Aliah University

Department of Mathematics and Statistics

Syllabus for

4 Year B.Sc. Honours in Mathematics

2023

4 Year B.Sc. Honours in Mathematics

Programme Outcomes (PO)

At the end of the programme, the students will be able to do the following:

PO1: Apply knowledge of mathematics in different field of science, engineering and technology.

PO2: To formulate and develop mathematical arguments.

PO3: To acquire knowledge and understanding in advanced areas of mathematics, chosen by the student from the courses available.

PO4: Understand the fundamental axioms in mathematics and grow capability of developing ideas based on them.

PO5: Understand, formulate and use mathematical models arising in science, technology and other related areas.

Programme Specific Outcomes (PSO)

PSO1: Prepare and motivate students for research studies in mathematics and allied disciplines.

PSO2: Provide knowledge of a wide range of mathematical techniques and application of mathematical methods/ tools in other scientific and engineering fields.

PSO3: Provide advanced knowledge on topics in mathematics, empowering the students to pursue higher studies at academic institutions of national and international importance.

PSO4: Strong foundation on different areas of mathematics which have strong links and applications in real life.

PSO5: Nurture problem solving skills, thinking, and creativity through assignments, project and research works.

PSO6: Assist students to make them competent for competitive exams, e.g., JAM, NET, GATE, SET, etc.

Semester – I

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC1101	Differential and Integral Calculus	Major	4	4	100	In Page 8
2	MATUGMCC1102	Algebra-I	Major	4	4	100	In Page 9
3	MATUGMIN1101	Differential and Integral Calculus	Minor	4	4	100	In Page 8
4	UCCUGMDC1101	Arabic and Islamic Studies	MDC	3	4	75	
5	MATUGSEC1101	Presentation Skill and Introduction to R Programming	SEC	3	4	75	In Page 11
6	UCCUGAEC1101	Bengali/Urdu/Hindi	AEC	4	4	100	

Semester – II

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC1203	Analytical Geometry and Matrix Algebra	Major	4	4	100	In Page 12
2	MATUGMCC1204	Theory of Real Functions-I	Major	4	4	100	In Page 14
3	MATUGMIN1202	Analytical Geometry and Matrix Algebra	Minor	4	4	100	In Page 12
4	MATUGMDC1202	Discrete Mathematics	MDC	3	4	75	In Page 15
5	MATUGSEC1202	Introduction to SageMath and MATLAB	SEC	3	4	75	In Page 16
6	UCCUGVAC1201	Environmental Science	VAC	4	4	100	

Semester – III

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC2305	Odinary Differential Equations	Major	4	4	100	In Page 19
2	MATUGMCC2306	Theory of Real Functions-II	Major	4	4	100	In Page 20
3	MATUGMIN2303	Odinary Differential Equations	Minor	4	4	100	In Page 19
4	MATUGMDC2303	Linear Optimization and Game Theory	MDC	3	4	75	In Page 21
5	MATUGSEC2303	Object Oriented Programming	SEC	3	4	75	In Page 23
6	UCCUGAEC2302	English	AEC	4	4	100	

Semester – IV

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC2407	Numerical Analysis	Major	4	4	100	In Page 24
2	MATUGMCC2408	Linear Algebra	Major	4	4	100	In Page 26
3	MATUGMCC2409	Metric Spaces	Major	4	4	100	In Page 27
4	MATUGMIN2404	Numerical Analysis	Minor	4	4	100	In Page 24
5	UCCUGVAC2402	Understanding India	VAC	4	4	100	

Semester – V

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC3510	System of ODEs and Introduction to PDE	Major	4	4	100	In Page 28
2	MATUGMCC3511	Multivariate Calculus	Major	4	4	100	In Page 30
3	MATUGMCC3512	Topology	Major	4	4	100	In Page 31
4	MATUGMIN3505	System of ODEs and Introduction to PDE	Minor	4	4	100	In Page 28
5	MATUGSIP3501	Summer Internship		4		100	

Semester – VI

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC3613	Complex Analysis	Major	4	4	100	In Page 32
2	MATUGMCC3614	Algebra-II	Major	4	4	100	In Page 33
3	MATUGMCC3615	Mechanics		4		100	In Page 40
4	MATUGMIN3606	Complex Analysis	Minor	4	4	100	In Page 32
5	MATUGMDS3601	Choose any one from DSE1 Course List	DSE	4	4	100	In Page 6

Honours without Research

Semester – VII

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC4716	Field Theory and Linear Algebra	Major	4	4	100	In Page 35
2	MATUGMCC4717	Measure Theory	Major	4	4	100	In Page 36
3	MATUGMIN4707	Field Theory and Linear Algebra	Minor	4	4	100	In Page 35
4	MATUGMDS4702	Choose any one from DSE2 Course List	DSE	4	4	100	In Page 6
5	MATUGMDS4703	Choose any one from DSE3 Course List	DSE	4	4	100	In Page 6
6	MATUGMDS4704	Choose any one from DSE4 Course List	DSE	4	4	100	In Page 6

Semester – VIII

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC4818	Integral Equation, Integral Transforms and Calculus of Variations	Major	4	4	100	In Page 37
2	MATUGMCC4819	Functional Analysis	Major	4	4	100	In Page 39
3	MATUGMIN4808	Integral Equation, Integral Transforms and Calculus of Variations	Minor	4	4	100	In Page 37
4	MATUGMDS4805	Choose any one from DSE5 Course List	DSE	4	4	100	In Page 7
5	MATUGMDS4806	Choose any one from DSE6 Course List	DSE	4	4	100	In Page 7
6	MATUGMDS4807	Choose any one from DSE7 Course List	DSE	4	4	100	In Page 7

Honours with Research

Semester – VII

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC4716	Field Theory and Linear Algebra	Major	4	4	100	In Page 35
2	MATUGMCC4717	Measure Theory	Major	4	4	100	In Page 36
3	MATUGMIN4707	Field Theory and Linear Algebra	Minor	4	4	100	In Page 35
4	MATUGMDS4702	Choose any one from DSE2 Course List	DSE	4	4	100	In Page 6
5	MATUGMDS4703	Choose any one from DSE3 Course List	DSE	4	4	100	In Page 6
6	MATUGPRJ4701	Research Project		4	4	100	

Semester – VIII

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATUGMCC4818	Integral Equation, Integral Transforms and Calculus of Variations	Major	4	4	100	In Page 37
2	MATUGMCC4819	Functional Analysis	Major	4	4	100	In Page 39
3	MATUGMIN4808	Integral Equation, Integral Transforms and Calculus of Variations	Minor	4	4	100	In Page 37
4	MATUGMDS4804	Choose any one from DSE5 Course List	DSE	4	4	100	In Page 6
5	MATUGPRJ4802	Research Project		8	8	200	

DSE1 Course List

DSE2 Course List

DSE3 Course List

DSE4 Course List

DSE5 Course List

DSE6 Course List

DSE7 Course List

MATUGMCCT1101 (Differential and Integral Calculus)

Learning Objectives

The student will –

- Revise their knowledge on limit; continuity; and differentiability.
- Study some application of differential calculus.
- Study the reduction formulae for integration of some special type of functions.
- Gain knowledge on the basic theory for definite integral.
- Gain knowledge on improper integral.
- Study some application of integral calculus.

Course Outcomes

Upon successful completion of this course, the students will –

- Develop a good understanding of differential and integral calculus and solve problems.
- Understand the application of calculus in different branches, also in real life.

Course Details

Unit I: Differential Calculcus

- Definition of limit, continuity and differentiability, some basic properties with applications. [6 – 8 Lectures]
- Higher order derivatives, Successive Differentiation, Leibnitz Theorem and problems there on. [6 – 8 Lectures]
- Rolle's Theorem, Lagrange's Mean Value Theorem, Their Geometrical interpretations, Cauchy's Mean Value Theorem, Taylor's Theorem, Indeterminate form and L'Hospital's rule. [8 – 10 Lectures]
- Extremum problems, Local maxima and local minima. [4 – 6 Lectures]
- Tangents and Normals to plane curves. Convexity and concavity, Curvature, Asymptotes and Envelopes. [6 – 8 Lectures]

Unit II: Integral Calculus

- Reduction Formulae, Derivations and Illustrations of Reduction Formulae. Definite Integral as a limit of a sum, Fundamental Theorem of Integral Calculus, Few Basic Properties of Definite Integral and Related Problems. [6 – 8 Lectures]
- Parametric equations, Parameterizing a Curve, Arc Length of a Curve, Arc Length of Parametric Curves. [6 – 8 Lectures]
- Improper Integral. [6 – 8 Lectures]
- Area under a Curve and Volume of Surface of Revolution. [6 – 8 Lectures]

References

[1] G. B. Thomas and R. L. Finney, Calculus, 9th Ed., Pearson Education, Delhi, 2005.

[2] T. Apostol, Calculus, Volume I and II.

[3] H. Anton, I. Bivens and S. Davis, Calculus, John Wiley and Sons (Asia) Pvt. Ltd. 2002.

[4] M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus, 3rd Ed., Dorling Kindersley (India) P. Ltd.(Pearson Education), Delhi, 2007.

[5] R. Courant and F. John, Introduction to Calculus and Analysis (Volumes I & II), Springer- Verlag, New York, Inc., 1989.

[6] B. C. Das and B. N. Mukherjee, Differential Calculus, U. N. Dhur and Sons (P) Ltd., 2010.

[7] B. C. Das and B. N. Mukherjee, Integral Calculus, U. N. Dhur and Sons (P) Ltd., 2010.

[8] S. Bandyopadhyay and S. K. Maity, Applications of Calculus, Problems and Solutions, Academic Publishers, 2011.

[9] K. C. Maity and R. K. Ghosh, Differential Calculus: An Introduction to Analysis Part I, New Central Book Agency, 2011.

[10] K. C. Maity and R. K. Ghosh, Integral Calculus: An Introduction to Analysis Part II, New Central Book Agency, 2011.

MATUGMCC1102 (Algebra-I)

Learning Objectives

The student will –

- Get an understanding of the common inequalities.
- Investigate polynomial expressions, reciprocal equations, common root finding techniques for cubic and biquadratic equations.
- Gain knowledge on some basic theory of the subject algebra, one of the pillars of modern mathematics.
- Study certain structures called groups with suitable examples and some related structures; and homomorphism.

Course Outcomes

Upon successful completion of this course, the students will –

- Demonstrate capacity to solve problems in inequalities and theory of equations.
- Acquire necessary skills to define, construct, analyze and compare algebraic structures – groups; and substructures – subgroups.
- State and prove Isomorphism Theorems, Cayley's theorem.
- Demonstrate capacity to solve problems and communicate mathematical ideas efficiently.

Course Details

Unit I: Classical Algebra

- Inequalities : The inequality involving AM ≥ GM ≥ HM, Cauchy-Schwartz inequality. [4 – 6 Lectures]
- Theory of equations : Relation between roots and coefficients, transformation of equation, Descartes rule of signs, Sturm's theorem,
  Relation between the roots and the coefficients of equations.
  Symmetric functions, Applications of symmetric function of the roots, Transformation of equations.
  Solutions of reciprocal and binomial equations.
  Algebraic solutions of the cubic equation (solution by Cardan's method) and biquadratic equation (solution by Ferrari's method). [18 – 20 Lectures]

Unit II: Group Theory I

- Definition and examples of groups including permutation groups and quaternion groups (through matrices), elementary properties of groups. Symmetries of a square, examples of commutative and non-commutative groups. [6 – 8 Lectures]
- Subgroups and examples of subgroups, necessary and sufficient condition for a nonempty subset of a group to be a subgroup, centralizer, normalizer, center of a group, product of two subgroups.
  Properties of cyclic groups, classification of subgroups of cyclic groups.
  Cycle notation for permutations, properties of permutations, even and odd permutations, alternating group. [10 – 12 Lectures]
- Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem.
  Normal subgroups, quotient groups. [6 – 8 Lectures]
- External direct product of a finite number of groups. [2 – 4 Lectures]
- Group homomorphisms, properties of homomorphisms, Cayley's theorem, properties of isomorphisms, First, Second and Third isomorphism theorems. [4 – 6 Lectures]

References

[1] W. S. Burnside and A. W. Panton, The Theory of Equations, Dublin University Press, 1954.

[2] J. B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.

[3] M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.

[4] J. A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

[5] J. J. Rotman, An Introduction to the Theory of Groups, 4th Ed., Springer Verlag, 1995.

[6] I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.

[7] D. S. Malik, J. M. Mordeson and M. K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill, 1997.

[8] M. K. Sen, S. Ghosh, P. Mukhopadhyay, S. K. Maity, Topics in Abstract Algebra, 4th Ed., Universities Press, 2022.

[9] S. K. Mapa, Higher Algebra : Classical, 9th Ed., Sarat Book House, 2019.

[10] S. K. Mapa, Higher Algebra : Abstract and Linear, 14th Ed., Sarat Book House, 2019.

MATUGSEC1101 (Presentation Skill and Introduction to R Programming)

Learning Objectives

In this course the students will –

- Learn different parts of a scientific report and how to prepare using Microsoft word/Google docs/Latex.
- Learn how to prepare a good presentation using power point/Beamer.
- Learn how to develop presentation skills.
- Learn and practice about Writing and compiling codes in R, writing outputs in word.
- Use loops and controls in R, functions in R.

Course Outcomes

At the end of the course a student will be able to –

- Prepare report using Microsoft office word/Google docs/Latex
- Deliver presentations using power point/Beamer.
- Exercise their report writing and presentation skills.
- Write and compile codes using R.
- Create different objects in R like scalar, vector, matrix, data frame, array, and their applications.
- Write codes based on controls, loops and create their own functions.

Course Details

Unit I:

- Formats of Reports: introduction, parts of a report, cover and title page, introductory pages, text, reference section, typing instructions, copy reading, proof reading. Presentation of a report: introduction, communication dimensions, presentation package, audio-visual aids, presenter's poise.
- Application to Microsoft office word, Google docs, Microsoft power point, Latex/Rmarkdown
- Project to create a report and give some presentation. [20–24 Lectures]

Unit II:

- Introduction to R; R help; help.search(), R mailing list, contributed documentation on CRAN. Saving workspace/history. Writing programs in R markdown. Data types in R: numeric/character/logical; real/integer/complex, strings and the paste command, R as a calculator: The four basic arithmetic operations. Use of parentheses nesting up to arbitrary level. The power operation. Evaluation of simple expressions. Quotient and remainder operations for integers. Standard functions, e.g., sin, cos, exp, log. Creating a vector using $c()$, $seq()$ and colon operator. Operations on vectors such as addition, subtraction, multiplication, Functions to summarize a vector: sum, mean, sd, median etc. [10–12 Lectures]

Unit III:

- Creation of matrix and operations on matrices such as addition, subtraction, multiplication, determinant, trace, matrix inverse, Solution of linear equations, eigenvalues and eigenvectors Other R objects: data frames, lists, factor, array etc. and their creation and various operations Uses of if, if-else and nested if-else and related exercises. Loops in R and writing codes using loops, Writing own functions in R and related exercises. [10–12 Lectures]

References

[1] M. Gardener, Beginning R: The Statistical Programming Language, Wiley Publications, 2012.

[2] W. J. Braun and D. J. Murdoch, A First Course in Statistical Programming with R, Cambridge University Press, New York, 2007.

[3] A simple introduction to R by Arnab Chakraborty (freely available at http://www.isical.ac.in/ arnabc/)

[4] R for beginners by Emmanuel Paradis (freely available at https://cran.r-project.org/doc/contrib/Paradisrdebuts_en.pdf).

MATUGMCC1203 (Analytical Geometry and Matrix Algebra)

Learning Objectives

The student will –

- Gather knowledge of General equation of second degree and its canonical form.
- Understand Pole, Polar, Conjugate Diameter.
- Understand the polar equations of different types of two dimensional objects.
- Gather knowledge of different types of three objects such as Plane, Straight line, Sphere, Cone, Cylinder etc.
- Get an understanding of different types of matrices, solution of system of linear equations by Gauss elimination and matrix inversion.
- Study the basic terminology of linear algebra in real vector spaces, including linear independence, spanning, basis to define row rank, column rank.
- Gain knowledge regarding finding eigenvalues and eigenvectors of a matrix.

Course Outcomes

Upon successful completion of this course, the students will –

- Be able to apply and understand Coordinate Geometry in different branch of mathematics.
- Demonstrate capacity to solve problems on determinants, and application of matrix algebra to solve system of linear equations.
- Demonstrate understanding of the concepts of vector spaces, linear independence and basis.

Course Details

Unit I: Geometry of 2 Dimensions

- General equation of Second Degree: Transformation of Coordinates, Classification of conic, Reduction to Canonical form, Pole and Polar, Conjugate Diameter. [8 – 10 Lectures]
- Polar equations: Polar Coordinates, Changes from Cartesian Coordinates to Polar Coordinates and Vice-versa, distance between two points, area of a triangle, polar equations of straight lines, circle, and conics. Equation of the directrices, chord of contact, tangent, normal of a conic. Polar of a point with respect to a conic. [8 – 10 Lectures]

Unit II: Geometry of 3 Dimensions

- Coordinate system: Rectangular coordinates in 3-dimension, projection, direction cosines, distance of a point from a line. [2 – 4 Lectures]

- Plane: General form, Normal form, Intercept form, Reduction of the general form to normal form, Equation of plane through three points, Angle between two planes, Parallel planes, Perpendicular distance of a point from the planes, Pair of the planes, Area of a triangle and Volume of a tetrahedron. [4 – 6 Lectures]
- Straight line: Various forms of equation of a line, straight line and the planes, conditions of parallelism and perpendicularity of a line and a plane, plane through a given line, condition of intersection of two lines, shortest distance between two lines, intersection of three planes. [4 – 6 Lectures]
- Sphere: General equation of a sphere, plane section of a sphere, intersection of two spheres, sphere through a given circle, intersection of a straight line and a sphere, equation of a tangent plane to a sphere, condition of tangency, plane of contact, angle of intersection of two spheres, length of tangent, radical plane, coaxial system of spheres. [4 – 6 Lectures]
- Cone: Equation of a cone whose vertex is at origin, equation of a cone with a given vertex and a guiding curve, condition that general equation of second degree represent a cone, intersection of a cone with a plane, Intersection of a cone with a line, right circular cone. [2 – 4 Lectures]
- Cylinder: Equation of a cylinder whose axis and guiding curve are given, Enveloping cylinder, right circular cylinder. [2 – 4 Lectures]
- General Equation of Second Degree in Three Variables: Central conicoids, paraboloids, plane sections of conicoids, Generating lines, Reduction of second degree equations to normal form; classification of quadrics. [2 – 4 Lectures]

Unit III: Matrix Algebra

- A brief introduction to matrices and determinants, Elementary row/column operations, row equivalence. Elementary matrices. Row-reduced matrices, row-reduced echelon matrices. [4-6 Lectures]
- Introduction to Real Vector Spaces(Up to concept of Row and Column space), System of linear equations as matrix equations and the invariance of its solution set under row-equivalence. Row rank and using these as tests for linear dependence. The dimension of the solution space of a system of independent homogeneous linear equations. [6-8 Lectures]
- Symmetric, skew-symmetric, hermitian, skew-hermitian, orthogonal and unitary matrices. [2-4 Lectures]
- Eigenvalues and eigenvectors. Cayley-Hamilton theorem. [2-4 Lectures]

References

[1] S. L. Loney, The Elements of Coordinate Geometry, Macmillan and Company, London.

[2] P. K. Jain and Khalil Ahmad, A Text Book of Analytical Geometry, 3rd edition, New Age International, 2014.

[3] C. G. Gibson, Elementary Euclidean Geometry: An introduction, Cambridge University Press, 2003.

[4] G. B. Thomas, Jr., Ross L. Finney, Calculus and Analytic Geometry, Pearson Education, 2001.

[5] R. J. T. Bill, Elementary Treatise on Coordinate Geometry of Three Dimensions, McMillan India Ltd., 1994.

[6] Ghosh and Chakravorty, Advanced analytical Geometry.

[7] S. H. Friedberg, A. J. Insel, L. E. Spence, Linear Algebra, 5th Ed., Pearson India, 2022.

[8] S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India,1999.

[9] J. H. Kwak and S. Hong, Linear Algebra. Birkhäuser Basel.

[10] K. Hoffman, R. A. Kunze, Linear Algebra, 2nd Ed., Prentice-Hall of India Pvt. Ltd., 1971.

[11] S. Lang, Introduction to Linear Algebra, 2nd Ed., Springer, 2005.

[12] A. R. Rao and P. Bhimasankaram : Linear Algebra, Hindustan Book agency.

[13] G. Strang, Linear Algebra and its Applications, Thomson, 2007.

[14] J. E. Gentle, Matrix Algebra, Sringer.

[15] S. K. Mapa, Higher Algebra : Abstract and Linear, 14th Ed., Sarat Book House, 2019.

MATUGMCC1204 (Theory of Real Functions-I)

Learning Objectives

In this course students are expected to be able to:

- Gather knowledge of sequence and series of real numbers.
- Gain the knowledge of continuous function, uniform continuous function and Lipschitz function and the comparison between them.
- Understand the deep results on continuity and uniform continuity.
- Gain the knowledge of differentiability and some related results.
- Understand the mean value theorem and its various applications.
- Determine the Tailor's series of various type of functions.

Course Outcomes

Upon the successful completion of the course a student will:

- Acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- Demonstrate capacity to solve problems and communicate mathematical ideas efficiently.

Course Details

Unit I: Basic Properties of $\mathbb{R}$

- Review of Algebraic and Order Properties of $\mathbb{R}$, Neighborhood of a point in $\mathbb{R}$, Idea of countable sets, uncountable sets and uncountability of $\mathbb{R}$. Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Suprema and Infima, The Completeness Property of $\mathbb{R}$, The Archimedean Property, Density of Rational (and Irrational) numbers in $\mathbb{R}$. [6-8 Lectures]

Unit II: Sequence of Real Numbers

- Sequences, Bounded sequence, Convergent sequence, Limit of a sequence. Limit Theorems, Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano Weierstrass Theorem for Sequences. Cauchy sequence, Cauchy's Convergence Criterion. [16-20 Lectures]

Unit III: Series of Real Numbers

- Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, Ratio Test, Cauchy's nth root test, Integral test, Alternating series, Leibniz test, Absolute and Conditional convergence. [10-12 Lectures]

Unit IV: Some Deeper Properties of Continuity

- Continuous functions, sequential criterion for continuity and discontinuity, Continuous functions on an interval, Boundedness theorem, Intermediate value theorem, Location of roots theorem, Preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem. [16-20 Lectures]

Unit V: Some Deeper Properties of Differentiability

- Differentiability of a function at a point and in an interval, Caratheodory's theorem, Intermediate value property of derivatives, Darboux's theorem. Applications of mean value theorem, Cauchy's mean value theorem, Taylor's theorem with Lagrange's form of remainder, Taylor's theorem with Cauchy's form of remainder, Application of Taylor's theorem, Taylor's series and Maclaurin's series expansions of some well known functions.[16-20 Lectures]

References

[1] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.

[2] K. A. Ross, Elementary Analysis: The Theory of Calculus, Springer, 2004.

[3] A. Mattuck, Introduction to Analysis, Prentice Hall, 1999.

[4] S. R. Ghorpade and B.V. Limaye, A Course in Calculus and Real Analysis, Springer, 2006.

[5] S. Shirali, H. L. Vasudeva, An Introduction to Mathematical Analysis, Alpha Science International Ltd, 2013.

[6] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill Education

[7] T. Apostol, Mathematical Analysis, Willey

MATUGMDC1202 (Discrete Mathematics)

Learning Objectives

The student will –

- be familiar with the philosophy of basic mathematical concepts and grammar like undefined terms, definitions, axioms and theorems etc.
- develop fluency in using basic logic.
- learn the basics of number theory, graph theory, combinatorics and Boolean Algebra.

Course Outcomes

Upon successful completion of this course, the students will –

- gain confidence towards dealing with natural language arguments by means of symbolic propositional logic.
- be able to construct truth tables and hence check the validity of statements.
- develop the skills required to study mathematics.
- understand what is meant by a proof and different methods of proof.
- be able to read, understand, assimilate and write down a proof.
- learn three basic Boolean algebra operations: or, and, complementation.
- be able to write down Boolean expression for logic gates and various logic circuit gates.
- be able to apply the knowledge of mathematical logic to validate an existing proof of a theorem or to provide an alternative proof of a theorem in any branch of mathematics.
- be able to apply the knowledge of discrete mathematical structures in some other courses like Graph Theory, Data Structure, Design and Analysis of Algorithms, Complexity Analysis, etc.

Course Details

Unit I: Logic and Proofs
Propositional logic, propositional equivalences, predicates and quantifiers, nested quantifiers, rules of inference, introduction to proofs, proof methods and strategy.

[15 – 18 Lectures]

Unit II: Boolean Algebra
Boolean functions, representing Boolean functions, logic gates, Normal forms, Karnaugh maps, minimization of circuits. Relations, Partial and linear orderings, Transitive closure, Chains and anti-chains, Lattices, Distributive lattices, Complementation, Topological sorting.

[20 – 22 Lectures]

Unit III: Counting
Basics of counting, pigeonhole principle, permutations and combinations, binomial coefficients, recurrence relations, solving linear recurrence relations, generating functions, principle of inclusion-exclusion. [6 – 8 Lectures]

Unit IV: Number System
Binary, octal and hexadecimal number system and their arithmetic, 1's complement, 2's complement. [4 – 6 Lectures]

References

[1] Kenneth H. Rosen, Discrete Mathematics and Its Applications, 6th edition, McGraw-Hill Education, 2007.

[2] Kolman, Busby, Ross, Discrete Mathematical Structures, 6th Edition, Pearson Education, 2015.

[3] Ralph P. Grimaldi, Discrete and Combinatorial Mathematics, 5th Edition, Pearson, 2003.

[4] Gary Chartrand, Ping Zhang, Discrete Mathematics, Waveland Pr Inc., 2011.

[5] V.K. Balakrishnan, Introductory Discrete Mathematics, Dover Publications, 2010.

[6] E.G. Goodaire, M.M. Parmenter, Discrete Mathematics with Graph Theory, Pearson Education Pvt. Ltd., Singapore, Indian Reprint, 2003.D. S. Malik, J. N. Mordeson, M. K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill International Editions, 1997.

MATUGSEC1202 (Introduction to SageMath and MATLAB)

Learning Objectives

The student will –

- be familiar with mathematical software like MATLAB and SageMath.
- be able to carry out mathematical computations using softwares.
- visualize mathematical concepts with the help of numerical examples.

Course Outcomes

At the end of the course a student will be able to

- be a able to solve algebraic and trancendental equations using MATLAB or SageMath.
- find out the determinant, eigenvalues and eigenvectors of matrices.
- plot given curves and surfaces.
- compute graph parameters and plot simple graphs.
- solve ODE and PDEs.

Course Details

(Students can choose Module 1 and Module 3 OR Module 2, Module 3 and Module 4 OR Module 2, Module 3 and Module 5)

Module I: Computational Mathematics Using SageMath

- Introduction: Exploring Integers, Solving Equations, 2d Plotting, 3d Plotting. [6 Lectures]
- Calculus: Calculus of One Variable, Applications of Derivatives, Integrals, Applications of Integrals, Partial Derivatives and Gradients, Jacobians, Local Maximum-Minimum. [18 Lectures]
- Linear Algebra: Working with Vectors, Solving System of Linear Equations, Vector Spaces, Linear Transformations, Eigenvalues and Eigenvectors. [18 Lectures]
- Graph Theory: Common Graphs, Basic Graph Operations,Graph Products, Paths and Cycles,Graph Properties (whether Eulerian, Hamiltonian, Regular, Planar, Bipartite, etc.), Spanning Trees, Graph Colouring, Independent Sets, Plotting of Graphs. [24 Lectures]

Module II:Introduction to MATLAB

- General Information, Data Types and Variables, Operators, Flow Control, Functions, Input/Output, Array Manipulation, Writing and Running Programs, Plotting. [21 Lectures]

Module III:Numerical Analysis

- Numerical Solution of Algebraic and transcendental Equations, Numerical Solutions of System of Linear Equations, Interpolations and curve fitting, Numerical Integration, Numerical Eigenvalues and eigenvectors, Numerical Solution of ODE: Initial Value Problems and Boundary Value Problems, Numerical Solution of Partial Differential Equations. [20 Lectures]

Module IV:Numerical Simulation in Dynamical Systems

- Determination of equilibrium positions by plotting nullclines of planner dynamical systems in a bounded region $E \subset \mathbb{R}^2$. Direction field and stability of limit cycles associated with Lienard systems. Phase diagrams of the chaotic dynamics of the Rössler system, Lorenz system and Chua's Circuit. [21 Lectures]

Module V:Numerical Inversion of the Laplace Transform

- Bellman Method, Gaver-Steh fest method, Talbots Method. [21 Lectures]

References

[1] Jaan Kiusalaas, Numerical Methods in engineering with MATLAB, Cambridge University Press, New York, 2005.

[2] Won Young Yang,Wenwu Cao, Tae-Sang Chung, John Morris, Applied Numerical Methods Using MATLAB, John Wiley & Sons, Inc., Hoboken, New Jersey, 2005.

[3] John HMathews, Kurtis D Fink, Numerical Methods using MATLAB, Prentice Hall, Upper Saddle River, 1999.

[4] Steven T. Karris, Numerical Analysis Using MATLAB and Spread sheets, Orchard Publications, Fremont, California.

[5] Dean G. Duffy, Advanced Engineering Mathematics with MATLAB, CRC Press, Boca Raton, 2017.

[6] Stormy Attaway,MATLAB: A Practical Introduction to Programming and ProblemSolving, Butterworth-Heinemann, New York, 2009.

[7] S.R. Otto, J.P. Denier, An Introduction to Programming and Numerical Methods in MATLAB, Springer-Verlag London Limited, 2005.

[8] Stephen Lynch, Dynamical Systems with Applications Using MATLAB, Birkhausar, 2014.

[9] Paul Zimmermann,Mathematical Computation with Sage, available from http://www.sagemath.org.

[10] Razvan AMezei, An Introduction to SAGE Programming: With Applications to SAGE Interacts for Numerical Methods, Wiley, 2016.

MATUGMCC2305 (Ordinary Differential Equations)

Learning Objectives

The student will –

- revise their knowledge on solving ordinary differential equations.
- be able to classify differential equations.
- solve analytically a wide range of ordinary differential equations (ODEs).
- understand that physical systems can be described by differential equations.
- understand the practical importance of solving differential equations.
- be able solve differential equations by using computational softwares (Matlab/Mathematica)

Course Outcomes

Upon successful completion of this course, the students will –

- gain a solid grasp of the methods used to solve differential equations.
- possess an understanding of how ordinary differential equations are used to describe mechanical and physical systems.

Course Details

Unit I:

- Classification of ordinary differential equations: Significance of differential equation, Formation of differential equation by elimination of arbitrary constants. Differential equation of first order and first degree. Exactness of the differential equation, Integrating factors, Determination of integrating factors. Separable, homogeneous and non-homogeneous equation. Bernoulli's equation. [14 – 16 Lectures]
- ODEs of first order but of higher degree solvable for x, y, p: Clairaut's equation, Singular solutions and Extraneous Loci. Applications: Orthogonal and oblique trajectories, Problems in mechanics, Rate problems. [10 – 12 Lectures]

Unit II:

- Linear higher order differential equations with constant coefficients. Complementary function, Particular integral: Method of undetermined coefficients, Method of variation of parameters. Homogeneous and non-homogeneous linear differential equations and solution by Euler method. Change of independent variables to make it an ODE with constant coefficients. [12 – 16 Lectures]
- Exactness of higher order differential equation: Criterion of an exact differential equation of higher order differential equation. Integrating factors for linear but not exact differential equation. Some special forms. [6 – 8 Lectures]

Unit III:

- Solution of linear differential equations of second order. Properties and applications of Wronskian. Second order linear differential equation with variable coefficients: Reduction of order when one solution of the homogeneous part is given. Reduction to Normal form. Change of dependent, independent variables. Operational factors. [12 – 16 Lectures]
- Sturm-Liouville problems: Simple eigenvalue problems. Applications. Equilibrium points, Interpretation of the phase plane, predatory-prey model and its analysis, epidemic model of influenza and its analysis, battle model and its analysis. [12 – 16 Lectures]

Unit IV: (List of Labs)

- Plotting of solutions of a family differential equations.
- Limited growth of population (with and without harvesting).
- Basic Lotka-Volterra model, with density dependence, effect of DDT.
- Battle model (basic battle model, jungle warfare).
- Epidemic model of influenza disease model with carriers. [10 – 12 Lectures]

References

[1] G. F. Simmons, Differential Equations.

[2] S. L. Ross, Differential Equation, 3rd Edition, John Wiley & Sons.

[3] E. A. Codington and Levinson, Theory of Ordinary Differential Equations.

[4] M. L. Abell and J.P. Braselton, Differential Equations with MATHEMATICA, 3rd Ed., Elsevier Academic Press, 2004.

[5] B. Barnes and G. R. Fulford, Mathematical modeling with case studies, A Differential Equation Approach using Maple and Matlab, 2nd Ed., Taylor and Francis group, London and New York, 2009.

[6] R. P. Agarwal and R. C. Gupta, Essentials of Ordinary Differential Equations.

MATUGMCC2306 (Theory of Real Functions-II)

Learning Objectives

In this course students are expected to be able to:

- Gather knowledge of Riemann Integration.
- Gather knowledge of sequence and series of real valued functions.
- Gain knowledge of power series.

Course Outcomes

Upon the successful completion of the course a student will:

- Acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- Demonstrate capacity to solve problems and communicate mathematical ideas efficiently.

Course Details

Unit I: Sequence and Series of Real Valued Functions

- Pointwise and uniform convergence of sequence of functions. Theorems on continuity, derivability and integrability of the limit function of a sequence of functions. Series of functions; Theorems on the continuity and derivability of the sum function of a series of functions; Cauchy criterion for uniform convergence and Weierstrass M-Test. [26-30 Lectures]

Unit II: Power Series

- Limit superior and Limit inferior. Power series, radius of convergence, Cauchy Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass Approximation Theorem. [10-12 Lectures]

Unit III: Functions of Bounded Variations
Unit IV: Riemann Integration

- Riemann integration; inequalities of upper and lower sums; Riemann conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums; equivalence of two definitions; Riemann integrability of monotone and continuous functions, Properties of the Riemann integral; definition and integrability of piecewise continuous and monotone functions. Intermediate Value theorem for Integrals; Fundamental theorems of Calculus. [26-30 Lectures]

References

[1] R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, 3rd Ed., John Wiley and Sons (Asia) Pvt. Ltd., Singapore, 2002.

[2] K. A. Ross, Elementary Analysis: The Theory of Calculus, Springer, 2004.

[3] Charles G. Denlinger, Elements of Real Analysis, Jones & Bartlett (Student Edition), 2011

[4] I. K. Rana, An Introduction to Measure and Integration, Narosa, 2007.

[5] S. C. Malik, S. Arora, Mathematical Analysis, 5th Ed., New Age International, 2018.

[6] W. Rudin, Principles of Mathematical Analysis, McGraw-Hill Education

MATUGMDC2303 (Linear Optimization and Game Theory)

Learning Objectives

- Theoretical foundation and computational procedures of linear programming problems will be covered in this course. Transportation problems and assignment problems will also be discussed. Games and basic terms related to game theory such as strategy, pure strategy, mixed strategy, saddle point, value of the game, etc. will be introduced.

Course Outcomes

Upon successful completion of this course students will be able to –

- Formulate a given real-world problem as a linear programming model in general, standard and canonical forms.
- Reduce a given feasible solution to a basic feasible solution.
- Use the simplex method and its variants to solve small linear programming models by hand.
- Understand the theories behind linear optimization problems.
- Model and solve conflicting real life problems as game.

Course Details

Unit I:

- Introduction to linear programming problem, graphical solution of L.P.P., Basic Solutions and Basic Feasible Solution (B.F.S) with reference to L.P.P., Degenerate and Non-degenerate B.F.S., convex sets, extreme points, convex hull and convex polyhedron, optimality and unboundedness, Reduction of a F.S. to a B.F.S., the simplex algorithm, simplex method in tableau format, introduction to artificial variables, two-phase method. Big-M method and their comparison. [16-20 Lectures]

Unit II:

- Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual, Dual Simplex method. Transportation problem and its mathematical formulation, northwest-corner method, least cost method and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, assignment problem and its mathematical formulation, Hungarian method for solving assignment problem, Travelling salesman problem. [16-20 Lectures]

Unit III:

- Game theory: Formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games. [16-20 Lectures].

References

[1] Mokhtar S. Bazaraa, John J. Jarvis and Hanif D. Sherali, Linear Programming and Network Flows, 2nd Ed., John Wiley and Sons, India, 2004.

[2] F.S. Hillier and G.J. Lieberman, Introduction to Operations Research, 9th Ed., Tata McGraw Hill, Singapore, 2009.

[3] Hamdy A. Taha, Operations Research, An Introduction, 8th Ed., Prentice-Hall India, 2006.

[4] G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 2002.

[5] K. Swarup, P. K. Gupta and M. Mohan, Operations Research, Sultan Chand & Sons, 1978.

MATUGSEC2303 (Object Oriented Programming)

Learning Objectives

- Object Oriented Programming with C++ will be taught in this course. Classes and objects will be introduced. Fundamental programming concepts and methodologies to write good C++ programmes will be discussed.

Course Outcomes

Upon successful completion of this course students will be able to –

- Demonstrate basic object oriented and structured programming concepts.
- Implement different functions for input and output, various data types, basic operators, files and functions.
- Implement programming techniques to solve problems in the C++ programming language.
- Apply the concepts and principles of the programming language to the real-world problems and solve the problems through project-based learning.

Course Details

Unit I:

- Introduction to C++, History of C++, Structure of C++, Basic data types, Derived data types, Symbolic constants. [2 Lectures]
- Dynamic initialization, Type modifiers, Type Casting, Operator and control statements, Input and Output statements in C++. [6 Lectures]

Unit II:

- Classes and objects, class specification, member function specification, scope resolution operator, Access qualifiers, Instance creation, Member functions. [6 Lectures]
- Function prototyping, Function components, Passing parameters, call by reference, Return by reference, Inline functions, Default arguments, Overloaded function. [6 Lectures]
- Array of objects, pointers to objects, this pointer, Dynamic allocation operators, Dynamic objects. [6 Lectures]

Unit III:

- Constructors, parameterized constructors, Overloaded constructors, Constructors with default arguments, copy constructors, static class members and static objects. [6 Lectures]
- Operator Overloading, Overloading unary and binary operator, Overloading the operator using friend function, stream operator overloading, data conversion. [6 Lectures]

Unit IV:

- Inheritance, Defining derived classes, Single inheritance, protected data with private inheritance, multiple inheritance, multi level inheritance, hierarchical inheritance, hybrid inheritance, multipath inheritance, Constructors in derived and base class, Abstract classes, virtual function and dynamic polymorphism, virtual destructor. [12 Lectures]

Unit V:

- Exception Handling, principle of Exception handling, Exception handling mechanism, multiple catch, Nested try, Rethrowing the exception. [4 Lectures]

Unit VI:

- Streams in C++, Stream classes, Formatted and Unformatted data, manipulators, User defined manipulators, file streams, file pointer manipulation, file open and close. [4 Lectures]
- Templates, Template functions and Template classes. [2 Lectures]

References

[1] BJarne Stroustrup, "The C++ Programming Language", Addison Wesley, 2004.

[2] Stanley B Lippman, "The C++ Primer", Addison Wesley, 2005.

[3] Grady Booch, James Rumbaugh, Ivar Jacobson, "Unified Modeling Language User Guide", Addison-Wesley Professional, 1998.

[4] Craig Larman, "Applying UML and Patterns : An Introduction to Object-Oriented Analysis and Design and Iterative Development", Prentice Hall Edition, 2004.

[5] E. Balagurusamy, Object-Oriented Programming with C++, McGraw Hill, 2017.

[6] Y. P. Kanetkar, Let Us C++, BPB Publications, 2003.

MATUGMCC2407 (Numerical Analysis)

Learning Objectives

The student will –

- Familiarize themselves with numerical methods based upon sound computational mathematics. Such methods include techniques for simple optimization, interpolation from the known to the unknown, linear algebra underlying systems of equations, ordinary differential equations to simulate systems, etc.

Course Outcomes

Upon successful completion of this course, the students will –

- Demonstrate understanding of common numerical methods and how they are used to obtain approximate solutions to otherwise intractable mathematical problems. Apply numerical methods to obtain approximate solutions to mathematical problems. Derive numerical methods for various mathematical operations and tasks, such as interpolation, differentiation, integration, the solution of linear and nonlinear equations, and the solution of differential equations. Analyse and evaluate the accuracy of common numerical methods

Course Details

Unit I:

- Representation of real numbers: Machine Numbers - floating point and fixed point. Sources of Errors, Rounding of numbers, significant digits and Error Propagation in machine arithmetic operations. Numerical Algorithms - stability and convergence.
- Interpolation : Lagrange and Newton's methods. Error bounds. Finite difference operators. Newton (Gregory) forward and backward difference interpolation.
- Approximation : Types of approximations- polynomial approximation, The Weierstrass polynomial approximation theorem (statement only). [8–10 Lectures]

Unit II:

- Numerical solution of Transcendental and polynomial equations : Bisection method, Secant method, Regula-falsi method, fixed point iteration, Newton-Raphson method.
- System of linear algebraic equations : Direct methods - Gaussian elimination and Gauss Jordan methods, Pivoting strategies. Iterative methods- Gauss Jacobi method, Gauss Seidel method. Matrix inversion- Gaussian elimination method. [8–10 Lectures]

Unit III:

- Numerical differentiation : Methods based on interpolations, methods based on finite differences. Numerical Integration : Newton Cotes formula, Trapezoidal rule, Simpson's 1/3-rd rule, Simpson's 3/8 - th rule, Weddle's rule. Composite trapezoidal rule, composite Simpson's 1/3-rd rule, composite Weddle's rule.
- Ordinary differential equations: The method of successive approximations (Picard), Euler's method, the modified Euler method, Runge-Kutta methods of orders two and four. [8-10 Lectures]

Numerical Analysis Lab using $C/C++$/ FORTRAN 90
List of practical using $C/C++$/ FORTRAN 90

- Calculate the sum $1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{N}$
- Enter 100 integers into an array and sort them in an ascending order.
- Solution of transcendental and algebraic equations by
  1. Bisection method
  2. Newton Raphson method.
  3. Secant method.
  4. Regula Falsi method.
- Solution of system of linear equations
  1. Gaussian elimination method
  2. Gauss-Jacobi method
  3. Gauss-Seidel method
- Interpolation
  1. Lagrange Interpolation
  2. Newton's forward, backward and divided difference interpolations
- Numerical Integration
  1. Trapezoidal Rule
  2. Simpson's one third rule
  3. Weddle's Rule
- Solution of ordinary differential equations
  1. Euler method
  2. Modified Euler method
  3. Runge Kutta method (order 4)

[24-30 Lab Classes]

References

[1] Brian Bradie, A Friendly Introduction to Numerical Analysis, Pearson Education, India, 2007.

[2] M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computation, 6th Ed., New age International Publisher, India, 2007.

[3] A. Gupta & S. C. Bose, Introduction to Numerical Analysis, Academic Publishers, Kolkata

[4] C. F. Gerald and P. O. Wheatley, Applied Numerical Analysis, Pearson Education, India, 2008.

[5] Uri M. Ascher and Chen Greif, A First Course in Numerical Methods, 7th Ed., PHI Learning Private Limited, 2013.

[6] John H. Mathews and Kurtis D. Fink, Numerical Methods using Matlab, 4th Ed., PHI Learning Private Limited, 2012.

[7] Scarborough, James B., Numerical Mathematical Analysis, Oxford and IBH publishing co.

[8] Atkinson, K. E., An Introduction to Numerical Analysis, John Wiley and Sons, 1978.

[9] Yashavant Kanetkar, Let Us C, BPB Publications.

[10] Balaguruswamy, E., Numerical Methods

MATUGMCC2408 (Linear Algebra)

Learning Objectives

The student will –

- Understand important concepts of vector spaces such as independence, basis, dimension.
- Study how to use characteristics of a matrix to solve a linear system of equations or study properties of a linear transformation.
- Gain knowledge of dual spaces and diagonalizability.
- Investigate inner product spaces and orthogonality.

Course Outcomes

Upon successful completion of this course, the students will –

- Develop capacity to find the dimension and basis of a a given vector space.
- Demostrate an understanding to write down the matrix representing a linear transformation (such as projection, rotation, etc.) under a given basis, and determine how the matrix changes if the basis is changed.
- Test for independence of vectors.
- Be able to find the Gram-Schmidt orthogonalization of a matrix.
- Gain an insight to perform diagonalization of a matrix.

Course Details

Unit I

- Vector spaces, subspaces, algebra of subspaces, quotient spaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces. [10-12 Lectures]
- Linear transformations, null space, range space, rank-nullity theorem. Matrix representation of a linear transformation with respect to an ordered basis, Algebra of linear transformations. Isomorphisms, Isomorphism theorems, invertibility and isomorphisms, Change of coordinate matrix. [18-20 Lectures]

Unit II

Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators, Eigen spaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator. [20-22 Lectures]

Unit III

Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel's inequality, Quadratic forms, the adjoint of a linear operator, Least Squares Approximation, minimal solutions to systems of linear equations, Normal and self-adjoint operators, Orthogonal projections and Spectral theorem. [20-22 Lectures]

References

[1] Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence, Linear Algebra, 5th Ed., Pearson India, 2022.

[2] S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India,1999.

[3] Jin Ho Kwak and Sungpyo Hong, Linear Algebra. Birkhäuser Basel.

[4] Kenneth Hoffman, Ray Alden Kunze, Linear Algebra, 2nd Ed., Prentice-Hall of India Pvt. Ltd., 1971.

[5] S. Lang, Introduction to Linear Algebra, 2nd Ed., Springer, 2005.

[6] A. Ramachandra Rao and P. Bhimasankaram : Linear Algebra, Hindustan Book agency.

[7] Gilbert Strang, Linear Algebra and its Applications, Thomson, 2007.

[8] J. E. Gentle, Matrix Algebra, Sringer.

[9] S. K. Mapa, Higher Algebra : Abstract and Linear, 14th Ed., Sarat Book House, 2019.

MATUGMCC2409 (Metric Spaces)

Learning Objectives

In this course students are expected to be able to:

- Gather knowledge of Metric Spaces.

Course Outcomes

Upon the successful completion of the course a student will:

- Acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- Demonstrate capacity to solve problems and communicate mathematical ideas efficiently.

Course Details

Unit I: Metric Spaces

- Definition and examples of Metric Spaces, Subspaces. [4-6 Lectures]
- Diameter and Boundedness of Sets. [4-6 Lectures]

Unit II: Topology in Metric Spaces

- Open and closed balls, neighbourhood, open set, interior of a set. Limit point of a set, closed set, Cantor's theorem. Equivalent Metric Spaces, dense sets, separable spaces. [10-12 Lectures]

Unit III: Complete Metric Spaces

- Sequences in Metric Spaces, Cauchy sequences. Complete Metric Spaces. [10-12 Lectures]

Unit III: Continuity in Metric Spaces

- Continuous mappings, sequential criterion and other characterizations of continuity. Uniform continuity. Homeomorphism, Contraction mappings, Banach Fixed point Theorem. [10-12 Lectures]

Unit III: Compact Metric Spaces

- Compactness in Metric Spaces, Total Boundedness, Sequential compcatness, BW compactness, Compactness and continuity. [10-12 Lectures]

Unit III: Connected Metric Spaces

- Connectedness in Metric Spaces, Continuity and Connectedness, Components. [10-12 Lectures]

References

[1] S. Shirali and Harikishan L. Vasudeva, Metric Spaces, Springer Verlag, London, 2006.

[2] S. Kumaresan, Topology of Metric Spaces, 2nd Ed., Narosa Publishing House, 2011.

[3] G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 2004.

[4] M.N. Mukherjee, Elements of Metric Spaces.

MATUGMCC3510 (System of ODEs and Introduction to PDE)

Learning Objectives

- The main objective of this course is to introduce concepts and techniques for the solution of partial differential equations (PDEs).
- In particular, the course aims are to present the method of characteristics for first-order PDEs.
- The theory of separation of variables for Laplace's and the linear wave equation.
- Solve linear systems of ordinary differential equations.
- The course will provide the basic theory of differential equations and practice for a several domains of subjects in Natural and Engineering Science and Technology.

Course Outcomes

Upon successful completion of this course, the students will –

- formulate and solve first and second order quasi-linear Partial Differential Equation(PDE).
- recognize the major classification of PDEs and the qualitative differences between the classes of equations, and be competent in solving linear PDEs using classical solution methods.
- be able to solve systems of linear differential equations by using eigenvalues and eigenvectors.
- determine the solutions of a certain classes of nonlinear differential equations by apply the linearization technique.

Course Details

Unit I:

- Introduction of Partial Differential Equation (PDE). Genesis of PDE: Construction of PDE by using the method of eliminating arbitrary functions. Lagrange's solution of a linear partial differential equation of the form $Pp+Qq=R$. Integral surfaces through a given curve. Surface orthogonal to a given system of surfaces. [14 – 16 Lectures]
- Solution of nonlinear first order PDE by using Charpit and Jacobi's methods. Integral surface passing through a curve. Compatibility of two first order linear and nonlinear PDEs. [10 – 12 Lectures]

Unit II:

- Derivation of heat equation, wave equation and Laplace equation. Classification of 2nd order linear equations as elliptic, parabolic, hyperbolic. Reduction of second order Linear Equations to canonical forms. Cauchy problem of an infinite string. Initial boundary value problems. Semi-infinite string with a fixed end. [12 – 16 Lectures]
- System of Linear Ordinary Differential Equations (ODE): Simultaneous equation of the form dx/P=dy/Q=dz/R. Pfaffian differential equations of the form $P(x,y,z)dx+Q(x,y,z)dy+R(x,y,z)dz=0$. Condition of integrability of Pfaffian equations and their solutions. [6 – 8 Lectures]

Unit III:

- Basic Theory of linear systems in normal form. Solution of homogeneous and non-homogeneous linear system of differential equations with constant coefficients. Two equations in two unknown functions. Matrix method for solving homogeneous linear system with constant coefficients consisting n equations in n unknown functions. [12 – 16 Lectures]

- Laplace transform of derivatives and its application to solving ordinary differential equation with constant coefficients and given initial conditions. [12 – 16 Lectures]

Unit IV: (List of Labs)

- Solution of Cauchy problem for first order PDE.

- Finding the characteristics for the first order PDE.

- Plot the integral surfaces of a given first order PDE with initial data.

- Solution of wave equation with varieties of initial and boundary conditions. [8 – 10 Lectures]

References

[1] T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Springer, Indian reprint, 2006.

[2] S. L. Ross, Differential equations, 3rd Ed., John Wiley and Sons, India, 2004.

[3] J. C. Burkill, The Theory of Ordinary Differential Equations [Oliver & Boyd, London].

[4] E. A. Codington and Levinson, Theory of Ordinary Differential Equations.

[5] G. F. Simmons, Differential Equations.

[6] G. Birkhoff and G. Rota, Ordinary Differential Equation.

[7] I. N. Sneddon, Elements of Partial Differential Equations [Dover].

[8] M. L. Abell and J. P. Braselton, Differential equations with MATHEMATICA, 3rd Ed., Elsevier Academic Press, 2004.

MATUGMCC3511 (Multivariate Calculus)

Learning Objectives

The student will –

- be introduced to the concepts how ideas of single variable Calculus is generalised towards higher variables.
- Understand how other branches of Mathematics like Linear Algebra, Geometry play a significant role in thev study of Calculus.

Course Outcomes

Upon successful completion of this course, the students will –

- Develop a good understanding of Total Derivative, and to be able to assimilate how derivative is considered as a linear map.
- Have an idea over integration over several geometric objects like curves, bounded portions of a plane, etc.

Course Details

Unit I: Differential Calculcus

- Functions of several variables, scalar and vector field, limit and continuity of functions of two (and more) variables. Partial derivative, directional derivatives, Frechet differentiability, sufficient condition for differentiability. Chain rule, the gradient, idea regarding extrema of functions of two variables, method of Lagrange multipliers (problems only), [20 - 24 Lectures]

  Unit II: Integral Calculcus

- Double integration over rectangular region, double integration over non-rectangular region, Fubini's Theorem, Double integrals in polar co-ordinates, Change of variables in double integrals . [8 - 10 Lectures]
- Curves and arc length, Reparametrisation, Line integrals, Fundamental theorem for line integrals, independence of path. Green's theorem. [8 - 10 Lectures]

References

[1] Tom M. Apostol, Calculus, Volume II, John Wiley and Sons Limited, 2003.

[2] W. Rudin, Principles of Mathematical Analysis, McGraw Hill Education, (Paperback), 2017.

[3] Tom M. Apostol, Mathematical Analysis, Narosa, 2002.

[4] E. Marsden, A.J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer (SIE), Indian reprint, 2005.

[5] M. Spivak, Calculus on Manifolds: A Modern Approach To Classical Theorems of Advanced Calculus, Westview Press.

MATUGMCC3512 (Topology)

Learning Objectives

The student will –

- This course is an introduction to topological spaces. It deals with constructions like subspaces, product spaces, and quotient spaces, and properties like completeness, compactness and connectedness.
- It deals with constructions like subspaces, product spaces, and quotient spaces, and properties like completeness, compactness and connectedness.

Course Outcomes

Upon successful completion of this course, the students will -

- can work with sets and functions, images and preimages, and you can distinguish between finite, countable, and uncountable sets
- know how the topology on a space is determined by the collection of open sets, by the collection of closed sets, or by a basis of neighborhoods at each point, and you know what it means for a function to be continuous
- know the definition and basic properties of connected spaces, path connected spaces, compact spaces, and locally compact spaces
- know what it means for a metric space to be complete, and you can characterize compact metric spaces
- are familiar with the Urysohn lemma and the Tietze extension theorem, and you can characterize metrizable spaces

Course Details

- Definition and examples of topological spaces. Basis and subbasis for a topology. Interior, closure and boundary of a set. The order Topology. Subspace and relative topology. Closed sets and Limit points. Hausdorff Spaces. Continuous Functions and topological properties. Construction of continuous functions. The Product topology. Metric topology and Uniform limit theorem. The quotient topology and quotient maps. Homeomorphisms. [24- 30 Lectures]
- Connectedness and Compactness: Connected spaces and its properties. Connected subspaces of the Real line .Components and local Connectedness. Compact spaces and their properties. Compact subspaces of the Real line. Limit point compactness. Sequentially compact spaces. Local compactness and properties. Countability and Separation Axioms. Ascoli-Arzela theorem, equicontinuity. [24 - 30 Lectures]

  Note : This course is based on the book (1), Chapters 1 - 7.

References

[1] C. Adams and R. Franzosa, Introduction to Topology Pure and Applied, Pearson, 2009.

[2] J. R. Munkres, Topology, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

[3] J. Dugundji, Topology, Allyn and Bacon, 1966.

[4] G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.

[5] J. L. Kelley, General Topology, Van Nostrand Reinhold Co., New York, 1995.

[6] L. Steen and J. Seebach, Counter Examples in Topology, Holt, Reinhart and Winston, New York, 1970

MATUGMCC3613 (Complex Analysis)

Learning Objectives

The student will –

- Determine continuity/differentiability/analyticity of a function of one complex variable.
- Gain the knowledge of Harmonic function.
- Understand Bilinear transformation and its basic properties.
- Find the radius of convergence of a power series.
- Gain the knowledge of Complex Integration and some basic properties on Complex Integration
- Evaluate a contour integral using parametrization, fundamental theorem of calculus and Cauchy's integral formula.
- Find the Taylor's series of a function and determine its circle or annulus of convergence.
- Compute the residue of a function and use the residue theorem to evaluate a contour integral or an integral over the real line.
- Determine the number of zeros of an analytic function using Rouche's theorem.

Course Outcomes

Upon successful completion of this course, the students will –

- Acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- Be efficient for further study of Advanced Real and Complex Analysis.

Course Details

- Introduction to Complex Numbers, Rectangular Representation, Point Set in Plane, Sequences, Compactness, Polar representation, Stereographic Projection. [4 - 6 Lectures]
- Functions of complex variable,Limits, Limits involving the point at infinity, Continuity. [4 - 6 Lectures]
- Derivatives, Differentiation formulas, Cauchy-Riemann equations, Sufficient conditions for differentiability. Analytic functions, Examples of analytic functions, Exponential function, Logarithmic function, Trigonometric function, Harmonic function. [6 - 8 Lectures]
- Bilinear Transformation, Multi-valued function, Branch point, Branch cut, Conformal mapping, Idea of Analytic Continuation. [8 - 10 Lectures]
- Convergence of Sequences and Series, Absolute and Uniform Convergence of Power Series. [4 - 6 Lectures]
- Complex Integration, Cauchy's Fundamental Theorem (statement only) and its consequences; Index of a closed curve, Cauchy's Theorem; Cauchy's Integral Formula, Derivative of an analytic function, Taylor's theorem, Morera's theorem, Cauchy's inequality, Liouville's theorem, Little Picard's Theorem; Zeros of an analytic function, Fundamental Theorem of Classical Algebra, Uniqueness Theorem, Identity Theorem; Maximum Modulus Theorem, Minimum Modulus Theorem, Schwarz's Lemma. [16 – 18 Lectures]
- Laurent series, classification of singularities, Riemann's theorem, Casorati-Weierstrass theorem; Residue theorem, Evaluation of some real integrals; Argument principle, Rouche's theorem, Open mapping theorem. [16 – 18 Lectures]

References

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[4] J. B. Conway, Functions of one Complex Variable, Springer

[5] L. V. Ahlfors, Complex Analysis, Mc GrawHill

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MATUGMCC3614 (Algebra-II)

Learning Objectives

The student will –

- Gain knowledge of direct product, group action, class equation.
- Study automorphism groups, finite abelian groups.
- State and apply Cauchy's theorem and Sylow Theorems.
- Identify and compare the properties of rings, ideals, quotient rings, integral domains, principal ideal domains, unique factorization domains.
- Understand polynomial rings and their irreducibility.

Course Outcomes

Upon successful completion of this course, the students will –

- Demonstrate capacity to apply group action and analyze simplicity of groups.
- Acquire necessary skills to define, construct, analyze and compare rings, integral domains and homomorphisms.
- Develop a profound grasp of the fundamental concepts of modern abstract algebra.
- Demonstrate mathematical reasoning abilities for evaluating, proving and describing basic algebra concepts.

Course Details

Unit I: Group Theory II

- Automorphism, inner automorphism, automorphism groups, automorphism groups of finite and infinite cyclic groups, applications of factor groups to automorphism groups, Characteristic subgroups, Commutator subgroup and its properties. Properties of external direct products, the group of units modulo n as an external direct product, internal direct products, Fundamental Theorem of finite abelian groups. [8–10 Lectures]
- Group actions, stabilizers and kernels, permutation representation associated with a given group action, Applications of group actions: Generalized Cayley's theorem, Index theorem. [8-10 Lectures]

- Groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n, p-groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of A_n for $n \geq 5$, non-simplicity tests. [8–10 Lectures]

Unit II: Ring Theory

- Definition and examples of rings, properties of rings, subrings, integral domains and fields, characteristic of a ring. Ideal, ideal generated by a subset of a ring, factor rings, operations on ideals, prime and maximal ideals. Ring homomorphisms, properties of ring homomorphisms, Isomorphism theorems I, II and III, field of quotients. [8-10 Lectures]

- Polynomial rings, division algorithm and consequences, principal ideal domains, factorization of polynomials, reducibility tests, irreducibility tests, Eisenstein criterion, unique factorization in Z[x]. Divisibility in integral domains, irreducibles, primes, unique factorization domains, Euclidean domains. [10–12 Lectures]

References

[1] John B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.

[2] M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.

[3] Joseph A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

[4] Joseph J. Rotman, An Introduction to the Theory of Groups, 4th Ed., Springer Verlag, 1995.

[5] I. N. Herstein, Topics in Algebra, Wiley Eastern Limited, India, 1975.

[6] David S. Dummit and Richard M. Foote, Abstract Algebra, 3rd Ed., John Wiley and Sons(Asia) Pvt. Ltd., Singapore, 2004.

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[8] D. A. R. Wallace, Groups, Rings and Fields, Springer Verlag London Ltd., 1998.

[9] D. S. Malik, John M. Mordeson and M.K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill, 1997.

[10] M. K. Sen, S. Ghosh, P. Mukhopadhyay, S. K. Maity, Topics in Abstract Algebra, 4th Ed., Universities Press, 2022.

[11] S. K. Mapa, Higher Algebra : Abstract and Linear, 14th Ed., Sarat Book House, 2019.

MATUGMCC4716 (Field Theory and Linear Algebra)

Learning Objectives

The student will –

- Understand the relationships among field extensions, polynomial rings and roots of polynomials, splitting fields.
- Apply the above in problems involving straight edge and compass constructions.
- Gain knowledge on splitting fields, separable extensions and cyclotomic polynomials.
- Understand the basics of Galois theory.
- Gain knowledge about important characteristics of matrices, its four fundamental subspaces.
- Study rational canonical form and Jordan canonical form in perspective of modules over principal ideal domains.

Course Outcomes

Upon successful completion of this course, the students will –

- Demonstrate proper use of advanced algebraic techniques in solving problems.
- Develop a deep understanding of the concepts and theorems in modules over PID and field theory.
- Apply knowledge acquired to solve problems and analyze examples.

Course Details

Unit I: Linear Algebra

- Brief concept of Inner Product Spaces, operators. [4–6 Lectures]
- LU and LDV factorization of matrices. [2–4 Lectures]
- Four Fundamental subspaces associated with a matrix, Fundamental Theorem of Linear Algebra, Its impact on system of equations and its solution; Hermitian, Self-Adjoint, Unitary and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms. The structure of orthogonal transformations in real Euclidean spaces. [10–12 Lectures]
- Finitely generated modules over PID. [2–4 Lectures]
- Existence and Uniqueness theorems on Invariant factors, elementary divisors (proof may be omitted). [4–6 Lectures]
- Companion matrix, Primary decomposition theorem, Rational Canonical form and Jordan Canonical form (through the R-module approach). Computational algorithm for these forms (upto 4×4 case) through similarity. [8–10 Lectures]

Unit II: Field Theory

- Basic Theory of Field Extensions. [2–4 Lectures]
- Algebraic Extensions. [4–6 Lectures]
- Classical Straightedge and Compass Constructions. [2–4Lectures]
- Splitting Fields and Algebraic Closures. [4–6 Lectures]
- Separable and Inseparable Extensions. [2–4 Lectures]
- Cyclotomic Polynomials and Extensions. [1–2 Lectures
- Galois Theory: Fundamental Theorem of Galois Theory, Finite fields, Galois Groups of polynomials, Solvability of polynomials by radicals, Insolvability of a Quintic. [8–10 Lectures]

References

[1] K. Hoffman, R. Kunze, Linear Algebra, 2nd Ed., Prentice Hall of India, 1999.

[2] G. Strang, Introduction to Linear Algebra, 5th Ed., Wellesley-Cambridge Press and SIAM, 2016.

[3] A. Ramachandra Rao, P. Bhimasankaran, Linear Algebra, Hindustan Book Agency, 2000.

[4] V. Sahai, V. Bist, Linear Algebra, Alpha Science International Ltd., 2002.

[5] J. H. Kwak, S. Hong, Linear Algebra, 2nd Ed., Birkhuser, 2004.

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[12] J. Rotman, Galois Theory, Springer, 2012.

[13] N. S. Gopalakrishnan, University Algebra, New Age International Pvt. Ltd., 2018.

MATUGMCC4717 (Measure Theory)

Learning Objectives

The student will –

- know about the measurable set and the measurable function;
- understand Lebesgue Integral and compare it with Riemann Integral.

Course Outcomes

Upon successful completion of this course, the students will –

- understand the fundamentals of measure theory and be acquainted with the proofs of the fundamental theorems underlying the theory of integration;
- understand how these underpin the use of mathematical concepts such as volume, area, and integration;
- develop a perspective on the broader impact of measure theory in ergodic theory;
- have the ability to pursue further studies in this and related areas.

Course Details

Lebesgue outer measure, Measurable sets, Regularity, Measurable functions, Borel and Lebesgue measurability. Integration of non-negative functions, Fatou's lemma, Lebesgue's monotone convergence theorem, Lebesgue dominated convergence theorem, Lebesgue and Riemann Integrals. The four derivatives, L^p spaces, completeness of L^p and denseness results in L^p, Radon-Nikodym theorem, Lebesgue decomposition theorem, dual of L^p spaces, Product measures and Fubini's Theorem, Measures on Infinite Product Spaces, Weak Convergence of measures.

References

[1] G. D. Barra, Measure Theory and Integration, 2nd Edition, New Age International Publishers, 2013.

[2] P. R. Halmos, Measure Theory, 2nd Edition, Springer, 2008.

[3] H .L. Royden and P. Fitzpatrick, Real Analysis, 4th Edition, Pearson India, 2009.

[4] R. B. Ash and C. A. Doleans-Dade, Probability andMeasure Theory, 2nd Edition, Academic Press, 1999.

MATUGMCC4818 (Integral Transform, Integral Equations and Calculus of Variations)

Learning Objectives

The student will –

- Achieve a good basic understanding of the theory and methods of finding solutions for integral equations and to introduce them to applications in modelling physical phenomena and mechanical systems.
- The course will provide both a comprehensive foundation of basic theory of integral equations and practice for a broad range of subjects in Science & Technology.
- Learn the methods of finding Laplace transform and Fourier Transforms as well as inverse transform of different functions.
- Be familiar with the methods of solving ordinary differential equations, partial differential equations, IVP and BVP, integral equations using Laplace transforms and Fourier transforms.

Course Outcomes

Upon successful completion of this course, the students will be able to–

- classify different kind of integral equations and to solve them.
- recognize the different methods of finding Laplace transforms and Fourier transforms of different functions.
- apply the knowledge of Laplace transform, Fourier transform in finding the solutions of differential equations, initial value problems, boundary value problems.

Course Details

Unit I: Integral Equations

- Integral Equations: Introductory concepts: Definition, Fredholm and Volterra integral equations. Non-linear Equations, Singular equations, Integro-differential equations. Relations between differential and integral equations. [4 – 6 Lectures]
- Fredholm integral equation: Equations of the first and second Kind. Solution with separable kernels. [6 Lectures]
- Volterra integral equation: First and second type. Solution with separable kernels. Characteristic numbers and eigenfunctions. Resolvent kernel. [6 Lectures]

Unit II: Integral Transform

- Fourier Transforms: Fourier integral Theorem. Definition and properties. Fourier transform of the derivative. Derivative of Fourier transform. Fourier transforms of some useful functions. Fourier cosine and sine transforms. Inverse of Fourier transforms. Convolution. Properties of convolution function. Parseval relation. Applications. [12 – 14 Lectures]

- Laplace Transforms: Definition and properties. Sufficient conditions for the existence of Laplace Transform. Laplace Transform of some elementary functions. Laplace transform of the derivatives. Inverse of Laplace transform. Bromwich Integral Theorem. Evaluation of inverse transforms by residue. Initial and final value theorems. Convolution theorem. Applications. [14 –16 Lectures]

Unit III: Calculus of Variations

- Calculus of Variation: Introduction, Euler's equation, different forms of Euler's equations, solutions of Euler's equation, geometrical problems, geodesics, minimum surface of revolution, isoperimetric problems, Brachistochrone problem. [8 Lectures]
- Variational problems involving several unknown functions, functionals dependent on higher order derivatives, variational problems involving several independent variables, constraints and Lagrange's multipliers, moving boundaries. [8 Lectures]

References

[1] S. M. Jemyan, The Classical Theory of Integral Equations, 12th edition, Birkhauser, 2012.

[2] W. V. Lovitt, Linear Integral Equations, 2nd edition, Dover Publications Inc., 2005.

[3] F. G. Tricomi, Integral Equations, 2nd edition, Dover Publications Inc., 1985.

[4] A. M. Wazwaz: A First Course in Integral Equations, 2nd edition, World Scientific, 2015.

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[6] L. Debnath, D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/ CRC, 2006.

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[8] F. C. Titchmash, Introduction to the theory of Fourier Integrals, Oxford Press, 1937.

[9] Peter K. F. Kuhfittig, Introduction to the Laplace Transform, Plenum Press, N.Y., 1980.

[10] E. J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.

[11] C. J. Tranter, Integral Transforms in Mathematical Physics, Methuen & Co., 1962.

[12] A. Pinkus, S. Zafrany, Fourier Series and Integral Transforms, Cambridge University Press, 1997.

MATUGMCC4819 (Functional Analysis)

Learning Objectives

Objective of the course is -

- To introduce students to the ideas and some of the fundamental theorems of functional analysis.
- To show students the use of abstract algebraic/topological structures in studying spaces of functions.
- To allow students to taste the subject with a view to further work as a postgraduate.
- To give students a working knowledge of the basic properties of Banach spaces, Hilbert spaces and bounded linear operators.
- To show students the idea of duals and adjoints.
- To show students the value of looking at the spectrum of a bounded linear operator.
- To demonstrate significant applications of the theory of functional analysis.

Course Outcomes

Upon successful completion of this course, the students will -

- Appreciate how functional analysis uses and unifies ideas from vector spaces, the theory of metrics, and complex analysis.
- Understand and apply fundamental theorems from the theory of normed and Banach spaces, including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem, and the Stone-Weierstrass theorem.
- Understand and apply ideas from the theory of Hilbert spaces to other areas, including Fourier series, the theory of Fredholm operators, and wavelet analysis.

Course Details

- Banach Spaces :
  Normed Linear Spaces, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Quotient Space of normed linear spaces and its completeness, Riesz Lemma, Fixed Point Theorems and its applications. Bounded Linear Transformations, Normed linear spaces of bounded linear transformations, Uniform Boundedness Theorem, Principle of Condensation of Singularities, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Hahn-Banach Theorem, Dual Space, Reflexivity of Banach Spaces. [20 – 28 lectures]
- Hilbert Spaces : Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Structure of Hilbert Spaces, Orthogonal Complement and Projection Theorem. Riesz Representation Theorem, Adjoint of an Operator on a Hilbert Space, Reflexivity of Hilbert Spaces, Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators, Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators. [22 – 28 lectures]

  Note : This course is based on the book (1) and (2).

References

[1] B. V. Limaye, Functional Analysis, New Age International Limited, Publishers, 2014

[2] E. Kreyszig, Introductory Functional Analysis with Applications, Wiley, 2014

[3] C. D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd., 1998.

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MATUGMCC3615 (Mechanics)

Learning Objectives

The student will be able to

- Learn about the basics of coplanar forces and three dimensional forces.
- Study about laws of friction and centre of gravity of different bodies.
- Acquire knowledge on principle of virtual work and stability of equilibrium.
- Study rectilinear motion and motion in three dimensions.
- Learn equation of motion referred to a set of rotating axis.
- Learn Kepler's laws and study planetary motion.
- Study the motion when the mass varies and in a resisting medium.
- Know about moments and products of inertia of a rigid body.
- Learn about D'Alemberts principle and its applications.
- Study motion of a rigid body about fixed axis and in two dimensions.
- Learn about translational and rotational motion of rigid bodies
- Learn how to calculate mementum and kinetic energy of rigid bodies
- Develop concept on conservation of momentum and energy

Course Outcomes

Upon successful completion of this course, the students will be able to

- Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces acting on a rigid body.
- Understand astatic equilibrium and astatic centre.
- Learn force of friction, angle of friction and cone of friction.
- Study equilibrium of a particle on a rough plane curve.
- Solve problems on friction.
- Understand conservative field of force, the energy test of stability.
- Study stability of equilibrium of different bodies.
- Find the equations of central axis of a given system of forces.
- Determine the centre of gravity of different bodies.
- Deal with the kinematics and kinetics of the rectilinear and planar motions of a particle including the constrained motions of particles on smooth sphere and cone.

- Learn simple harmonic motion and determine velocity and acceleration in different coordinates.
- Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions.
- Solve problems involving the motion in resisting medium and motion of particle of varying mass.
- Understand the motion of artificial satellite.
- Understand the degrees of freedom of rigid bodies.
- Determine momemt of inertia, product of inertia and momental ellipsoid of different bodies.
- Find equations of motion of any rigid body.
- Understand the motion of compound pendulum.
- Determine kinetic energy and moment of momentum of rigid bodies moving in two dimensions.
- Deal with the impact of rotating sphere on the ground.

Course Details

Unit I: Statics

- Co-planar forces, Astatic equilibrium; Friction, Equilibrium of a particle on a rough curve; Virtual work, The principle of virtual work; Stability of equilibrium, Conservative field of force, Energy test of stability; Centre of gravity for different bodies; Forces in three dimensions, General conditions of equilibrium. [16 Lectures]

Unit II: Particle Dynamics

- Simple Harmonic Motion; Velocities and accelerations in Cartesian, polar, and intrinsic coordinates; Equations of motion referred to a set of rotating axes; Central forces, Stability of nearly circular orbits; Motion under the inverse square law, Keplar's laws of motion, Slightly disturbed orbits, Motion of artificial satellites; Motion of a projectile in a resisting medium; Varying mass; Motion of a particle in three dimensions; Motion on a smooth sphere, cone. [26 Lectures]

Unit III: Rigid Dynamics

- Degrees of freedom, Moments and products of inertia, Momental Ellipsoid, Principal axes; D'Alembert's Principle, Impulsive forces; Motion about a fixed axis, Compound pendulum; Motion of a system of particles, Motion of a rigid body in two dimensions under finite and impulsive forces; Conservation of momentum and energy. [18 Lectures]

References

[1] I.H. Shames and G. Krishna Mohan Rao, Engineering Mechanics: Statics and Dynamics, 2006. Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2009.

[2] R.C. Hibbeler and Ashok Gupta, Engineering Mechanics: Statics and Dynamics, 11th Ed., Dorling Kindersley (India) Pvt. Ltd. (Pearson Education), Delhi, 2010.

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[5] S. L. Loney, Elements of Statics and Dynamics I and II, 2004.

[6] M. C. Ghosh, Analytical Statics. Verma, R. S., A Textbook on Statics, Pothishala, 1962 .

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Mathematical Modeling

Learning Objectives

- A primary purpose of any modeling course should be to develop students' capacity to solve problems through the use of mathematical models as a transferable process that will equip them to address novel problems in the future.
- While specific objectives and cognitive learning goals may vary with the context of each course, a few relatively universal aims emerge in accord with the essential goal of teaching modeling as a transferable process.

Course Outcomes

Upon successful completion of this course, the students will –

- typically strive to introduce the elements of mathematical modeling process.
- present the application-driven mathematics motivated by problems from the different domain of science and technology.
- exemplify the value of mathematics in problem solving and demonstrate the possible connections among different mathematical domain of research.
- effectively modeling everyday's situations into mathematical statements, which can be analyzed, validated, and interpreted in context of problem.

Course Details

Unit I:

- Perturbation methods: Introduction to modeling concepts, dimensional analysis, perturbation techniques, applications of techniques of nonlinear dynamics.
- Flow on the circle: Introduction, Examples and definitions, Uniform oscillator, non-uniform oscillator, over-damped pendulum, Josephson Junctions. [14 – 16 Lectures]

Unit II:

- Modelling of Social Dynamics: Mathematical theories of war, Richardson's theory of conflict,Lancaster's combat models. Modeling of environmental relatedphenomena. Modeling of intoxicants. Modelling of Electrical Circuits: Oscillations in RLC circuits, response of RLC circuits to sinusoidalsquare, pulse, ramp and burst. [12 – 16 Lectures]
- Models for inventory controls: Inventor, Demand, Holding cost, shortage cost, Setup cost, Leadtime, Deterioration, models for cost minimization and profit maximization. Sensitivity analysis. [12 – 14 Lectures]

Unit III:

- Power series solution of a differential equation about an ordinary point, solution about a regular singular point, Bessel's equation and Legendre's equation. [6 – 8 Lectures]
- Monte Carlo Simulation Modeling: Simulating deterministic behavior (area under a curve, volume under a surface), Generating Random Numbers: middle square method, linear congruence. [8 – 10 Lectures]

References

[1] W. Meyer, Concepts of Mathematical Modeling, McGraw Hill, New York, 1994.

[2] F. R. Giordano, M. D. Weir and W. P. Fox, A First Course in Mathematical Modeling,Thomson Learning.

[3] Hinch, Perturbation methods, Cambridge Texts in Applied Mathematics.

[4] Kevorkian and Cole, Perturbation methods in applied mathematics, Applied Mathematical Sciences,Springer.

[5] Bender and Orszag, Advanced mathematical methods for scientists and engineers, AsymptoticMethods and Perturbation Theory: v.1, Springer.

[6] S. H. Stogatz, Nonlinear Dynamics and Chaos.

[7] G. Hadeley and T. M. Whitin, Analysis of Inventory Systems, Prentice Hall, 1963.

Control Theory

Learning Objectives

- To provide fundamental knowledge in analysis and design of dynamical system.
- To learn about the dynamical system and control method.
- To implement the feedback control system with performance specifications.

Course Outcomes

Upon successful completion of this course, the students will –

- understand fundamental limits of control and estimation.
- be able to use advanced mathematical techniques to formulate and solve control problems.
- appreciate issues of robustness, optimality, architecture and uncertainty in control problems.
- identify practical challenges in posing control problems.

Course Details

- **Introduction:** Motivation, examples of control systems, feedback control systems. [6 – 8 Lectures]
- **Mathematical modelling:** Mathematical modelling of mechanical systems, Laplace transforms, transfer functions, State-space modelling of dynamical systems. Linearity, time-invariance versus nonlinearity and time-variance. Linearization. [6 – 8 Lectures]
- **Impulse response of dynamical systems:** Obtaining solutions from mathematical models. Poles and zeros and their effects on solutions, Transfer function. [6 – 8 Lectures]
- **Stability:** Definition of stability: continuous/discrete, Input-output stability, Routh-Hurwitz test. Lyapunov theory (Function, Equation) . [6 – 8 Lectures]
- **Controllability and Observability:** Definition, analysis, example. [6 – 8 Lectures]
- **Feedback control:** Basic idea of feedback control systems. Error analysis. P, PI, PD, PID controllers. [6 – 8 Lectures]
- **Frequency domain analysis:** Nyquist plot, Nyquist stability criterion, gain and phase margins, robustness. [6 – 8 Lectures]
- **Optimal control:** Basic problem, necessary condition, Pontryagin's maximum principle, application in ecology. [6 – 8 Lectures]

References

[1] J-J E. Slotine and W. Li, Applied Nonlinear Control, Englewood Cliffs, NJ: Prentice-Hall, 1991.

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[7] M. Gopal, Digital Control & State VariableMethods, Tata McGraw Hill Education, 2008.

[8] T. Kailath, Linear Systems by , Prentice-Hall Inc.,1980.

Number Theory

Learning Objectives

The student will –

- get elementary ideas from number theory which will have applications in cryptography and coding theory;
- identify and apply various properties of and relating to the integers including the Well-Ordering Principle, primes, unique factorization, the division algorithm, understand the concept of a congruence;
- impart the knowledge of encryption and decryption techniques and their applications in managing the security of data.

Course Outcomes

Upon successful completion of this course, the students will –

- solve problems in elementary number theory.
- apply elementary number theory to cryptography.
- develop a deeper conceptual understanding of the theoretical basis of number theory and identify how number theory is related to and used in cryptography.

Course Details

Linear Diophantine equation, prime counting function, statement of prime number theorem, Goldbach conjecture, linear congruences, complete set of residues, Chinese Remainder theorem, Fermat's Little theorem, Wilson's theorem. [16-18 Lectures]

Number theoretic functions, sum and number of divisors, totally multiplicative functions, definition and properties of the Dirichlet product, the Mobius Inversion formula, the greatest integer function, Euler's phi-function, Euler's theorem, reduced set of residues, some properties of Euler's phi-function. [18-20 Lectures]

Order of an integer modulo n, primitive roots for prime, composite numbers having primitive roots, Euler's criterion, the Legendre symbol and its properties, quadratic reciprocity, quadrati ccongruences with composite moduli. Public key encryption, RSA encryption and decryption, the equation $x^2 + y^2 = z^2$, Fermat's Last theorem. [20-22 Lectures]

References

[1] David M. Burton, Elementary Number Theory, 6th Ed., Tata McGraw-Hill, Indian reprint,2007.

[2] Neville Robinns, Beginning Number Theory, 2nd Ed., Narosa Publishing House Pvt. Ltd.,Delhi,2007

[3] Neal Koblitz, A Course in Number Theory and Cryptology, Graduate Texts in Mathematics, Springer, 1994.

Introduction to Mathematical Logic and Automata Theory

Learning Objectives

The student will –

- get basic concepts and techniques of mathematical logic;
- understand the basics of automata theory.

Course Outcomes

Upon successful completion of this course, the students will –

- give correct logical arguments, find errors in incorrect arguments Generell competence;
- discuss logical arguments and their correctness with others communicate the basic concepts of logic and their relevance for computer science;
- develop a perspective on the broader impact of automata theory;
- have the ability to pursue further studies in this and related areas.

Course Details

Classical propositional calculus (PC): Syntax. Valuations and truth tables, Truth functions, Logical equivalence relation. Semantic consequence and satisfiability. Compactness theorem with application. Adequacy of connectives. Normal forms. Applications to Circuit design.

Axiomatic approach to PC: soundness, consistency, completeness. Other proof techniques: Sequent calculus, Computer assisted formal proofs: Tableaux. Decidability of PC, Boolean algebras: Order relations. Boolean algebras as partially ordered sets. Atoms, Homomorphism, sub-algebra. Filters. Stone's representation (sketch). Completeness of PC with respect to the class of all Boolean algebras. Classical first order logic (FOL) and first order theories, Syntax. Satisfaction, truth, validity in FOL. Axiomatic approach, soundness. Computer assisted formal proofs: Tableaux. Consistency of FOL and completeness (sketch). Equality. Examples of first order theories with equality.

Introduction: Alphabets, strings, and languages. Finite Automata and Regular Languages: deterministic and non-deterministic finite automata, regular expressions, regular languages and their relationship with finite automata, pumping lemma and closure properties of regular languages.

Context Free Grammars and Pushdown Automata: Context free grammars (CFG), parse trees, ambiguities in grammars and languages, pushdown automaton (PDA) and the language accepted by PDA, deterministic PDA, Non- deterministic PDA, properties of context free languages; normal forms, pumping lemma, closure properties, decision properties. Turing Machines: Turing machine as a model of computation, programming with a Turing machine, variants of Turing machine and their equivalence. Undecidability: Recursively enumerable and recursive languages, undecidable problems about Turing machines: halting problem, Post Correspondence Problem, and undecidability problems about CFGs.

References

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[3] E. Mendelson, Introduction to Mathematical Logic, CRC Press, 2015

ODE and Special Functions

Learning Objectives

- The objective of the course is to study Ordinary Differential Equations and find solutions with special properties.
- To learn about Pickard's theorem
- To learn about Sturm' theory
- To learn about the properties of Legendre polynomial
- To learn about Bessel's solutions
- To learn about Green functions

Course Outcomes

Upon successful completion of this course, the students will –

- Acquire necessary skills about Pickard's theorem on existence and uniqueness of solution of ODE.
- Able to apply the Sturm's seperation and comparison theory in relevants fields.
- Know the use of Legendre polynomial and Bessel's functions to solve pertial differential equation.
- Acquire the idea of Green functions and its application to some relevan field of science and engineering.

Course Details

Unit I:

- First order system of equations: The fundamental existence and uniqueness theorem, simple illustrations. Dependence of solution on initial conditions. Peano's and Picard's theorems. [10 – 12 Lectures]
- Definition of adjoint of differential equation. Self adjoint equation. Lagrange's identity. Sturm's theory: Some basic results on Sturm's theory.Sturm's separation theorem, Sturm's comparison theorem. [12 – 14 Lectures]

Unit II:

- Ordinary point and singularity of a second order linear differential equation in the complex plane. Classification of singularities of a second order differential equation. Solution about an ordinary point. Regular singularity, Frobenius' method to solve second order differential equations about a regular singular points. Solution of Hermite and Chebyshev's equations as examples. [12 – 14 Lectures]

Unit III:

- Legendre polynomial: its generating function; Rodrigue's formula, recurrence relations and differential equations satisfied by it; Its orthogonality, expansion of a function in a series of Legendre Polynomials. Bessel's functions and its orthogonality. Legendre and Bessel Fourier series. [10-12 Lectures]
- Green's function for finding nontrivial solution of initial value problems. [6 – 8 Lectures]

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[1] J. C. Burkill, The Theory of Ordinary Differential Equations, 3rd edition, Prentice Hall Press, 1975,

[2] E. A. Codington, N. Levinson, Theory of Ordinary Differential Equations, 3rd edition, Krieger Publishing Company, 1984.

[3] G. F. Simmons, S.G. Krantz, Differential Equations, Indian edition, McGraw Hill Education, 2017.

[4] G. Birkhoff, G.C. Rota, Ordinary Differential Equations, 4th edition, Willey, 1991.

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Graph Theory

Learning Objectives

- Students will achieve command of the fundamental definitions and concepts of graph theory.
- This covers basic elements of graphs such as degree, walk, path, cycle, distance, eccentricity, radius, diameter, etc.
- Different types of graphs such as simple graph, multigraph, directed graph, Eulerian graph, Hamiltonian graph, bipartite graph, trees, etc. will be discussed.
- Concepts of connectivity, colouring, covering, matching and planarity will be introduced.
- Students will be able to apply their knowledge of graph theory to problems in other areas.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- write precise and accurate mathematical definitions of objects in graph theory.
- use mathematical definitions to identify and construct examples and counter-examples.
- use a combination of theoretical knowledge and independent mathematical thinking in creative investigation of questions in graph theory.
- apply graph theoretical knowledge in solving real life problems.

Course Details

- **Fundamental concepts of graphs:** Basic definitions of graphs and multigraphs, digraphs and relations, simple graph, weighted graph, matrix representation of graphs: adjacency matrix and incidence matrix, regular graph, bipartite graph, sub-graph, complete graph, complement of a graph, graph operations: cartesian product of graphs, line graphs, isomorphism, girth, decompositions, important graph like cubes and the Petersen graph. [8 Lectures]
- **Connectivity:** Walks, trails, paths and cycles, connectedness of a graph, disconnected graphs and their components, distance, centre, radius, cut-vertices, cut-edges, blocks, vertex connectivity, edge connectivity, shortest path problem: Dijkstra's algorithm.

  [6 Lectures]
- **Eulerian and Hamiltonian Graphs:** Motivation and origin, necessary conditions and sufficient conditions for existence of Eulerian trail and Hamiltonian cycle.

  [4 Lectures]

- **Trees:** Definition and characterizations, rooted and binary trees, spanning trees, counting of trees: Cayley's formula, breadth first and depth first search, minimum spanning tree problem: Prim's algorithm and Kruskal's algorithm. [10 Lectures]

- **Independent Set, Cover and Matching:** Independent set, vertex cover, edge cover, matching in bipartite graphs, perfect matching, M-alternating path, Hall's Theorem, König-Egeváry Theorem, Gallai's Theorem . [10 Lectures]

- **Graph Colouring:** Vertex colouring, cliques and chromatic number, chromatic polynomial, Vizing's Theorem, k-Chromatic graphs: Mycielski's construction, Turán's Theorem . [10 Lectures]

- **Planarity:** Planar graphs, Euler's formula, Kuratowski's graphs, detection of planarity, geometric dual, combinatorial dual. [8 Lectures]

- **Directed Graphs:** Underlying graph of a digraph, out-degrees and in-degrees, tournaments. [4 Lectures]

References

[1] Robin J. Wilson, Introduction to Graph Theory, 4th Edition, Pearson, 2007.

[2] D.B. West, Introduction to Graph Theory, 2nd Edition, Pearson, 2015.

[3] Gary Chartrand and Ping Zhang, Introduction to Graph Theory, McGraw-Hill, 2004.

[4] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Springer, 2008.

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Introduction to Coding Theory

The student will –

- get idea to some of the classical methods in coding theory,
- While mathematical background on linear algebra and probability is assumed, coverage of necessary background on finite fields is included as part of the course.
- through concrete examples of code construction, where simple, yet powerful mathematical tools are put to use, the course is expected to improve students' insights into the mathematical foundations.

Course Outcomes

Upon successful completion of this course, the students will –

-
- use algebraic techniques to construct efficient codes
- identify the parameters of a given code the quality of a given code;
- develop a perspective on the broader impact of coding theory;
- have the ability to pursue further studies in this and related areas.

Course Details

Communication channels, error detection, maximum likelihood decoding, Hamming distance, nearest neighbour/minimum distance decoding, distance of a code. Linear codes: Definition, Hamming weight, bases for linear codes, generator matrix and parity-check matrix, equivalence of linear codes, encoding with a linear code, decoding of linear codes, nearest neighbour decoding for linear codes, syndrome decoding

The main coding theory problem, lower bounds, sphere-covering bound, Gilbert–Varshamov bound, Hamming bound and perfect codes, Binary Hamming codes, error detection, correction and decoding if q-ary Hamming codes, Golay codes, some remarks on perfect codes, singleton bound and MDS codes, Plotkin bound.

Constructions of linear codes: Propagation rules, Reed–Muller codes, subfield codes. Cyclotomic cosets. Cyclic codes: Definition, generator polynomials, generator and parity-check matrices, decoding of cyclic codes, Burst-error-correcting codes. Some special cyclic codes: BCH codes, Reed–Solomon codes, Quadratic-residue codes.

References

[1] S. Ling and C. Xing, Coding Theory: A first course, Cambridge University Press.

[2] J.H. Lint, Introduction to Coding Theory, Spinger.

[3] R. Hill, A first course in coding theory, Oxford Applied Mathematics and Computing Science Series.

[4] F.J. MacWilliams and N.J.A. Sloane,The theory of error correcting codes, North-Holland Mathematical Library.

[5] E. Berlekamp, Algebraic coding theory, World Scientific Publishing Co Pvt Ltd; Revised edition.

Advanced Algebra

Learning Objectives

The student will –

- Study solvable and nilpotent groups, semidirect products and free groups.
- Gain knowledge on the basic theory for noetherian and artinian rings, rings of fractions and the Jacobson radical.
- Understand and investigate basic results in module theory.

Course Outcomes

Upon successful completion of this course, the students will –

- Develop a good understanding of groups and rings, and to be able to prove important theorems and solve problems.
- Have an insight of module theory as vector spaces over rings, having a well developed theory.
- Demonstrate proper use of advanced algebraic techniques in solving problems.

Course Details

Unit I: Group Theory III

- Structure theorem for finitely generated Abelian groups. [6–8 Lectures]
- Normal and subnormal series, composition series, Jordan–Hölder theorem, solvable groups and nilpotent groups. [8–10 Lectures]
- Semi direct products. [4–6 Lectures]
- Free groups. [4–6 Lectures]

Unit II: Ring Theory II

- Noetherian and Artinian rings, Hilbert Basis Theorem. [10–12 Lectures]
- Ring of fractions, Extension and contraction of ideals. [8–10 Lectures]
- The Jacobson radical, Jacobson semisimple ring. [8–10 Lectures]

Unit III: Module Theory

- Modules: Basic definitions and examples, submodules, quotient modules and module homomorphisms. [14–16 Lectures]
- Generation of modules, direct sum and free modules. [10–12 Lectures]
- Exact sequences, Projective modules. [12–14 Lectures]

References

[1] D. S. Malik, J. N. Mordeson, M. K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill International Editions, 1997.

[2] D. S. Dummit, R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 1999.

[3] N. S. Gopalakrishnan, University Algebra, New Age International Pvt. Ltd., 2018.

[4] T. W. Hungerford, Algebra, Springer, 1980.

[5] I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.

[6] J. A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

[7] M. Artin, Algebra, Prentice Hall of India,1994.

[8] S. Lang, Algebra I, 3rd Ed., Addison Wesley, 2005.

[9] G. Birkhoff, S. Mac Lane, A Survey of Modern Algebra, Taylor and Francis, 2008.

[10] J. Rotman, An Introduction to the Theory of Groups, Springer-Verlag, 1990.

[11] J. Rotman, Galois Theory, Springer, 2012.

Classical Mechanics

Learning Objectives

The objective of the course is to enable the students

- to distinguish between 'inertial frame of reference' and 'non-inertial frame of reference'.
- to know how to impose constraints on a system in order to simplify the methods to be used in solving physics problems.
- to know the importance of concepts such as generalized coordinates and constrained motion.
- understand Poisson brackets, understand canonical transformations.
- to find the linear approximation to any dynamical system near equilibrium and also know how to derive and solve the wave equation for small oscillations.

Course Outcomes

Upon successful completion of this course, the students will –

- learn about Lagrangian and Hamiltonian formulation of Classical Mechanics.
- be able to state the conservation principles involving momentum, angular momentum and energy and understand that they follow from the fundamental equations of motion.
- have a deep understanding of Newton's laws.

Course Details

- Fundamental of mechanics: Mechanics of a system of particles: Constraints; Generalized coordinates; Degrees of freedom; Virtual displacement and principle of virtual work; D'Alembert's principle; Generalized forces; Generalized momentum. [4 – 6 Lectures]
- Rotating frames of reference: Frames of reference rotating with constant angular velocity; Coriolis forces; Virial Theorem. Two- and Three-Body Motions; Foucault's pendulum. [4 – 6 Lectures]

  Lagrange's equation: Lagrangian; Lagrange's equations of motion; cyclic coordinates; Routh's process for the ignoration of co-ordinates; Conservative system; Natural System; Liouville's system; velocity dependent potential; Principle of energy; Rayleigh's dissipation function. [10 – 12 Lectures]
- Hamilton's equations: Calculus of variations and its applications in shortest distance, minimum surface of revolution, Euler-Lagrange equation, Brachistochrone problem, geodesic. Hamilton's principle. Lagrange's undetermined multipliers. Derivation of Hamilton's Equations; Hamiltonian; The form of the Hamiltonian function; Legendre transformation; Hamilton's principle from D'Alembert's principle; Lagrange's equations of motion from Hamilton's principle; Hamilton's equations of motion from Hamilton's principle. Principle of least action. Symmetry properties and conservation laws; Noether's theorem. [6 – 8 Lectures]
- Canonical Transformations: Canonical coordinates and canonical transformations. Principal forms of Generating function; Lagrange's and Pois- son's brackets and their relation under canonical transformations, Hamilton's equations of motion in Poisson's bracket. Canonical transformations in Poisson's bracket; constant of motion. [4 – 6 Lectures]
- Hamilton-Jacobi equation: Hamilton's principal function; Jacobi's Theorem. Hamilton's Principal Function. Hamilton's Characteristic Function; Action-angle variables; Adiabatic Invariance. [4 – 6 Lectures]
- Theory of Small Oscillations (Conservative System): Normal Coordinates. Oscillations under constraints. Stationary Character of Normal Modes. Elements of Non-linear Oscillations. [4 – 6 Lectures]
- An Introduction to Special Theory of Relativity. [4 – 6 Lectures]

References

[1] D. T. Green Wood, Classical Dynamics, Dover Publications Inc., 1977.

[2] K. C. Gupta, Classical Mechanics of Particles and Rigid Bodies, New Age International, 2018.

[3] F. A. Chorlton, Text Book of Dynamics, CBS, 2002.

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[15] V. I. Arnold, Mathematical Methods of Classical Mechanics, 2nd ed., Springer-Verlag, 1997.

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Advanced Topology

Learning Objectives

- This course deals with deeper aspects of Point Set Topology.
- It consists of concepts like Separability, Metrizability, Compactification, Nets and Filters etc.
- In this course, the student comes to know the difference between metric spaces and spaces which does not 'admit' a metric; they even come to know under what conditions a topological space can be endowed with a metric.
- Once studied sequences, students can enjoy its generalization nets and filters.
- Students will be delighted to observe how basic geometric structures may be studied by transforming them into algebraic questions.
- Studying geometric objects by associating algebraic invariants to them is a powerful idea which influenced many areas of mathematics. As for example, deciding about the existence of a map between spaces (often a difficult task) may be translated into deciding whether an algebraic equation has a solution (which is often quite simple).

Course Outcomes

Upon successful completion of this course, the students will –

- naturally develop geometric maturity.
- Apart from gaining knowledge in topological concepts, students learn analyzing logic while trying to prove a theorem.

Course Details

- Countability and Separation Axioms: Countability Axioms, The Separation Axioms, Equation spaces, Lindelöf spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze Extension Theorem. [10 Lectures]
- Nets and Filters: Directed Sets, Nets and Subnets, Convergence of a net, Ultranets, Partially Ordered Sets and Filters, Convergence of a filter, Ultrafilters, Basis and Subbase of a filter, Nets and Filters in Topology. [8 Lectures]
- Tychonoff Theorem and Compactification: Tychonoff Theorem, Completely Regular spaces, Local Compactness, One-point compactification, Stone-Cech Compactification. [8 Lectures]
- Metrization: Urysohn Metrization Theorem, Topological Imbedding, Imbedding Theorem of a regular space with countable base in $\mathbf{R}^n$ Partitions of Unity, Topological m-Manifolds, Imbedding Theorem of a compact m-manifold in $\mathbf{R}^n$. . Local Finiteness, Nagata-Smirnov Metrization Theorem, Para-compactness, Stone's Theorem, Local Metrizability, Smirnov Metrization Theorem. Uniform Spaces. [10 Lectures]
- Complete Metric Spaces and Function Spaces: (Optional) Complete Metric Spaces, The Peano Space-Filling Curve, Hahn-Mazurkiewicz Theorem (statement only). Compactness in Metric Spaces, Equicontinuity, Pointwise and Compact Convergence, The Compact-Open Topology, Stone-Weierstrass Theorem, Ascoli's Theorem, Baire Spaces, A Nowhere Differentiable Function. [10 Lectures]

References

[1] J.R. Munkres, Topology: A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

[2] J. Dugundji, Topology, Allyn and Bacon, 1966.

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Mathematical Biology-I

Learning Objectives

The student will –

- To understand the qualitative properties of dynamical system like stability, bifurcation and chaos.
- To know the linear, nonlinear differential equations and difference equations and their solutions.
- To formulate simple growth problems and study of population models.
- To study the continuous population models for single species and interacting population.

Course Outcomes

Upon successful completion of this course, the students will –

- formulate many biological and ecological systems by mathematical modelling.
- study several qualitative properties of the systems.
- making of predictions about the biological systems.
- simulate real-world problems in biological sciences and healthcare industry.

Course Details

Unit I Mathematical Biology and the modeling process An overview. Continuous models: Malthus model, logistic growth, Gompertz growth, Michaelis-Menten Kinetics, Bacterial growth in a Chemostat, Harvesting in a single natural population. [12-16 Lectures]

Unit II: Lotka-Volterra prey-predator interaction Dynamics of interacting populations subject to competition and disease. Holling type growth function, Gauss exclusion principle. Qualitative analysis of continuous models. Steady-state solutions, stability and linearisation. Routh-Hurwitz Criteria. Phase plane analysis. Qualitative study of solutions in terms of limit cycles and local bifurcations with examples in the context of biological scenario. [20-24 Lectures]

Unit III: Difference Equation Overview of difference equations, steady state solution and linear stability analysis, Discrete single population model. Discrete Prey-Predator models. Density-dependent growth models with linear harvesting. Host-Parasitoid systems (Nicholson-Bailey model). Numerical solution of the models and its graphical representation. Case Studies: Optimal exploitation models, Models in Genetics. [16-20 Lectures]

References

[1] L. E. Keshet, Mathematical Models in Biology, SIAM, 1988.

[2] J. D. Murray, Mathematical Biology, Springer, 1993.

[3] Y. C. Fung, Biomechanics, Springer-Verlag, 1990.

[4] F. Brauer, P.V.D. Driessche and J. Wu, Mathematical Epidemiology, Springer, 2008.

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[8] H. I. Freedman, Deterministic mathematical models in population ecology, New York, 1980.

[9] J. M. Smith, Models in Ecology, CUP, 1978.

[10] C. W. Clark, Mathematical Bioeconomics. The optimal management of renewable resources. II Ed., John Wiley and Sons. New York, 1990.

[11] S. Elaydi, An Introduction to Difference Equations, Springer,2004.

Differential Geometry

Learning Objectives

The student will –

- The syllabus is designed to discuss the basic concepts of Differential Geometric proprties of smooth curves and surfaces in 3 dimensional Euclidean space.
- As a prerequisite the reader is supposed to have done a course in Calculus of Several Real Variables and Elementary ideas in Linear Algebra.
- First objective of this course is to introduce the student with the concepts of curves, surfaces and mainly the notion of Curvatures.

Course Outcomes

Upon successful completion of this course, the students will –

- After going through the course, a student will come to know what is meant by curves and Surfaces.
- They will also be able to understand tangent vectors and normal vectors etc.
- In short, they will be matured enough to understand how the notions of curvature helps us to characterise objects geometrically.

Course Details

- Parametric Curves, Arc Length Function, Reparametrisation. [6 – 8 Lectures]
- Curvature of Unit speed Space Curves, Serret Frenet Formulae. [6 – 8 Lectures]
- Smooth Surfaces. Smooth Maps. Tangents and Derivatives, Normals and Orientability. [8 – 10 Lectures]
- First and Second Fundamental Forms, Lengths of Curves on Surfaces. Isometries and Conformal Mappings. [10 - 12 Lectures]
- Normal, Geodesic, Gaussian, Mean and Principal Curvatures. [8 - 10 Lectures]

References

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[2] John McCleary, Geometry from a Differentiable Viewpoint, Cambridge University Press, 1994.

[3] Manfredo P. do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, NJ, 1976.

Mathematical Biology-II

Learning Objectives

The student will –

- To formulate infectious disease modelling and study
- To study the delay differential equation and apply it in the formulation of biological phenomena
- To learn the pattern formation of diffusive model.

Course Outcomes

Upon successful completion of this course, the students will be able to -

- To formulate an infectious disease model and know the study of dynamic phenomena
- To study the diffusion model describing population dispersal
- To formulate the delay model and make predictions of future population dynamics
- To have predictions for future policy making.

Course Details

Unit I Diseases modelling Introduction, simple, general and recurring epidemic. Stochastic epidemic models: with/without removal, with carrier and/or infective. [8-10 Lectures]
Unit II Delay Differential Equations Single species with time lag. Analysis of Oscillatory phenomena, Stability switching. [8-10 Lectures]
Unit III Spatial Models One species model with diffusion. Two species model with diffusion, Conditions for diffusive instability. Formation of Turing Pattern via diffusion-driven instability. [6-8 Lectures]
Unit IV Bio-Mathematics Lab Graphical Interpretations using MatLab/Mathematica for the following model systems:

- Growth model.
- Decay model.
- Limited growth of population with and without harvesting.
- Lotka-Volterra predator-prey model with density dependence, effect of DDT.
- Epidemic model of influenza
- Battle model (basic battle model, jungle warfare, long range weapons). [16-20 Lectures]

References

[1] L. E. Keshet, Mathematical Models in Biology, SIAM, 1988.

[2] J. D. Murray, Mathematical Biology, Springer, 1993.

[3] Y. C. Fung, Biomechanics, Springer-Verlag, 1990.

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[10] C. W. Clark, Mathematical Bioeconomics. The optimal management of renewable resources. II Ed., John Wiley and Sons. New York, 1990.

Dynamical System

Learning Objectives

- acquire knowledge on existence and uniqueness of solutions of dynamical systems.
- be able use the direction fields and isoclines to draw various solution plots for dynamical systems.
- determine the qualitative behaviors of nonlinear dynamical systems around the critical points.
- determine the long term behavior of model systems as the parameters change.
- understand how does the stability change and give rise to bifurcations.

Course Outcomes

Upon successful completion of this course, the students will –

- find the fixed points and linearize the nonlinear system about such equilibria and discuss their stability.
- understand the distinction between hyperbolic and non-hyperbolic fixed points. Sketch a phase portrait of linear and nonlinear dynamical systems.
- use the Poincaré-Bendixson theorem and transformation of Cartesian to polar coordinates to investigate the existence of limit cycles.
- find fixed points and periodic orbits of iterated one-dimensional mappings.

Course Details

Unit I:

- Fundamental theorem for existence and uniqueness of Nonlinear ODEs, Gronwall's inequality, Dependence on initial conditions and parameters, maximal interval of existence, Global existence of solutions, vector fields and flows, Topological conjugacy and equivalence. [16 – 18 Lectures]

Unit II:

- Linear flows on $\mathbb{R}^n$. The matrix exponential.Jordan canonical forms. Linear autonomous dynamical systems, invariant subspaces, stability theory, classification of linear flows, fundamental matrix solution. Non-homogeneous linear systems, periodic linear systems and Floquet theory on non-autonomous dynamical systems. [12 – 12 Lectures]
- Alpha and omega limit sets of an orbit, attractors, periodic orbits and limit cycles. Local structure of critical points (the local stable manifold theorem, the Hartman-Grobman theorem, the center manifold theorem), idea of normal form theory. [12 – 14 Lectures]

Unit III:

- Lyapunov function, local structure of periodic orbits (Poincare map and Floquet theory), the Poincare-Benedixson theorem. Benedixson's nonexistence criterion of linit cycle. Dulac's extension theorem. Lienard systems and number of limit cycles. [12 – 16 Lectures]
- Numerical simulations and sensitivity analysis for the periodic orbits of planer dynamical systems by MATLAB software. [6 – 8 Lectures]

References

[1] L. D. Perko: Differential Equations and Dynamical Systems.

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[4] D. W. Jordan and P. Smith (1998): Nonlinear Ordinary Equations- An Introduction to Dynamical Systems (Third Edition), Oxford Univ. Press.

[5] F. Verhulust, Nonlinear Differential Equations and Dynamical Systems, Springer Verlag, 1996.

[6] G. Iooss and D.D. Joseph, Elementary Stability and Bifurcation Theory, Springer Verlag, 1997.

Partial Differential Equations

Learning Objectives

The student will –

- The main objective of this course is to introduce concepts and techniques for the solution of partial differential equations (PDEs).
- In particular, the course aims to present the method of characteristics for first-order PDEs.
- The theory of separation of variables for Laplace's and the linear wave equation in cylindrical and spherical polar coordinates.
- Green's function methods for the one-dimensional heat equation and two- and three-dimensional waves.

Course Outcomes

Upon successful completion of this course, the students will –

- Formulate and solve the Monge equations for a first order quasi-linear PDE in 2D. Calculate the earliest time at which solutions become multi-valued.
- Construct shock solutions and study their behaviour, in particular calculate shock speeds. Apply the theory to model specific problems in traffic flow.
- Apply separation of variables to construct solutions to Laplace's equation and the wave equation in cylindrical and spherical polar coordinates in terms of Bessel functions and Legendre polynomials.
- Formulate and solve initial value problems for the wave equation in 1D, 2D and 3D.

Course Details

Unit I: Classification of second order Partial Differential Equations (PDEs) The Canonical forms for Hyperbolic, Parabolic and Elliptic equations. Existence, uniqueness and continuous dependence of the solution to the initial and boundary conditions. Dirichlets, Neumann's and mixed problems. Derivation of Laplace, Wave and Diffusion equation. Adjoint operator, Green's Divergence theorem, Lagrange's and Green's identities. [12 - 16 Lectures]

Unit II: Laplace equation

Fundamental solution. Mean value properties of solutions. Maximum-minimum principle and its consequences. Laplace equation in polar, Spherical polar and cylindrical polar coordinates. Standard methods of solution: Separation of variables (Fourier method). Stability theory. Method of solution: theory of Green's function method. [10 - 12 Lectures]

Unit III: Wave equation Occurrence of wave equation in physics. Elementary solution of one-dimensional equation, Riemann method of solution, vibrating membrane: application of calculus of variation. Uniqueness of the solution. Non-homogeneous wave equation. Duhamel's principle. [8 - 10 Lectures]

Unit IV: Diffusion equation Fundamental solution. Occurrence in physics. Elementary solution of diffusion equation. General solution, Cauchy. Method of Characteristics. Maximum-minimum principle and consequences. Green's function method. Application in ecology. [8 - 10 Lectures]

Unit V: Boundary value problems of Dirichlet and Neumann Dirichlet's principle, Dirichlet problem for a rectangle, The Neumann problem for a rectangle, Interior Dirichlet problem for a circle, Exterior Dirichlet problem for a circle, Interior Neumann problem for a circle, Green's function solution of Dirichlet's and Neumann's problem for sphere. [10 - 12 Lectures]

References

[1] I. N. Sneddon, Elements of Partial Differential Equations, McGraw Hill, 1957.

[2] F. John, Partial Differential Equations, Springer, New York, 2012.

[3] T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Pub. House, 2003.

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Advanced Functional Analysis

Learning Objectives

The student will –

- This course is an introduction to L_p- spaces.
- It deals with Banach lattice, Ascoli-Arzel$\grave{a}$ Theorem and its application, The Stone-Weierstrass Theorem and its different versions like algebraic and lattice version.
- Final object is to characterize the compact subsets of $L_p[0,1]$ spaces.

Course Outcomes

Upon successful completion of this course, the students will –

- After going through the course, a student will be familiar with Banach Lattice structure.
- They will also be able to characterize the compact subsets of $C[a,b]$ and L_p spaces.
- In short, they will be able to compare l_p as well as L_p spaces.

Course Details

- A Brief Review of Topology and Continuity: Topological Spaces, Continuous Real-Valued Functions, Separation Properties of Continuous Functions, Ascoli-Arzel$\grave{a}$ Theorem and it's application to Differential Equations, Separation Properties of Continuous Functions, The Stone-Weierstrass Approximation Theorem. [16 – 18 Lectures]
- Normed Spaces And L_p Spaces: Normed Spaces and Banach Spaces, Operators Between Banach Spaces, Linear Functionals, Banach Lattices, L_p - Spaces and Kolmogorov Theorems. [28 – 32 Lectures]

References

[1] C. D. Aliprantis and O. Burkinshaw, Principles of Real Analysis, Harcourt Asia PTE Ltd, 2000.

[2] C. Goffman and G. Pedrick, First Course in Functional Analysis, Prentice Hall of India Pvt, 1996.

[3] Erwin Kreyszig, Introductory Functional Analysis with Applications, Wiley, 2014.

Introduction to Smooth Manifold

Learning Objectives

- The syllabus is designed to discuss the basic concepts of Smooth Manifolds.
- As a prerequisite the reader is supposed to have done a course in Differential Geometry of Curves and Surfaces; and a course in Several Variable Calculus. A smooth manifold is an object which locally looks like n- dimensional Euclidean space and over which we can do Calculus. The motivation lies in the study of Smooth Surfaces.
- First objective of this course is to introduce the student with the concepts of manifolds and submanifolds with several examples. The syllabus focus on extension of calculus from curves and surfaces to its higher dimensional analogues, i.e. manifolds. The notion of vector fields is discussed in several ways. The concept of differential forms is also discussed. Notions of Lie Groups is also to be discussed so that a student can interlink the topological and algebraic structures on Manifolds.

Course Outcomes

Upon successful completion of this course, the students will –

- After going through the course, a student will come to know what is meant by manifolds, submanifolds.
- They will also be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate free approach.
- In short, they will be matured enough to enter into the realm of Riemannian Geometry..

Course Details

- A Brief Review of Calculus of Several Real Variables: Partial derivatives and Directional Derivatives, Differentiability and Chain Rule, Real Analytic Functions, Taylor's Theorem with Remainder. [6 – 8 Lectures]
- Tangent Vectors in Euclidean Spaces as Derivations Germs of Functions, Derivations at a Point, Vector Fields, Vector Fields as Derivations. [6 – 8 Lectures]
- The Exterior Algebra of Multi-covectors Dual Space, Multilinear Functions, Permutation Action on Multilinear Functions, Symmetrizing and Alternating Operators, Tensor Product and Wedge Product, Anti-commutativity and Associativity of Wedge Product, Basis for k-vectors. [12 – 14 Lectures]

- Differential Forms on Euclidean Spaces Differential 1-forms, Differential of a function, Differential k-forms, Differential Forms as Multilinear Functions on Vector Fields, The Exterior Derivative, Closed Forms and Exact Forms, Convention on Subscripts and Superscripts. [12 – 14 Lectures]

- Manifolds Topological Manifolds, Compatible Charts, Smooth Manifolds, Examples of Smooth Manifolds. [8 Lectures]

- Smooth Maps on a Manifold Smooth Functions on a Manifold, Smooth Maps between Manifolds, Diffeomorphisms, Smoothness in terms of Components, Partial Derivatives, The Inverse Function Theorem. [8 Lectures]

References

[1] Loring W. Tu, An Introduction to Manifolds, Springer, 2011.

[2] S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindustan Book Agency (Text and Readings in Mathematics 22), 2002.

[3] John M. Lee, Introduction to Manifolds, 2nd edition, Springer, 2013.

Mechanics of Continua

Learning Objectives

The objective of the course is

- To expose the students to the foundation in Continuum Mechanics.

- To learn the conservation principles and derive the equations governing the mechanics of continuum

- To learn the constitutive equations for solid and fluid

- To develop practical skills in working with tensors

- To develop problem solving skills, applying the conservation principles and the constitutive equations to solve practical engineering problems.

Course Outcomes

- After attending this course, the students will be able to appreciate a wide variety of advanced courses in solid and fluid mechanics.

Course Details

- Analysis of Stress: The Continuum Concept , Homogeneity. Isotropy. Mass-Density, Body Forces. Surface Forces, Cauchy's Stress Principle. The Stress Vector, State of Stress at a Point. Stress Tensor, The Stress Tensor—Stress Vector Relationship, Force and Moment. Equilibrium. Stress Tensor Symmetry, Stress Transformation Laws , Stress Quadric of Cauchy, Principal Stresses. Stress Invariants. Stress Ellipsoid, Maximum and Minimum Shear Stress Values, Mohr's Circles for Stress, Plane Stress, Deviator and Spherical Stress Tensors. [10-12 Lectures]

- Deformation and Strain: Particles and Points, Continuum Configuration. Deformation and Flow Concepts, Position Vector. Displacement Vector, Lagrangian and Eulerian Descriptions, Deformation Gradients. Displacement Gradients, Deformation Tensors. Finite Strain Tensors, Small Deformation Theory. Infinitesimal Strain Tensors, Relative Displacements. Linear Rotation Tensor. Rotation Vector, Interpretation of the Linear Strain Tensors, Stretch Ratio. Finite Strain Interpretation, Stretch Tensors. Rotation Tensor, Transformation Properties of Strain Tensors, Principal Strains. Strain Invariants. Cubical Dilatation, Spherical and Deviator Strain Tensors, Plane Strain. Mohr's Circles for Strain, Compatibility Equations for Linear Strains. [14-16 Lectures]

- Motion and Flow: Motion. Flow. Material Derivative, Velocity. Acceleration. Instantaneous Velocity Field, Path Lines. Stream Lines. Steady Motion, Rate of Deformation. Vorticity. Natural Strain, Physical Interpretation of Rate of Deformation and Vorticity Tensors, Material Derivatives of Volume, Area and Line Elements, Material Derivatives of Volume, Surface and Line Integrals. [10-12 Lectures]

- Fundamental Laws and of Continuum Mechanics: Conservation of Mass. Continuity Equation, Linear Momentum Principle. Equations of Motion. Equilibrium Equations, Moment of Momentum (Angular Momentum) Principle, Conservation of Energy. First Law of Thermodynamics. Energy Equation, Equations of State. Entropy. Second Law of Thermodynamics, The Clausius-Duhem Inequality. Dissipation Function, Constitutive Equations. Thermomechanical and Mechanical Continua. [14-16 Lectures]

References

[1] G.E. Mase, Theory and Problems of Continuum Mechanics, Schaums Outline Series, Mcgraw Hill Book Company, 1970.

[2] R.N. Chatterjee, Mathematical Theory of Continuum Mechanics, Narosa, 2002.

[3] J.N. Reddy, Principles of Continuum Mechanics, Cambridge University Press, 2010.

[4] Y.C. Fung, A First Course in Continuum Mechanics, Prentice Hall, 1977.

[5] R.C. Batra, Elements of Continuum Mechanics, AIAA, 2005.

[6] W.M. Lai, D. Rubin, E. Krempl, Continuum Mechanics, Butterworth Heinemann, 1999.

[7] S. Nair, Introduction to Continuum Mechanics, Cambridge University Press, 2009.

[8] J. L. Wegner, J. B. Haddow, Elements of Continuum Mechanics and Thermodynamics, Cambridge University Press, 2009.

[9] D.S. Chandrasekharaiah, L. Debnath, Continuum Mechanics, Academic Press, 1994.

[10] T. J. Chung, Applied Continuum Mechanics, Cambridge University Press, 1996.

[11] A.C. Eringen, Mechanics of continua, Robert E. Krieger Publishing Company, 1980.

[12] L.E. Malvern, Introduction to the Mechanics of a Continuous Medium, Prentice-Hall, Inc, 1977.

[13] L.I. Sedov, Introduction to the Mechanics of a Continuous Medium, Addison Wesley Publishing Company, 1965.

Introduction to Operator Theory

The student will –

- be introduced to some topics of operator theory (with an emphasis on spectral theory) and to the fundamentals of Banach algebra theory.

Course Outcomes

Upon successful completion of this course, the students will –

- This is an introductory course in Operator Theory. It will introduce the student to terms, concepts and results for bounded linear operators which are commonly used in this particular area of Mathematics.
- Prove the continuity of concrete linear operators between topological vector spaces.
- Given a linear operator, understand whether or not it is compact.
- The essential spectra of linear operators.
- Find the maximal spectra of concrete commutative Banach algebras.
- Describe the functional calculi and the spectral decompositions of concrete selfadjoint operators and Compact Operators.
- Able to apply spectral theorems of bounded selfadjoint linear operators, Spectral theorem of normal operators.
- Able to apply Spectral theorem for unitary and selfadjoint operators, Multiplication operator and differentiation operators.
- Efficient to calculate the numerical range of some special operators.
- It will also introduce the students which are relevant to current research and prepare the student to persue such a career.

Course Details

- Bounded linear Operators: Resolvent set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Specrtral properties of a bounded linear operator, Spectral mapping theorem for polynomials. Numerical range, Numerical radius, Convexity of numerical range, Closure of numerical range contains the spectrum, Relation between the numerical radius and norm of a bounded linear operator. [12 – 14 Lectures]
- Banach Algebra: Definition of normed and Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an element, The spectral radius. [8 – 12 Lectures]
- Compact linear operators: Spectral properties of compact linear operators on a normed linear space, Operator equations involving compact linear operators, Fredholm alternative theorem, Fredholm alternative for integral equations. Specrtral theorem for compact normal operators. [12 – 16 Lectures]

References

[1] Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and sons, 2014

[2] G. Bachman and L. Narici, Functional Analysis, Dover Publications, 2012

[3] A . Taylor and D. Lay, Introduction to Functional Analysis, John Wiley and Sons, 1980

[4] N. Dunford and J.T. Schwarts, Linear Operators – Part III, Wiley, 1988

[5] Y. Eidelman, V. Milman, and A. Tsolomitis, Functional Analysis An Introduction, American Mathematical Society, 2004

Advanced Complex Analysis

Learning Objectives

- Understand normal families and related theorems.
- Study entire functions and meromorphic functions.
- Study the Nevanlinna's Theory

Course Outcomes

Upon the successful completion of the course a student will

- Acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- Develop a deep understanding of the concepts and theorems in Complex Analysis.
- Acquire necessary skills to research on various topic of Complex Analysis such as Nevanlinna's Theory, Order and Grouth properties of meromorphic functions, Uniqueness theory of meromorphic functions etc.

Course Details

- Harmonic Functions, Comparison with Analytic Function, Poisson Integral Formula, Positive Harmonic Functions. [10-12 Lectures]
- Conformal Mappings, Normal Families, Riemann Mapping Theorem, The Class $\mathbb{S}$. [10-12 Lectures]
- Entire and Meromorphic Functions, Nevalinna's Theory, Infinite Products, Weierstrass' Product Theorem, Mittag-Leffler Theorem. [20-24 Lectures]
- Analytic Continuation and Special Functions. [10-12 Lectures]

References

[1] S. Ponnusamy and H. Silverman, Complex Variable with Applications, Birkhauser

[2] J. W. Brown and R. V. Churchill, Complex Variables and Applications, 8th Ed., McGraw – Hill International Edition, 2009.

[3] J. Bak and D. J. Newman, Complex Analysis, 2nd Ed., Undergraduate Texts in Mathematics, Springer-Verlag New York, Inc., NewYork, 1997.

[4] J. B. Conway, Functions of one Complex Variable, Springer

[5] L. V. Ahlfors, Complex Analysis, Mc GrawHill

[6] S. Ponnusamy, Foundation of Complex Analysis, Narosa Publications

[7] H. S. Kasana, Complex Variables: Theory and Applications, PHI.

[8] A. I. Markushevich, Theory of functions of a Conrplex Variable, (Vol. I, II, III),2nd Edition, AMS,200l.

[9] W. K. Hayman, Meromorphic Functions, New Edition, Oxford University Press, 1968.

[10] L. Yang, Value Distribution Theory, First Edition, Springer, 1993.

[11] C. C. Yang and Y. X. Yi,Uniqueness Theory of Meromorphic Functions,3rd Edition, Kluner Academic Publishers,2003.