Aliah University
Department of Mathematics and Statistics

Syllabus for Ph.D. Course work

2020

Sl. No.	Course Code	Course Title	Classes per week			Credit	Full Marks
			L	T	P		
1	PHD/RM-01	Research Methodology	4	0	0	4	50
2	PHD/RPE-02	Research and Publication Ethics	1	0	1	2	50
3	PHD/LR-03	Literature Review, Report and Seminar Presentation	1	2	1	4	50
4	PHD/SP-04	Subject Paper	4	0	0	4	50

List of Subject Papers:

1. Advanced Differential Equations
2. Advanced Dynamical Systems
3. Advanced Linear Algebra
4. Advanced Reliability Theory
5. Algebraic Graph Theory
6. Algorithms and Complexity
7. An Introduction to Riemannian Geometry
8. Analysis
9. Banach Algebras, C^* algebras and Spectral Theory
10. Calculus of Several Variables
11. Commutative Algebra
12. Complex Analysis
13. Fixed Point Theorems
14. Functional Analysis
15. Introduction to Smooth Manifolds
16. Mathematical Biology
17. Semigroup Theory
18. Solution Methods in Generalized Thermoelasticity
19. Statistics – I
20. Statistics – II
21. Survival Analysis

Detailed Syllabus

Research Methodology (PHD/RM-01)

Introduction: meaning of research, objectives of research, motivation in research, research approaches, significance of research.

Foundations of Research: Empiricism, deductive and inductive theory.

Types of research: descriptive vs. analytical, applied vs. fundamental, quantitative vs. qualitative, conceptual vs. empirical.

Research methods versus methodology: research and scientific method, importance of knowing how research is done, research process, criteria of good research, problems encountered by researchers in India.

Research problem: defining a research problem, selecting the problem, necessity of defining the problem technique involved in defining a problem; Hypothesis – qualities of a good hypothesis, null hypothesis and alternative hypothesis, testing of hypothesis.

Data collection and analysis: Observation and collection of data, method of data collections; Sampling method: concepts of statistical population, sample, sampling frame, sampling error, sample size, non response; Univariate analysis: frequency tables, bar charts, pie charts, percentages; Chi-square test including testing hypothesis of association.

Research tools: searching Google (query modifiers), MathSciNet, ZMATH, Scopus, ISI Web of Science, Impact Factor, h-index, Google Scholar, ORCID, JStor, Online and open access Journals, ArXiv and other repositories.

Software for Mathematics/ Statistics: Mathematica/ Matlab/ Scilab/ SAS/ SPSS.

References:

1. C.R Kothari, Research Methodology - Methods and Techniques, Second revised Edition, New Age International Publishers, 2004.

2. Rama Nand Singh, Research Methodology and Techniques in Mathematics, Centrum Press, 2012.

3. Alexander Zaretsky, Research Methodology and Techniques in Mathematics, Delve Publishing LLC, 2015.

4. Brian R. et al., A Guide to Matlab for Beginners and Experienced Users, Cambridge University Press, 2001.

5. Stephen J. Chapman, Matlab Programming for Engineers, Cengage Learning, 2003.

Research and Publication Ethics (PHD/RPE-02)

Course Objective: This course has 6 modules mainly focusing on basics of philosophy of science and ethics, research integrity, publication ethics. Hands on sessions are designed to identify research misconduct and predatory publications. Indexing and citation databases, open access publications, research metrics and plagiarism tools will be introduced in the course.

Theory

Philosophy and Ethics (3 Hrs)

1. Introduction to philosophy: definition, nature and scope, concept, branches.
2. Ethics: definition, moral philosophy, nature of moral judgments and reactions.

Scientific Conduct (5 Hrs)

1. Ethics with respect to science and research.
2. Intellectual honest and research integrity.
3. Scientific misconducts: falsification, fabrication, and plagiarism.
4. Redundant publications: duplicate and overlapping publications, salami slicing.
5. Selective reporting and misrepresentation of data.

Publication Ethics (7 Hrs)

1. Publication ethics: definition, introduction and importance.
2. Best practices/ standards setting initiatives and guidelines: COPE, WAME, etc.
3. Conflicts of interest.
4. Publication misconduct: definition, concept, problems that lead to unethical behavior and vice versa, types.
5. Violation of publication ethics, authorship and contributorship.
6. Identification of publication misconduct, complaints and appeals.
7. Predatory publishers and journals.

Practice

Open Access Publishing (4 Hrs)

1. Open access publications and initiatives.
2. SHERPA/RoMEO online resource to check publisher copyright and self-archiving policies.
3. Software tool to identify predatory publications developed by SPPU.

4. Journal finder/ journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.

Publication Misconduct (4 Hrs)

A. Group Discussions (2 Hrs)

1. Subject specific ethical issues, FFP, authorship
2. Conflicts of interest
3. Complaints and appeals: examples and fraud from India and abroad

B. Software tools (2 Hrs)

Use of plagiarism software like Turnitin, Urkund and other open source software tools.

Databases and Research Metrics (7 Hrs)

A. Databases (4 hrs)

1. Indexing databases.
2. Citation databases: Web of Science, Scopus, etc.

B. Research Metrics (3 hrs)

1. Impact Factor of journals as per journal citation report: SNIP, SJR, IPP, Cite Score.
2. Metrics: h-index, g index, i10 index, alt-metrics.

References:

1. Bird, A. (2006). Philosophy of Science. Routledge.
2. MacIntyre, Alasdair (1967). A Short History of Ethics. London.
3. Chaddah, P. (2018). Ethics in Competitive Research: Do not get Scooped; do not get Plagiarized. ISBN: 978-938748086
4. National Academy of Sciences, National Academy of Engineering and Institute of Medicine (2009). On Being a Scientist: A Guide to Responsible Conduct in Research: Third Edition. National Academies Press.
5. Resnik, D.B. (2011). What is Ethics in Research & Why is it Important. National Institute of Environmental Health Sciences, 1-10.https://www.niehs.nih.gov/research/resources/bioethics/whatis/index.cfm
6. Beall, J. (2012). Predatory publishers are corrupting open access. Nature, 489(7415), 179-179. https://doi.org/10.1038/489179a
7. Indian National Science Academy (INSA) (2019). Ethics in Science Education, Research and Governance. ISBN: 978-81-939482-1-7. http://www.insaindia.res.in/pdf/Ethics_Book.pdf

Literature Review, Report and Seminar Presentation (PHD/LR-03)

Review of a scientific research paper: Studying a research paper and writing a review of the same, identifying any new problem, question, and direction emanating from the paper.

Scientific writing and presentation: Writing a research paper, survey article, thesis writing.

Scientific document preparation software: LaTeX, PSTricks, Beamer, HTML and MathJaX .

Research proposal: Writing a detailed proposal of research including a thorough review of literature on a topic of choice and presentation of the same.

References:

1. L. Lamport, LaTeX, a Document Preparation System, 2nd ed, Addison-Wesley, 1994.
2. Norman E. Steenrod, Paul R. Halmos, Menahem M. Schiffer, Jean A. Dieudonne, How to Write Mathematics, American Mathematical Society, 1973.
3. Nicholas J. Higham, Handbook of Writing for the Mathematical Sciences, Second Edition, SIAM, 1998.
4. Donald E. Knuth, Tracy L. Larrabee, and Paul M. Roberts, Mathematical Writing, Mathematical Association of America Washington, D.C., 1989.
5. Frank Mittelbach, Michel Goossens, Johannes Braams, David Carlisle, Chris Rowley, The LaTeX Companion, 2nd edition (TTCT series), Addison-Wesley, 2004.
6. Michel Goossens, Frank Mittelbach, Sebastian Rahtz, Denis Roegel, Herbert Voss, The LaTeX Graphics Companion, 2nd edition (TTCT series), Addison-Wesley, 2004
7. Mathtools documentation (http://mirrors.ctan.org/macros/latex/contrib/mathtools/mathtools.pdf)
8. Pstricks documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)
9. MathJax documentation (http://tug.org/PSTricks/main.cgi?file=doc/docs)

Advanced Differential Equations (PHD/SP-04)

Ordinary Differential Equations:

Review of existence and uniqueness of solutions of Initial value problems for system of first order differential equations. Existence and Uniqueness theorem for a linear system; homogeneous and inhomogeneous linear systems; linear equations with constant coefficients; Fundamental matrix Linear differential equations with periodic coefficients: Floquet theory Stability for linear systems. Principle of linearised stability. Stability for autonomous systems. liapunov functions. Plane autonomous systems Periodic solutions of plane autonomous systems.

Partial Differential Equations:

Review of method of characteristics for first order partial differential equations and classification of second order partial differential equations Nonlinear first order partial differential equations. Conservation laws. Lax-Oleinik . formula. Riemann's problem.Long time behavior Separation of variables. Similarity solutions. Transform methods. Converting nonlinear partial differential equations into linear partial differential equation. Asymptotics. Maximum principles.

References:

1. R. Grimshaw, Nonlinear ordinary differential equations, Blackwell Scientific publications, 1990.
2. Lawrence C. Evans, Partial Differential Equations, American Mathematical Society, 1991.
3. David Betounes, Differential equations: Theory and applications, Springer, 2010.
4. L. Perko, Differential equations and dynamical systems, Springer, 2001.
5. Mathew P. Coleman, An introduction to partial differential equations with Matlab, CRC Press, 2005.
6. Sandro Salsa, Partial differential equations in action: From modelling to theory, Springer, 2008.

Advanced Dynamical Systems (PHD/SP-04)

Linear dynamical systems: The Fundamental Theorem for Linear Systems. Linear Systems in $\boldsymbol{R}^n$. Stability Theory. Nonhomogeneous Linear Systems and their solutions. [12 -16 Lectures]

Nonlinear dynamical systems: The fundamental existence-uniqueness theorem. Dependence on initial conditions and parameters. The maximal interval of existence. The flow defined by a differential equation. The stable manifold theorem. The Statement and proof of Hartman-Grobman theorem. Global stability and Liapunov functions. Qualitative natures: Saddles, Nodes, Foci, Centers, Saddle-Node, Center-Focus, Cusp singularities, Periodic orbits and Limit Cycles of nonlinear dynamical systems. [20-22 Lectures]

Global theory of nonlinear systems: Hyperbolic and non-hyperbolic critical points in $\boldsymbol{R}^n$ ($n > 2$). Stable, unstable and center spaces for dynamical systems. Stable, unstable and center manifold theory on $\boldsymbol{R}^n$. Normal Form Theory. Periodic orbits. Classifications of Limit cycles and separatrix cycles. The Poincare map. The stable and unstable manifold theorems for periodic orbits. The Poincare-Bendixson theory in $\boldsymbol{R}^2$. Lienard systems. Bendixson's criteria. The Poincare sphere and the behavior at infinity. [16-18 Lectures]

Bifurcation theory of nonlinear systems: Structural stability and Peixoto's theorem. Bifurcations at non-hyperbolic equilibrium points. Higher co-dimension bifurcations at non-hyperbolic equilibrium points. Hopf bifurcations and bifurcations of limit cycles from a multiple focus. Bifurcations at non-hyperbolic periodic orbits. [12-16 Lectures]

References:

1. Guckenheimer, J., Holms, P. (1983). Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer, New York.
2. Zhi-fen, Z., Tong-ren, D., Wen-zao, H., Zhen-xi, D. (1992). Qualitative Theory of Differential Equations, American Mathematical Society, Rhode Island.
3. Perko, L. (1996). Differential Equations and Dynamical Systems, Springer, New York.
4. Robinson, C. (1998). Dynamical Systems: Stability, Symbolic Dynamics, and Chaos (Studies in Advanced Mathematics), CRC Press, Boca Raton.
5. Wiggins, S. (2003). Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer, New York.
6. Kuznetsov, Y.A. (2013). Elements of applied bifurcation theory, Springer, New York.

Advanced Linear Algebra (PHD/SP-04)

Eigenvalues and eigenvectors, Schur's theorem - real and complex versions, Spectral theorems for normal and Hermitian matrices - real and complex versions. Gerschgorin discs with associated perturbation theorems and inclusion results. Jordan canonical forms with application, minimal polynomials, companion matrices. Functions of matrices via spectral decompositions. Variational characterizations of eigenvalues of Hermitian matrices, Rayleigh-Ritz theorem, Courant-Fischer theorem, Weyl theorem, Cauchy interlacing theorem, Inertia and congruence, Sylvester's law of inertia. Matrix norms, spectral radius formula, relationships between matrix norms. Singular value decomposition, polar decomposition. Positive definite matrices, characterizations of definiteness, polar form and singular value decompositions, congruence and simultaneous diagonalization.

References:

1. R. A. Horn and C. R. Johnson, Matrix Analysis, CUP, 1985.
2. S. Axler, Linear Algebra Done Right, 2nd Edition, UTM, Springer, Indian Edition, 2010.
3. P. Lancaster and M. Tismenetsky, The Theory of Matrices, Second edition, Academic Press, 1985.
4. F. R. Gantmacher, The Theory of Matrices, Vol-I, Chelsea, 1959.

Advanced Reliability Theory (PHD/SP-04)

Life-distributions, reliability function, hazard rate, common univariate life distributions – exponential, Weibull, Gamma, Generalized Exponential, Lomax, Burr – XII, Birnbaum-Saunders, Rayleigh, Bivariate Exponential, Marshall-Olkin Bivariate Exponential, Marshall-Olkin Bivariate Weibull. Different hybrid and progressive censoring schemes – Inference of parametric lifetime models under different censoring schemes – Dependent and independent competing risk model and the related classical and Bayesian inferences.

Accelerated Life Test - Step stress model – Models to analyze step-stress data – Cumulative exposure model, tempered failure rate model, tempered random variable model – Classical and Bayesian inference of step-stress models under different censoring scheme – Step-stress competing risk model – Optimal design of step-stress model, A-optimality, D-optimality, Bayes-optimality.

References:

1. P.J. Smith: Analysis of Failure and Survival Data.
2. J.D. Kalbfleisch & R.L. Prentiice: The Statistical Analysis of Failure Time Data, 2nd ed.

3. D. Kundu & A. Ganguly: Analysis of step-stress models: existing methods and recent developments, Elsevier/ Academic Press.
4. Wayne Nelson: Accelerated Testing, Statistical Models, Test Plans and Data Analysis, John Wiley & Sons.

Algebraic Graph Theory (PHD/SP-04)

Elements of Linear Algebra: Simultaneous diagonalization, Perron-Frobenius theory, Equitable partitions, Equitable and almost equitable partitions of graphs, Rayleigh quotient, Interlacing, Schur's inequality, Schur complements, Courant-Weyl inequalities, Gram matrices, Diagonally dominant matrices, Geršgorin circles.

Graph Spectrum: Matrices associated to a graph, spectrum of a graph, characteristic polynomial, spectrum of – complete graph, complete bipartite graph, cycle, path, line graphs, Cartesian products, strong products.

Eigenvalues and eigenvectors of graphs: Largest eigenvalue, bounds, energy of a graph, graphs with largest eigenvalue at most 2, interlacing, co-spectral graphs – Siedel switching, Godsil-MacKay switching.

Laplacian matrix of a graph: Laplacian spectral radius, Algebraic connectivity, bounds, Laplacian energy, Laplacian eigenvalues and degrees, Grone-Merris conjecture, threshold graph.

Strongly regular graphs: Adjacency matrix and eigenvalues of a strongly regular graph, examples, Paley graph, Latin square graph.

Distance matrix of a graph: distance regular graph, distance spectrum of a tree.

Graphs from algebraic structures: Cayley graph from groups - Construction and Recognition, Isomorphism, Graphs from commutative rings - zero divisor graph, Cayley graph, Cayley sum graph, Total graph, Unit graph, Standard comaximal graph.

References:

1. R. B. Bapat, Graphs and Matrices, Universitext, Springer, Hindustan Book Agency, New Delhi, 2010.
2. A. E. Brouwer, W. H. Haemers, Spectra of graphs, Springer, 2011.
3. N. Biggs, Algebraic Graph Theory, Cambridge University Press, 1993.
4. C. Godsil, G. Royle, Algebraic Graph Theory, Graduate Texts in Mathematics 207, Springer-Verlag, 2001.

Algorithms and Complexity (PHD/SP-04)

Searching and Sorting: Insertion search, binary search, selection sort, Quicksort, mergesort, heapsort, comparison of sorting algorithms.

Data structures and algorithms for sets: Balanced binary trees, 2-3 trees, *B*-trees, structures for sets, Hashing.

Graph algorithms: Definition and implementations, searching – BFS, DFS, pre-order, in-order and post-order traversal, connected components, topological sort, shortest path problems, spanning trees, network flow, matching.

Algorithm Design techniques: Greedy algorithms, divide and conquer, dynamic programming, randomized algorithm.

Introduction to complexity Theory: Asymptotic notations - o, O, θ, Θ, ω, Ω and their properties, P, NP, NP-hard, NP-completeness.

Interval and related graphs: Perfect elimination orders, algorithms for graphs with a p.e.o., chordal graphs, comparability graphs, circular-arc graphs, boxicity graphs, *t*-interval graphs, perfect graphs.

Tree-width and relatives: tree-decompositions, tree-width, computing tree-width, algorithms, branch-width, path-width.

Planar and related graphs: Clique and colouring problems in planar graphs, NP-hard problems on planar graphs, series parallel graphs, outerplanar graphs.

References:

1. Martin Charles Golumbic, Algorithmic Graph Theory and Perfect Graphs, 2nd Edition, Annals of Discrete Mathematics Series: Volume 57, North-Holland, 2004.
2. Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest, Clifford Stein, Introduction to Algorithms, MIT Press, Cambridge, 2009.
3. Sanjoy Dasgupta, Christos Papadimitriou, and Umesh Vazirani. Algorithms. McGraw-Hill, 2008.
4. Steve Kleinberg and Eva Tardos, Algorithm Design, Pearson Education, 2014.

An Introduction to Riemannian Geometry (PHD/SP-04)

An Introduction to Curvature: Curvature for Plane Curves and Space Curves, Curvature of Surfaces: Principal Curvature, Gaussian Curvature, Gauss's Theorema Eggregium, Gauss Bonnet Theorem, Curvature in Higher Dimensions.

Tensors, Manifolds and Vector Bundles: Tensors as Multilinear Maps, Tensor Product and Wedge Product, Manifolds, Smooth Functions on a Manifold, Smooth Maps between Manifolds, Diffeomorphisms, Smoothness in Terms of Components, Partial Derivatives, The Inverse Function Theorem, Vector Bundles and Fibres, Tensor Bundles and Tensor Fields.

Riemannian Metrics: Riemannian Metrics, Raising and Lowering Indices, Inner Product of Tensors, Volume Element and Integration, Models of Riemannian Manifolds.

Connections: Connections, Linear Connections, Existence of Connections, Vector Fields along Curves, Covariant Derivatives along Curves, Geodesics – their Existence and Uniqueness, Parallel Translation, Existence and Uniqueness of Solutions of Linear ODEs.

Riemannian Geodesics: the Riemannian connection, fundamental lemma of Riemannian geometry, naturality of the Riemannian connection, the exponential map, rescaling lemma, normal neighbourhoods and normal components.

Geodesics and Distance: Lengths of Curves, The Distance Function, Geodesics and Minimising Curves, Hopf Rinow Theorem.

Curvature: local invariants, curvature endomorphism, flat manifolds, symmetries of the curvature tensor, bianchi identity, ricci and scalar curvatures, contracted bianchi identity.

References:

1. Introduction to Riemannian Manifolds by John M. Lee, Springer, 2018.
2. An Introduction to Differentiable Manifolds and Riemannian Geometry (Revised 2e) by William M. Boothby, Elsevier Academic Press, 2003.

Analysis (PHD/SP-04)

Metric spaces, Open and closed sets, Compactness and connectedness, Completeness, Continuous functions (several variables and on metric spaces), uniform continuity C(X), X, compact metric space, Uniform convergence, compactness criterion, Differentiation, Inverse and Implicit function theorems. Riemann Integration, Lebesgue Integration, L^p spaces. Complex Analysis: Analytic functions, Harmonic conjugates, Cauchy theorems and consequences, Power series, Zeros of analytic functions, Maximum modulus theorem, Singularities, Laurent series, Residues. Mobius transformations. Hilbert spaces: Inner product, Orthogonality, Orthonormal bases, Riesz Lemma, The space L^2 as a Hilbert space.

References:

1. R. Bhatia: Notes on Functional Analysis (Hindustan Book Agency)
2. J. B. Conway: A Course in Functional Analysis, 2nd edition (Springer low price edition)
3. B. V. Limaye: Functional Analysis, 3rd edition (New Age Publishers)
4. M. Schechter: Principles of Functional Analysis, 2nd edition (Universities Press)
5. S. Lang, Complex Analysis, 4th Ed., Springer, 1999.

Banach Algebras, C^* Algebras and Spectral Theory (PHD/SP-04)

Basics of Banach Algebras and C*-algebras, examples, spectrum. Commutative Banach Algebra and C*-algebras; maximal ideal spaces, Gelfand Transform; normal elements, continuous functional calculus. Representations of C*-algebras, Von Neumann Algebras, WOT and SOT, density theorems, double commutant theorem. Spectral Theorem for normal operators. Abelian Von Neumann algebras.

References:

1. Conway: Operator Theory.
2. Davidson: C*-algebras by Examples.

Calculus of Several Variables (PHD/SP-04)

Functions on Euclidean Spaces: Norm and Inner Product, Subsets of Euclidean Spaces, Functions and Continuity.

Differentiation: Notions of Differentiability, Partial Derivatives, Jacobian matrix, Chain rule, Inverse Function Theorem, Implicit Function Theorem.

Integration: Partitions, Upper and Lower Sums, Measure Zero and Content Zero, Integrable Functions, Fubini’s Theorem, Partitions of Unity, Change of Variables.

Integration on Chains: Multilinear Functions, Tensor Product, Orientability, Wedge Product, Fields and Forms, Differentiable Forms, Differentials, Closed and Exact Forms, Poincare Lemma, Singular n-cube, Boundary, Fundamental Theorem of Calculus.

Smooth Manifolds: Topological Manifolds, Smooth Charts, Smooth Manifolds, Examples of Smooth Manifolds.

Integration On Manifolds: Fields and Forms on Manifolds, Stoke's Theorem on Manifolds, The Volume Element, Green's Theorem, Stoke's Theorem.

References:

1. Calculus on Manifolds by Michael Spivak, Westview Press, 1998.
2. A Course in Multivariable Calculus and Analysis by Sudhir R. Ghorpade and Balmohan V. Limaye, Springer, 2010.
3. Analysis on Manifolds by James R. Munkres, Westview Press, 1991.

Commutative Algebra (PHD/SP-04)

Prime and maximal ideals, Chinese Remainder theorem, nilradical and Jacobson radical, prime avoidance. Free modules, projective modules, tensor product of modules, flat modules. Localisation and local rings. Polynomial and power series rings. Noetherian rings and modules, primary decomposition. Artininan modules, modules of finite length. Integral extensions, Going-up theorem, integrally closed domains, Going-down theorem. Hilbert's Nullstellensatz. Valuation rings, discrete valuation Rings, Dedekind domains.

References :

1. Introduction to Commutative Algebra , M. F. Atiyah and I. G. Macdonald, Addison-Wesley, 1994.
2. Undergraduate Commutative Algebra, Miles Reid, Cambridge University Press, 1995.
3. Commutative Algebra, N. S. Gopalakrishnan, Universities Press, 2016.

Complex Analysis (PHD/SP-04)

Module-I: Equicontinuity, Arzela-Ascoli theorem, Schwartz's lemma, Montel's theorem, normal families, Riemann mapping theorem. [10 Lectures]

Module-II: Dirichlet problem for harmonic functions, Poisson integral formula, Jensen's formula, infinite products, Hadamard factorization theorem. [10 Lectures]

Module-III: Meromorphic functions, Definition of $m(r,a)$, $N(r,a)$ and $T(r)$. Nevanlinna's first fundamental theorem. Cartan's identity and convexity theorems. Order of a meromorphic function. Comparative growth of $\log M(r)$ and $T(r)$. Nevanlinna's second fundamental theorem. Estimation of $S(r)$. Nevanlinna's theorem on deficient functions. Nevanlinna's five-point uniqueness theorem. Milloux theorem. Milloux basic result. [20 Lectures]

Module-IV: Functions of several complex variables. Power series in several complex variables. Region of convergence of power series. Associated radii of convergence. [4 Lectures]

Module-V: Analytic functions of several complex variables, Cauchy-Riemann equations, Cauchy's integral formula, Taylor's expansion, Cauchy's inequalities. Zeros and Singularities of analytic functions of several complex variables, Maximum modulus theorem, Weierstrass preparation theorem. Analytic sets, Analytic set germs, Regular and singular points of analytic sets.

[16 Lectures]

References:

1. Ahlfors, Complex Analysis

2. J.B.Conway, Functions of One Complex Variable.

3. A. I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III).

4. W. K. Hayman, Meromorphic Functions.

5. L. Yang, Value Distribution Theory.

6. C.C. Yang and Y. X. Yi, Uniqueness theory of meromorphic functions.

7. An Introduction to Complex Analysis in Several Variables, 3rd Edition L. Hörmander, North Holland Ed.

8. Holomorphic Functions and Integral Representations in Several Complex Variables, M. Range, Springer-Verlag Ed.

9. Function Theory of Several Complex Variables, S. G. Krantz, AMS Chelsea Publishing, 2001.

10. R. Narasimhan, Several Complex Variables.

11. M. Herve, Several Complex Variables.

12. Partial differential equations in several complex variables, So-Chin Chen and Mei-Chi Shaw, American Mathematical Society, Providence, RI, 2001.

13. Bochner and Martin. Several Complex Variables.

14. R. C. Gunning and H. Rossi, Analytic Functions of Several Complex Variables.

15. B. A. Fuks. An Introduction to the Theory of Analytic Functions of Several Complex Variables.

16. Functions of several complex variables and their singularities, Wolfgang Ebeling.

Fixed Point Theorems (PHD/SP-04)

Contractions, Banach Contraction Principle, Theorem of Edelstein, Picard–Lindelof Theorem. Non expansive Maps, Schauder'sTheorem for non–expansive maps, Continuation Methods for Contractive and non–expansive mappings.

Some Applications of The Banach Contraction Principle, Some Extensions of Banach Contraction Principle for Single – Valued Mappings, Generalized distances, Some Extensions of Banach Contraction Principle under Generalized Distances, Multivalued versions of Banach Contraction Principle.

References:

1. Vittorino Patta, Fixed Point Theorems and Applications, Spinger, 2019.

2. Y. J. Chao, Fixed Point Theory and Applications, Nova Publishers, 2007.

Functional Analysis (PHD/SP-04)

Review of normed linear spaces, Hahn-Banach theorems, uniform boundedness principle, open mapping theorem, closed graph theorem, Riesz representation theorem on Hilbert spaces. Weak and weak* convergence, reflexivity in the setting of normed linear spaces. Compact operators, Sturm-Liouville problems. Spectral projections, spectral decomposition theorem, spectral theorem for a bounded normal operator, unbounded operators, spectral theorem for an unbounded normal operator.

References:

1. M. Ahues, A. Largillier and B. V. Limaye, Spectral Computations for Bounded Operators, Chapman & Hall/CRC, 2001.

2. J. B. Conway, Functional Analysis, 2nd Ed., Springer-Verlag, 1990.

3. B. V. Limaye, Functional Analysis, 2nd Ed., New Age International Publishers, 1996.

4. F. Riesz and B. Sz.-Nagy, Functional Analysis, Dover Publications, 1990.

5. W. Rudin, Functional Analysis, Tata McGraw Hill, 1974.

6. K. Yosida, Functional Analysis, 5th Ed., Narosa, 1979.

Introduction to Smooth Manifolds (PHD/SP-04)

Smooth Manifolds: Topological Manifolds, Smooth Charts, Smooth Manifolds, Examples of Smooth Manifolds.

Smooth Maps on a Manifold: Smooth Functions on a Manifold, Smooth Maps between Manifolds, Diffeomorphisms, Smoothness in Terms of Components, Examples of Smooth Maps, Partial Derivatives, The Inverse Function Theorem.

Tangent Vectors: Tangent Vectors, The Differential of a Smooth Map, Computations in Coordinates, Velocity Vectors of Curves, Alternative Approaches to the Notion of Tangent Spaces.

Submanifolds: Embedded and Immersed Submanifolds, Tangent Space to a Submanifold.

Vector Fields and Integral Curves: Vector Fields on Manifolds, Smooth Maps, Lie Brackets, Integral Curves and Flows.

References:

1. Introduction to Manifolds (2e) by John M. Lee, Springer, 2013.
2. A Course in Differential Geometry and Lie Groups by S. Kumaresan, Hindusthan Book Agency (Text and Readings in Mathematics 22), 2002.
3. An Introduction to Manifolds by Loring W. Tu, Springer, 2011.

Mathematical Biology (PHD/SP-04)

Brief overview of linear algebra.

Qualitative analysis of a system of linear and non-linear differential equations: existence and uniqueness of solutions, flow, qualitative properties, viz-stability, bifurcation (saddle node, transcritical, pitchfork, Hopf etc), limit cycle.

Qualitative analysis of a system of linear and nonlinear difference equations: Steady states and their stability, bifurcation, chaos, creation of fractal.

Basic concept of the interaction of biological systems: autotroph-based ecosystem, population growth, community dynamics- succession and community responses.

Stability analysis of predator-prey models; Lotka-Volterra system; Gauss's model, Gause model, Kolmogorov model, Leslie Gower model, etc.

Competition model: Gauss exclusion principle and stability analysis. Models on mutualism.

Delay differential equations: linearization of delay population model; Insect out break model- Spruce-Budworm model.

Models of migration of population and developmental pattern formation

Infectious Disease Modelling: elementary epidemic models. Determinations of Threshold Values and Critical Parameters and inferences

Pontryagin's principle: modeling the harvest of economic population and determination of strategy fishery management

References:

1. L. Perko, Differential Equation and Dynamical Systems, (3rd Ed), 2002
2. J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1983.
3. J. Marsden and M. McCracken, The Hopf Bifurcation and Its Applications, Springer-Verlag, 1976.
4. J.D.Murray: Mathematical Biology, Springer and Verlag, 1999
5. H. I. Freedman - Deterministic Mathematical Models in Population Ecology
6. Mark Kot, Elements of Mathematical Ecology, Cambridge Univ. Press, 2001
7. Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, Mathematics in Science and Engineering, vol. 191, Academic Press, Massachusetts, 1993.
8. C. W. Clark, Bioeconomics Modeling and Fisheries Management, John Wiley & Sons, New York, 1985

Semigroup Theory (PHD/SP-04)

Basic Definitions and Examples, Monogenic semigroups, Periodic semigroups, Semilattices. Congruences.Ideals and Rees congruences. Green's equivalences, regular D-classes and regular semigroups. Simple and 0- simple semigroup, primitive idempotents, union of groups, semilattices of groups, bands. Completely regular semigroups. Clifford semigroups. Orthodox semigroups. Completely 0-simple semigroups, Rees' theorem. Inverse semigroups, Congruences on inverse semigroups, Fundamental inverse semigroups.

References :

1. Fundamentals of semigroup theory, J. M. Howie, Clarendon Press, Oxford, 1995.
2. An introduction to semigroup theory, J. M. Howie, Academic Press, London, 1976.
3. The algebraic theory of semigroups, A. H. Clifford and G. B. Preston, Amer. Math. Soc., Math Surveys No. 7, Providence, Vol I, 1961, Vol II, 1967.
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Solution Methods in Generalized Thermoelasticity (PHD/SP-04)

Basic Laws of Thermoelasticity; Thermodynamics of Elastic Continuum; Introduction to Generalised Thermoelasticity; Fundamentals of linear thermoelasticity with finite wave speeds; Formulations of initial-boundary value problems; Existence and uniqueness theorems; Convolutional variational principles; Methods in Generalized Thermoelasticity: Finite element methods, State space approach, Eigenvalue approach; Normal mode analysis.

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Statistics – I (PHD/SP-04)

Mathematical Analysis: Analytic Function, Cauchy-Riemann Equations, Pole, Singularity, Residuals and its usage in evaluating integrals, Cauchy's integral formula.
Data Mining: Classification and Regression Tree, ANN, Pattern Recognition.
Stochastics Process: Brownian Bridge, Martingales and Stopping time.

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3. P.-N. Tan, M. Steinbach, and V. Kumar, Introduction to Data Mining, Addison Wesley, 2005.
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Statistics – II (PHD/SP-04)

Statistical Inference: Semi-parametric, SPRT, concept of asymptotic Power under Pitman's alternative.
Design of experiments: Introduction of response Surface Designs, First Order Regression Designs, first Order Rotatability, BIBD/PBIBD.

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Survival Analysis (PHD/SP-04)

Introduction. Basic functions and Models. Censoring and Truncation. Parametric univariate estimation, Standard models – exponential, Weibull, log-logistic, lognormal and Gamma.
Nonparametric univariate estimation: Actuarial, Kaplan-Meier and Nelson-Aalen estimators. Tests of equality of survival functions: Gehan's and Mantel-Haenszel tests. Semiparametric regression models: Cox proportional hazard model – estimation, tests, diagnostics. Additive Models. Accelerated Models, Competing Risk and Multivariate Survival models. Frailty Models.

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2. P.J. Smith: Analysis of Failure and Survival Data
3. J.D. Kalbfleisch & R.L. Prentiice: The Statistical Analysis of Failure Time Data, 2nd ed.
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