4.8 Principle of operation of a D. C. Motor

Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by **Fleming's Left Hand Rule** which is as follows:

1. Hold the thumb, first finger and second finger of the left hand in such a way that they are at right angles at each other (see Fig.4.3).
2. If the forefinger represents the direction of the field and the second finger the direction of the current, then the direction of the force is indicated by the thumb.

The magnitude of this force is given by

$$F = BIl \text{ newtons}$$

where B = field strength in teslas (Wb/m^2)

I = current flowing through the conductor (Amps)

l = length of the conductor in metres

In a practical d.c. motor, the field winding produces the required magnetic field; the current carrying armature conductors are placed in this magnetic field and so experience a force. As conductors are placed in slots which are on the periphery, the individual force experienced by the conductors acts like a twisting or turning force on the armature which is called a **torque**. The torque is the product of the force and the radius at which this force acts. We shall now consider motoring action is some detail.

Consider a d.c. motor having North and South Poles, represented by *N* and *S* as shown in Fig. 4.5. Here, conductors are placed uniformly in the slots of the armature.

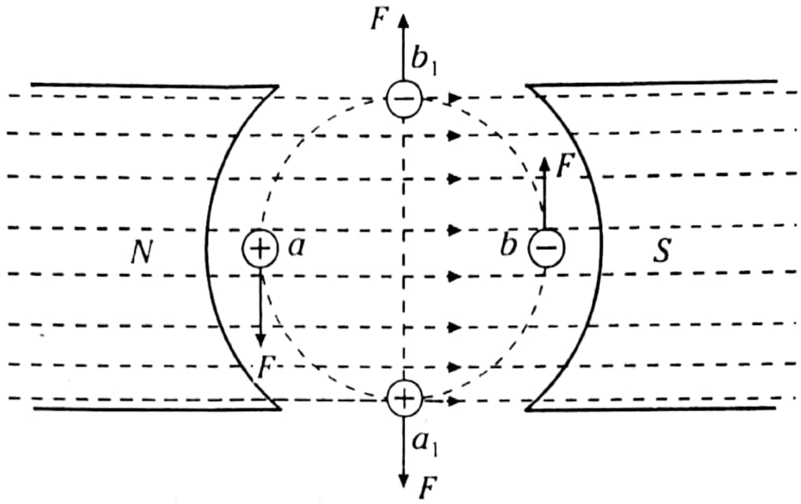


Fig.4.5

For the purpose of explaining the principle of working of *a d. c.* motor, only two conductors *a* and *b*, which come under the influence of the North and South pole respectively, are considered. These two conductors are joined together by an end connection at the rear end of the armature, and to the commutator segments at the front end of the armature. When a d.c. supply is made available at the motor terminals, current passes through the conductors *a* and *b* via the commutator. The +ve sign marked on conductor *a*, shows that the current is flowing inwards and the ve sign marked on conductor *b* shows that the current is flowing outwards. Horizontal dotted lines indicate the lines of magnetic force which originate from the North Pole *N* and terminate on the South Pole *S* as shown in Fig. 4.7.

As per Fleming's Left Hand Rule, the conductor *a* is subjected to a force *F* acting in the downward direction and the conductor *b* experiences an equal Force *F* acting in the upward direction. As the two conductors are connected together the two equal and opposite forces *F* acting on them constitute a couple, which rotates the armature in the anticlockwise direction through an angle of 90^0 and the conductor *a* and *b* occupy the positions a_1 and b_1 respectively.

In this new position, these conductors experience a force *F* in opposite directions along the *same line*, and hence the torque experienced by them is zero. Had the armature contained just these two conductors, the armature would have stopped in the position a_1, b_1. However, as the armature has several other conductors, which are uniformly distributed in the slots of the armature and which are interconnected, they experience a torque in the anticlockwise direction.

As it is necessary that the armature experiences a continuous anticlockwise torque, the direction of currents in the conductors a and b must be reversed as soon as they cross the positions a_1 and b_1 respectively. Otherwise the armature would experience a pulsating torque in the clockwise direction in the position a_1, b_1. This reversal of current in conductor a and b, after they cross the positions a_1 and b_1 respectively is brought about by the commutator, thus *making the armature experience a continuous anticlockwise torque, resulting in continuous rotation of the armature in the anticlockwise direction.*

4.9 Back E.M.F. in a D.C. Motor

As soon as the armature of a D.C. Shunt Motor (described in Sec. 4.10) starts rotating, dynamically induced e.m.f. is produced in the armature conductors. The direction of this induced e.m.f as found by Fleming's Right- Hand Rule, is such that it opposes the applied voltage (Fig. 4.6).

This induced e.m.f is known as ***Back e.m.f.*** E_b. It has the same value as that of the motionally induced e.m.f. in the generator.

So, $$E_b = \frac{\phi ZN}{60} \times \frac{P}{A} \text{ Volts}$$

The applied voltage V has to force current through the armature against this back e.m.f. The electrical work performed in over-coming this opposition is converted into mechanical energy developed in the armature.

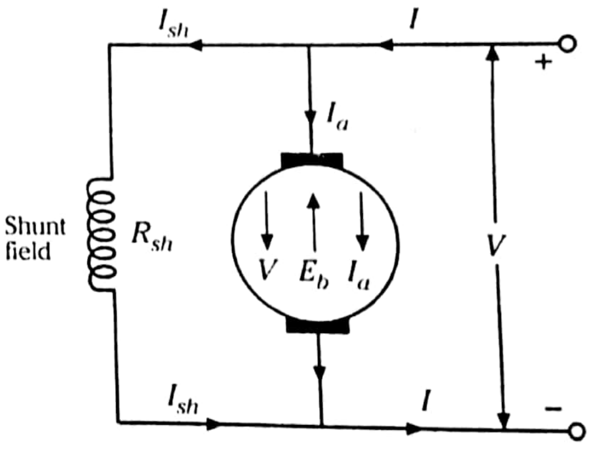


Fig.4.6

4.9.1 Value of Back E.M.F. ; Voltage & Current Relations

The back e.m.f. E_b is always less than the applied voltage V, although the difference is small when the motor is running under normal conditions.

The net voltage across the armature circuit = $V - E_b$

If armature resistance is R_a,

$$\text{Armature current}, I_a = \frac{\text{Net voltage in armature circuit}}{\text{Armature Resistance}}$$

or $$I_a = \frac{V - E_b}{R_a}$$

Since V and R_a are usually fixed, the value of E_b will determine the armature current drawn by the motor. If the speed of the motor is high, $E_b = \left(\frac{\phi ZN}{60} \times \frac{P}{A}\right)$ large and hence the motor will draw less armature current and vice-versa.

4.9.2 Significance of Back-E.M.F.

Due to the presence of back e.m.f., the d.c. motor becomes a self-regulating machine *i.e.*, the motor is made to draw as much armature current as is just sufficient to develop the torque required by the load.

When the motor runs on no load, a small amount of torque is required to overcome the friction and windage losses. So, a small amount of armature current flows and the back e.m.f is almost equal to the applied voltage.

If load is suddenly brought on to the motor, the first effect is to slow down the armature. Therefore, the speed at which the armature conductors move through the field is reduced and so there is a fall in the back e.m.f. E_b. The decreased e.m.f. allows a larger current to flow through the armature, and a larger current means an increased driving torque. Thus, the driving torque increases as the speed of the motor reduces, and the motor will stop slowing down when the armature current is just sufficient to produce the reduced torque required by the load.

Taking another case, when the load on the motors is decreased, the driving torque is momentarily in excess of the requirement, so that the armature is accelerated. As the armature speed increases, the back e.m.f. E_b also increases and causes the armature current I_a to decrease. The motor will stop accelerating when the armature current is just sufficient to produce the reduced torque required by the load.

So, we conclude that back *e.m.f. in a d.c. motor regulates the flow of armature current i.e., it automatically alters the armature current to meet the load requirement.*

4.10 Types of D.C. Motors and their Representation

Depending upon how the field winding is placed in relation to the armature, d.c. motors are of three types; Shunt Motors, Series Motors and Compound Motors.

(i) D.C. Shunt Motors

Fig. 4.7 symbolically represents a D.C. Shunt Motor, where

V = applied voltage

E_b = back e.m.f.

I_a = armature current

R_a = armature circuit resistance

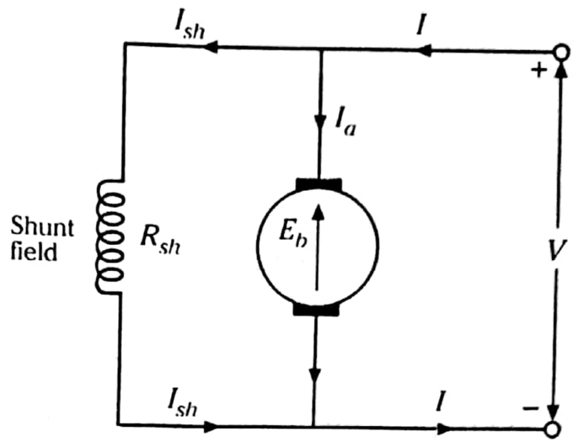


Fig.4.7

Voltage V has to

a) overcome the back e.m.f E_b and

b) supply the armature ohmic drop $I_a R_a$.

Thus, voltage equation of a motor is given by

$$V = E_b + I_a R_a \quad \text{... (i)}$$

Since the emf E_b generated in the armature of the motor is in opposition to the applied voltage V, it is called back e.m.f., which is explained in detail in the earlier Sec. 4.9.

Multiplying both L.H.S. and R.H.S. of eqn(i) by I_a, we obtain

$$VI_a = E_b I_a + I_a^2 R_a$$

VI_a = electrical input to the armature (armature input)

$E_b I_a$ = electrical equivalent of mechanical power developed by the armature *(total armature output)*

$I_a^2 R_a$ = Electrical power lost in the armature (armature copper loss).

Thus, out of the armature input, a small portion (about 5 %) is wasted as $I_a^2 R_a$ loss and the remaining portion $E_b I_a$ is converted into mechanical energy within the armature.

The motor develops mechanical power given by

$$P_m = VI_a - I_a^2 R_a$$

If we differentiate both L.H.S. and R.H.S. with respect to I_a, and equate to zero, we have

$$\frac{dP_m}{dI_a} = V - 2I_a R_a = 0$$

$V - 2 I_a R_a$

$V = 2 I_a R_a$

or

$$R_a I_a = \frac{V}{2}$$

$I_a = \frac{V}{2R_a}$

But, we have seen that $V = E_b + I_a R_a$ ... (iii)

Substituting the value of $I_a R_a$ given in eqn (ii) in eqn (iii), we obtain

$$V = E_b + \frac{V}{2} \text{ or } \mathbf{E_b + \frac{V}{2}}$$

Thus, mechanical power developed by a motor is maximum when the back e.m.f. is equal to half the applied voltage.

This is a purely theoretical condition. In practice, the current will be far greater than the normal current of the motor. Besides, half the input is wasted in the form of heat and other losses, bringing down the motor efficiency to less than 50 %.

(ii) D.C. Series Motors

In a series wound motor, the field winding is connected in series with the armature as shown in Fig.4.8 The series field winding consists of a few turns of thick wire having low resistance. It is apparent that the same current flows through both the field winding and the armature. If the mechanical load on the motor increases, the armature current also increases.

Therefore, the flux in the series motor increases with the increase in armature current and vice-versa.

From the circuit diagram of Fig. 4.10, we have

(i) $V = E_b I_a (R_a + R_{se})$

where V = Applied voltage

E_b = Back e.m.f

I_a = armature resistance

R_{se} = series field resistance

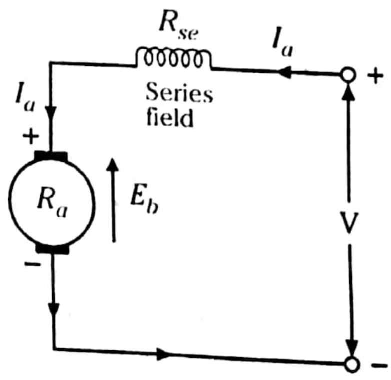


Fig.4.8

(iii) D.C. Compound Motor

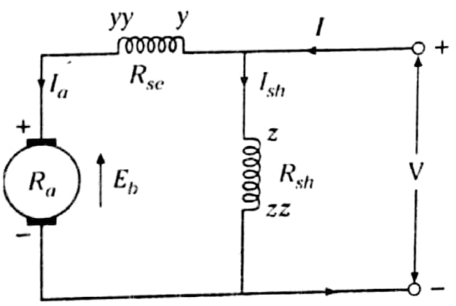
A D.C. Compound Motor has a shunt field winding and a series field winding. If the fluxes ϕ_{sh} produced by the shunt field winding and ϕ_{se} produced by the series field winding are in the same direction and are additive, then the motor is said to be *cumulatively compounded*. If the two fluxes oppose each other, then the motor is said to be *differentially compounded*. Depending on the way in which the two field windings are connected, the compound motors can be either long *shunt* or *short shunt*. The four types of D.C. compound motors are shown in Figs. 4.8(a), (b), (c) and (d).

In the case of cumulatively compounded motors, it is seen that the currents enter the positive terminals of the two field windings and hence the fluxes produced by them are in the same direction and they are additive. For the differentially compounded motors, the current through the series winding enters the negative terminal and the current through the shunt field winding enters the positive terminal. Hence the two fluxes produced are in opposite directions and hence they oppose each other.

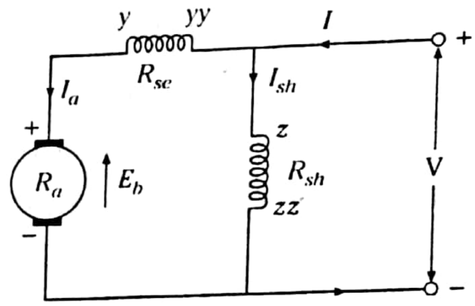
The following equations are applicable to cumulatively or differentially compounded long shunt D.C. Motor.

$$I_{sh} = \frac{V}{R_{sh}} \quad \text{and } I_a - I_{sh}$$

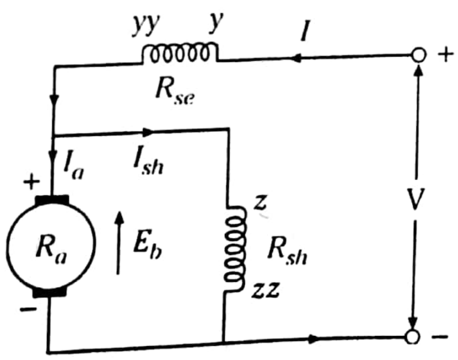
Also, $$E_b = V - I\,R_{se}\,I_a R_a.$$



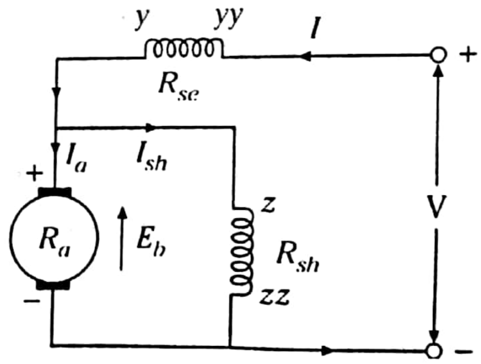
(a) Cumulatively Compounded Long Shunt D.C. Motor



(b) Differentially Compounded Long Shunt D.C. Motor



(c) Cumulatively Compounded Short Shunt D.C. motor



(d) Differentially Compounded Short Shunt D.C. motor

Fig.4.9

4.11 Production of Torque and Torque Equations

4.11.1 What is Torque?

The measure of causing the rotation of a wheel or the turning or twisting moment of a force about an axis is called the torque. It is measured by the product of force and the radius at which this force acts.

Let us take a wheel of radius R metres acted upon by a circumferential force F newtons, making it rotate at N r.p.s. (Fig. 4.11).

So, torque $T = F \times R$ newton-metres

Work done by this force in one revolution

$$= \text{Force} \times \text{distance}$$

$$= F \times 2\pi R \text{ joules}$$

Work done per second

$$W = F \times 2\pi R \times N \text{ joules/second}$$

$$= (F \times R) \times 2\pi N \text{ joules/second}$$

But $2\pi N$ = angular velocity = ω (radians/second)

Also, torque = $T = F \times R$

Work done per second, $W = T \times \omega$ joules/second

Also, power developed,

$$P = T \text{ watts}$$

where T = torque in newton-metres

ω = angular velocity in radians/second

4.11.2 Armature Torque

Let T_a = torque developed by a motor armature (N-m)

N = speed of rotation (r.p.s.)

Angular velocity, $\omega = 2\pi N$ rad/sec

Power developed, $P = T_a$

$$= T_a \, 2\pi N \text{ watts} \qquad \text{... (i)}$$

The electrical power converted into mechanical power in the armature

$$= E_b I_a \text{ watts} \qquad \text{... (ii)}$$

Equating eqns (i) and (ii),

$$T_a \times 2\pi N = E_b \, I_a \qquad \text{... (iii)}$$

But we know that

$$E_b = \phi ZN \times \frac{P}{A} \times$$

So

$$T_a = \frac{\phi Z I_a}{2\pi} \times \frac{P}{A} \times$$

or

$$T_a = \frac{\phi Z I_a}{2\pi} \times \frac{P}{A} \times \text{N-m}$$

or

$$\boldsymbol{T_a = 0.159 \phi Z I_a \times \frac{P}{A} \text{ N-m}} \qquad \text{... (iv)}$$

$$= \left(\frac{0.159}{9.81}\right) \phi Z I_a \times \frac{P}{A} \text{ kg-m}$$

$$= 0.0162 \, \phi \, Z \, I_a \frac{P}{A} \text{ kg-m}$$

Since Z, P and A are constant for a particular machine, $\boldsymbol{T_a \propto \phi I_a}$. ---(v)

Hence, torque in a d.c. motor is directly proportional to flux per pole and armature current.

a) For a series motor, flux is directly proportional to armature current I_a, prior to saturation, as full armature current flows through the field windings.

$$\therefore \qquad T_a \propto I_a^2$$

b) In the case of a shunt motor, is almost constant, so

$$T_a \propto I_a$$

From eqn (iii) we have

If N is in r.p.m., then armature torque (refer eqn (iii))

$$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}} = \frac{60 E_b I_a}{2\pi N}$$

$$= 9.55 \frac{E_b I_a}{N} \text{ N - m}$$

4.11.3 Shaft Torque

The torque which is available at the motor shaft for doing useful work is called shaft torque (T_{sh}).

The total torque T_a developed in the armature is not available at the shaft, as part of it is lost in overcoming the iron and frictional losses. Therefore, shaft torque T_{sh} is somewhat less than the total armature torque T_a.

$$\text{Output} = T_{sh} \times 2\pi N \text{ watts}$$

where T_{sh} is in *N-m*, and N is in r.p.s.

$$T_{sh} = \frac{\text{output in watts}}{2\pi N} \text{ N - m}$$

If N is in r.p.m., then

$$T_{sh} = \frac{\text{output in watts}}{\frac{2\pi N}{60}} \text{ N - m}$$

$$= \frac{60}{2\pi} \cdot \frac{\text{output in watts}}{N}$$

$$= 9.55 \frac{\text{output in watts}}{N} \text{ N - m}$$

4.12 Speed of a D.C. Motor

From the voltage equation (i) of Sec. 4.7(i), we get

$$E_b = V - I_a R_a$$

or

$$\frac{\phi Z N}{60}\left(\frac{P}{A}\right) = V - I_a R_a$$

$\therefore$

$$N = \frac{V - I_a R_a}{\phi}\left(\frac{60A}{ZP}\right) r.p.m$$

Now $\quad V - I_a R_a = E_b$

$\therefore$

$$N = \frac{E_b}{\phi} \times \left(\frac{60A}{ZP}\right)$$

As, Z, A and P are constant for a particular machine, the quantities within the brackets above can be considered as constant K.

$\therefore$

$$N = K\frac{E_b}{\phi}$$

or

$$N \propto \frac{E_b}{\phi}$$

Thus, speed is directly proportional to the back e.m.f. E_b and inversely proportional to the flux ϕ.

4.12.1 For Series Motor

Let N_1, I_{a1} and ϕ_1 be the speed, armature current and flux per pole in the first case.

Let N_2, I_{a2} and ϕ_2 be the speed, armature current and flux per pole in second case

Then, using the above relation, we get

$$N_1 \propto \frac{E_{b1}}{\phi_2}, \quad \text{where } E_{b1} = V - I_{a1}R_a$$

and

$$N_2 \propto \frac{E_{b1}}{\phi_2}, \quad \text{where } E_{b2} = V - I_{a2}R_a$$

Before saturation of the magnetic poles occurs, $\propto I_a$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

4.12.2 For Shunt Motor

In this case too, the same equation is used

i.e., $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

if $$\phi_1 = \phi_2, \quad \text{then } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

4.13 D.C. Motor Characteristics

D.C. motor characteristics depict the relationships between the following quantities :

i) **Torque and armature current or T_a/I_a Characteristic** (also called electrical characteristic).

ii) **Speed and armature current i.e., N/I_a Characteristic.**

iii) **Speed and torque or N/T_a Characteristic** (also called mechanical characteristic). This can also be ascertained from (i) and (ii) above.

4.13.1 Characteristics of Series Motors

In a series wound motor, the field winding is connected in series with the armature as shown in Fig.4.10. The series field winding consists of a few turns of thick wire having low resistance. It is apparent that the same current flows through both the field winding and the armature. If the mechanical load on the motor increases, the armature current also increases. Therefore, the flux in the series motor increases with the increase in armature current and vice-versa.

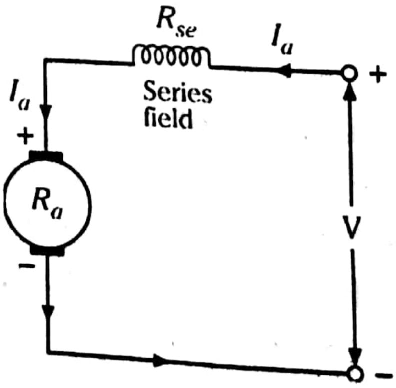


Fig. 4.10

1. T_a/I_a Characteristic:

We know that $T_a \propto \phi I_a$ (Eqn(v) of Sec 4.8.2). Also, in a series motor, as the field windings also carry armature current, $\phi \propto I_a$ till the point of magnetic saturation is reached. Thus $T_a \propto I_a^2$.

At light loads, I_a and hence ϕ, is small. But, as I_a increases, T_a increases as the square of the current in a parabolic manner, till the point of saturation A is reached (see Fig. 4.13).

After saturation, is practically independent of I_a , hence $T_a \propto I_a$, and so the characteristic becomes a straight line. (portion AB of the characteristic).

The shaft torque T_{sh} is less than the armature torque because of stray losses, a dotted curve depicting it in Fig.4.11.

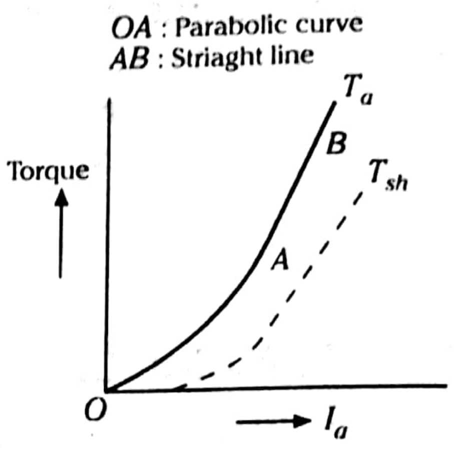


Fig. 4.11

So, we reach the conclusion that, on heavy loads, before the onset of magnetic saturation, the armature torque is proportional to the square of armature current. Therefore, if a large starting torque is required for accelerating heavy masses quickly; (e.g. in electric locomotives, hoists etc.) series motors are ideal.

2. N/I_a Characteristic:

Looking back at Sec 4.12, we know that changes in speed can be determined from the formula :

$$N \propto \frac{E_b}{\phi}$$

Variation of E_b for different load currents is so negligible that E_b may be treated as a constant. If I_a is increased, flux ϕ too increases. So, speed is inversely proportional to the armature current, as shown in Fig. 4.12.

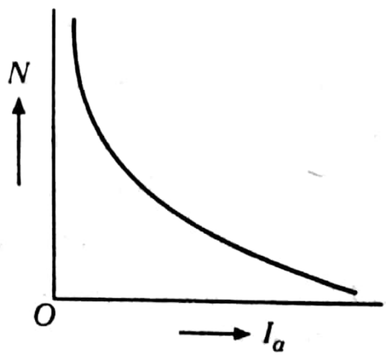


Fig. 4.12

When there is a heavy load, I_a is large. But when the load, and consequently I_a , slumps to a low value, the speed becomes dangerously high. Hence, a series motor should invariably be started with some mechanical load on it, to prevent excessive speed and damage due to the heavy centrifugal forces produced.

3. N/T_a Characteristic

The N/T_a characteristic of a series motor is shown in Fig. 4.13. From the curve, it is apparent that the series motor develops a high torque at low speed and vice-versa. This is because an increase in torque requires an increase in armature current, which is also the field current.

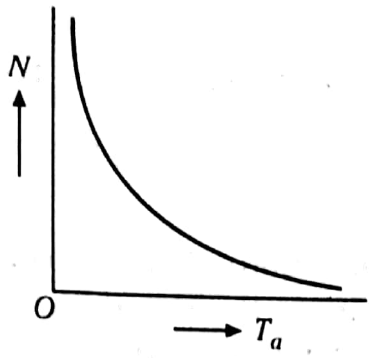


Fig. 4.13

The result is that the flux is strengthened and hence speed drops as $\left(\text{as } N \propto \frac{1}{\phi}\right)$. Similarly, at low torque, the motor speed is high.

4.13.2 Characteristics of Shunt Motors

In the shunt wound motor, the field winding is connected in parallel with the armature, as shown in Fig.4.14. The line current I divides into two parallel paths : I_{sh} flows in the shunt field circuit and I_a in the armature circuit. It is to be kept in mind that the field current is constant, since the field winding is directly connected to the supply voltage V, which is assumed to be constant.

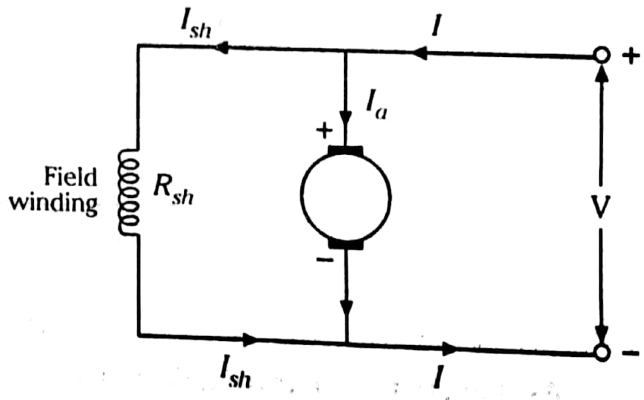


Fig. 4.14

Hence, the flux in a shunt motor is approximately constant,

1. T_a/I_a Characteristic:

As we have assumed flux ϕ to be practically constant (neglecting armature reaction), we see that $T_a \propto I_a$. This implies that this characteristic is practically a straight line through the origin. (Fig. 4.15).

The shaft torque vs armature current is also shown (dotted). It is clear from the curve that a larger armature current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

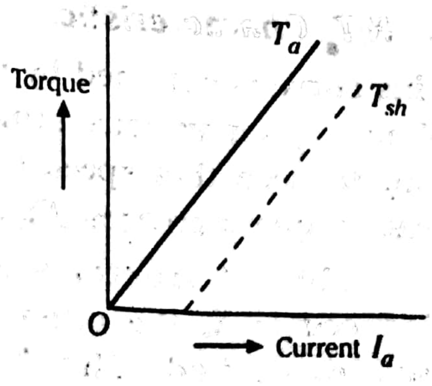


Fig. 4.15

2. N/Ia Characteristic:

We have seen that $N \propto \frac{E_b}{\phi}$. As is assumed to be constant, $N \propto E_b$. As E_b is also practically constant, the speed too is practically constant (Fig. 4.16), as indicated by dotted line *AB*.

However, to be accurate, both E_b and decrease with increasing load. But E_b decreases somewhat more than so that, all considered, there is some decrease in speed, the drop ranging from 5 to 15 % of full load, depending on certain other conditions. Thus, the actual speed curve will be somewhat drooping, as shown by line *AC*. It may be noted that the characteristic does not have a point of zero armature current, because a small current (no-load current I_o) is necessary to maintain the rotation of the motor at no-load.

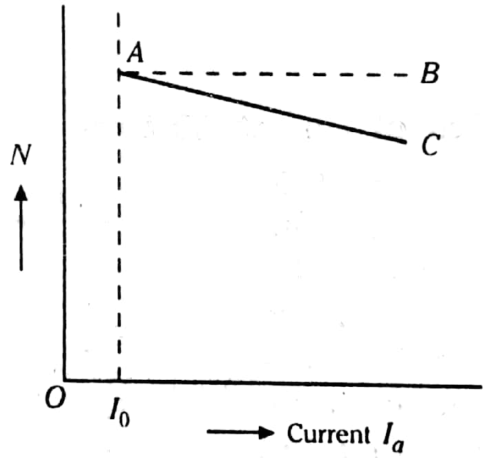


Fig. 4.16

As there is no marked change in the speed of a shunt motor, during the transition from no-load to full load, it may be connected to loads which can be suddenly disconnected without fear of excessive speeding. Because of this virtue of constant speed, shunt motors can be usefully employed for driving shafts, lathes, machine tools and other applications where an approximately constant speed is desired.

3. N/T_a Characteristic:

This curve is obtained by plotting the values of N and T_a for various armature currents I_a. It may be seen that speed falls somewhat as the load torque increases (Fig. 4.17).

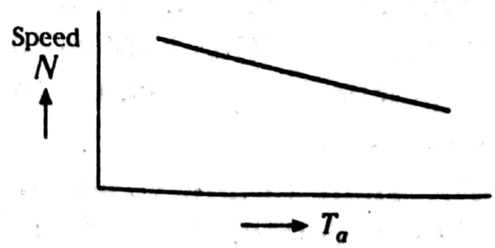


Fig. 4.17

This characteristic can be also be deduced from the other two characteristics just described. The N/T_a characteristic is of great importance in determining which type of motor is best suited to drive a given load.

4.12.2 For Shunt Motor

In this case too, the same equation is used

i.e., $$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1}{\phi_2}$$

if $$\phi_1 = \phi_2, \quad \text{then } \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

4.13 D.C. Motor Characteristics

D.C. motor characteristics depict the relationships between the following quantities :

i) **Torque and armature current or T_a/I_a Characteristic** (also called electrical characteristic).

ii) **Speed and armature current i.e., N/I_a Characteristic.**

iii) **Speed and torque or N/T_a Characteristic** (also called mechanical characteristic). This can also be ascertained from (i) and (ii) above.

4.13.1 Characteristics of Series Motors

In a series wound motor, the field winding is connected in series with the armature as shown in Fig.4.10. The series field winding consists of a few turns of thick wire having low resistance. It is apparent that the same current flows through both the field winding and the armature. If the mechanical load on the motor increases, the armature current also increases. Therefore, the flux in the series motor increases with the increase in armature current and vice-versa.

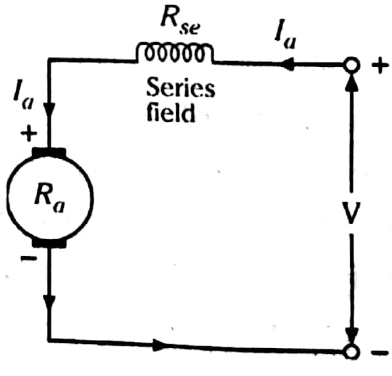


Fig. 4.10

Ta ∝ Ja φ
Ja ∝ φ

1. T_a/I_a Characteristic:

We know that $T_a \propto \phi I_a$ (Eqn(v) of Sec 4.8.2). Also, in a series motor, as the field windings also carry armature current, $\phi \propto I_a$ till the point of magnetic saturation is reached. Thus $T_a \propto I_a^2$.

Ta ∝ Ja²

At light loads, I_a and hence ϕ, is small. But, as I_a increases, T_a increases as the square of the current in a parabolic manner, till the point of saturation A is reached (see Fig. 4.13).

4.14 Applications of D.C. Motors

Some applications of the 3 types of D.C. motors, viz, shunt, series and compound motors have already been mentioned under their respective headings. However, we may summarize the applications as follows:

Shunt Motors

1. Blowers and fans.
2. Centrifugal and Reciprocating pumps.
3. Lathes.
4. Machine tools.
5. For driving constant speed line shafting.

Series Motors

1. Traction purposes : Electric Locomotives, Trolley cars, etc.
2. Hoists and Cranes
3. Conveyors.

Cumulative Compound Motors

(Also see last paragraph of Sec. 2.12.2)

1. Elevators
2. Conveyors
3. High-torque loads of intermittent nature
4. Punches
5. Shears
5. Heavy machine tools
7. Heavy planers.

P = 4
V = 250
Z = 1254