together, [either starting ends or finishing ends are joined together]
- The common point at which these ends are connected together is called Neutral or starpoint denoted by N
- Normally only 3 wires carried out for external circuit giving 3 ϕ -3 wire star connected system.
- Sometimes a fourth wire known as Neutral wire is carried to neutral point of external load circuit giving rise to 3 ϕ -4 wire star connected system.
- "The voltage b/w any line and neutral point is called Phase voltage"
- "The voltage b/w any two lines is called line voltage"



- In Fig a) (+)ve directions of EMF are taken from starpoint outward
- The arrow heads on EMF and current indicate the (+)ve direction
- Here 3 phases are numbered as usual R, Y, B indicates Red, Yellow and Blue

tion, sequence R/Y/B is taken as positive and R/B/Y as negative

in Figure b) EMF induced in the 3 phases are shown vectorially

in star connection there are two winding b/w each pair of center

Due to joining of similar ends together the EMF induced in them are in opposition.

- The voltage b/w R and Y line is line voltage E_{RY} it is the vector difference of phase EMFs E_R and E_Y or it is the vector sum of phase EMFs E_R & E_Y

$$\text{i.e. } E_{RY} = E_R - E_Y \quad [\text{vector difference}]$$

$$E_{RY} = E_R + (-E_Y) \quad [\text{vector sum}]$$

$$\sqrt{a^2 + b^2 + 2ab \cos \theta}$$

- Phase angle b/w E_R & E_Y is 60°

From vector diagram $E_{RY} = \sqrt{E_R^2 + E_Y^2 + E_R E_Y \cos 60^\circ}$

$$E_R = E_Y = E_B = E_p \quad (\text{phase voltage})$$

$$E_{RY} = \sqrt{E_p^2 + E_p^2 + 2E_p^2 \times 0.5}$$

$$= \sqrt{3E_p^2}$$

$$E_{RY} = \sqrt{3}E_p$$

Then line voltage $E_{RY} = \sqrt{3}E_p$

$$\text{Similarly } E_{YB} = E_Y - E_B = \sqrt{3}E_p$$

$$E_{BR} = E_B - E_R = \sqrt{3}E_p$$

- In a balanced star system E_{RY} , E_{YB} , and E_{BR} are equal in magnitude and are called line voltages

$$E_L = \sqrt{3}E_p$$

- Since in a star connection each line conductor is connected to separate phase so current flowing through line and phases are the same.

$$\text{Line Current } I_L = \text{Phase Current } I_p$$

I_p phase current has phase difference of ϕ with the phase voltage

$$\text{Power output per phase} = E_p I_p \cos \phi$$

$$\begin{aligned} \text{Total power output } P &= 3 E_p I_p \cos \phi \\ &= 3 \frac{E_L}{\sqrt{3}} I_p \cos \phi \end{aligned}$$

$$P = \sqrt{3} E_L I_L \cos \phi$$

Power = $\sqrt{3}$ x line voltage x line current x power factor

Apparent power of 3-phase star-connected system

$$= 3 \times \text{apparent power per phase}$$

$$= 3 \times E_p I_p$$

$$= 3 \times \frac{E_L}{\sqrt{3}} \times I_L \quad [I_L = I_p]$$

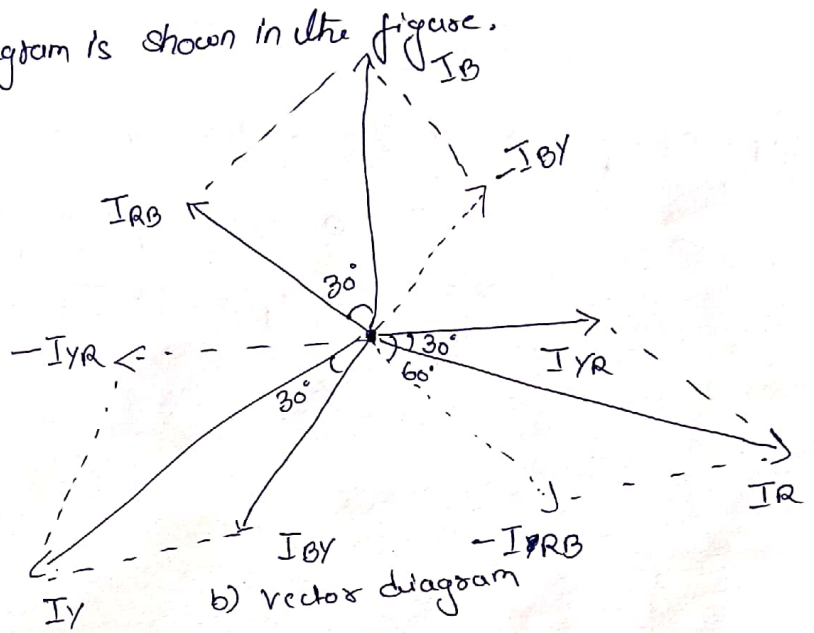
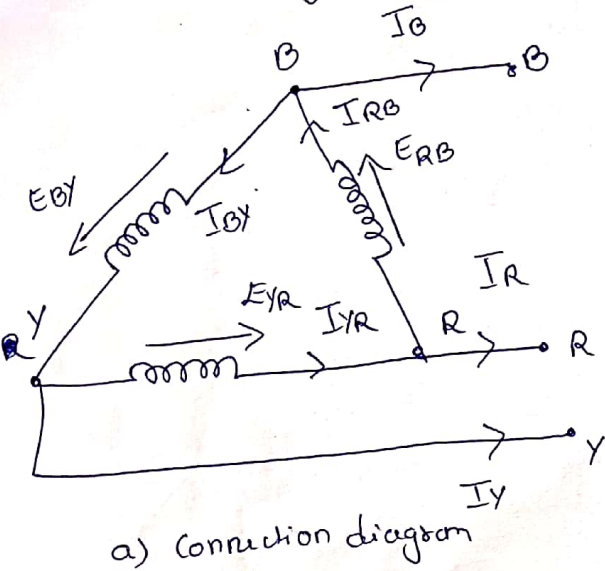
$$P = \sqrt{3} E_L I_L$$

3 ϕ Star
3 ϕ Delta

3 ϕ Delta Connection and Phase Quantities & Expression for power for a 3 ϕ Delta Connection.

Delta Connection is obtained when starting end of one coil is connected to finishing end of another coil.

The connection diagram and vector diagram is shown in the figure.



From the vector diagram line current is the vector sum of two phase currents

i.e. I_R line current is vector sum of I_{YR} and I_{RB}

$$\text{Line Current } I_R = I_{YR} - I_{RB} \text{ (vector difference)}$$

$$I_R = I_{YR} + (-I_{RB}) \text{ (vector sum)}$$

The phase angle b/w currents I_{YR} & I_{RB} is 60°

$$I_R = \sqrt{I_{YR}^2 + I_{RB}^2 + 2 I_{YR} I_{RB} \cos 60^\circ}$$

For balanced load phase current in each winding is equal and let it be I_p

$$\text{Line Current } I_Y = I_{BY} - I_{YR} = \sqrt{3} I_p$$

$$I_B = I_{RB} - I_{BY} = \sqrt{3} I_p$$

In delta network there is only one phase b/w any pair of lines so the potential difference b/w the line center called line voltage

In delta net line voltage = phase voltage

$$E_L = E_p$$

- Power output per phase = $E_p I_p \cos \phi$

- Total power output $P = 3 E_p I_p \cos \phi$
 $= 3 E_L \frac{I_L}{\sqrt{3}} \cos \phi$

$$P = \sqrt{3} E_L I_L \cos \phi$$

- Apparent power of 3 ϕ delta-connected system

$$= 3 \times \text{apparent power per phase}$$

$$= 3 E_p I_p = 3 E_L \frac{I_L}{\sqrt{3}} = \sqrt{3} E_L I_L$$

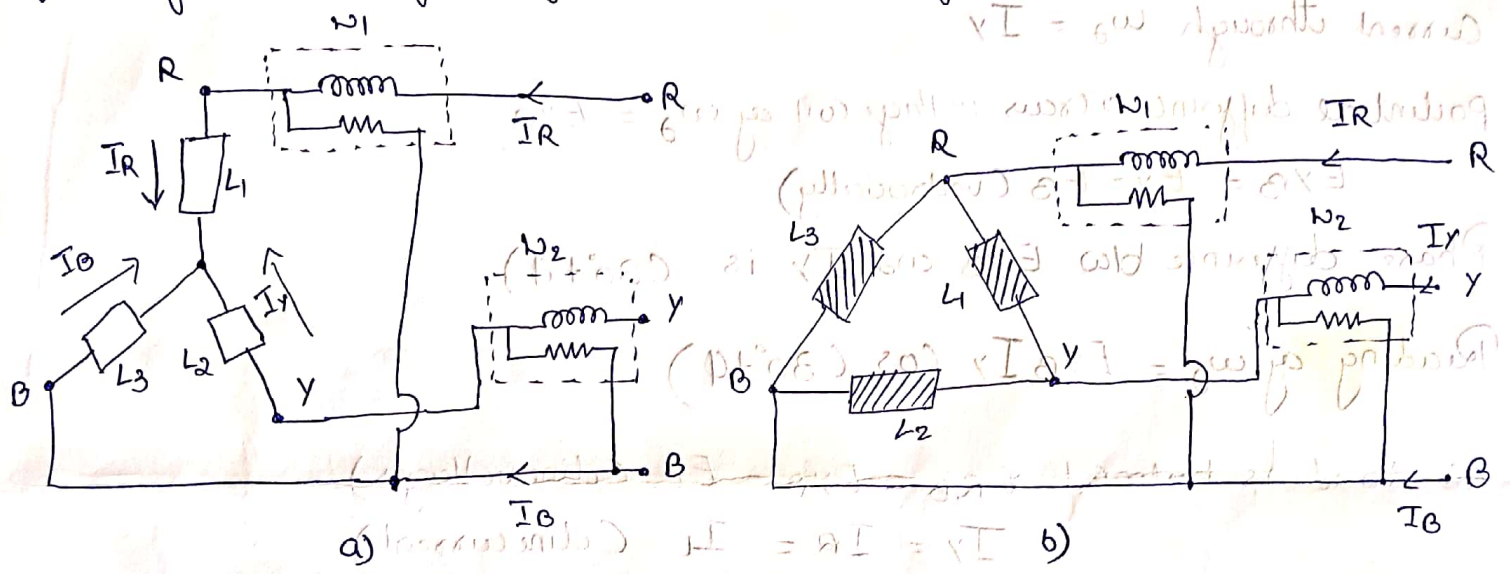
of power in

the meter Method :- This 3 phase

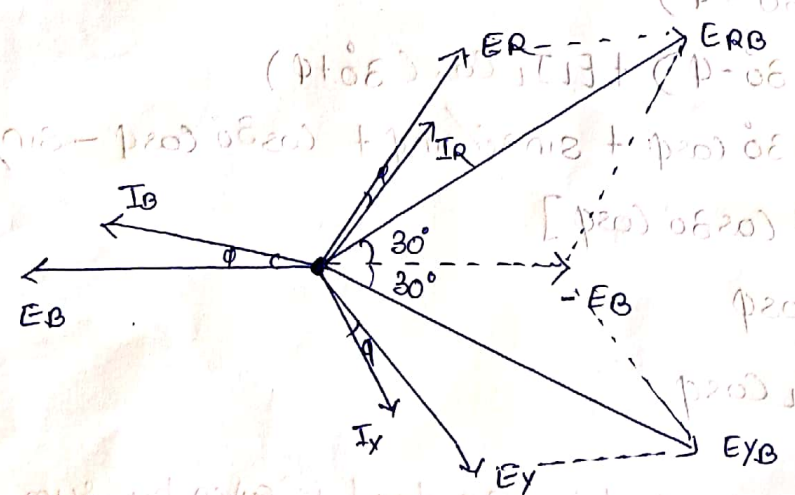
of power in 3-phase circuits :-

Wattmeter Method :- This method is normally used for measuring power in 3-phase - 3-wire balanced load circuits.

- Consider the figure shown, which consist of two wattmeter, in which current coil are inserted in any two lines and potential coil of each wattmeter is joined to third line.
- In case of balanced load impedances of all the 3 phases are equal and power factor of load can be found by two wattmeter readings.



- Let the star connected load be inductive the vector diagram is shown in the figure



- Let 3 phase voltage have rms value E_R, E_Y or E_B
- Let 3 phase current have rms value I_R, I_Y and I_B
- Currents are lagging behind their respective phase angle ϕ

• Current through $W_1 = I_R$

• Potential difference across voltage coil of $W_1 = E_{RB}$

$$E_{RB} = E_R - E_B \text{ (vectorially)}$$

• Phase difference b/w E_{RB} and I_R is $(30^\circ - \phi)$

• Reading of $W_1 = E_{RB} I_R \cos(30^\circ - \phi)$

• Current through $W_2 = I_Y$

• Potential difference across voltage coil of $W_2 = E_{YB}$

$$E_{YB} = E_Y - E_B \text{ (vectorially)}$$

• Phase difference b/w E_{YB} and I_Y is $(30^\circ + \phi)$

• Reading of $W_2 = E_{YB} I_Y \cos(30^\circ + \phi)$

• As load is balanced $E_{RB} = E_{YB} = E_L$ (line voltages)
 $I_Y = I_R = I_L$ (line current)

$$W_1 = E_L I_L \cos(30^\circ - \phi)$$

$$W_2 = E_L I_L \cos(30^\circ + \phi)$$

$$\begin{aligned} W_1 + W_2 &= E_L I_L \cos(30^\circ - \phi) + E_L I_L \cos(30^\circ + \phi) \\ &= E_L I_L [\cos 30^\circ \cos \phi + \sin 30^\circ \sin \phi + \cos 30^\circ \cos \phi - \sin 30^\circ \sin \phi] \\ &= E_L I_L [2 \cos 30^\circ \cos \phi] \\ &= E_L I_L \sqrt{3} \cos \phi \end{aligned}$$

$$W_1 + W_2 = \sqrt{3} E_L I_L \cos \phi$$

"Thus total power absorbed in 3ϕ load is given by sum of two wattmeter reading"

It is shown that two wattmeter are sufficient to measure 3ϕ power

Power factor - Balanced 3-phase load

Case 1 :- lagging power factor :-

$$W_1 + W_2 = E_L I_L \cos(30^\circ - \phi) + E_L I_L \cos(30^\circ + \phi)$$
$$= E_L I_L \sqrt{3} \cos \phi$$

$$W_1 + W_2 = \sqrt{3} E_L I_L \cos \phi \rightarrow (1)$$

$$W_1 - W_2 = E_L I_L \cos(30^\circ - \phi) - E_L I_L \cos(30^\circ + \phi)$$

$$= E_L I_L \cancel{2} \times \sin \phi \times \frac{1}{\cancel{2}}$$

$$W_1 - W_2 = E_L I_L \sin \phi \rightarrow (2)$$

÷ Eqn (2) by (1)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{E_L I_L \sin \phi}{\sqrt{3} E_L I_L \cos \phi}$$

$$\frac{\sqrt{3} (W_1 - W_2)}{(W_1 + W_2)} = \tan \phi \rightarrow (3)$$

If W_2 reading is negative, its reading is taken after reversing the pressure coil, then Eqn (3) would become

$$\tan \phi = \frac{\sqrt{3} (W_1 - (-W_2))}{W_1 + (-W_2)}$$

$$\tan \phi = \frac{\sqrt{3} (W_1 + W_2)}{(W_1 - W_2)}$$