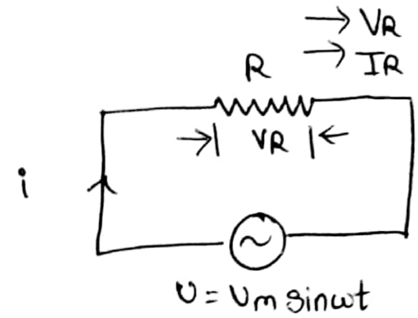
Module 2 a) Single phase AC Circuits

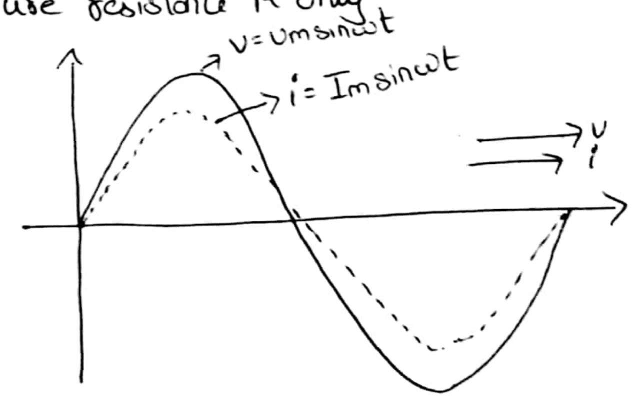
- The path for the flow of alternating current is called ac circuits
- In a dc circuit the current flowing through circuit is given by $I = \frac{V}{R}$
- However in ac circuit voltage and current changes from instant to instant and so give rise to magnetic (inductive) and electrostatic (capacitive) effects
- So in an ac circuit inductance and capacitance must be considered in addition to resistance

AC Circuit containing pure ohmic Resistance :-

- When alternating voltage is applied across a pure ohmic resistance current flows in one direction during 1st ½ cycle and opposite direction in next ½ cycle thus constituting alternating current in circuit.

Let us consider an ac circuit with pure resistance R only





Let the applied voltage be given by the Eqn

$V = V_m \sin\theta$

$v = V_m \sin\omega t \rightarrow (1)$

as a result of this alternating voltage and alternating current will flow through circuit

The applied voltage has to supply the drop in resistance

$V = iR$

Substituting value of v in Eqn (1)

$iR = V_m \sin\omega t$

$i = \frac{V_m}{R} \sin\omega t$ [The value of i is maximum when $\sin\omega t = 1$] $\rightarrow (2)$

$I_m = \frac{V_m}{R}$

Eqn (2) becomes $i = I_m \sin\omega t$

From Eqn ① & ② it is apparent that voltage and current are in phase with each.
This is indicated by wave & vector diagram.

Inductive
however
negative

Power:- voltage and current are changing at every instant

Then Instantaneous power - $p = VI$

$$= V_m \sin\omega t \cdot I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t$$

$$= V_m I_m \frac{(1-\cos 2\omega t)}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

- instantaneous part consist of
  constant part $= \frac{V_m I_m}{2}$
  Fluctuating part $= \frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of voltage and current wave

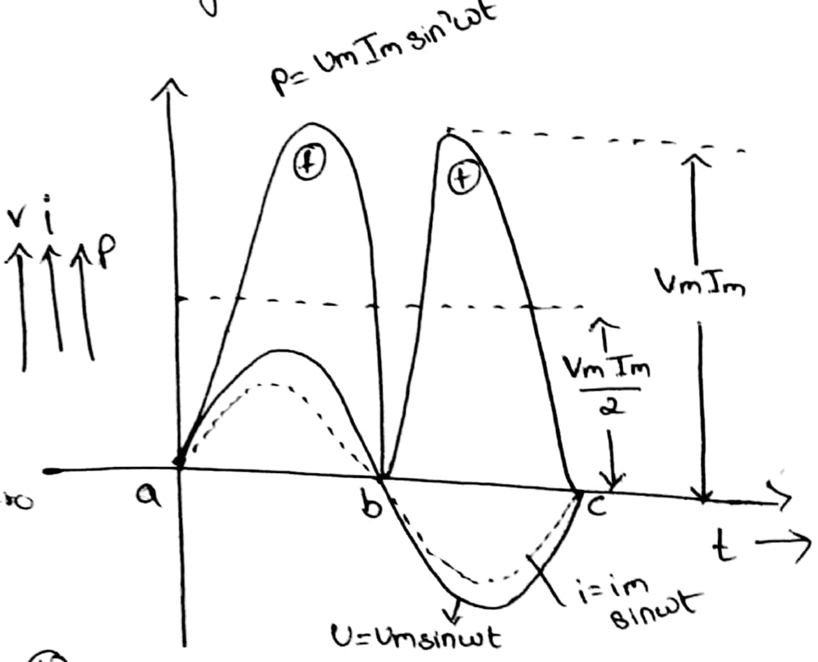
The average value of $\frac{V_m I_m}{2} \cos 2\omega t$ over complete cycle is zero.

hence power for complete cycle is $P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} =$

$$P = VI \text{ watts}$$

Where V = rms value of applied voltage
I = rms value of current.

Power Curve:-



- power curve for purely resistive CKT is shown
- It is apparent that power in such circuit is zero
- only at the instant a, b, and c when both voltage & current are zero
- but it's positive at all other instant
- In other words power is always ⊕ve so that power is always lost in resistive ac circuit.
- The power is dissipated as heat.

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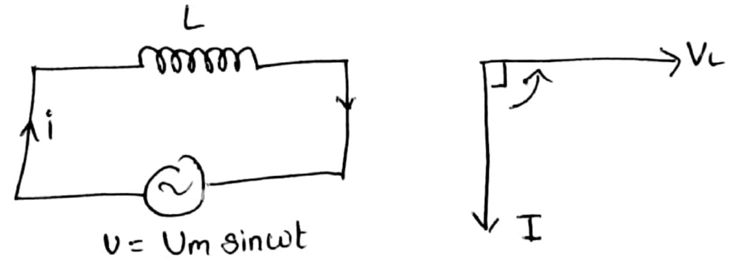
A.C Circuit Containing pure Indutance :-

Inductive Coil is a coil with or without an iron core and negligible resistance
However a coil of thick copper wire wounded on a laminated iron core has negligible resistence

For the purpose of our study we will consider purely inductive coil.



- When ever alternating voltage is applied to a circuit containing a pure inductance
- the back emf is produced because of self indutance of the coil
- This back emf oppose the rise and fall of current at every stage.
- Because of absence of voltage drop. applied voltage has the overcome this self-induced emf.

Let the applied voltage be $V = V_m \sin\omega t$

self indutance of coil = L henry

self induced e.m.f in the coil $e_L = -L\dfrac{di}{dt}$

since applied voltage at every instant is equal and opposite to self-indutance e.m.f i,e $V = e_L$

$$V_m \sin\omega t = -\left(-L\frac{di}{dt}\right)$$

$$di = \frac{V_m}{L}\sin\omega t\, dt$$

on integrating b.s we get

$$\int di = \int \frac{V_m}{L}\sin\omega t\, dt$$

$$i = \frac{V_m}{L}\int \sin\omega t\, dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t) + A$$

Where A is a constant of integration which is found to be zero from initial condition.

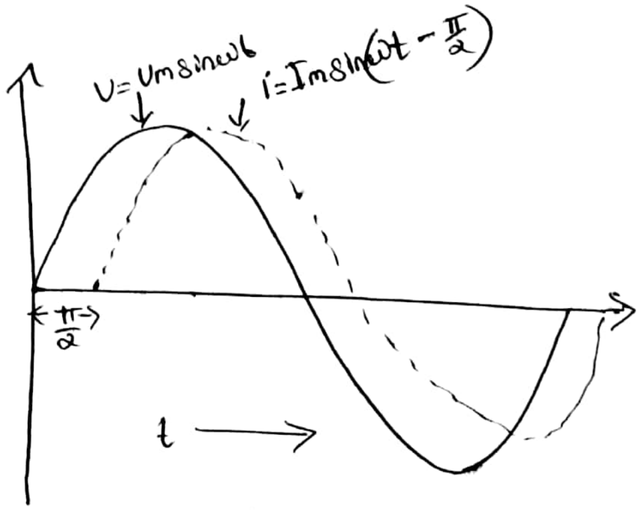
$$i = -\frac{V_m}{\omega L}\cos\omega t$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

- The current will be max when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$
- hence value of max current $I_m = \frac{V_m}{\omega L}$
- and instantaneous current may be expressed as $i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

~~Current will be maximum when~~ $\cancel{\sin\left(\omega t - \frac{\pi}{2}\right) = 1}$

~~hence value of maximum current~~ $\cancel{I_m = \frac{V_m}{\omega L}}$



- From the expression of instantaneous applied voltage $V = V_m \sin\omega t$ and instantaneous current flowing through a purely inductive coil
- It is cleared that current lags behind voltage by $\frac{\pi}{2}$

Inductive Reactance :-

'ωL'

In expression $I_m = \frac{V_m}{\omega L}$ is known as inductive reactance denoted by X_L

unit of L = henry

ω = radians/sec

X_L = ohms

$X_L = \omega L$

$X_L = 2\pi f L$

hence inductive reactance play's the part of resistance

Power :- Instantaneous power :-

$$p = v \times i$$

$$= V_m \sin\omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin\omega t \cos\omega t$$

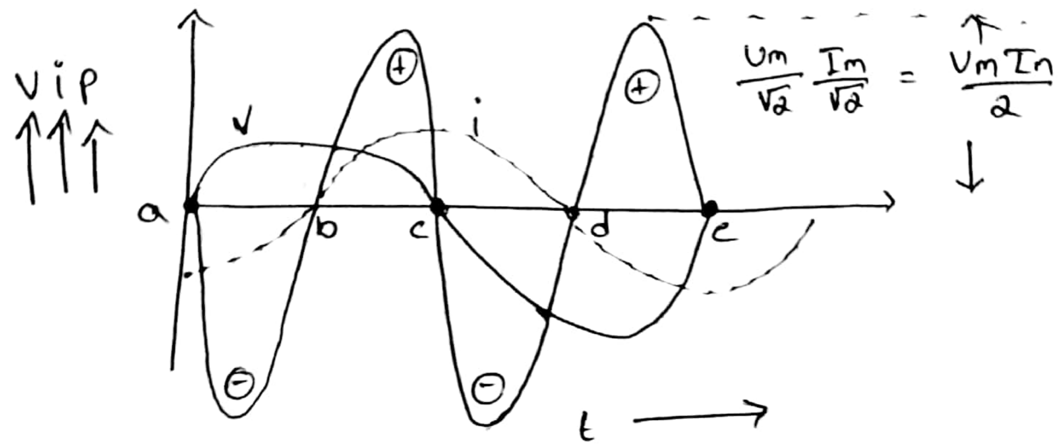
$$= -\frac{V_m I_m}{2}\sin 2\omega t$$

ower Measured by a Wattmeter is average value of P which is zero ie average of a sinusoidal quanity of double frequency over a complet cycle by put this in Mathematical terms.

ower for whole cycle $p = -\frac{V_m I_m}{2} \int_0^{2\pi} \sin 2\omega t \, dt = 0$

hence power absorbed in pure inductive circuit is zero.

Power Curve :-



- Power Curve for pure inductive circuit is shown
- This indicates that power absorbed in the circuit is zero
- at the instant a, c and e voltage is zero. hence power is zero.
- when current is at b and d power is also zero
- Between a and b voltage and current are in opposite direction
- Between b and c voltage and current are in same direction
- Between c and d power is negative and Energy is taken from the circuit
- Between d and e power is positive and Energy is put in the circuit

hence net power is zero

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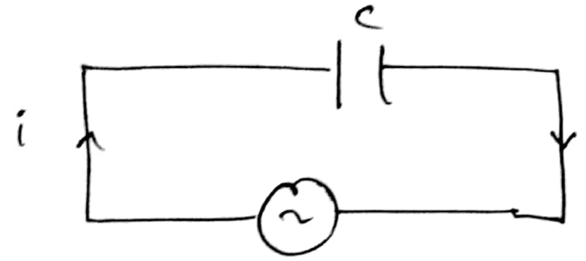
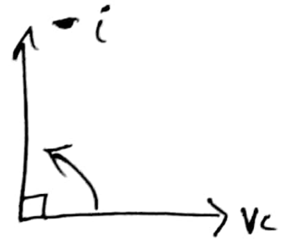
i. C Circuit Containing Pure Capacitance :-



$v = V_m \sin\omega t$



- When an alternating voltage is applied across the plates of a capacitor
- Capacitor is charged in one direction and then in opposite direction as the voltage reverses.

- Let alternating voltage represented by $v = V_m \sin\omega t$ applied across the capacitor of capacitance C Farads.

Instantaneous Charge $q = Cv$
$= C V_m \sin\omega t$

Capacitor Current is equal to rate of change of charge

$$i = \frac{dq}{dt} = \frac{d[C V_m \sin\omega t]}{dt}$$

$$= \omega C V_m \cos\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \cos\omega t$$

$$i = \frac{V_m}{\frac{1}{\omega C}} \sin(\omega t + \frac{\pi}{2}) \rightarrow (1)$$

The Current is maximum when $t = 0$

$$I_m = \frac{V_m}{\frac{1}{\omega C}} \rightarrow (2)$$

Sub(stit)uting Eqn (2) in Eq (1)

$$i = I_m \sin(\omega t + \frac{\pi}{2})$$

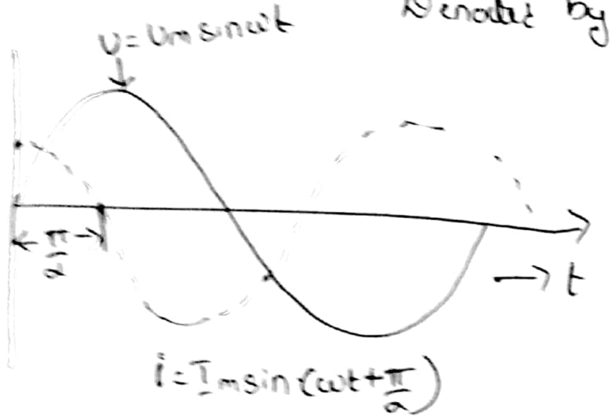
Capacitive Reactance :- $\frac{1}{\omega C}$ in the Expression

$I_m = \frac{V_m}{\frac{1}{\omega C}}$ is known as capacitive Reactance and Denoted by X_C i,e $X_C = \frac{1}{\omega C}$

Where unit of C = Farad

ω = radians

X_C = ohms



- here applied voltage $v = V_m \sin\omega t$
- Current $i = I_m \sin(\omega t + \frac{\pi}{2})$
- Current in pure capacitor leads the voltage by quater cycle
- phase difference b/w voltage & current is $\frac{\pi}{2}$, with current leading

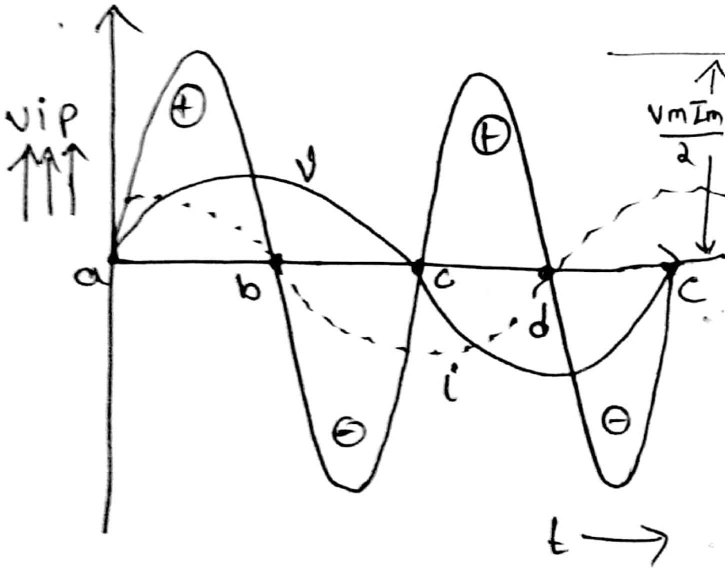
Power :- Instantaneous power

$$P = vi$$
$$= V_m \sin\omega t \; I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$
$$= V_m I_m \sin\omega t \cos\omega t$$
$$= \frac{1}{2} V_m I_m \sin 2\omega t$$

Power for complet cycle $= \frac{1}{2} V_m I_m \int_0^{2\pi} \sin 2\omega t \, dt = 0$

"Average power consumed by pure capacitor is zero"

Power Curve :-



- at instant b, d current is zero hence power is zero
- at instant a, c, c voltage is zero
- b/w a & b voltage & current are in same direction hence power is ⊕ve power is put back the circuit
- b/w b & c voltage & current are in opposite direction hence power is ⊖ve power is taken from circuit
- same b/w c & d and d & c

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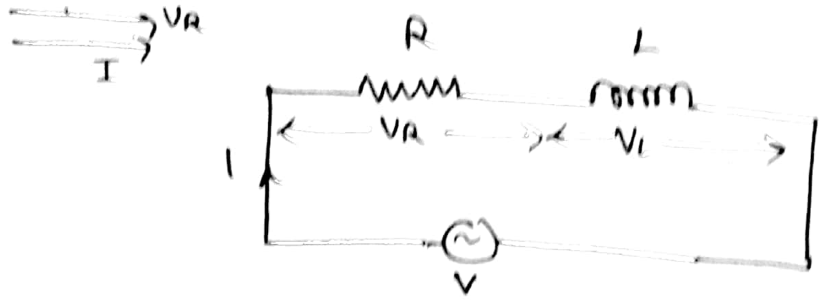
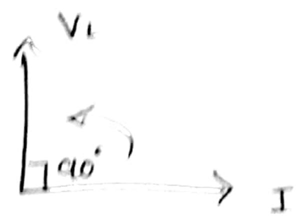
Series R-L Circuit :

Consider an a.c circuit containing pure Resistance "R" ohms and pure inductance L henry as shown





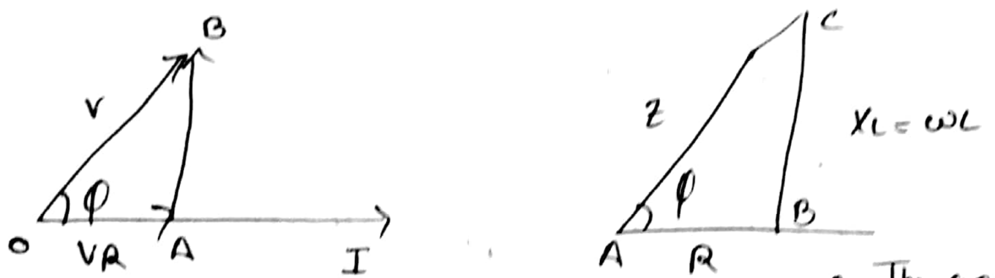
Let V = r.m.s value of applied voltage

I = rms value of the current

Voltage drop across Resistance $V_R = IR$ [voltage inphase with I]

Voltage drop across inductance $V_L = IX_L$ [leading I by 90°]

The voltage drop across these two circuit components are shown in the figure



- Vector OA indicates V_R
- Vector AB indicates V_L
- The applied voltage V is the vector sum of two

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

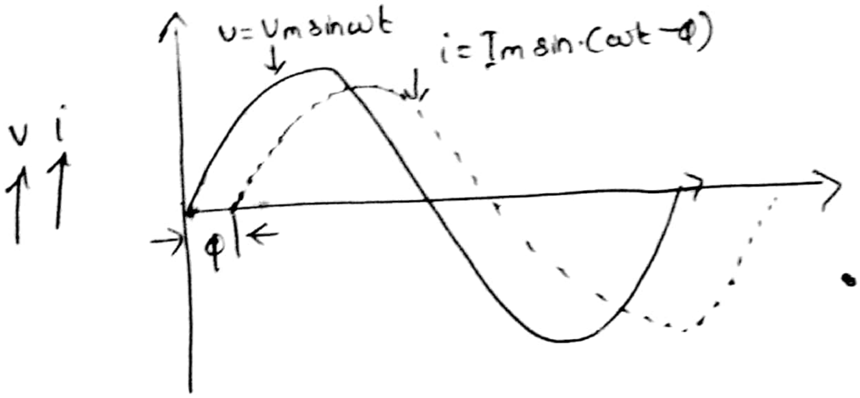
$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$ [∴ The term $\sqrt{R^2 + X_L^2}$ offer opposition for flow of current is called impedance]

$$I = \frac{V}{Z}$$

Referring the impedance triangle

$$Z^2 = R^2 + X_L^2$$

$$(\text{impedance})^2 = (\text{Resistance})^2 + (\text{reactance})^2$$



- applied voltage leads the current I by angle ϕ

- $\tan\phi = \frac{V_L}{V_R} = \frac{I X_L}{I \cdot R} = \frac{X_L}{R}$

$= \frac{\text{reactance}}{\text{resistance}}$

- From the wave form we observe that current lags behind applied voltage by angle "ϕ"

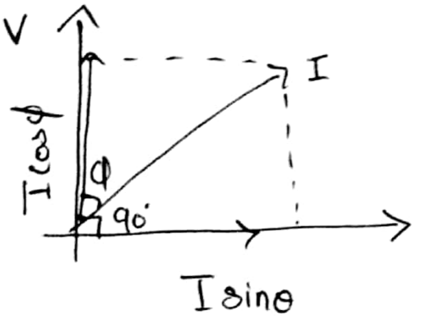
$\phi = \tan^{-1} \frac{X_L}{R}$

- If applied voltage is expressed as $V = V_m \sin\omega t$ and current is expressed as $i = I_m \sin(\omega t - \phi)$

Where $I_m = \frac{V_m}{Z}$

Real power :-, Reactive power, apparent power and power factor

- Consider a series R-L circuit drawing a current I (rms value) when alternating voltage V (rms value) is applied to it. Suppose the current lags behind the applied voltage by angle ϕ



\# power factor and its significance :-

"Power factor is the cosine of angle of lead or lag"

the angle of lag is shown in above fig hence power factor $\cos\phi$

addition to having numerical value.

- The power factor of circuit carries a notation that signifies the nature of circuit
- Weather equivalent circuit is resistive, inductive or capacitive
- Thus the power factor might be expressed as
  0.8 lagging
- The lagging and leading refers to the phase of current vector w.r.t voltage vector.
- lagging power factor means current lags the voltage and circuit is inductive in nature
- leading power factor the current leads the voltage and circuit is capacitive in nature

Apparent power:- The product of Rms value of current & voltage is called apparent power

- Measured in terms of volt amper (VA) or
- kilo volt amper (KVA)

Real power:- The product of apparent power and power factor is called real power

- Expressed in watts or
- kilo watts "KW"

Real power = Volt amper × Power factor
"W" "VA" "$\cos\phi$"

$$\text{Watts} = VA\cos\phi$$

It should be noted that power is only consumed by ohomic resistance only as pure inductance doesnot consume any power

$$P = VI\cos\phi$$

$$\cos\phi = \frac{R}{Z}$$

...age power

$P = VI \times \cos\phi$

$= VI \times \frac{R}{Z}$ But $Z := \frac{V}{I}$

$= \cancel{V}I \times \frac{RI}{\cancel{V}}$

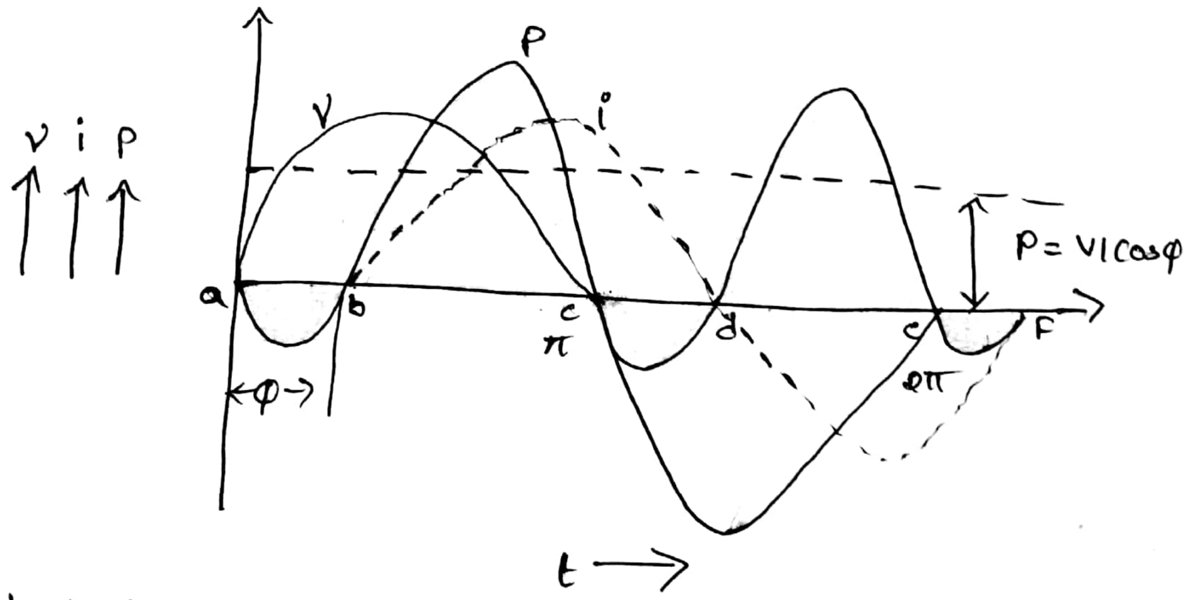
$P = I^2R$ watts

Reactive power:- It is the power developed in the inductive reactance of the Circuit

The quantity VI sin is called reactive power

it is measured in reactive volt-amper or (VAr)

The power consumed can be represented by means of wave form



instantaneous power $p = vi$

$= V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$= V_m I_m \sin\omega t \sin(\omega t - \phi)$

$= \frac{1}{2} V_m I_m [\cos\phi - \cos(2\omega t - \phi)]$

The power consist of two parts :-

1. Constant part :- $\frac{1}{2} V_m I_m \cos\phi$

2. Sinosidally varying part $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$

- Frequency is twice of voltage and current
- average value over complet cycle is zero

...rage power consumed $P = \frac{1}{2} V_m I_m \cos\phi$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P = VI\cos\phi$$

Where V & I are r.m.s values.

Power Curve :-

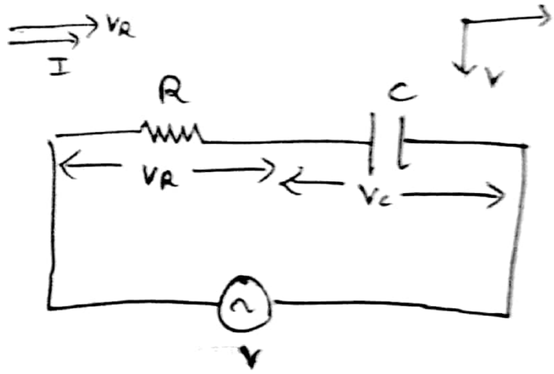
- From the curve greater part is (+)ve and smaller part is (−)ve so that the net power over the cycle is (+)ve

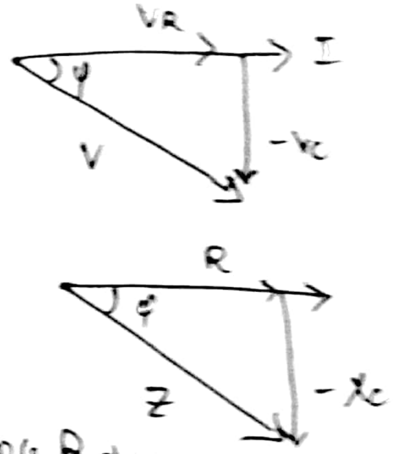
- During time interval "a to b" applied voltage & current are in opposite direction hence power is negative
Under such condition inductance L returns power to circuit

- During time interval "b to c" applied voltage & current are in same direction hence power is positive
under such condition power is put in the circuit

- in the similar way at instant "c to d" inductance L returns power to circuits and at instant d to e power is put in the circuit
- The power absorbed by resistance R is converted in the heat and Not returned.

Series R-C circuit :-





- Consider an ac circuit consist of resistance R ohms capacitance C Farady

Let V = rms value of voltage
I = rms value of current

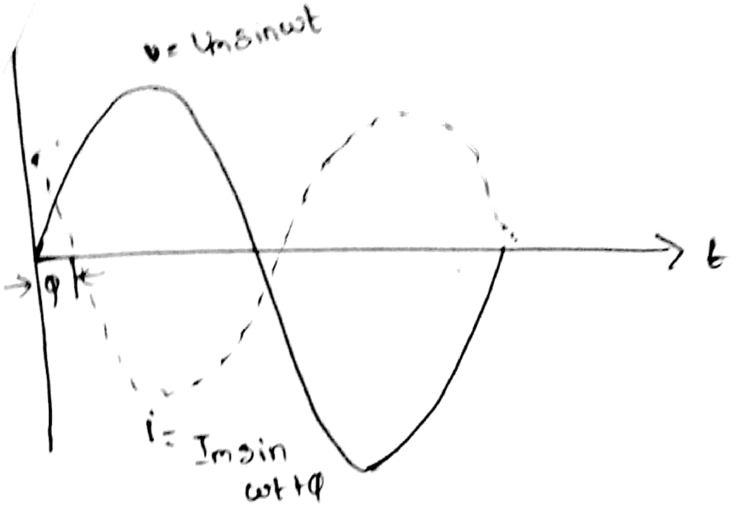
Voltage drop across resistance :- $V_R = IR$
Voltage drop across capacitor :- $V_C = IX_C$

The capactive Reactance is negative so V_C is in negative direction of Y axis which is shown in above vector diagram

We have
$$V = \sqrt{V_R^2 + (-V_c)^2}$$
$$= \sqrt{(IR)^2 + (-IX_c)^2}$$
$$V = I\sqrt{R^2 + X_c^2}$$
$$I = \frac{V}{\sqrt{R^2+X_c^2}} = \frac{V}{Z}$$

The denominator in above Eqn is impendance of circuit $Z = \sqrt{R^2+X_c^2}$ which depiles impendence triangle

Power factor $\Rightarrow \cos\phi = \frac{R}{Z}$

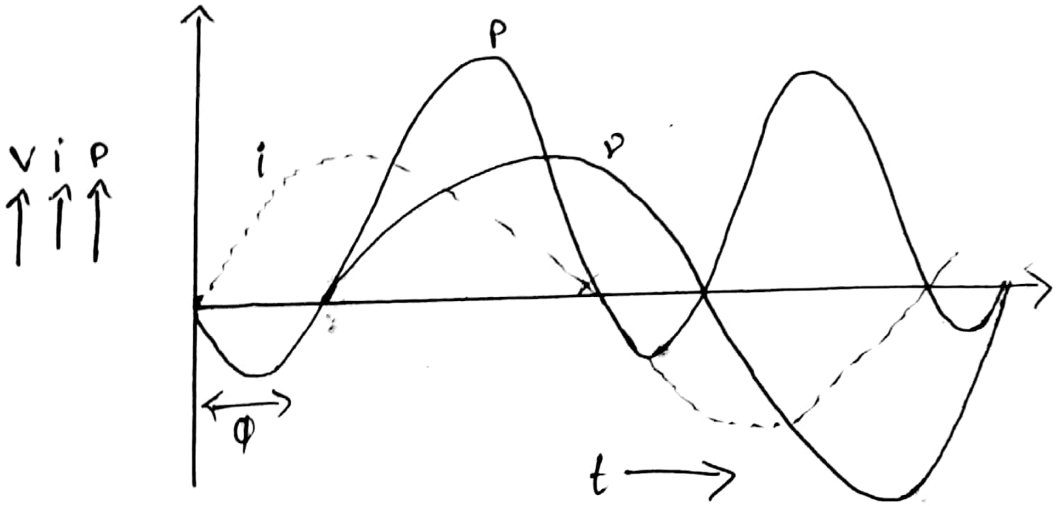


- This implies that if the alternating Voltage is $v = V_m \sin\omega t$
- The resultant alternating Current in R-C circuit is $I = I_m \sin\omega t + \phi$
- Such that Current leads the voltage by an angle ϕ

Power : Average power $P = v \times i$
$= VI\cos\phi$

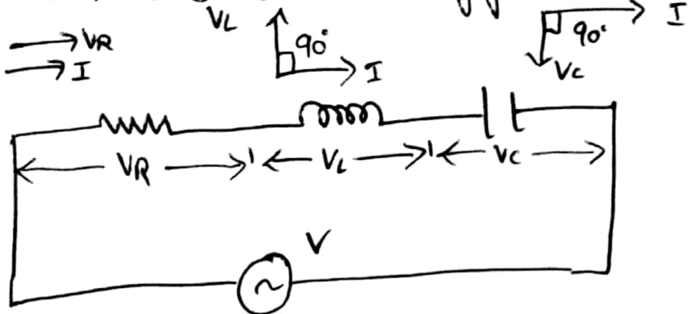
Power Curve :- The power Curve for R-C series circuit is shown in below figure

- The Curve indicate that the greater part is positive and the Smaller part is Negative
- So that net power is positive



istance, Inductance and Capacitance in Series :-

Consider an AC series circuit containing resistance R ohms, inductance L henries and Capacitance C Farads as shown in figure.



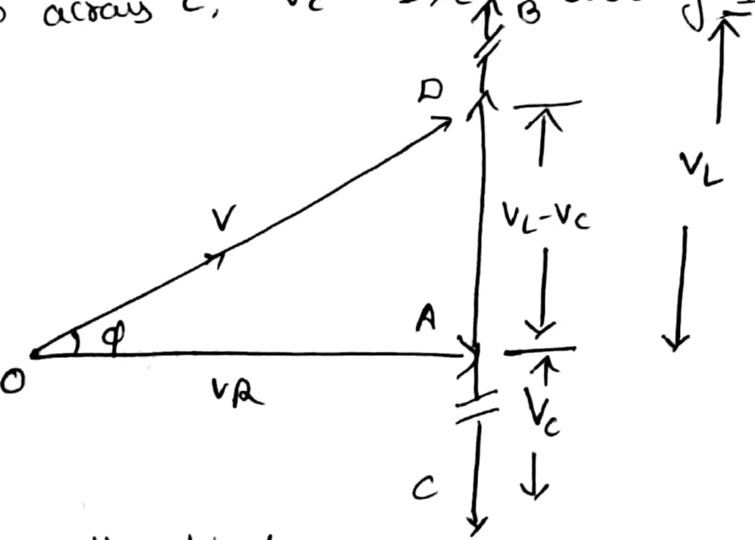
V = r.m.s value of applied voltage

I = r.m.s value of applied Current

Voltage drop across R, $V_R = IR$ "in phase with I"

Voltage drop across L, $V_L = I \cdot X_L$ "leading I by 90°"

Voltage drop across C, $V_C = IX_C$ "leading I by 90°"



in the above voltage triangle

$OA = V_R$

$AB = V_L$

$AC = V_C$

Thus the net reactive drop across the combination is

$$AD = AB - AC \qquad [\because AC = BD]$$
$$= AB - BD$$
$$= V_L - V_C$$
$$= IX_L - IX_C$$
$$= I(X_L - X_C)$$

OD represent applied voltage V
it is the vector sum of OA and AD

$$OD = \sqrt{OA^2 + AD^2}$$

$$V = \sqrt{V_R^2 + (IX_L - IX_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - X_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I = \frac{V}{\sqrt{R^2 + X^2}} = \frac{V}{Z}$$

The denominator $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X^2}$ is impendence of circuit
Where X = net reatance

- Phase angle ϕ :- $\tan^{-1}\frac{(X_L - X_C)}{R} \Rightarrow \frac{X}{R}$ $\tan^{-1}\left(\frac{X}{R}\right)$
- Power facter $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$.
- power $= VI\cos\phi$

If applied voltage is represented by a sign $V = V_m \sin\omega t$
then Resultant Current in an R-L-C Circuit is given by

$$i = I_m \sin(\omega t \pm \phi)$$

- If $X_C > X_L$; then Current leads ; ⊕ve sign should be used
- If $X_L > X_C$; then Current lags ; ⊖ve sign should be used

In any case Current leads or lags the Supply voltage by an angle ϕ
i.e $\tan\phi = \left[\frac{X}{R}\right]$.

we Employ the j operator then we have

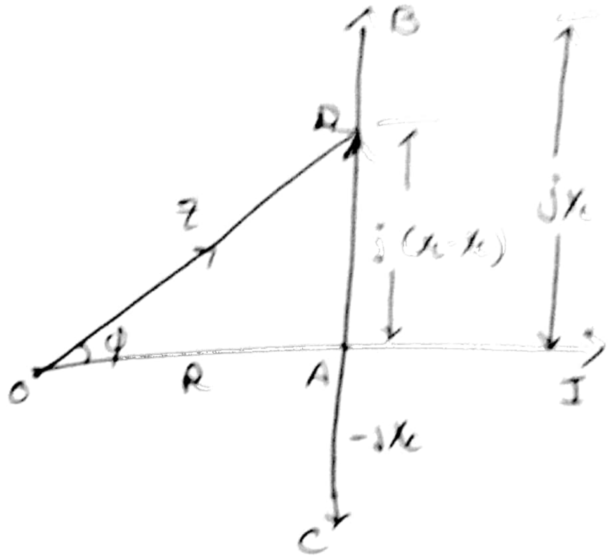
$$Z = R + j(X_L - X_C)$$

Value of impendance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase angle $\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right]$

$$Z\angle\phi = Z\angle\tan^{-1}\left[\frac{X_L - X_C}{R}\right]$$



Parallel AC Circuits :-

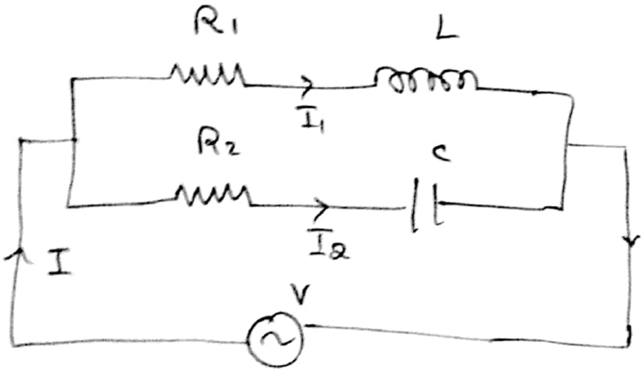
- In parallel a.c circuits voltage across each branch of circuit will be sa
- Current in each branch depends upon the branch impendence
- Since alternating currents are vector quantities.
- total line current is the vector sum of branch current.

The Following are the 3 Methods used to solve parallel AC circuits

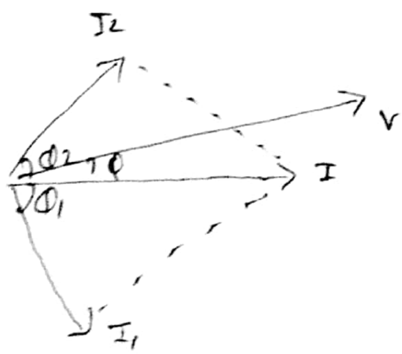
i. Vector Method
ii. Admittance Method
iii. Symbolic or j-Method

* Vector Method :-

- In this Method line current is found by drawing the vector diagram of circuit
- As voltage is common / same in parallel circuit hence it's taken as reference vector
- various branch currents are represented vectorially
- Total line current can be determined from the vector diagram either by the parallelogram Method or by Method of components



- Let us take parallel a.c circuit which consisting of two branch impendence $Z_1(R_1, L)$ and $Z(R_2, C)$ connected in parallel across alternating voltage "V" volts
- The total current draw is vector sum of branch current I_1 & I_2



Branch 1 :- Impedance $Z_1 = \sqrt{R_1^2 + X_L^2}$

Current $I_1 = \frac{V}{Z_1}$

$\cos\phi_1 = \frac{R_1}{Z_1}$ or $\phi_1 = \cos^{-1}\left[\frac{R_1}{Z_1}\right]$

Current I_1 lags behind the applied voltage by ϕ_1

Branch 2:- Impendance $Z_2 = \sqrt{R_2^2 + X_c^2}$

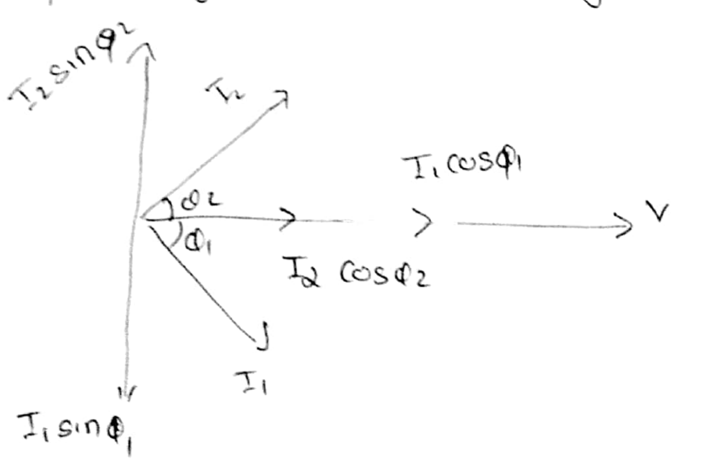
Current $I_2 = \frac{V}{Z_2}$

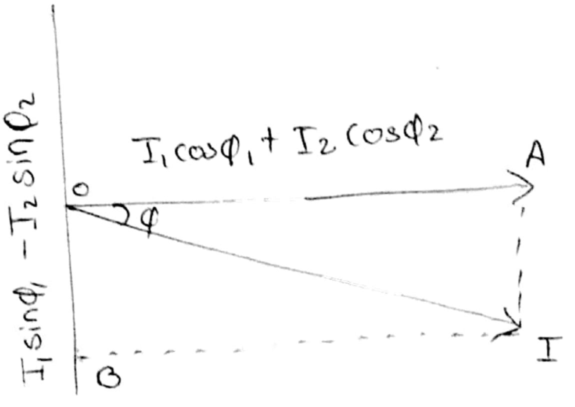
$\cos\phi_2 = \frac{R_2}{Z_2}$ or $\phi_2 = \cos^{-1}\left[\frac{R_2}{Z_2}\right]$

Current I_2 leads the applied voltage by ϕ_2

Resultant Current :- The total line current I is vector sum of branch current I_1 and I_2 which is found by law of parallelogram

- The second method is the method of components i.e.... resolving the branch current I_1 and I_2 along X axis and Y axis then finding the resultant of these components.
- Let the resultant current I and ϕ be the phase angle. Then the components of I along X-axis is equal to algebraic sum of components of branch current I_1 & I_2 along X-axis
- Illy the component of I along Y axis is equal to the algebric sum of component of I_1 and I_2 along Y axis





Component of resultant current along X axis :-

= algebric sum of I_1 and I_2 along X-axis

$$I \cos\phi = I_1 \cos\phi_1 + I_2 \cos\phi_2$$

Component of resultant current along Y-axis :-

= algebric sum of I_1 and I_2 along Y-axis

$$I \sin\phi = I_1 \sin\phi_1 \pm I_2 \sin\phi_2$$

Total Current $I = \sqrt{(I\cos\phi)^2 + (I\sin\phi)^2}$

$$= \sqrt{(I_1\cos\phi_1 + I_2\cos\phi_2)^2 + (I_1\sin\phi_1 \pm I_2\sin\phi_2)^2}$$

$$\tan\phi = \frac{I_1\sin\phi_1 - I_2\sin\phi_2}{I_1\cos\phi_1 + I_2\cos\phi_2}$$

Phase angle $\phi = \tan^{-1}\left[\frac{I_1\sin\phi_1 - I_2\sin\phi_2}{I_1\cos\phi_1 + I_2\cos\phi_2}\right]$

If $\tan\phi$ = (+)ve current leads

If $\tan\phi$ = (−)ve current lags

Power factor of entire circuit

$$\cos\phi = \frac{I_1\cos\phi_1 + I_2\cos\phi_2}{I}$$

ttance Method :-

reciprocal of impendence of a circuit it is called as admittance

it is represented by $Y = \frac{1}{Z} = \frac{I}{V}$

$$Y = \frac{r.m.s \text{ amperes}}{r.m.s \text{ Volts}}$$ unit is siemens "S"

Inductive circuit :- Capacitive circuit :-

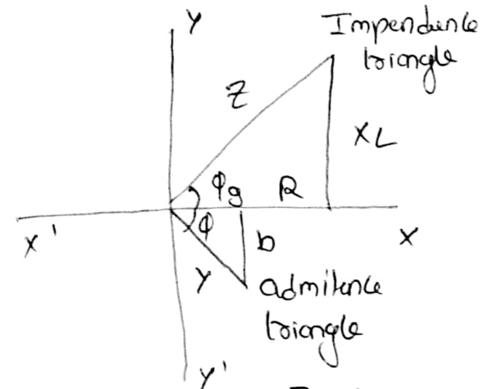


Fig a)

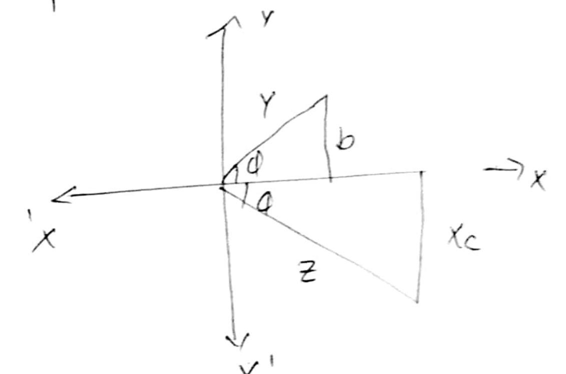


fig b)

- Impendence of circuit has two ~~rectangular~~ components
  - → Resistance "R"
  - → Reatance "X"
- admitance of circuit has two retangular components
  - → conductance "g"
  - → susceptance "b"
- In the above figure The impendence triangle and admitance triangle.
- In the Fig a impendence and admitance triangle for inductive circuit shown
- it is apparent that ~~sub~~ susceptence b is negative being below X axis hence inductive susceptence is negative
- In the Fig b impendence and admitence triangle for capactive circuit shown it is Evident that susceptance is positive, being above X axis hence capacitive susceptance in positive.

Relations :-

Condutence :- $g = Y\cos\phi$

$$g = \frac{1}{Z}\frac{R}{Z} \Rightarrow \frac{R}{Z^2} \Rightarrow \frac{R}{R^2+X^2}$$

:- Conductence is always positive

Susceptence :- $b = Y\sin\phi$

$$b = \frac{1}{Z}\frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2+X^2}$$

:- susceptence b is positive If reactence X is capacitive

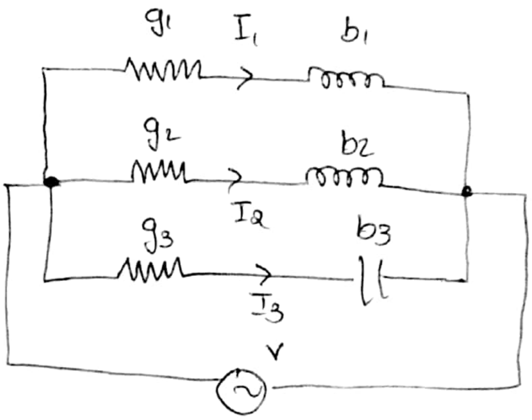
:- susceptence b is negative If reactence X is inductive

Admitence $Y = \sqrt{g^2+b^2}$

units of g, b and Y are in siemens.

Application of Admittance Method :-

- Let us consider a parallel circuit with 3 branches as shown



- We can determine conductence of just by adding the conductence of 3 branches
- In the same way susceptence is determined by algebraic addition of susceptence of diffrent branches.

Total conductence :- $G = g_1 + g_2 + g_3$

Total susceptence :- $B = (-b_1) + (-b_2) + b_3$

Total admittence $Y = \sqrt{G^2 + B^2}$

Total Current $I = VY$

Power factor $\cos\phi = \frac{G}{Y}$

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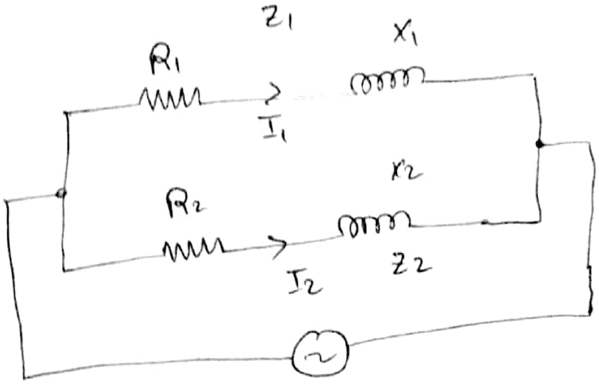
mbolic or j Method :-

sider parallel two-branch circuit shown in Figure ith some p.d across the two impendence Z_1 and Z_2



$$I_1 = \frac{V}{Z_1} \quad \text{and} \quad I_2 = \frac{V}{Z_2}$$

Total Current $I = I_1 + I_2$

$$= \frac{V}{Z_1} + \frac{V}{Z_2}$$

$$= V\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$$

$$= V\,(Y_1 + Y_2)$$

$$I = VY$$

where the total admittance $Y = Y_1 + Y_2$

Note :- admittances are added for parallel branches.
impendence are added for series branches.

Both admittance and impendence are complex quantities hence all addition have to be performed in complex form.

In the case of two parallel branches

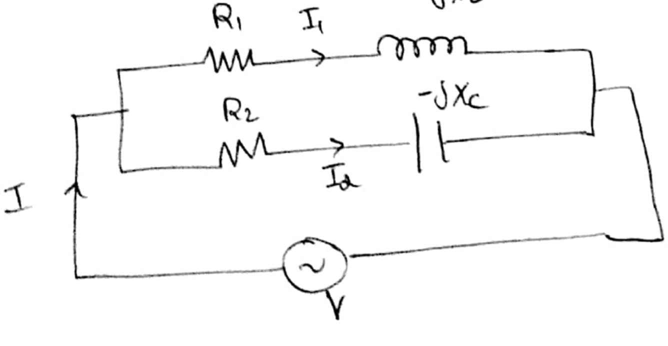
$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{R - jX_L}{(R_1 + jX_L)(R - jX_L)} = \frac{(R_1 - jX_L)}{R_1^2 + X_L^2} = \frac{R_1}{R_1^2 + X_L^2} - \frac{jX_L}{R_1^2 + X_L^2}$$

$$= g_1 - jb_1$$

where $g_1 = \frac{R_1}{R_1^2 + X_L^2} \Rightarrow$ conductance of top branch

$b_1 = \frac{X_L}{R_1^2 + X_L^2} \Rightarrow$ susceptance of top branch.

$$Y_2 = \frac{1}{Z_2} = \frac{1}{(R_2 - jX_c)} = \frac{(R_2 + jX_c)}{(R_2 + jX_c)(R_2 - jX_c)}$$



$$\Rightarrow \frac{R_2 + jX_c}{R_2^2 + X_c^2} = \frac{R_2}{R_2^2 + X_c^2} + \frac{jX_c}{R_2^2 + X_c^2}$$

$$= g_2 + jb_2$$

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:- Total admittance :- $Y = Y_1 + Y_2$

$$= (g_1 - jb_1) + (g_2 + jb_2)$$

$$= (g_1 + g_2) - j(b_1 - b_2)$$

$$Y = G - jB$$

$$Y = \sqrt{(g_1+g_2)^2 + (b_1-b_2)^2}$$

$$\phi = \tan^{-1}\left[\frac{b_1 - b_2}{g_1 - g_2}\right]$$

:- In polar form, admittance $Y = Y\angle\phi^\circ$

$$Y = \sqrt{G^2 + B^2} \angle \tan^{-1}\left(\frac{B}{G}\right)$$

:- Total current $I = VY$

$$I_1 = VY_1$$

$$I_2 = VY_2$$

$V = V\angle 0^\circ$ and $Y = Y\angle\phi$

$$I = VY$$

$$V\angle 0^\circ \times Y\angle\phi = VY\angle\phi$$

:- Considering general case :-

$$V = V\angle\alpha$$

$$Y = V\angle\beta$$

$$I = VY$$

$$= V\angle\alpha \times Y\angle\beta = VY\angle\alpha+\beta$$