PHYUGMCC1101: Mechanics

Tutotrial Assignment-1

1. State and prove Work-energy theorem.
2. What are conservative and non-conservative forces. Give examples.
3. What do you understand by central force and non-central force? Give examples of central force and non-central force.
4. Obtain expressions for the radial and transverse (tangential) components of velocity and acceleration in plane polar coordinate system.
5. Write down the equation of motions of a system of two-particles under the action of a central force. Now reduce the two-body central force problem to a one-body problem.
6. Give examples of inverse square law forces. Show that inverse square law forces are central forces. Check whether such forces are conservative or not.
7. Prove the law of conservation of energy for an object moving under the action of a central force. Or, Prove that $\frac{1}{2}mv^2+U+\frac{L^2}{2mr^2}=E$=Constant, where symbols have their usual meanings.
8. Discuss the nature of path of particle moving under the action of central force.
9. Show that the differential equation of motion of a particle of mass m under the control of a central isotropic force force can be written as

$$\frac{d^2u}{d\theta^2}+u=-\frac{m}{L^2u^2}F(\frac{1}{u})$$

10. State and prove Kepler's laws of planetary motion.
11. Derive Newton's law of gravitation from Kepler's laws.
12. The minimum and maximum separation of a comet from the sun are 7×10^{10} m and 1.4×10^{12} m respectively. If the speed of the comet at the nearest point is 6×10^4 m/s, determine the speed of the comet at the farthest position. [Ans: 3000 m]
13. The moon moves in a circular orbit of 3.8×10^5 km around the earth in 27 days and the earth takes 365 days to complete one round in circular orbit of radius 1.5×10^8 km about the Sun. Estimate mass of the Sun in terms of the mass of the earth.
14. Identify the conservative and non-conservative forces in the following:
    (a) $\vec{F}=(2xy+z^2)\hat{i}+x^2\hat{j}+2xz\hat{k}$
    (b) $\vec{F}=(y^2-x^2)\hat{i}+3xy\hat{j}$
    (c) $\vec{F}=y\hat{i}+x\hat{j}+xy\hat{k}$
15. Here is an interesting problem that is a key to the nuclear reactor physics. In nuclear reactor uranium produces high energy neutrons which need to be slowed down using moderators implanted in the reactor. The first nuclear reactor built i Chicago USA 1942 used graphite (carbon) as moderators. One such neutron (mass 1.0 u) travelling at 2.6×10^7 m/s undergoes

Khan Sir

a head-on elastic collision with a carbon (graphite) nucleus (mass 12 u) initially at rest.

(a) Neglecting external forces during the collision, find the velocities after collision. (1 u is the atomic mass unit equalling 1.66×10^{-27} kg.). [Ans: $v_n = -2.2 \times 10^7$ m/s, $v_C = 0.4 \times 10^7$ m/s.]

(b) Estimate the energy transferred from the projectile (neutron) to the target (uranium nucleus) in this collision. [Ans: 28% transferred]

(c) What amount of energy will be transferred if the collision happens inelastically?

(d) Will there be any energy transfer to the target if the neutron was at rest and uranium nucleus moving with same kinetic energy as that of the neutron (in the previous cases) collides elastically with the neutron?

(e) Calculate the energy transfer (if any) assuming the collision as an inelastic one. Summarize the results in tabular form, analyse them and make your own conclusion based on your findings.

16. On a frictionless table, a billiard ball (mass $m_A = 0.5$ kg) moving in the positive x-direction (velocity $u_A = 4.0$ m/s) collides elastically with another billiard ball (mass $m_B = 0.3$ kg) initially at rest (velocity $u_B = 0$). Ball A has final velocity $v_A = 2$ m/s in an unknown direction θ. Find the final velocity v_B of the ball B moving in the unknown direction ϕ. Also find the directions θ and ϕ. [Ans: $v_B = 4.47$ m/s; $\theta = 36.9°$; $\phi = 26.6°$]

17. A neutron of mass 1.67×10^{-27} kg and speed 10^5 m/s collides with a stationary deuteron of mass 3.34×10^{-27} kg. The particles does not stick together and no kinetic energy is lost in collision. What are the subsequent speeds of the particles? [Ans. $v_n = 3.33 \times 10^4$ m/s, $v_d = 6.67 \times 10^4$ m/s in opposite direction]

18. Define centre of mass of a system of particles. Write down the most general expression for the position of the centre of mass of a system of particles using vector algebra. Using the above expression write down expressions for the Cartesian components of the centre of mass.

19. In a water molecule, the oxygen-hydrogen separation $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the centre of mass of the water molecule. Angle between the O-H bonds is 105°. [Ans: (6.5×10^{-12} m, 0 m)]

20. What is a rocket? Describe rocket motion and derive an expression for its acceleration assuming a gravity free region.

21. A bomb weighing 50 kg explodes into three pieces in flight when its velocity is $20\hat{i} + 22\hat{j} + 10\hat{k}$ m/s. Two fragments of the bomb weighing 10 and 20 kg, are found to have velocities $100\hat{i} + 50\hat{j} + 20\hat{k}$ m/s and $30\hat{i} + 20\hat{j} + 10\hat{k}$ m/s. Find the velocity of the third piece.

22. (a) What is driven (or forced harmonic) oscillator? How it differs from the simple harmonic, and damped harmonic oscillators?

(b) A particle of mass m is subjected to a damping force proportional to its instantaneous velocity and a driving force $F_0 \cos \omega t$. Write down

Khan Sir

the equation of motion. Solve the equation to obtain instantaneous displacement and velocity.

(c) Graphically show nature of displacement-time graph assuming large, small and critical damping.

KhanSir