## PRINCIPLE OF OPERATION OF THREE-PHASE INVERTERS

A three-phase inverter can be formed by connecting three single-phase bridge inverters in parallel. Such a system requires twelve switch-diode pairs and three single-phase transformers. On the other hand, a single three-phase inverter unit consists of six switching devices and six diodes. A three-phase inverter is classified as 180°-conduction or 120°-conduction modes inverter, according to the period of conduction of each switch. The circuit configuration of a three-phase inverter is shown in Fig. 8.17.

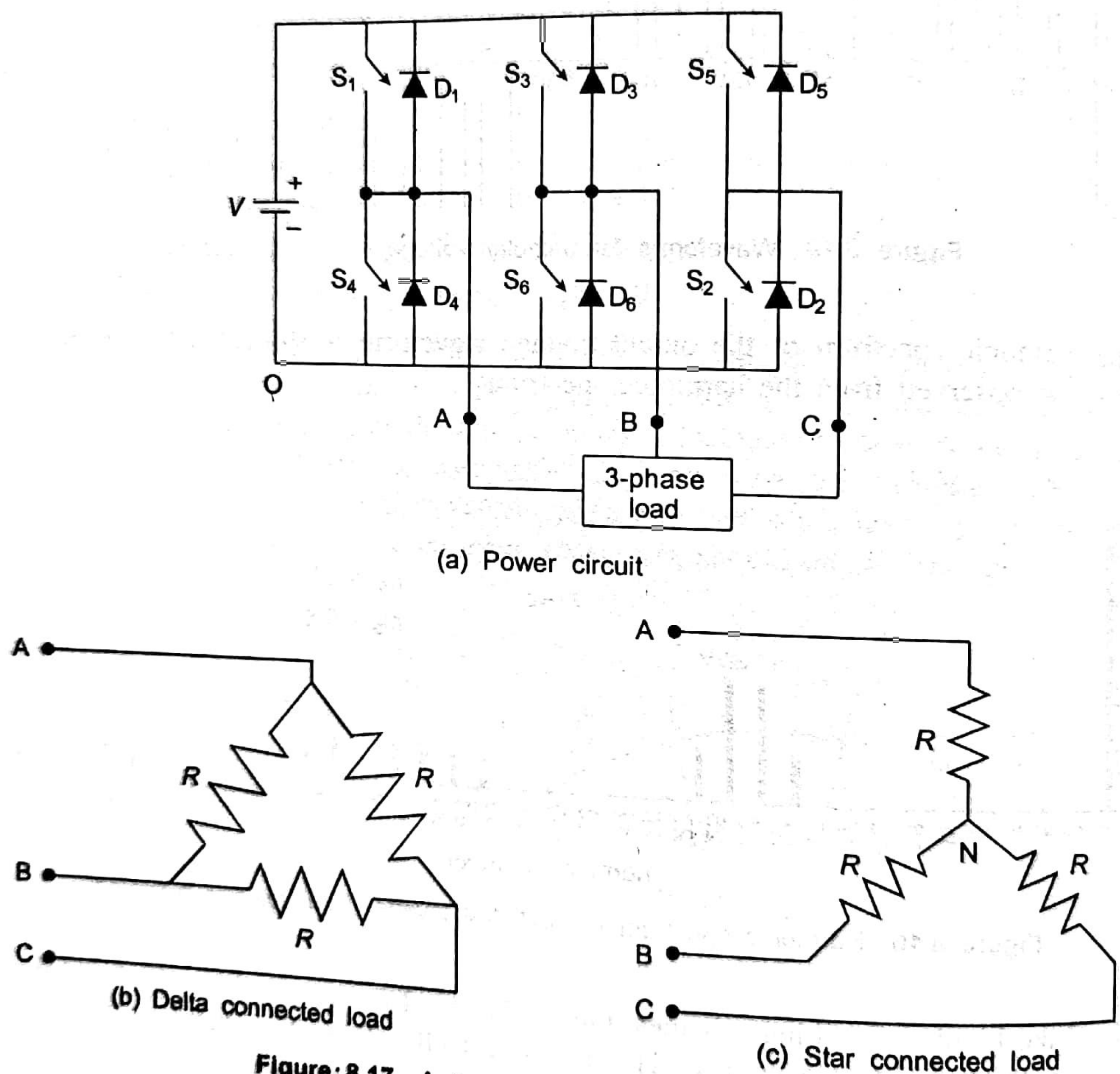


**Figure 8.17** A three-phase full-bridge inverter.

### 8.9.1 180°-CONDUCTION MODE

In this control scheme, each switch conducts for a period of 180° or half-cycle electrical. Switches are triggered in sequence of their numbers with an interval of 60°. The gating signals for switches are shown in Fig. 8.18. At a time, three switches (one from each leg) conduct. Thus two switches of the same leg are prevented from conducting simultaneously. One complete cycle is divided into six modes, each of 60° intervals.

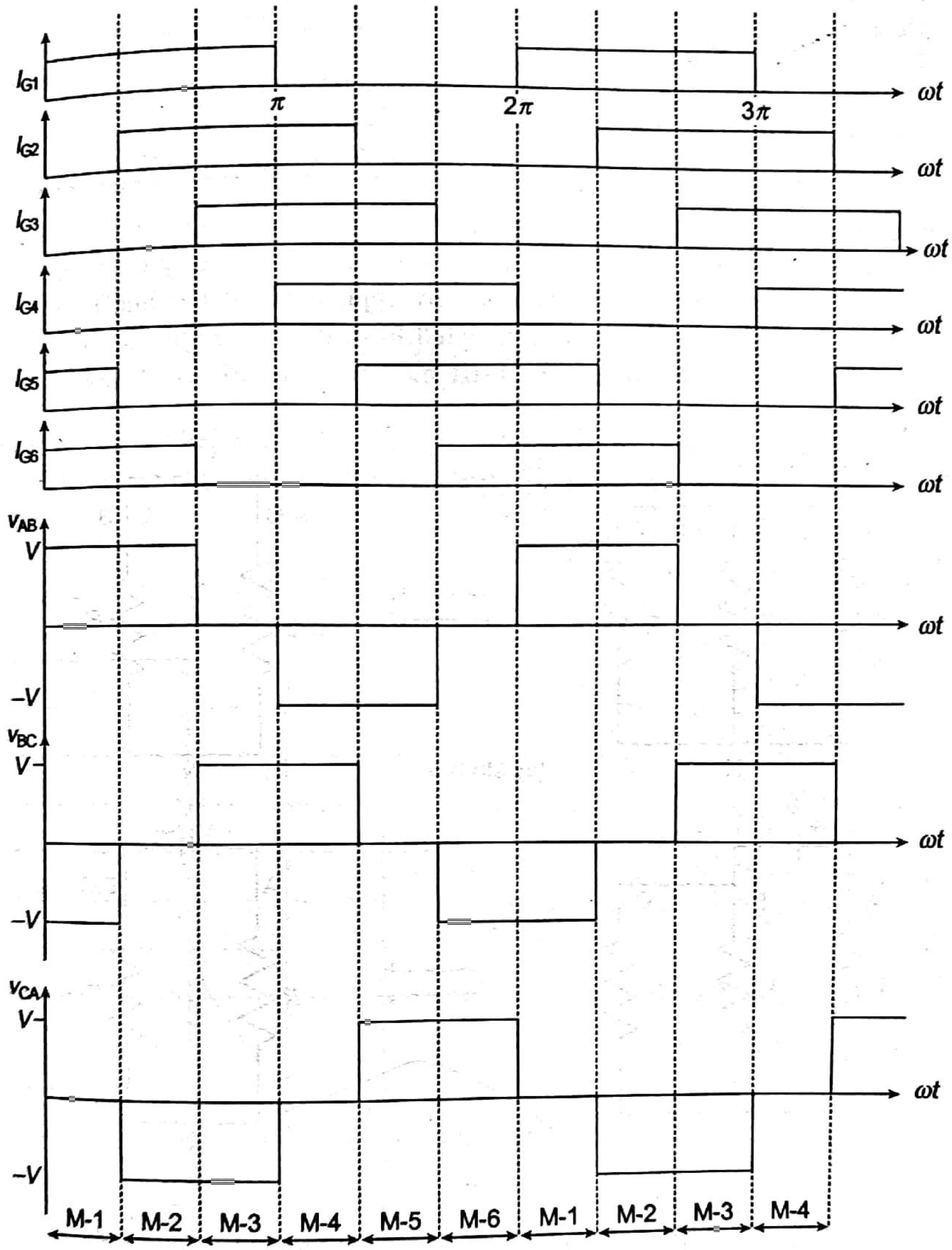


**Figure 8.18** Gating signals and output voltages of a three-phase inverter for 180° conduction.

Potential of a load terminal, with respect to the common reference point O, is +V when the upper switch of the corresponding arm conducts and it is zero when the lower switch conducts. The potentials of the three load terminals A, B, and C with respect to the reference point O and the line voltages $V_{AB}$, $V_{BC}$ and $V_{CA}$ are given in Table 8.1. Waveforms of line voltages are shown in Fig. 8.18.

**Table 8.1** Conducting Switches, Load Terminals and Line Voltages

| Modes | Conducting switches | Load terminal potentials | | | Line voltages | | |
|---|---|---|---|---|---|---|---|
| | | $V_{AO}$ | $V_{BO}$ | $V_{CO}$ | $V_{AB} = V_{AO} - V_{BO}$ | $V_{BC} = V_{BO} - V_{CO}$ | $V_{CA} = V_{CO} - V_{AO}$ |
| I | $S_5, S_6, S_1$ | $V$ | 0 | $V$ | $V$ | $-V$ | 0 |
| II | $S_6, S_1, S_2$ | $V$ | 0 | 0 | $V$ | 0 | $-V$ |
| III | $S_1, S_2, S_3$ | $V$ | $V$ | 0 | 0 | $V$ | $-V$ |
| IV | $S_2, S_3, S_4$ | 0 | $V$ | 0 | $-V$ | $V$ | 0 |
| V | $S_3, S_4, S_5$ | 0 | $V$ | $V$ | $-V$ | 0 | $V$ |
| VI | $S_4, S_5, S_6$ | 0 | 0 | $V$ | 0 | $-V$ | $V$ |

The inverter may supply power to a delta-connected or a star-connected load, as shown in Fig. 8.17. For a delta-connected load, the phase currents can be directly obtained from the line voltages. The line currents can simply be obtained by application of Kirchhoff's current law. To evaluate the phase or line currents for a star-connected load, the phase voltage of the load must be determined. The equivalent circuits for modes I–III may be obtained, as illustrated in Fig. 8.19.

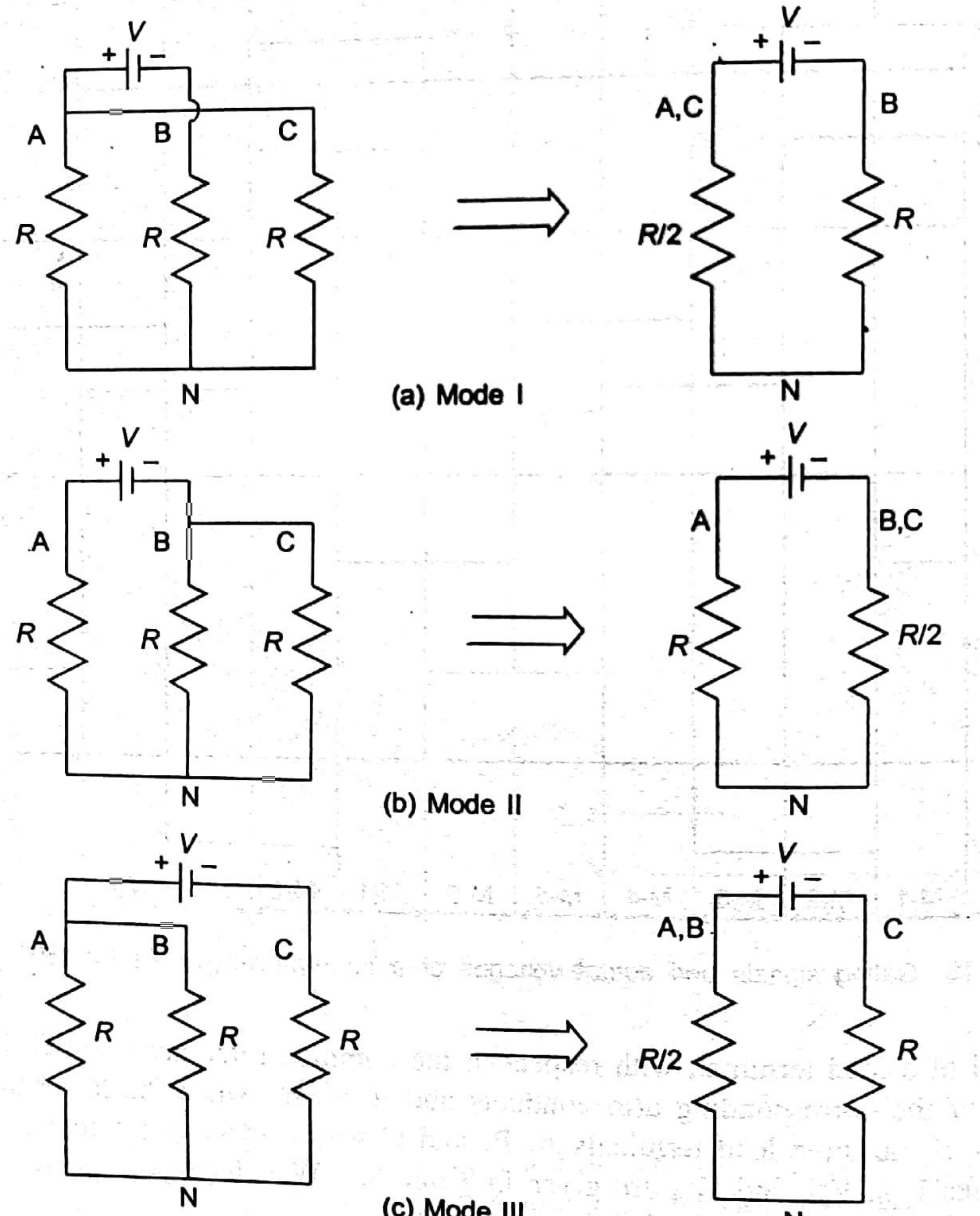


**Figure 8.19** Equivalent circuits of star-connected load for 180° conduction.

From the equivalent circuits, phase voltages is given by

*During Mode I:* $(0 \le \omega t < \pi/3)$ From Fig. 8.19a, we have

$$v_{AN} = v_{CN} = \frac{R/2}{R + (R/2)} V = \frac{V}{3} \quad \text{and} \quad v_{BN} = -\frac{R}{R + (R/2)} V = -\frac{2}{3} V$$

*During Mode II:* $(\pi/3 \le \omega t < 2\pi/3)$ From Fig. 8.19b, we have

$$v_{AN} = \frac{R}{R + (R/2)} V = \frac{2}{3} V \quad \text{and} \quad v_{BN} = v_{CN} = -\frac{R/2}{R + (R/2)} V = -\frac{1}{3} V$$

*During Mode III:* $(2\pi/3 \le \omega t \le \pi)$ From Fig. 8.19c, we obtain

$$v_{AN} = v_{BN} = \frac{R/2}{R + (R/2)} V = \frac{V}{3} \quad \text{and} \quad v_{CN} = -\frac{R}{R + (R/2)} V = -\frac{2}{3} V$$

Equivalent circuits for modes IV, V and VI may be obtained by reversing the polarity of the voltage source in the equivalent circuits for modes I, II and III, respectively. The phase voltages are shown in Fig. 8.20.

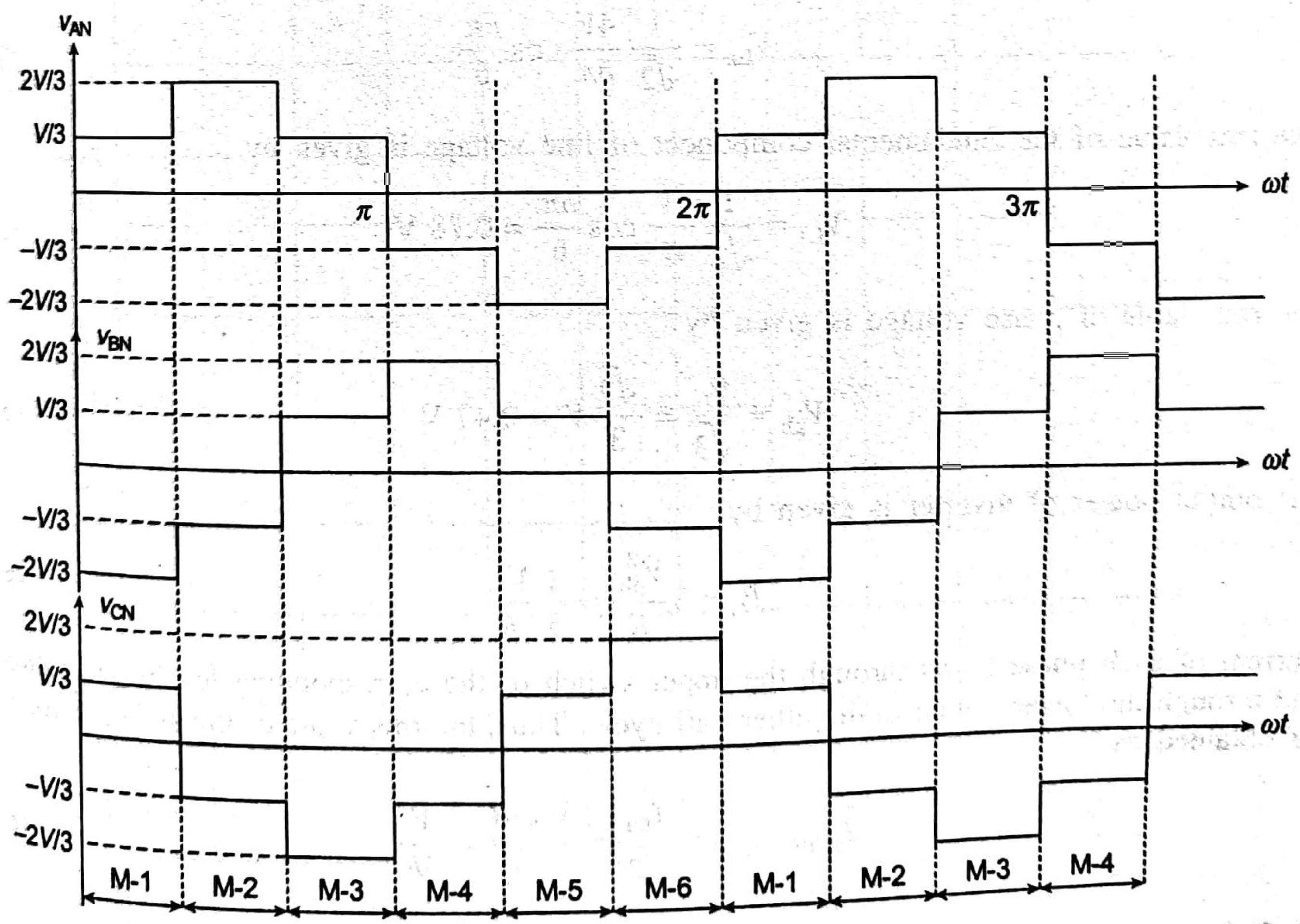


**Figure 8.20** Phase voltages of star-connected load for 180° conduction.

The output line voltages can be expressed by the following Fourier expansion:

$$v_{AB} = \sum_{n=2k+1}^{\infty} \frac{4V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{n\pi}{6}\right) \quad (8.37)$$

$$v_{BC} = \sum_{n=2k+1}^{\infty} \frac{4V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t - \frac{n\pi}{2}\right) \quad (8.38)$$

$$v_{CA} = \sum_{n=2k+1}^{\infty} \frac{4V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{5n\pi}{6}\right) \quad (8.39)$$

where $k = 0, 1, 2, 3, \ldots.$

From equations (8.37)–(8.39), it is obvious that triplen harmonics ($n = 3, 9, \ldots$) are zero.

The line-to-line rms voltage is given by

$$V_L = \left[\frac{2}{2\pi}\int_0^{2\pi/3} V^2 d(\omega t)\right]^{1/2} = \frac{\sqrt{2}}{\sqrt{3}} V = 0.816\ V \quad (8.40)$$

The rms value of the $n$th harmonic component of the line voltage is given by

$$V_{Ln} = \frac{1}{\sqrt{2}} \frac{4V}{n\pi} \cos\frac{n\pi}{6} \quad (8.41)$$

The rms value of the fundamental component of line voltage is given by

$$V_{L1} = \frac{1}{\sqrt{2}} \frac{4V}{\pi} \cos\frac{n\pi}{6} = 0.78\ V \quad (8.42)$$

The rms value of phase voltage is given by

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{\sqrt{2}}{3} V = 0.47\ V \quad (8.43)$$

The output power of inverter is given by

$$P_L = 3\frac{V_{ph}^2}{R} = \frac{2}{3}\frac{V^2}{R} \quad (8.44)$$

Current of each phase flows through the upper switch of the corresponding leg in one half-cycle and through the lower switch in the other half-cycle. Thus, the rms value of the switch current can be obtained as

$$I_{switch\ (rms)} = \frac{I_{ph}}{\sqrt{2}} = \frac{V_{ph}/R}{\sqrt{2}} = \frac{V}{3R} \quad (8.45)$$

### 8.9.2 120°-CONDUCTION MODE

In this scheme, each switch conducts for 120° duration. Two switches conduct simultaneously in the sequence $S_6S_1$, $S_1S_2$, $S_2S_3$, $S_3S_4$, $S_4S_5$, $S_5S_6$. The switching scheme is shown in Fig. 8.21. A delay of 60° is provided between the turning-on and turning-off of two switches of each leg. Thus short-circuiting of the dc source is avoided. As only two switches conduct at a time, one load terminal remains floating in this conduction scheme.

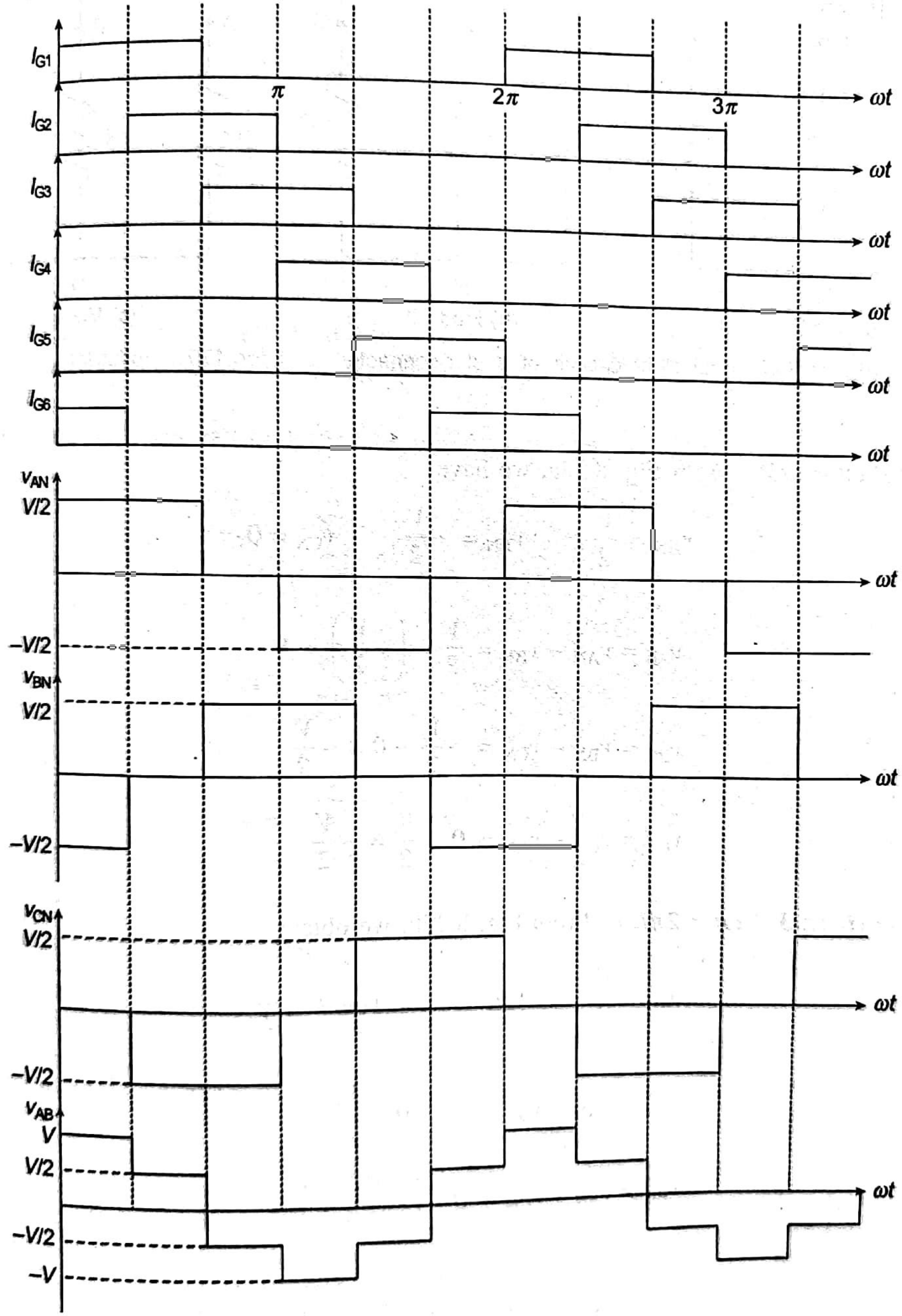


Figure 8.21 Gating signals and output voltages of a three-phase inverter for 120° conduction.

For a balanced Y-connected resistive load, the equivalent circuits are shown in Fig. 8.22. The analysis is presented for half-cycle.

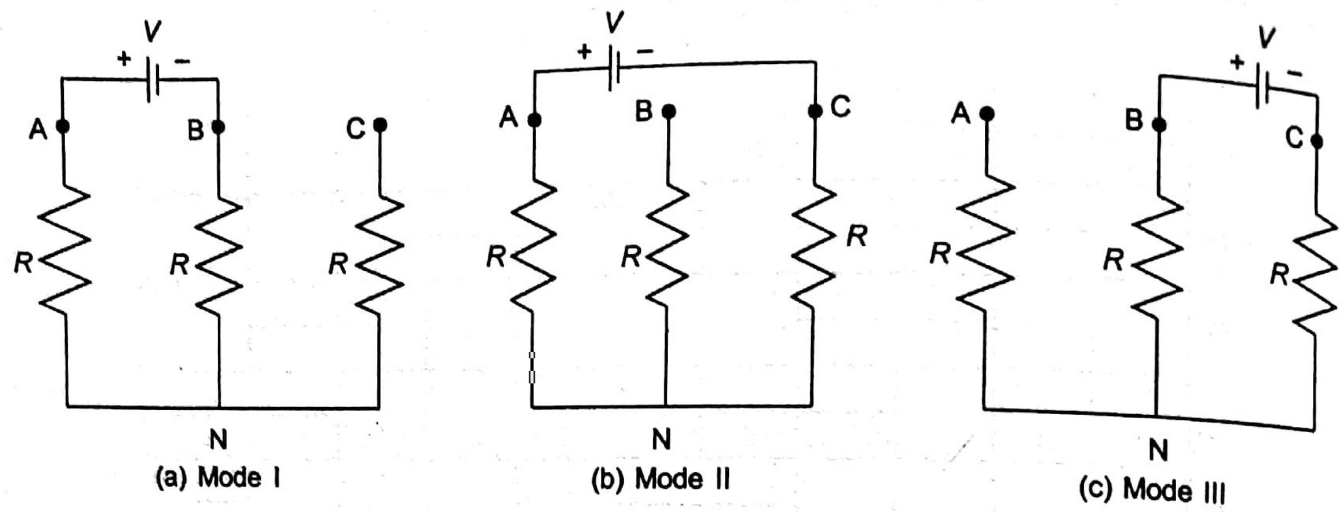


**Figure 8.22** Equivalent circuits of a star-connected load for 120° conduction.

***Mode I:* $(0 \le \omega t < \pi/3)$** From Fig. 8.22a, we have

$$v_{AN} = \frac{V}{2}, \qquad v_{BN} = -\frac{V}{2}, \qquad v_{CN} = 0$$

Then

$$v_{AB} = v_{AN} - v_{BN} = \frac{V}{2} - \left(-\frac{V}{2}\right) = V$$

$$v_{BC} = v_{BN} - v_{CN} = -\frac{V}{2} - 0 = -\frac{V}{2}$$

$$v_{CA} = v_{CN} - v_{AN} = 0 - \frac{V}{2} = -\frac{V}{2}$$

***During mode II:* $(\pi/3 \le \omega t < 2\pi/3)$** From Fig. 8.22b, we obtain

$$v_{AN} = \frac{V}{2}, \qquad v_{BN} = 0, \qquad v_{CN} = -\frac{V}{2}$$

Then

$$v_{AB} = v_{AN} - v_{BN} = \frac{V}{2} - 0 = \frac{V}{2}$$

$$v_{BC} = v_{BN} - v_{CN} = 0 - \left(-\frac{V}{2}\right) = \frac{V}{2}$$

$$v_{CA} = v_{CN} - v_{AN} = -\frac{V}{2} - \frac{V}{2} = -V$$

*Mode III:* $(2\pi/3 \le \omega t \le \pi)$ From Fig. 8.22c, we have

$$v_{AN} = 0, \qquad v_{BN} = \frac{V}{2}, \qquad v_{CN} = -\frac{V}{2}$$

Then

$$v_{AB} = v_{AN} - v_{BN} = 0 - \frac{V}{2} = -\frac{V}{2}$$

$$v_{BC} = v_{BN} - v_{CN} = \frac{V}{2} - \left(-\frac{V}{2}\right) = V$$

$$v_{CA} = v_{CN} - v_{AN} = -\frac{V}{2} - 0 = -\frac{V}{2}$$

Equivalent circuits for modes IV, V and VI may be obtained by reversing the polarity of voltage source in the equivalent circuits for modes I, II and III, respectively. The phase voltages are shown in Fig. 8.21.

The output voltages can be expressed in Fourier series as:

$$v_{AN} = \sum_{n=2k+1}^{\infty} \frac{2V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{n\pi}{6}\right) \tag{8.46}$$

$$v_{BN} = \sum_{n=2k+1}^{\infty} \frac{2V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t - \frac{n\pi}{2}\right) \tag{8.47}$$

$$v_{CN} = \sum_{n=2k+1}^{\infty} \frac{2V}{n\pi} \cos\frac{n\pi}{6} \sin\left(n\omega t + \frac{5n\pi}{6}\right) \tag{8.48}$$

where $k = 0, 1, 2, 3, \ldots$

The rms value of fundamental phase voltage,

$$V_{ph1} = \frac{2V}{\sqrt{2}\pi} \cos\frac{\pi}{6} = \frac{\sqrt{3}}{\sqrt{2}\,\pi} V = 0.39\text{ V} \tag{8.49}$$

The rms value of fundamental line voltage,

$$V_{L1} = \sqrt{3} V_{ph1} = \frac{3V}{\sqrt{2}\pi} = 0.675\text{ V} \tag{8.50}$$

The rms value of phase voltage,

$$V_{ph} = \left[\frac{1}{\pi}\int_0^{2\pi/3} \left(\frac{V}{2}\right)^2 d(\omega t)\right]^{1/2} = \frac{V}{\sqrt{6}} = 0.408\text{ V} \tag{8.51}$$

The rms value of line voltage,

$$V_L = \sqrt{3} V_{ph} = 0.707 \text{ V} \tag{8.52}$$

The output power of the inverter,

$$P_L = 3\frac{V_{ph}^2}{R} = \frac{V^2}{2R} \tag{8.53}$$

The rms value of the switch current can be obtained as

$$I_{\text{switch (rms)}} = \frac{I_{ph}}{\sqrt{2}} = \frac{V_{ph}/R}{\sqrt{2}} = \frac{V}{2\sqrt{3}\,R} \tag{8.54}$$

The load power is given by

$$P_L = 3I_{ph}^2 R = 6I_{\text{switch (rms)}}^2 R$$

which gives

$$I_{\text{switch (rms)}} = \frac{1}{2\sqrt{3}}\frac{V}{R}$$

### 8.9.3 Comparison of 180°- and 120°-conduction Schemes

It may be observed that, for 180°-conduction scheme, there is no delay between switching-on and switching-off of the switches in the same arm. As the turn-off time of a semiconductor switch is more than the turn-on time, both switches of same arm remain conducting for some time and short circuiting of the dc source takes place. This problem is overcome in 120°-conduction scheme by providing a time lag of $T/6$ seconds or $\pi/3$ radians between turn-on and turn-off of two switches of same arm. This results in reliable and safe operation of the inverter, at the cost of poor utilization of switches (i.e. their capacity).