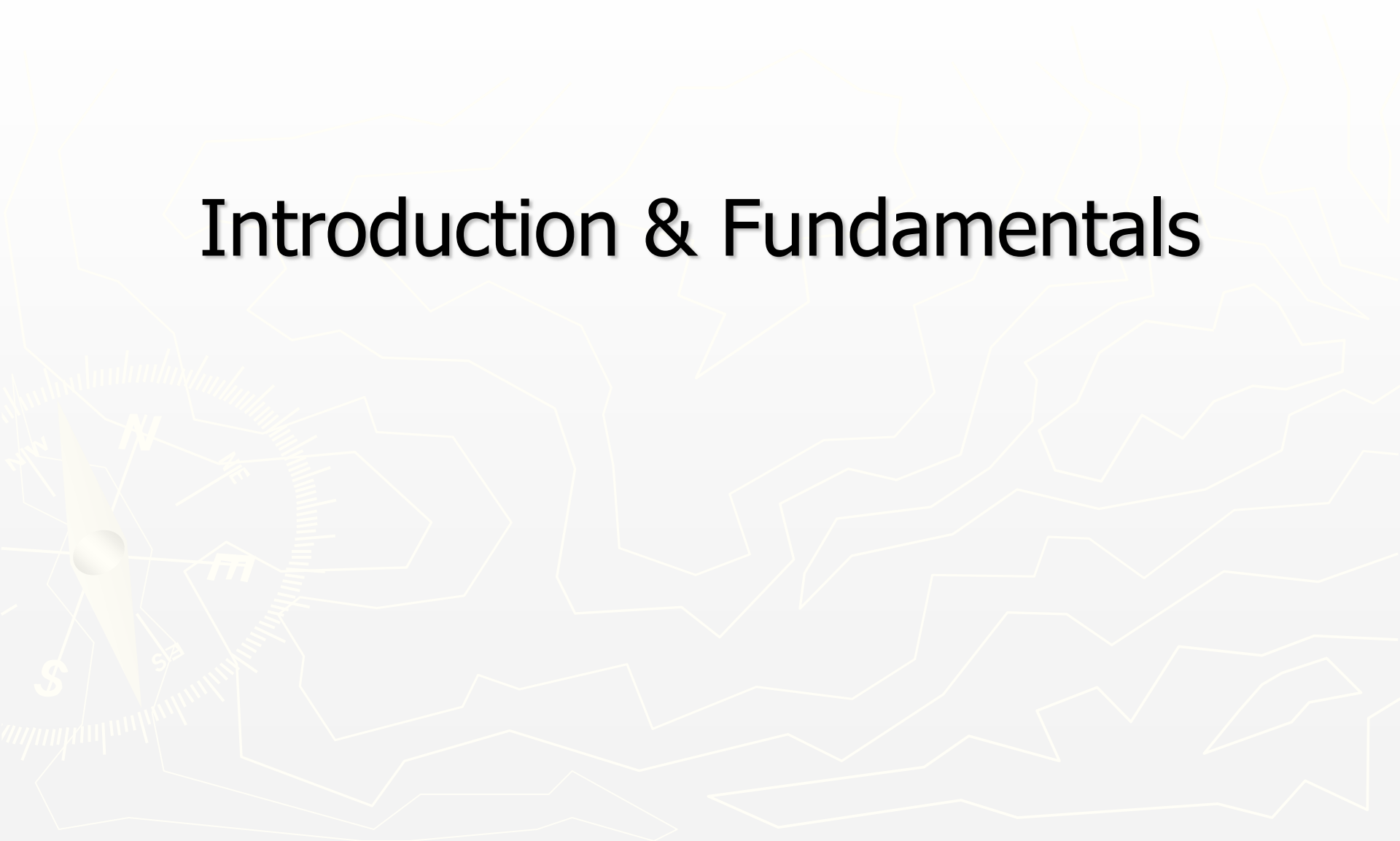


Digital Image Processing

Introduction & Fundamentals



Scope of the Subject

► Application Area

- Medical image analysis (MRI/PET/CT/X-ray tumor detection/classification)
- Face, fingerprint, and other object recognition
- Document Image Analysis
- Image and/or video compression
- Image segmentation and/or denoising
- Digital image/video watermarking/steganography and detection

Introduction

► What is Digital Image Processing?

Digital Image

- a two-dimensional function $f(x, y)$
 x and y are spatial coordinates

The amplitude of f is called **intensity** or **gray level** at the point (x, y)

Digital Image Processing

- process digital images by means of computer, it covers low-, mid-, and high-level processes
- low-level: inputs and outputs are images
- mid-level: outputs are attributes extracted from input images
- high-level: an ensemble of recognition of individual objects

Pixel

- the elements of a digital image

Digital Image Processing Categorization

Low Level

Source- Image

Output- Image

Operations- Image processing to reduce noise, contrast enhancement, sharpening etc.

Mid Level

Source- Image

Output- Different attributes like edges, contour, segments of source Image

Operations- Segmentation, classification etc.

High Level

Source- Image

Output- Some useful information which will be used for machine vision

Operations- Image analysis algorithms to extract useful information from image

Origins of Digital Image Processing



FIGURE 1.1 A digital picture produced in 1921 from a coded tape by a telegraph printer with special type faces. (McFarlane.[†])

Sent by submarine cable between London and New York, the transportation time was reduced to less than three hours from more than a week

Origins of Digital Image Processing

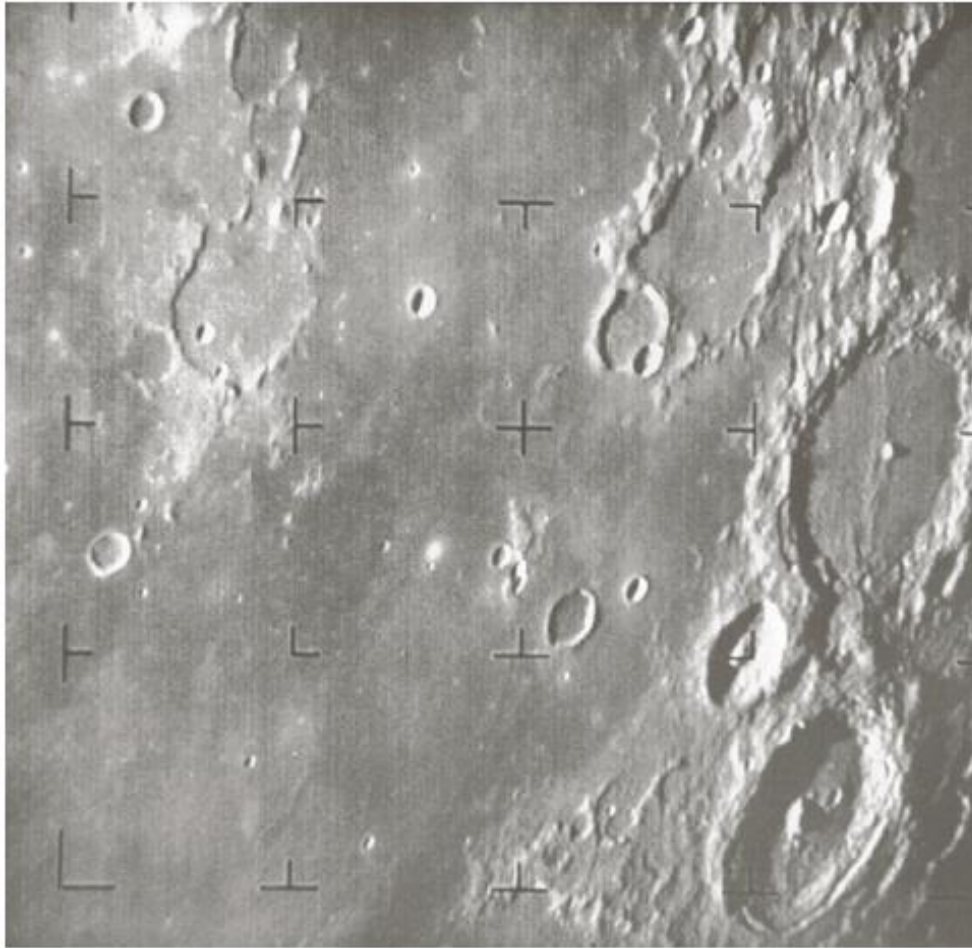


FIGURE 1.4 The first picture of the moon by a U.S. spacecraft. *Ranger 7* took this image on July 31, 1964 at 9 : 09 A.M. EDT, about 17 minutes before impacting the lunar surface. (Courtesy of NASA.)

Sources for Images

- ▶ Electromagnetic (EM) energy spectrum
- ▶ Acoustic
- ▶ Ultrasonic
- ▶ Electronic
- ▶ Synthetic images produced by computer

Electromagnetic (EM) energy spectrum

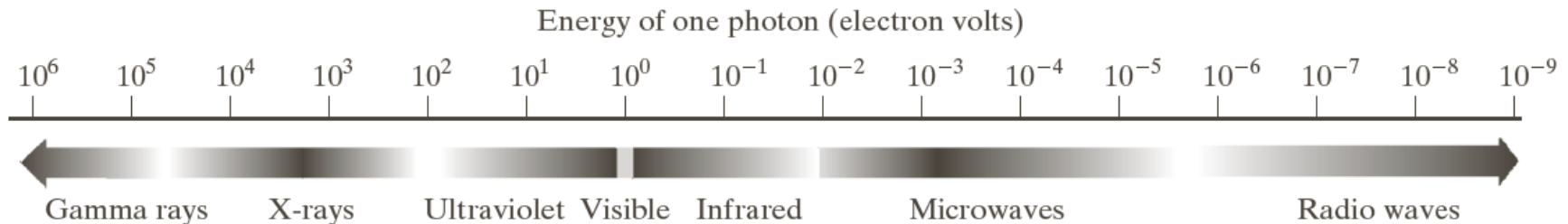


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

Major uses

Gamma-ray imaging: nuclear medicine and astronomical observations

X-rays: medical diagnostics, industry, and astronomy, etc.

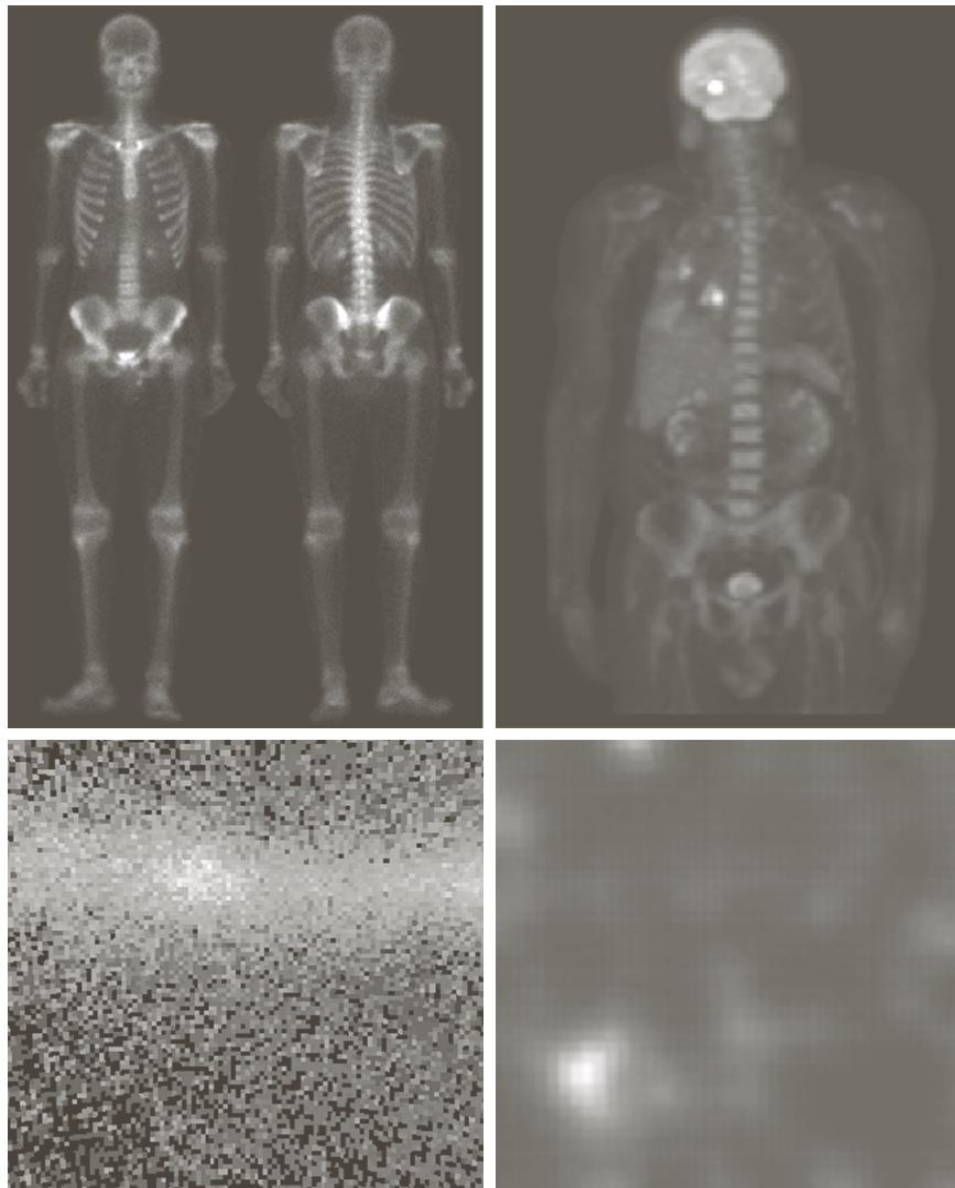
Ultraviolet: lithography, industrial inspection, microscopy, lasers, biological imaging, and astronomical observations

Visible and infrared bands: light microscopy, astronomy, remote sensing, industry, and law enforcement

Microwave band: radar

Radio band: medicine (such as MRI) and astronomy

Examples: Gama-Ray Imaging



a	b
c	d

FIGURE 1.6

Examples of gamma-ray imaging. (a) Bone scan. (b) PET image. (c) Cygnus Loop. (d) Gamma radiation (bright spot) from a reactor valve.

(Images courtesy of (a) G.E.

Medical Systems,

(b) Dr. Michael

E. Casey, CTI

PET Systems,

(c) NASA,

(d) Professors

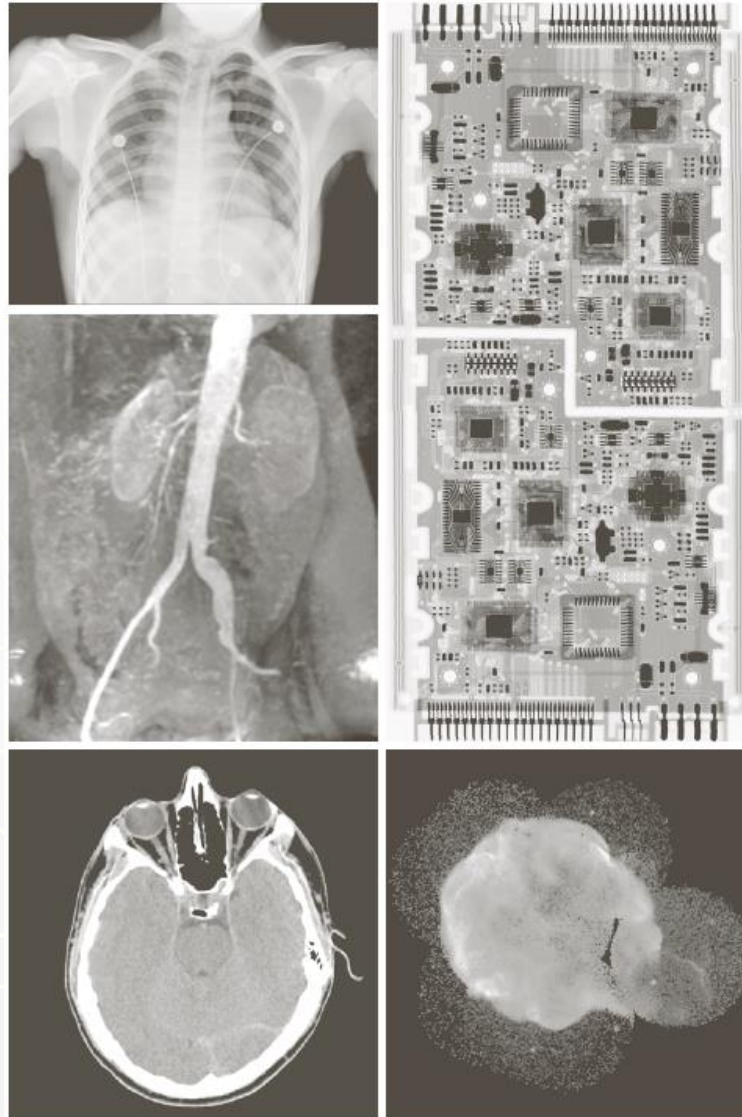
Zhong He and

David K. Wehe,

University of

Michigan.)

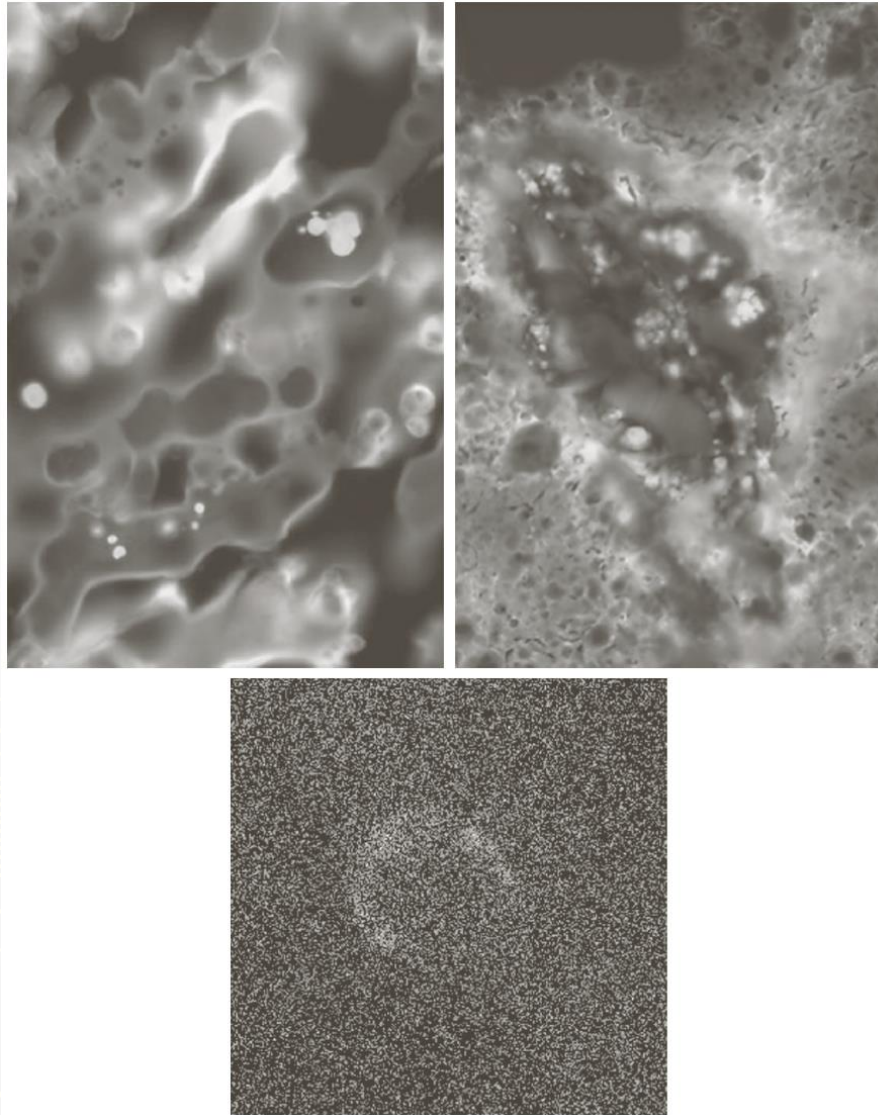
Examples: X-Ray Imaging



a	d
b	e
c	e

FIGURE 1.7 Examples of X-ray imaging. (a) Chest X-ray. (b) Aortic angiogram. (c) Head CT. (d) Circuit boards. (e) Cygnus Loop. (Images courtesy of (a) and (c) Dr. David R. Pickens, Dept. of Radiology & Radiological Sciences, Vanderbilt University Medical Center; (b) Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School; (d) Mr. Joseph E. Pascente, Lixi, Inc.; and (e) NASA.)

Examples: Ultraviolet Imaging



a b
c

FIGURE 1.8

Examples of ultraviolet imaging.

(a) Normal corn.

(b) Smut corn.

(c) Cygnus Loop.

(Images courtesy of (a) and

(b) Dr. Michael

W. Davidson,

Florida State

University,

(c) NASA.)

Examples: Light Microscopy Imaging

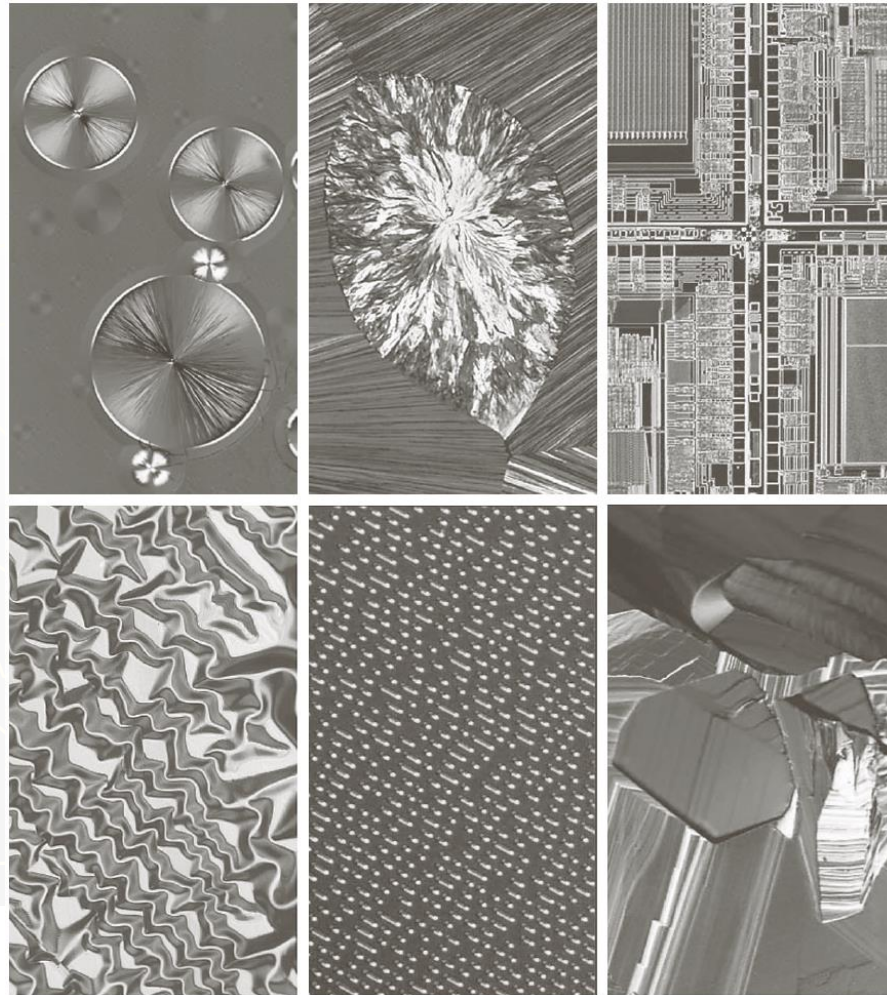


FIGURE 1.9 Examples of light microscopy images. (a) Taxol (anticancer agent), magnified 250 \times . (b) Cholesterol—40 \times . (c) Microprocessor—60 \times . (d) Nickel oxide thin film—600 \times . (e) Surface of audio CD—1750 \times . (f) Organic superconductor—450 \times . (Images courtesy of Dr. Michael W. Davidson, Florida State University.)

Examples: Visual and Infrared Imaging

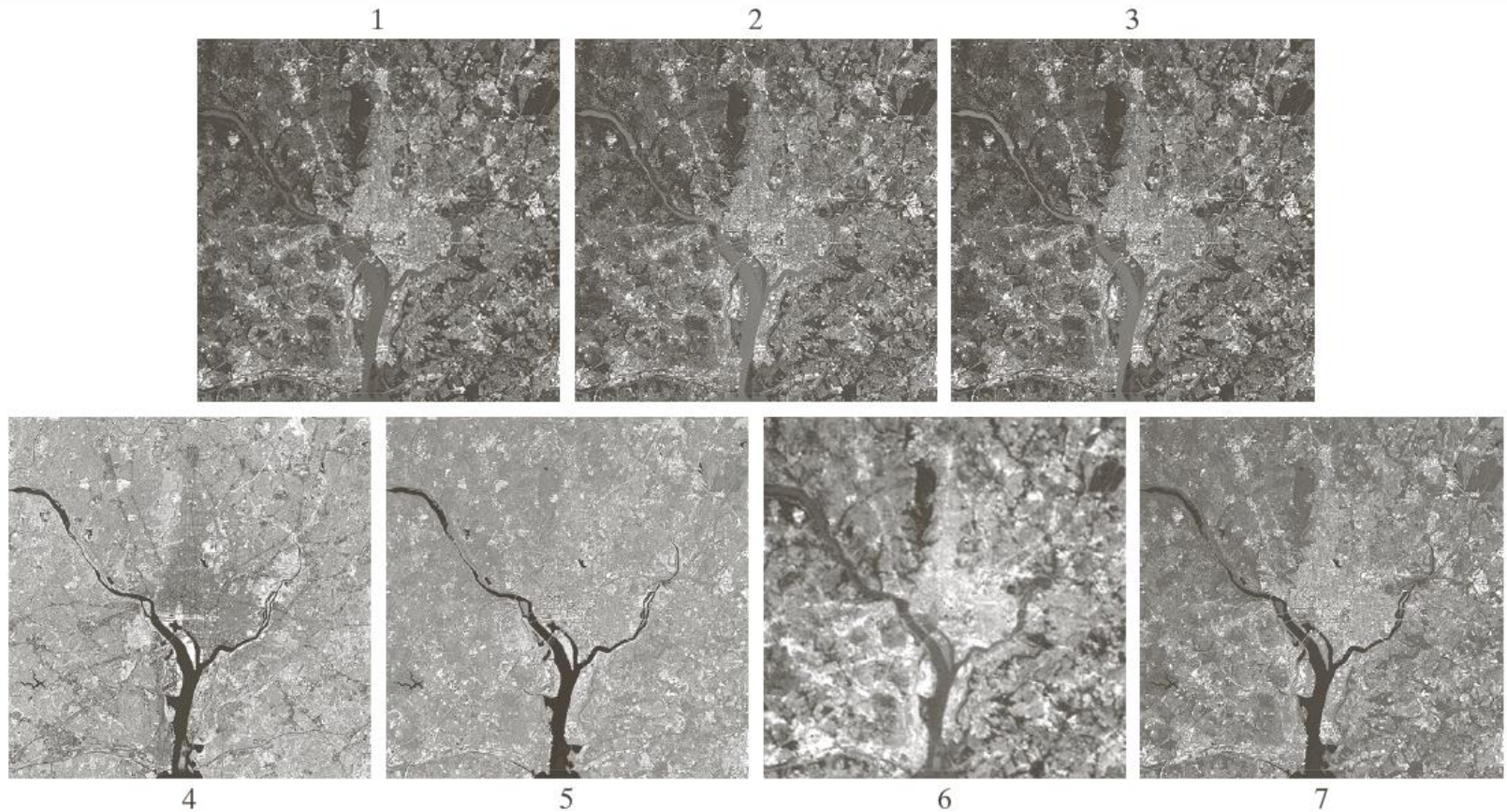


FIGURE 1.10 LANDSAT satellite images of the Washington, D.C. area. The numbers refer to the thematic bands in Table 1.1. (Images courtesy of NASA.)

Examples: Visual and Infrared Imaging

TABLE 1.1

Thematic bands
in NASA's
LANDSAT
satellite.

Band No.	Name	Wavelength (μm)	Characteristics and Uses
1	Visible blue	0.45–0.52	Maximum water penetration
2	Visible green	0.52–0.60	Good for measuring plant vigor
3	Visible red	0.63–0.69	Vegetation discrimination
4	Near infrared	0.76–0.90	Biomass and shoreline mapping
5	Middle infrared	1.55–1.75	Moisture content of soil and vegetation
6	Thermal infrared	10.4–12.5	Soil moisture; thermal mapping
7	Middle infrared	2.08–2.35	Mineral mapping

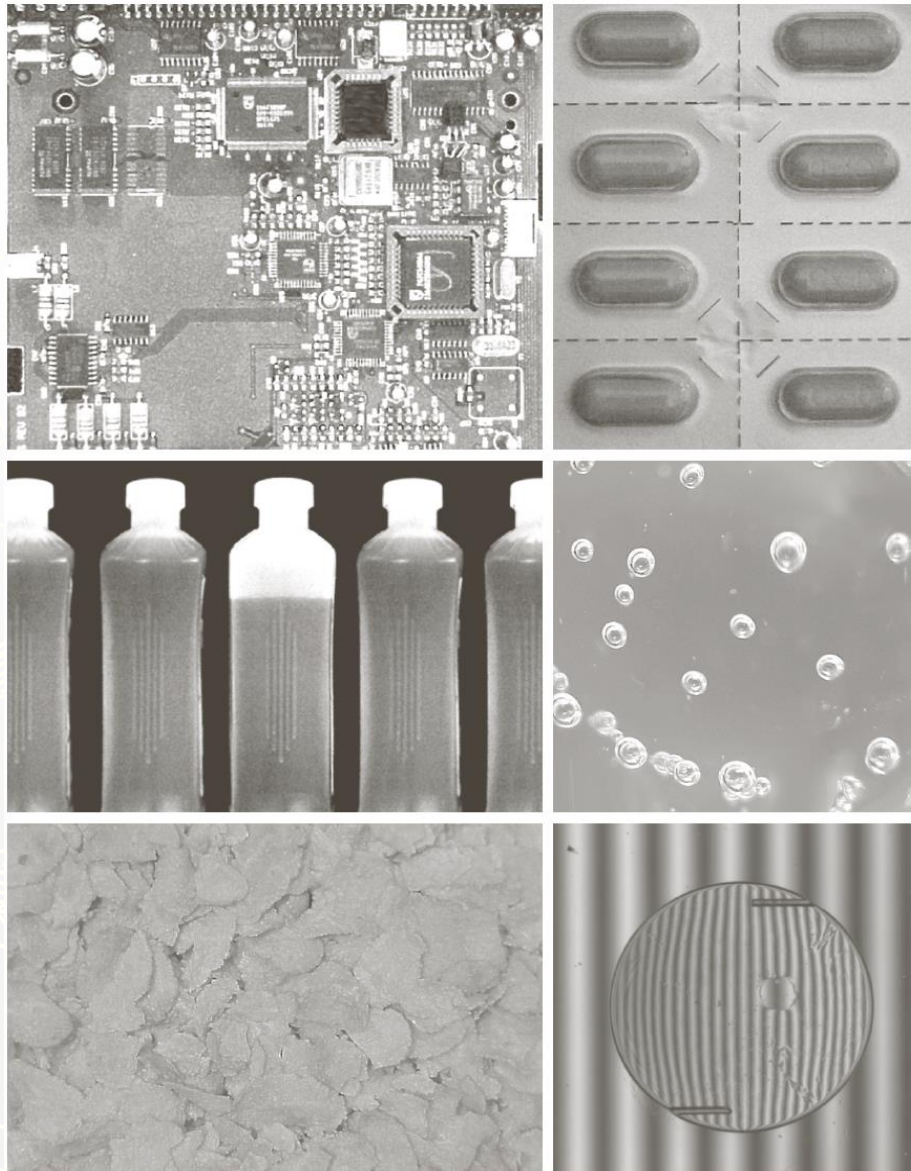


USA 2003

Examples: Infrared Satellite Imaging



Examples: Automated Visual Inspection



a	b
c	d
e	f

FIGURE 1.14

Some examples of manufactured goods often checked using digital image processing.

(a) A circuit board controller.

(b) Packaged pills.

(c) Bottles.

(d) Air bubbles in a clear-plastic product.

(e) Cereal.

(f) Image of intraocular implant.

(Fig. (f) courtesy of Mr. Pete Sites, Perceptics Corporation.)

Examples: Automated Visual Inspection



a b
c
d

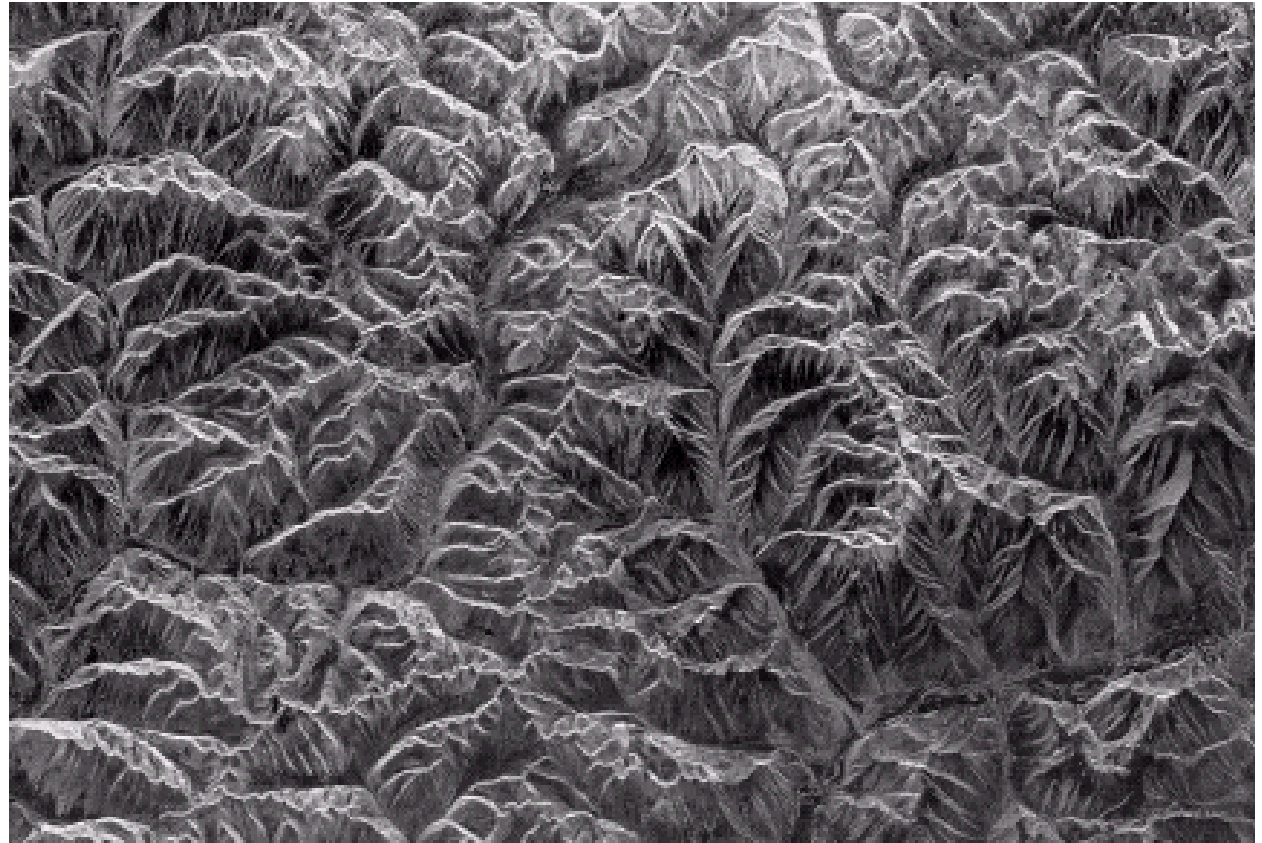
FIGURE 1.15

Some additional examples of imaging in the visual spectrum. (a) Thumb print. (b) Paper currency. (c) and (d) Automated license plate reading. (Figure (a) courtesy of the National Institute of Standards and Technology. Figures (c) and (d) courtesy of Dr. Juan Herrera, Perceptics Corporation.)

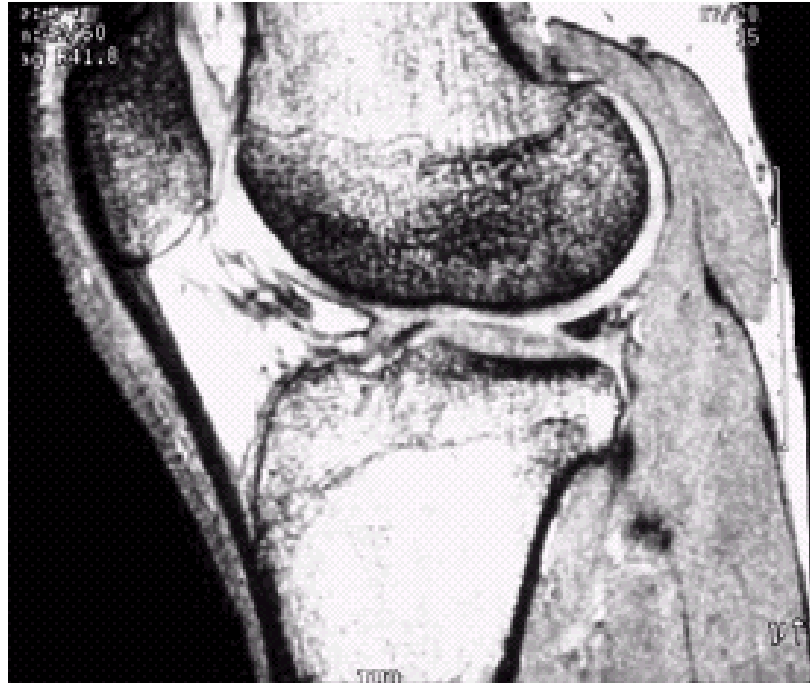
Example of Radar Image

FIGURE 1.16

Spaceborne radar image of mountains in southeast Tibet. (Courtesy of NASA.)



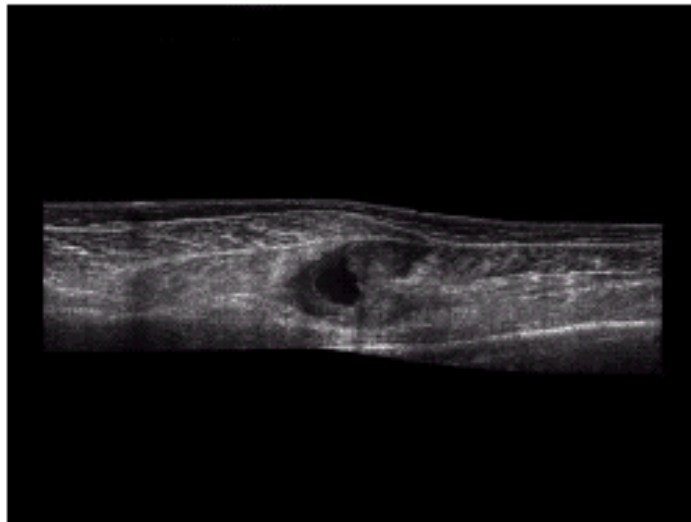
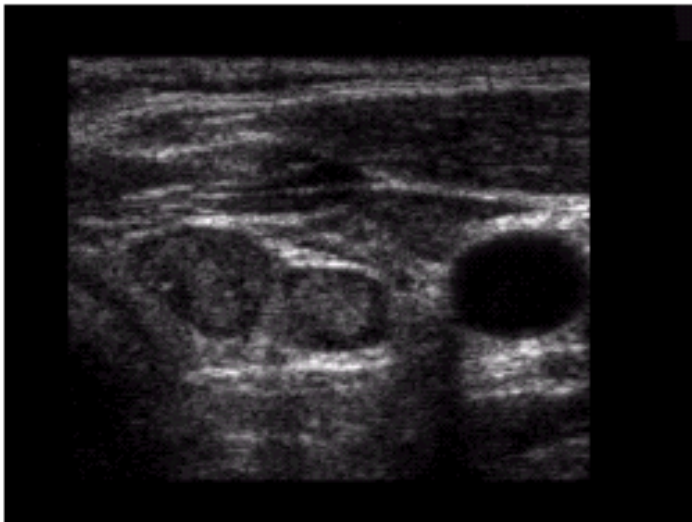
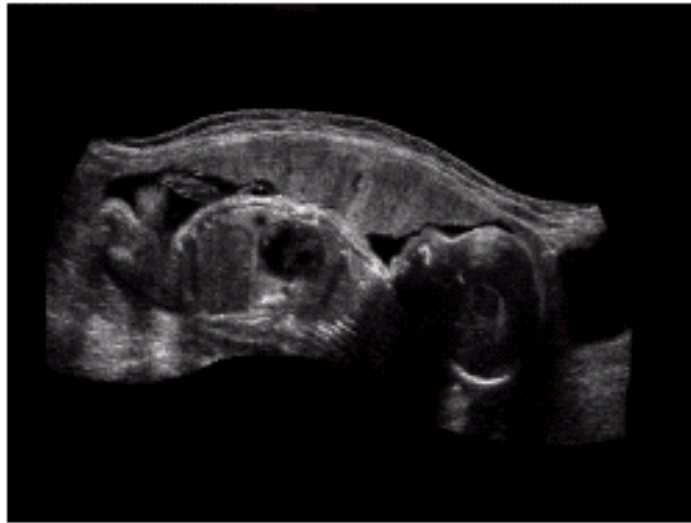
Examples: MRI (Radio Band)



a b

FIGURE 1.17 MRI images of a human (a) knee, and (b) spine. (Image (a) courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School, and (b) Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Examples: Ultrasound Imaging



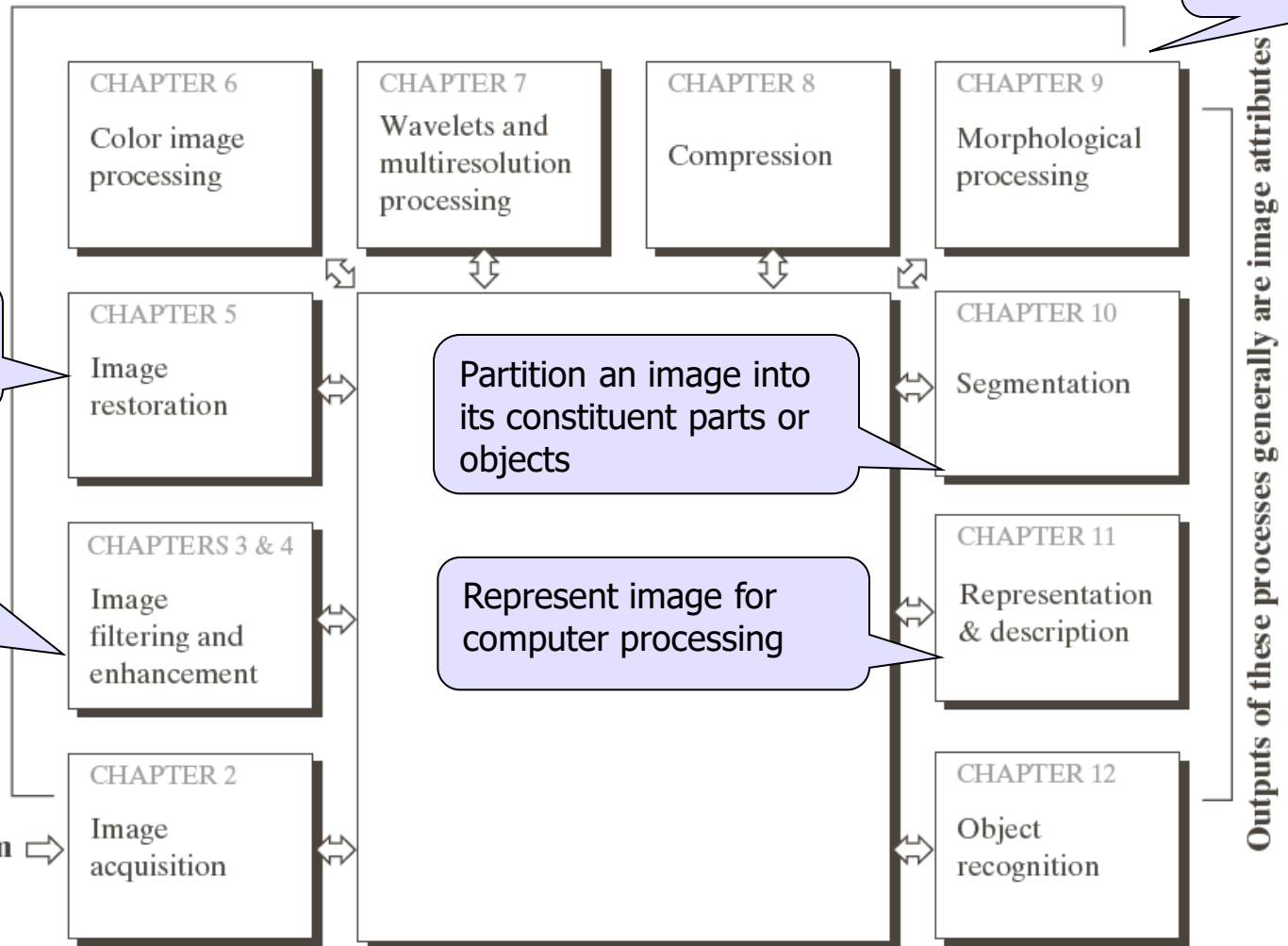
a	b
c	d

FIGURE 1.20
Examples of
ultrasound
imaging. (a) Baby.
(2) Another view
of baby.
(c) Thyroids.
(d) Muscle layers
showing lesion.
(Courtesy of
Siemens Medical
Systems, Inc.,
Ultrasound
Group.)

Fundamental Steps in DIP

Outputs of these processes generally are images

Extracting image components



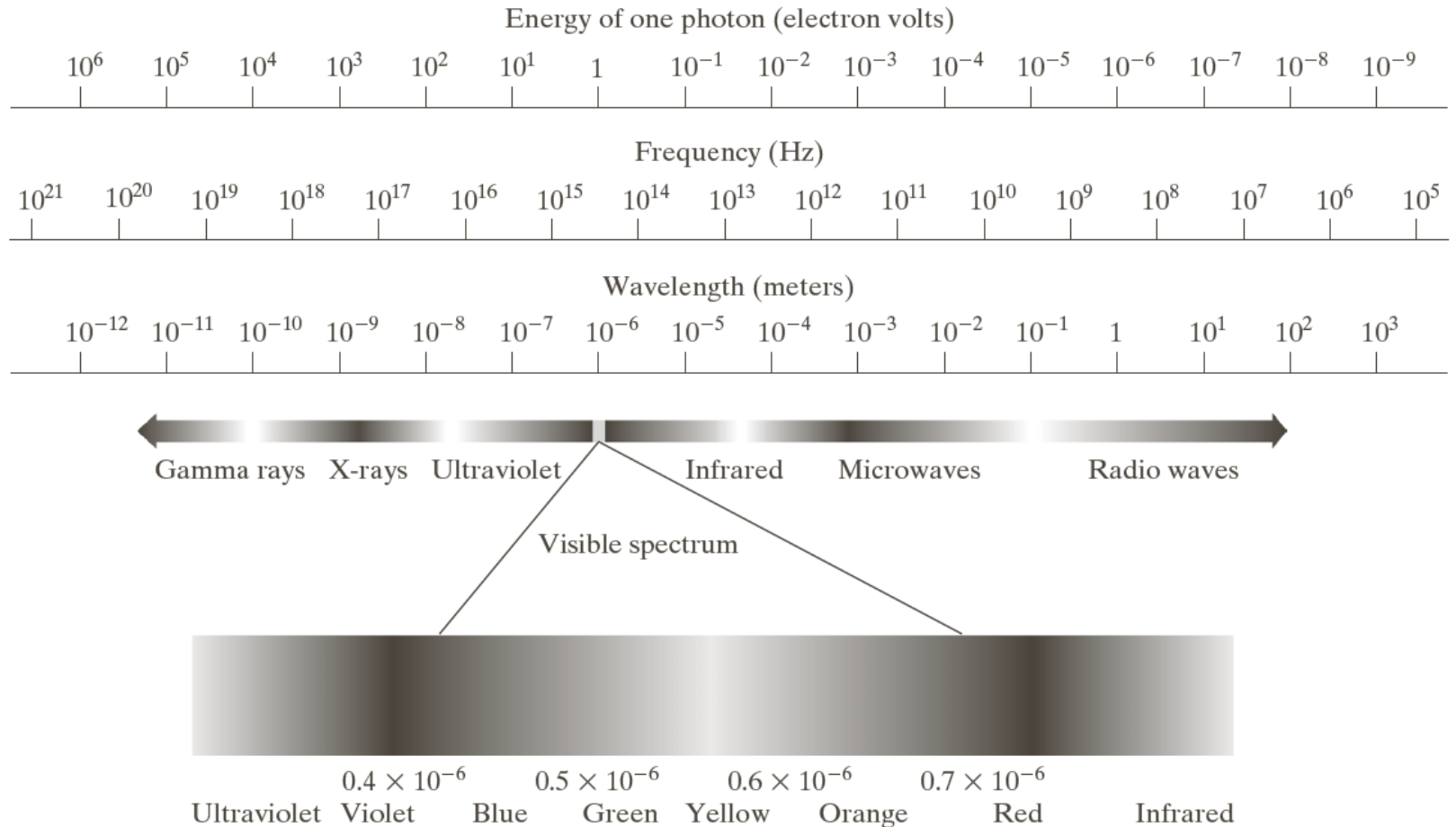
Improving the appearance

Result is more suitable than the original

Partition an image into its constituent parts or objects

Represent image for computer processing

Light and EM Spectrum



$$c = \lambda \nu$$

$$E = h\nu, \quad h: \text{Planck's constant.}$$

Light and EM Spectrum

- ▶ The colors that humans perceive in an object are determined by the nature of the light reflected from the object.

e.g. green objects reflect light with wavelengths primarily in the 500 to 570 nm range while absorbing most of the energy at other wavelength

Light and EM Spectrum

- ▶ Monochromatic light: void of color

Intensity is the only attribute, from black to white

Monochromatic images are referred to as **gray-scale** images

- ▶ Chromatic light bands: 0.43 to 0.79 μm

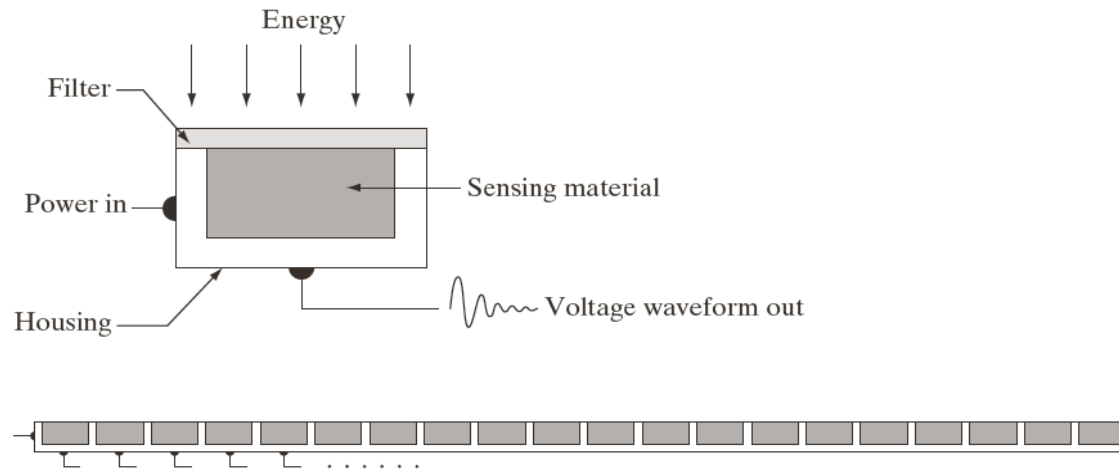
The quality of a chromatic light source:

Radiance: total amount of energy

Luminance (I_m): the amount of energy an observer perceives from a light source

Brightness: a subjective descriptor of light perception that is impossible to measure. It embodies the achromatic notion of intensity and one of the key factors in describing color sensation.

Image Acquisition



a
b
c

FIGURE 2.12

(a) Single imaging sensor.

(b) Line sensor.

(c) Array sensor.

Transform
illumination
energy into
digital images

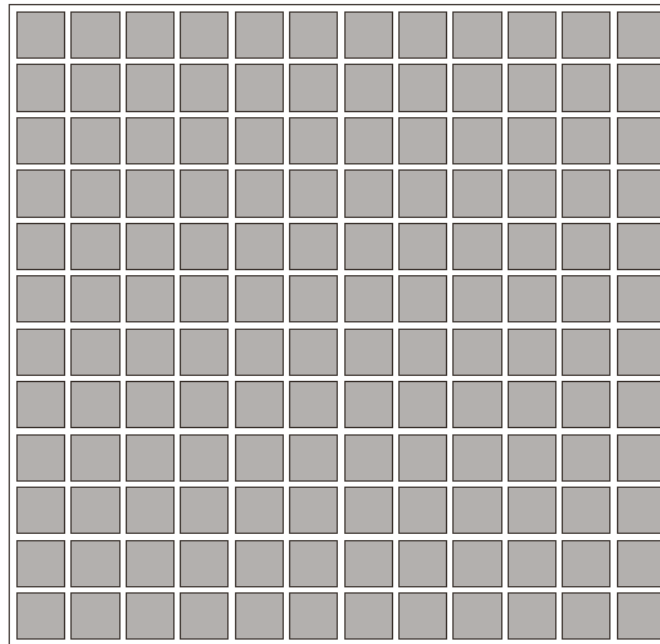


Image Acquisition Using a Single Sensor

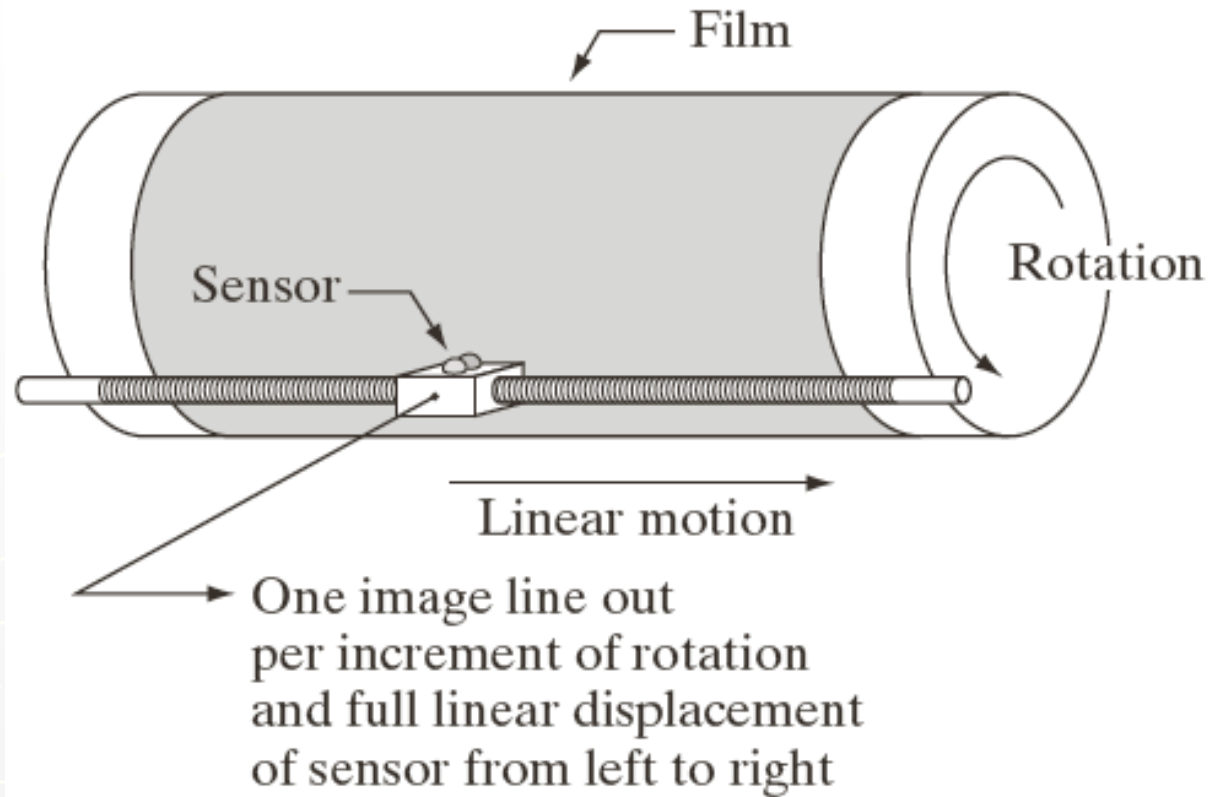
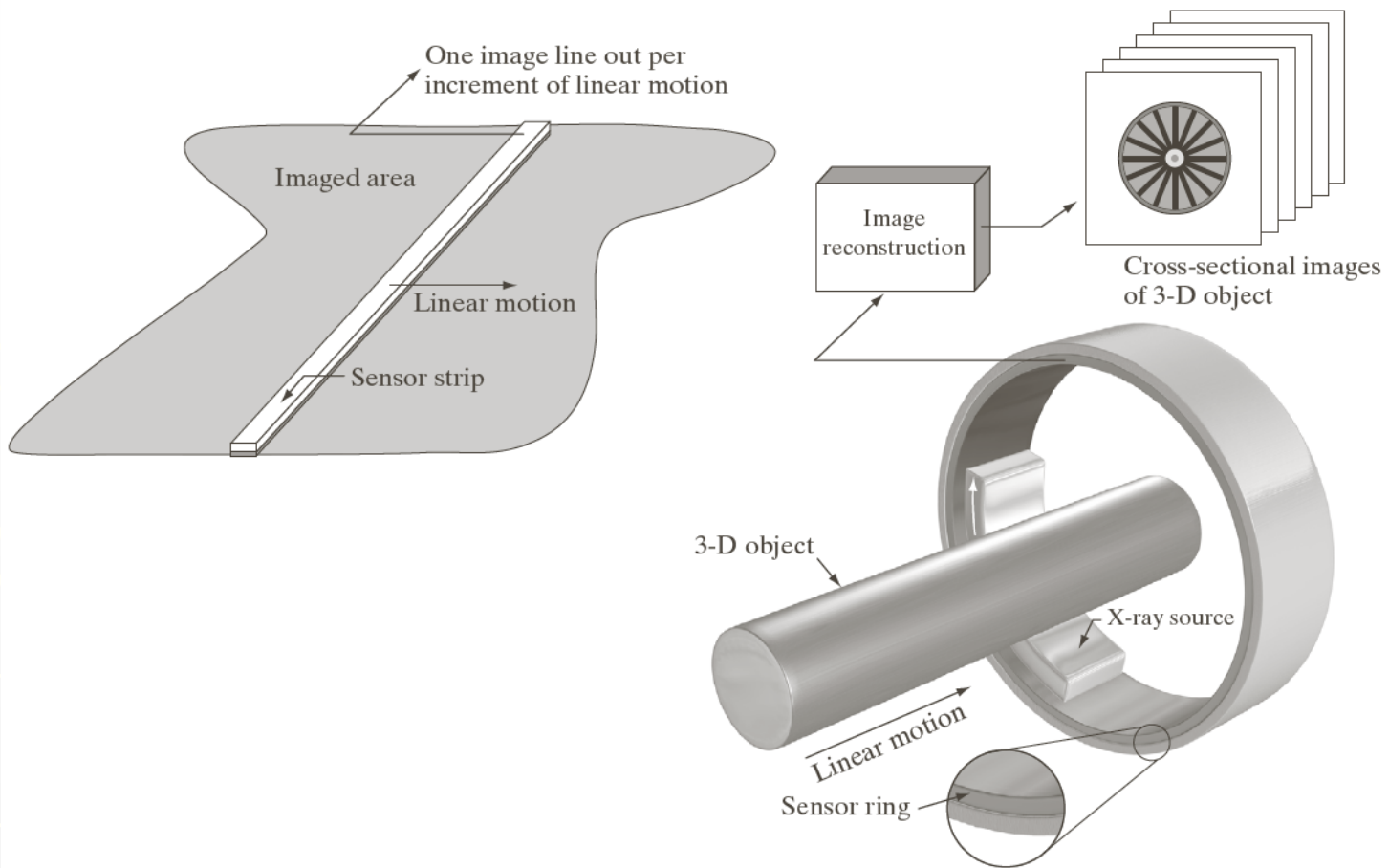


FIGURE 2.13
Combining a single sensor with motion to generate a 2-D image.

Image Acquisition Using Sensor Strips



a b

FIGURE 2.14 (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

Image Acquisition Process

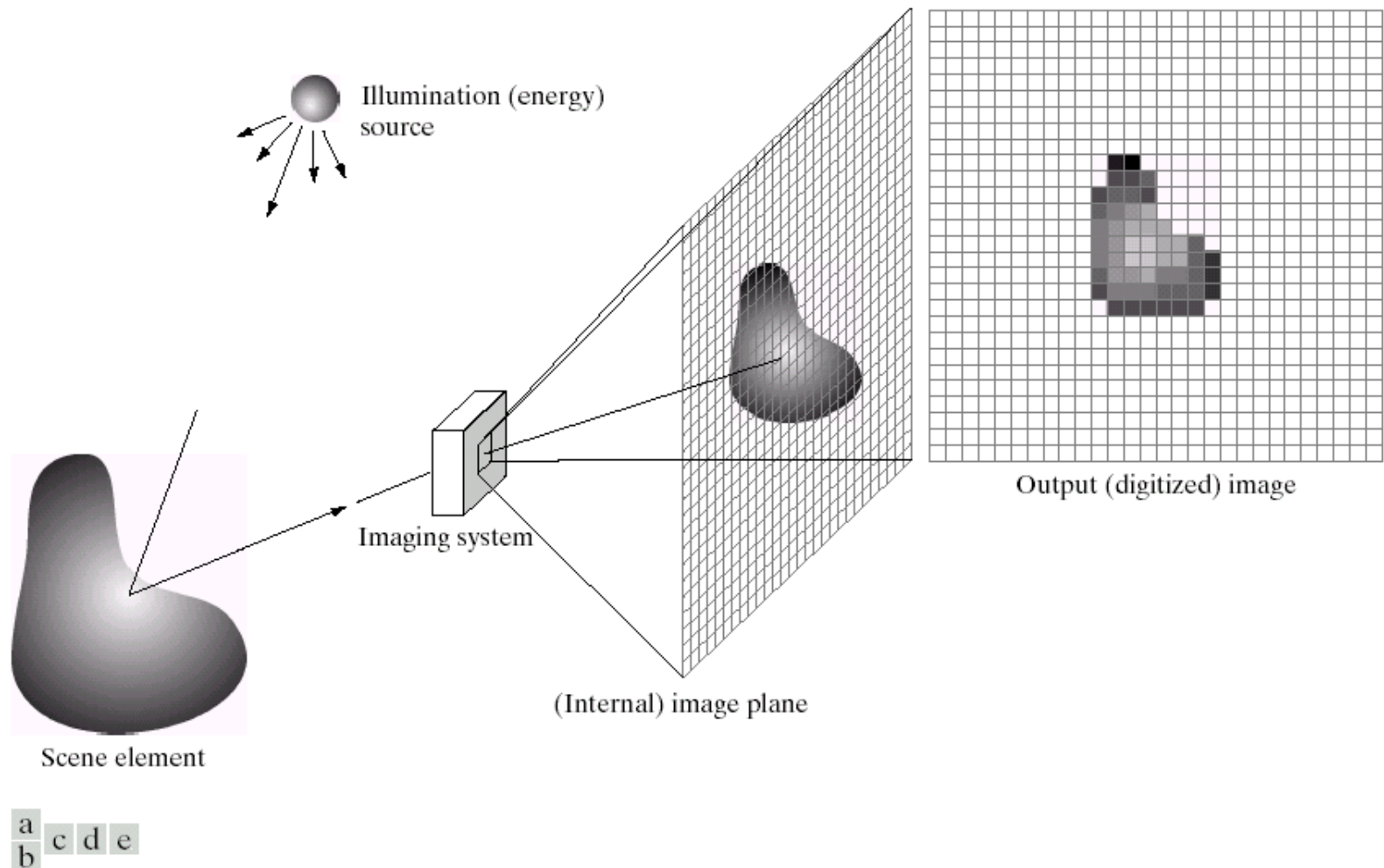


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Formation Model

$$f(x, y) = i(x, y) \cdot r(x, y)$$

$f(x, y)$: intensity at the point (x, y)

$i(x, y)$: illumination at the point (x, y)

(the amount of source illumination incident on the scene)

$r(x, y)$: reflectance/transmissivity at the point (x, y)

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x, y) < \infty$ and $0 < r(x, y) < 1$

Some Typical Ranges of illumination

► Illumination

Lumen — A unit of light flow or luminous flux

Lumen per square meter (lm/m^2) — The metric unit of measure for illuminance of a surface

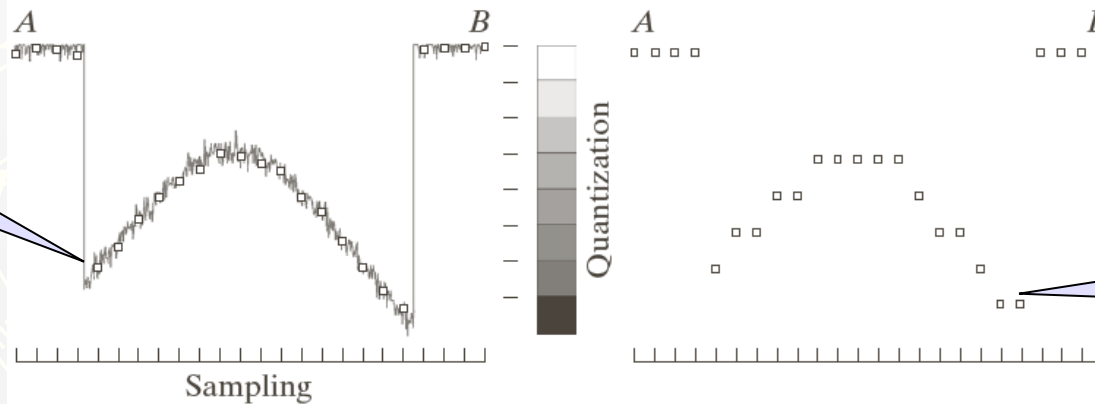
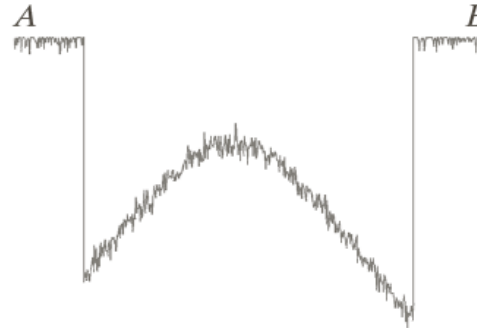
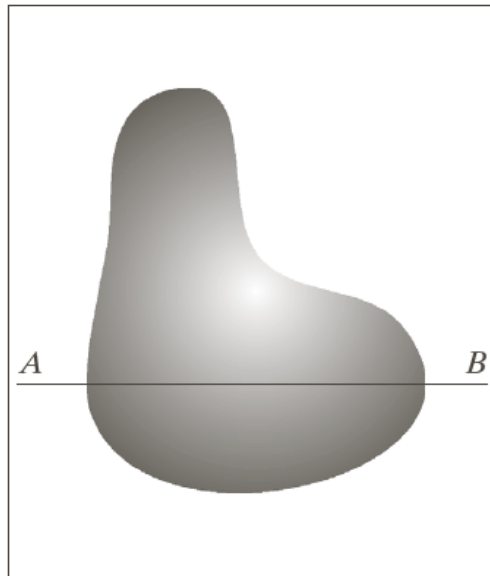
- On a clear day, the sun may produce in excess of $90,000 \text{ lm/m}^2$ of illumination on the surface of the Earth
- On a cloudy day, the sun may produce less than $10,000 \text{ lm/m}^2$ of illumination on the surface of the Earth
- On a clear evening, the moon yields about 0.1 lm/m^2 of illumination
- The typical illumination level in a commercial office is about 1000 lm/m^2

Some Typical Ranges of Reflectance

► Reflectance

- 0.01 for black velvet
- 0.65 for stainless steel
- 0.80 for flat-white wall paint
- 0.90 for silver-plated metal
- 0.93 for snow

Image Sampling and Quantization



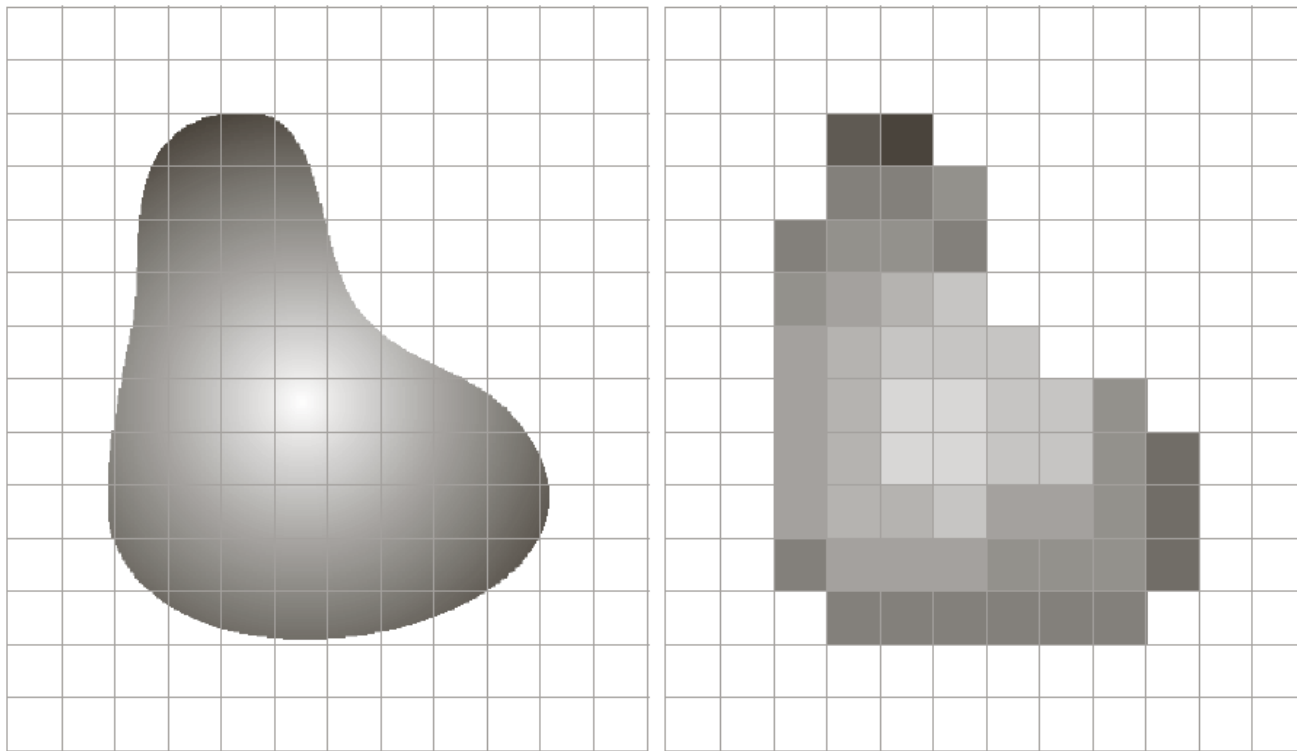
a	b
c	d

FIGURE 2.16
Generating a digital image.
(a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Digitizing the coordinate values

Digitizing the amplitude values

Image Sampling and Quantization



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

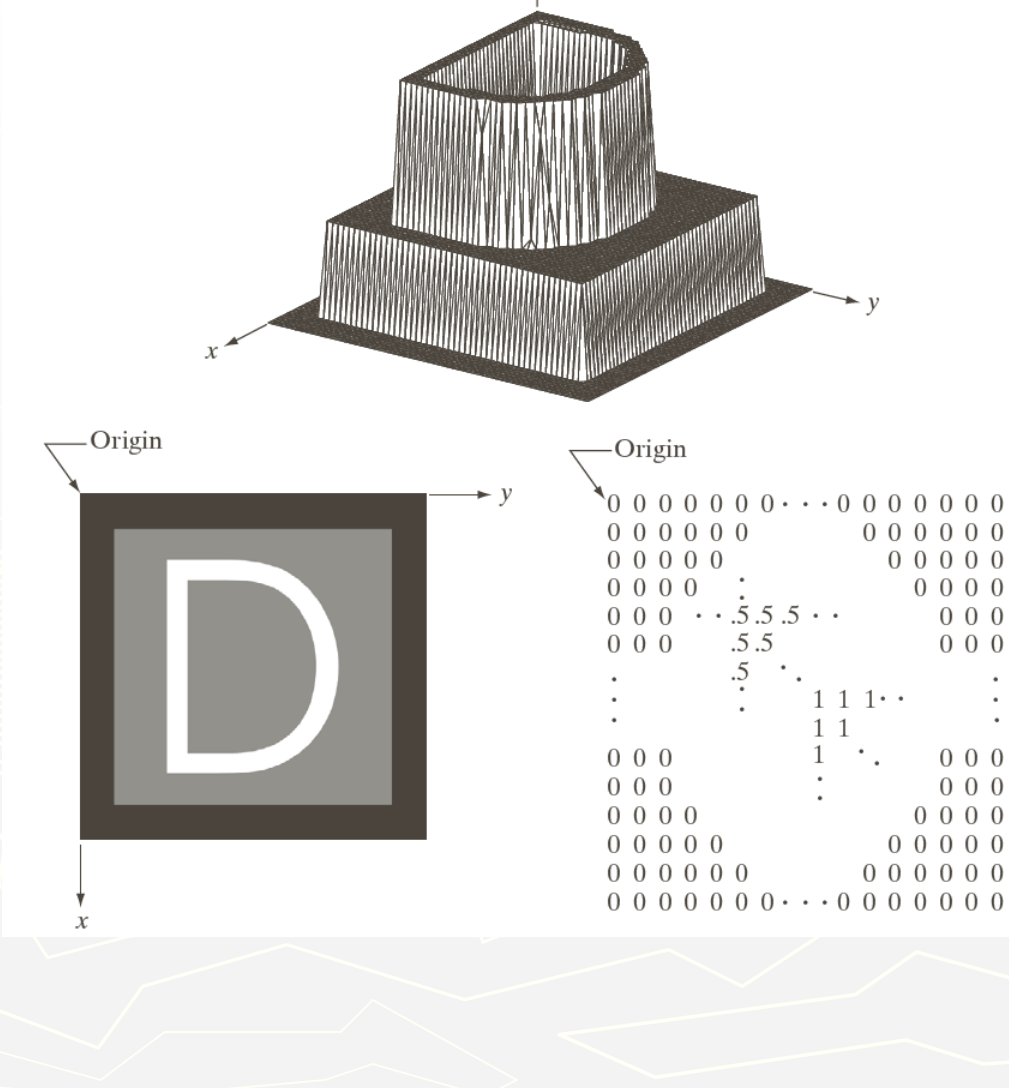


FIGURE 2.18
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

Representing Digital Images

- The representation of an $M \times N$ numerical array as

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

Representing Digital Images

- The representation of an $M \times N$ numerical array as

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \dots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \dots & a_{1,N-1} \\ \dots & \dots & \dots & \dots \\ a_{M-1,0} & a_{M-1,1} & \dots & a_{M-1,N-1} \end{bmatrix}$$

Representing Digital Images

- The representation of an $M \times N$ numerical array in MATLAB

$$f(x, y) = \begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

Representing Digital Images

- ▶ Discrete intensity interval $[0, L-1]$, $L=2^k$
- ▶ The number b of bits required to store a $M \times N$ digitized image

$$b = M \times N \times k$$

Representing Digital Images

TABLE 2.1

Number of storage bits for various values of N and k .

N/k	1 ($L = 2$)	2 ($L = 4$)	3 ($L = 8$)	4 ($L = 16$)	5 ($L = 32$)	6 ($L = 64$)	7 ($L = 128$)	8 ($L = 256$)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Spatial and Intensity Resolution

► Spatial resolution

- A measure of the smallest discernible detail in an image
- stated with *line pairs per unit distance, dots (pixels) per unit distance, dots per inch (dpi)*

► Intensity resolution

- The smallest discernible change in intensity level
- stated with *8 bits, 12 bits, 16 bits, etc.*

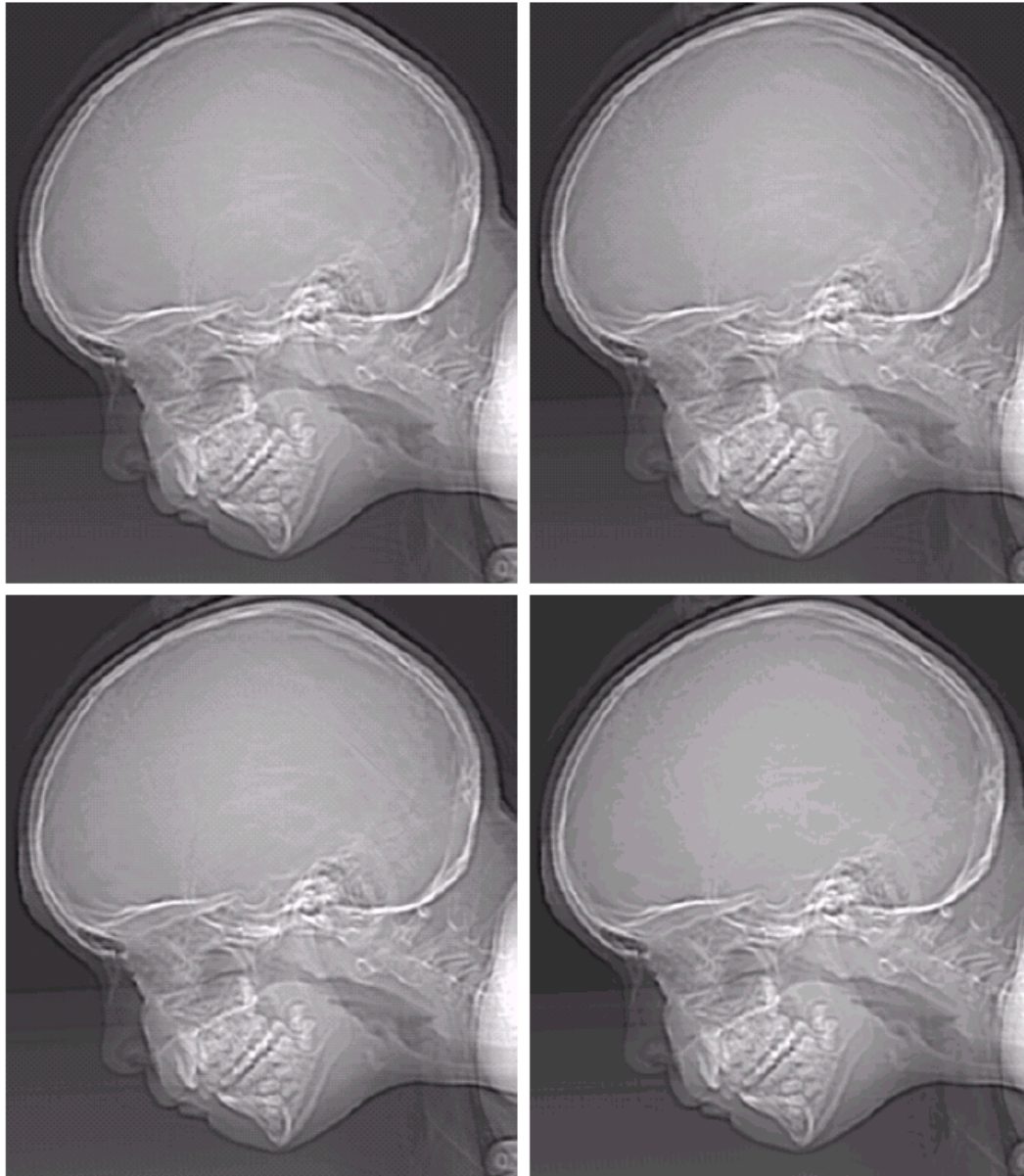
Spatial and Intensity Resolution



a b
c d

FIGURE 2.20 Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

Spatial and Intensity Resolution



a	b
c	d

FIGURE 2.21

(a) 452×374 , 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Spatial and Intensity Resolution

e f
g h

FIGURE 2.21
(Continued)
(e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

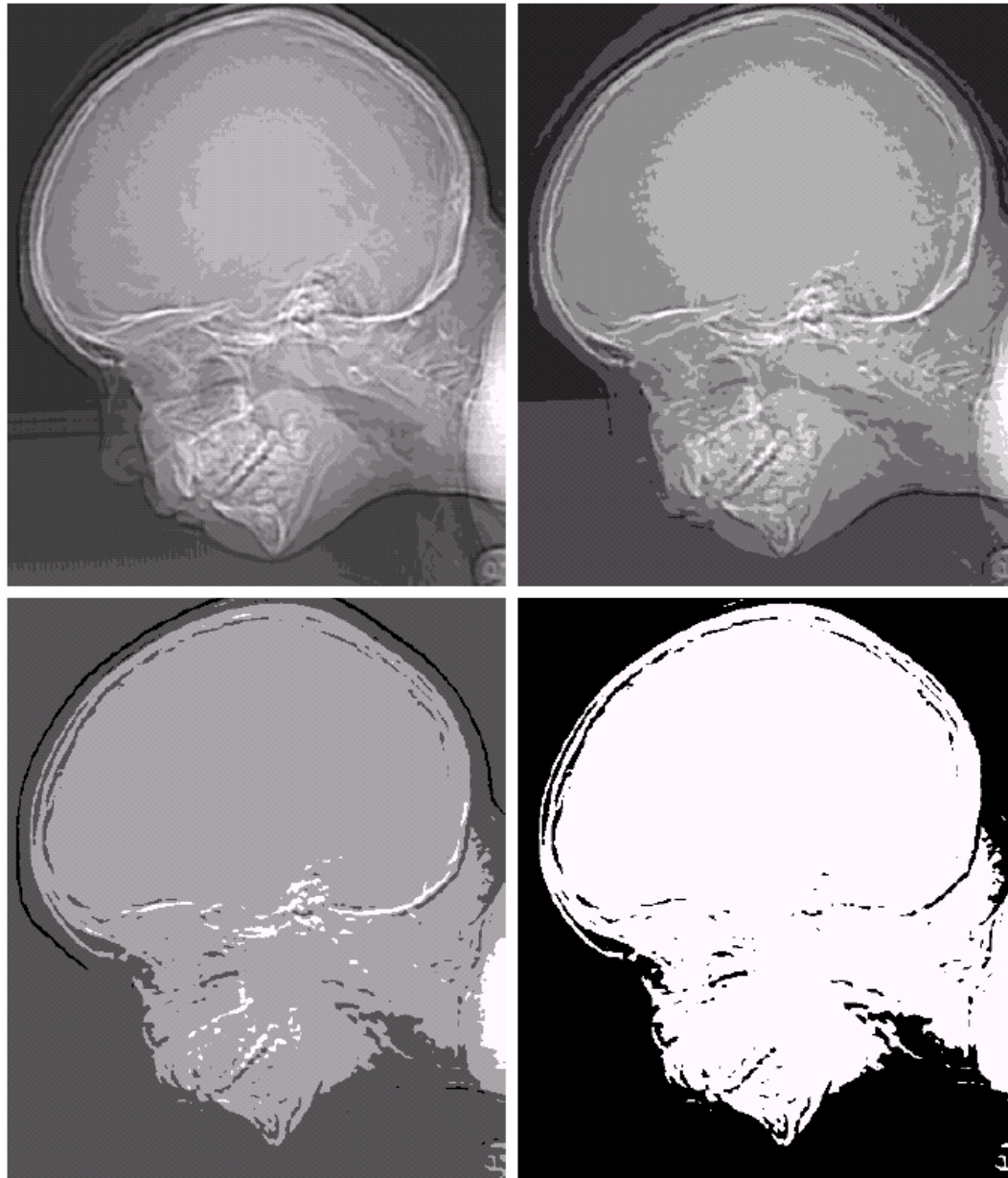


Image Interpolation

- ▶ **Interpolation** — Process of using known data to estimate unknown values

e.g., zooming, shrinking, rotating, and geometric correction

- ▶ **Interpolation** (sometimes called *resampling*) — an imaging method to increase (or decrease) the number of pixels in a digital image.

Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

Image Interpolation: Nearest Neighbor Interpolation

$$\begin{aligned} f_1(x_2, y_2) &= \\ f(\text{round}(x_2), \text{round}(y_2)) &= \\ &= f(x_1, y_1) \end{aligned}$$

$$f(x_1, y_1)$$

$$\begin{aligned} f_1(x_3, y_3) &= \\ f(\text{round}(x_3), \text{round}(y_3)) &= \\ &= f(x_1, y_1) \end{aligned}$$

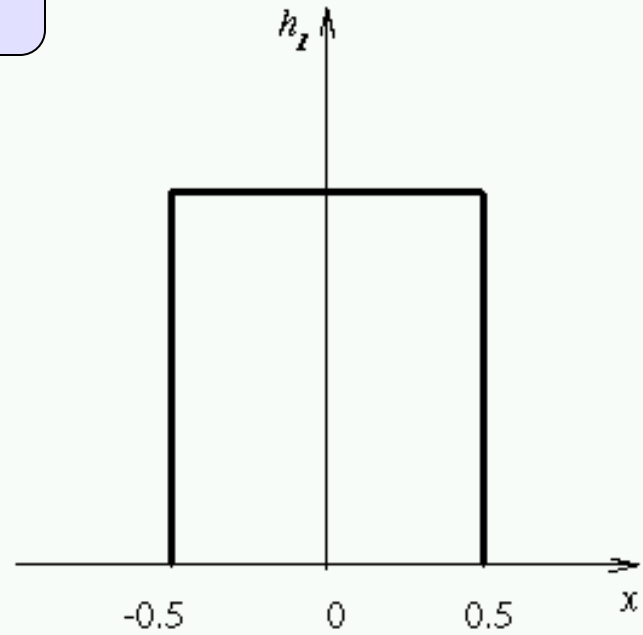
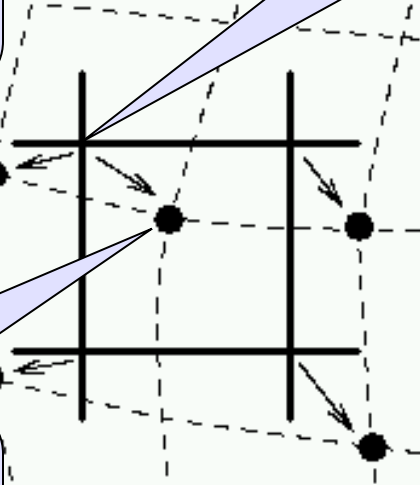
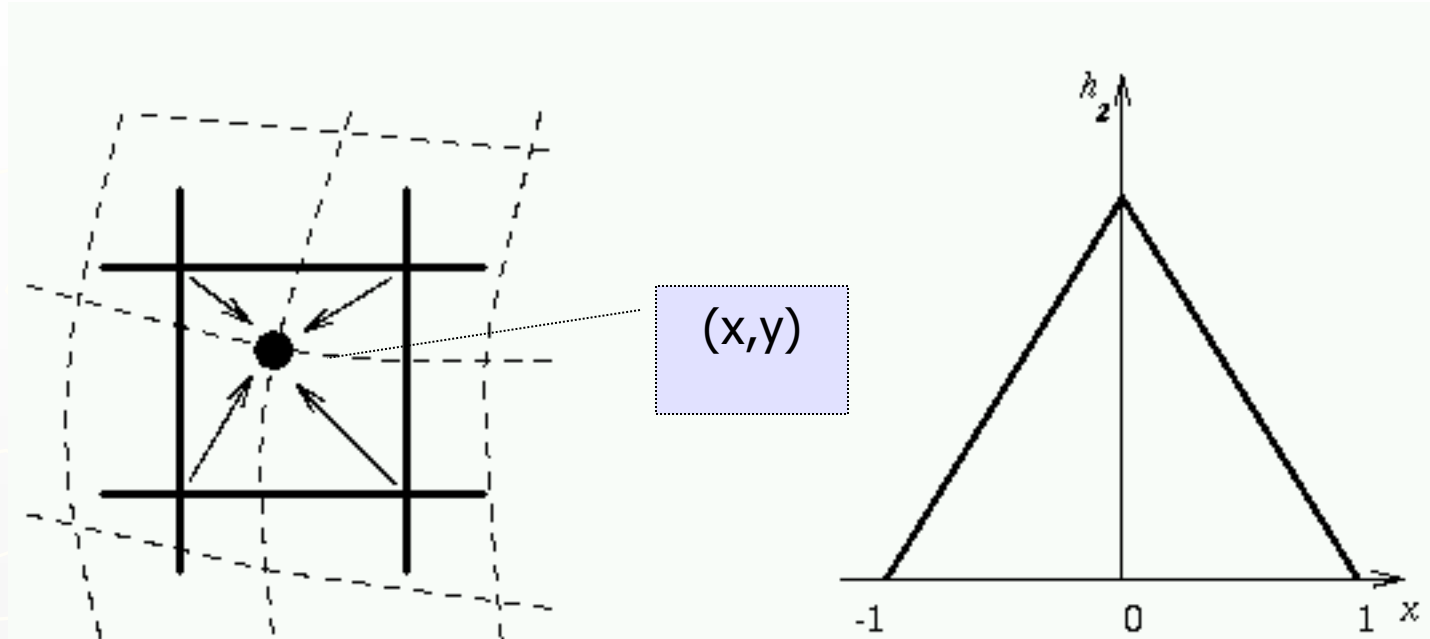


Image Interpolation: Bilinear Interpolation



$$f_2(x, y)$$

$$= (1-a)(1-b)f(l, k) + a(1-b)f(l+1, k)$$

$$+ (1-a)b f(l, k+1) + a b f(l+1, k+1)$$

$$l = \text{floor}(x), k = \text{floor}(y), a = x - l, b = y - k.$$

Image Interpolation: Bicubic Interpolation

- ▶ The intensity value assigned to point (x,y) is obtained by the following equation

$$f_3(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j$$

- ▶ The sixteen coefficients are determined by using the sixteen nearest neighbors.

Examples: Interpolation

Original Image



Examples: Interpolation

Nearest Neighbor Interpolation



Examples: Interpolation

Bilinear Interpolation



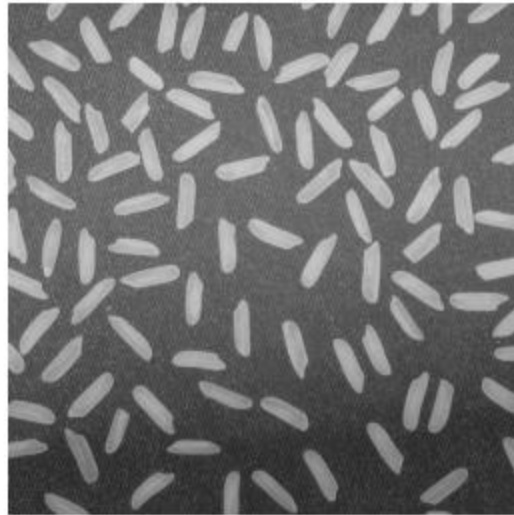
Examples: Interpolation

Bicubic Interpolation



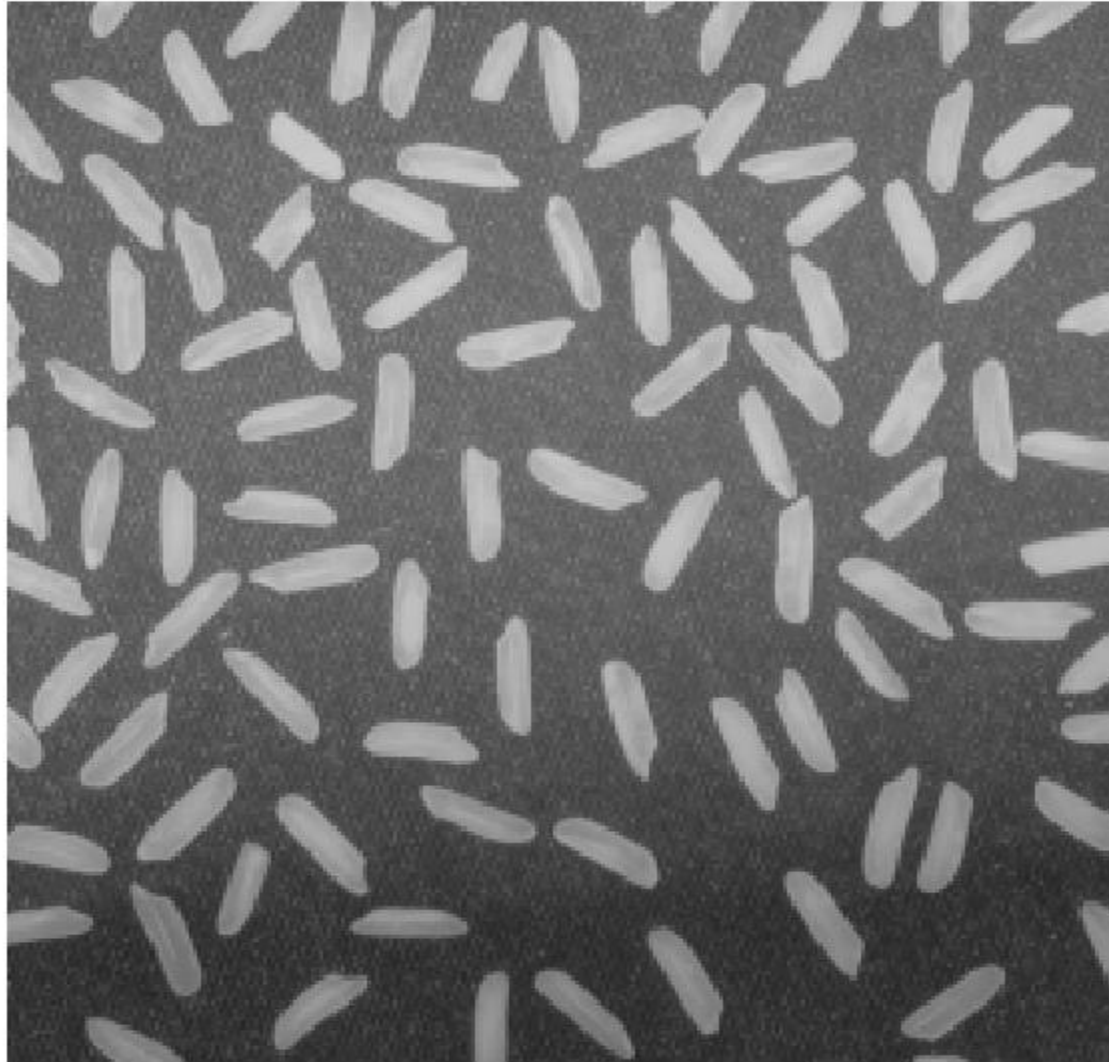
Examples: Interpolation

original image



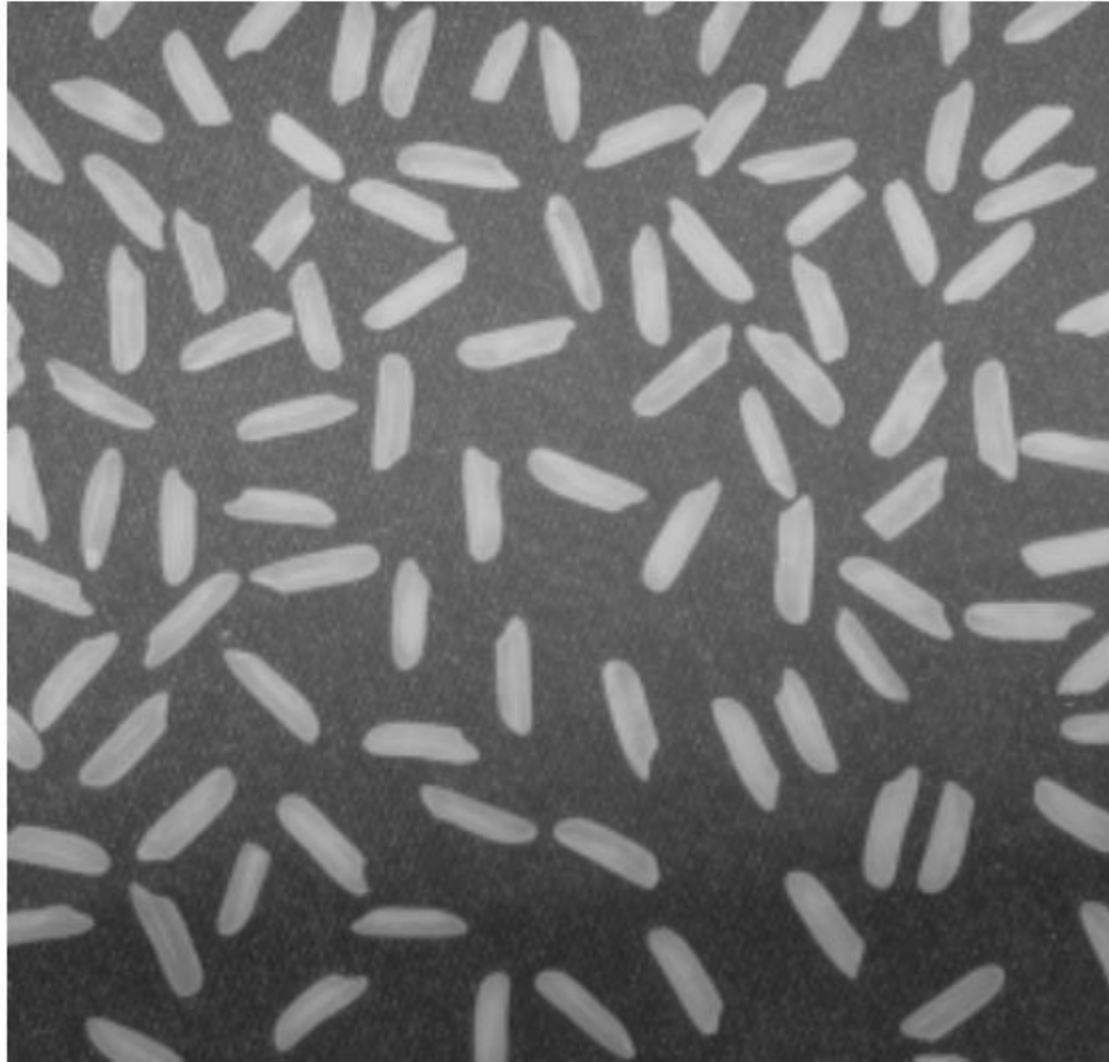
Examples: Interpolation

nearest



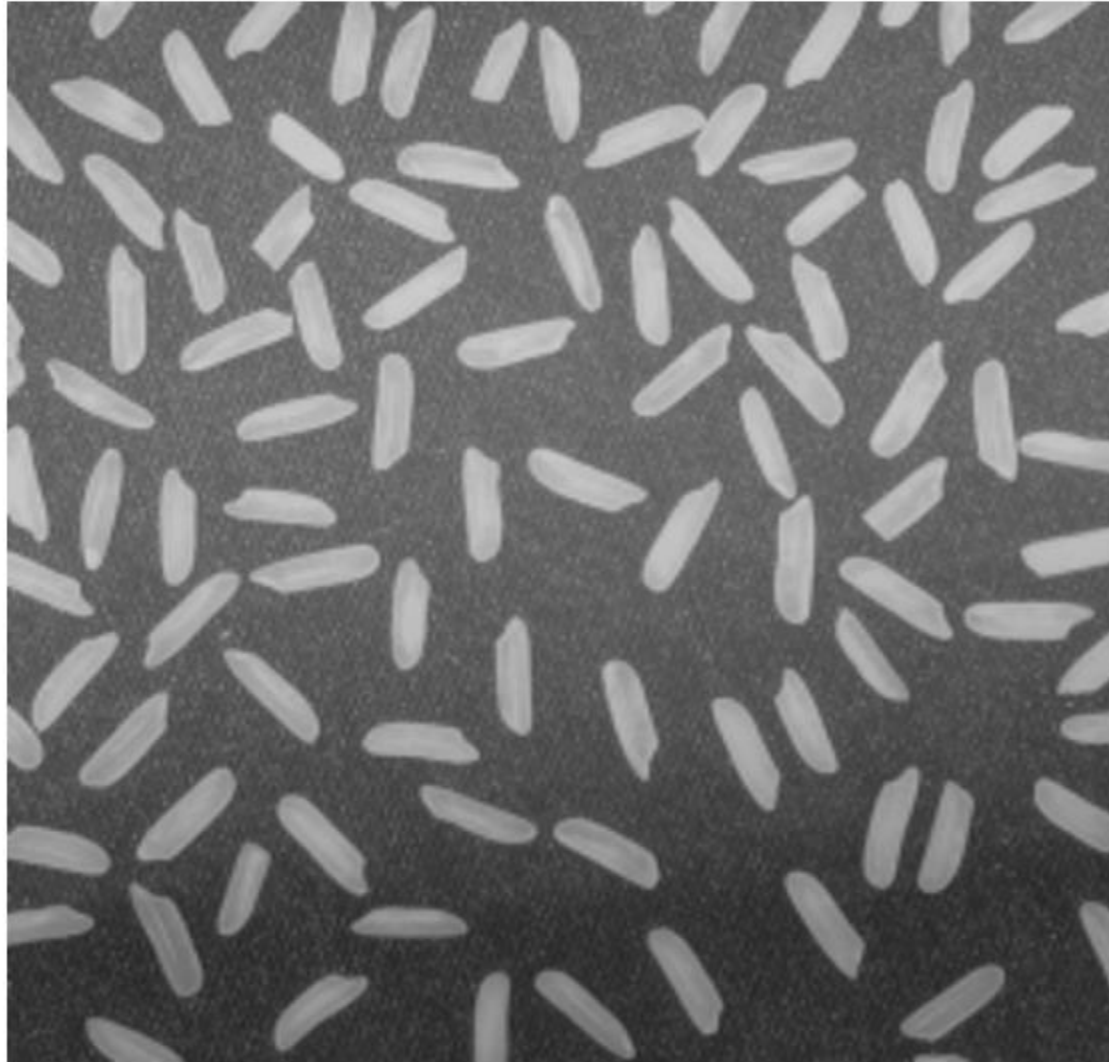
Examples: Interpolation

bilinear



Examples: Interpolation

bicubic



Basic Relationships Between Pixels

- ▶ Neighborhood
- ▶ Adjacency
- ▶ Connectivity
- ▶ Paths
- ▶ Regions and boundaries

Basic Relationships Between Pixels

- ▶ **Neighbors** of a pixel p at coordinates (x,y)
 - **4-neighbors of p** , denoted by $\mathbf{N}_4(p)$:
 $(x-1, y)$, $(x+1, y)$, $(x, y-1)$, and $(x, y+1)$.
 - **4 diagonal neighbors of p** , denoted by $\mathbf{N}_D(p)$:
 $(x-1, y-1)$, $(x+1, y+1)$, $(x+1, y-1)$, and $(x-1, y+1)$.
 - **8 neighbors of p** , denoted $\mathbf{N}_8(p)$
$$\mathbf{N}_8(p) = \mathbf{N}_4(p) \cup \mathbf{N}_D(p)$$

Basic Relationships Between Pixels

► **Adjacency**

Let V be the set of intensity values

- **4-adjacency:** Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
- **8-adjacency:** Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

► Adjacency

Let V be the set of intensity values

► **m-adjacency:** Two pixels p and q with values from V are m-adjacent if

(i) q is in the set $N_4(p)$, or

(ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V .

Basic Relationships Between Pixels

► Path

- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \leq i \leq n$.

- Here n is the *length* of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is **closed** path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

$$\mathbf{V} = \{1, 2\}$$

0 1 1

0 2 0

0 0 1

0 1 1

0 2 0

0 0 1

0 1 1

0 2 0

0 0 1



Examples: Adjacency and Path

$$V = \{1, 2\}$$

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

8-adjacent

Examples: Adjacency and Path

$$V = \{1, 2\}$$

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

8-adjacent

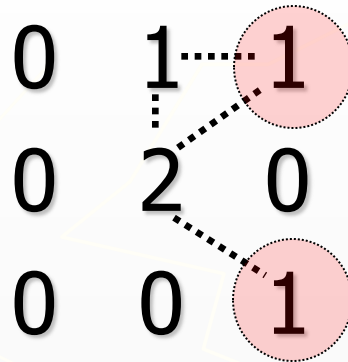
0	1	1
0	2	0
0	0	1

m-adjacent

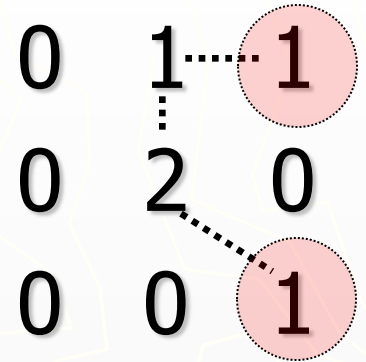
Examples: Adjacency and Path

$$V = \{1, 2\}$$

0 _{1,1}	1 _{1,2}	1 _{1,3}
0 _{2,1}	2 _{2,2}	0 _{2,3}
0 _{3,1}	0 _{3,2}	1 _{3,3}



8-adjacent



m-adjacent

The 8-path from (1,3) to (3,3):

- (i) (1,3), (1,2), (2,2), (3,3)
- (ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):

- (1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

► Connected in S

Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Where $\forall i, 0 \leq i \leq n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- ▶ For every pixel p in S , the set of pixels in S that are connected to p is called a ***connected component*** of S .
- ▶ If S has only one connected component, then S is called ***Connected Set***.
- ▶ We call R a **region** of the image if R is a connected set
- ▶ Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- ▶ Regions that are not to be adjacent are said to be ***disjoint***.

Basic Relationships Between Pixels

► **Boundary (or border)**

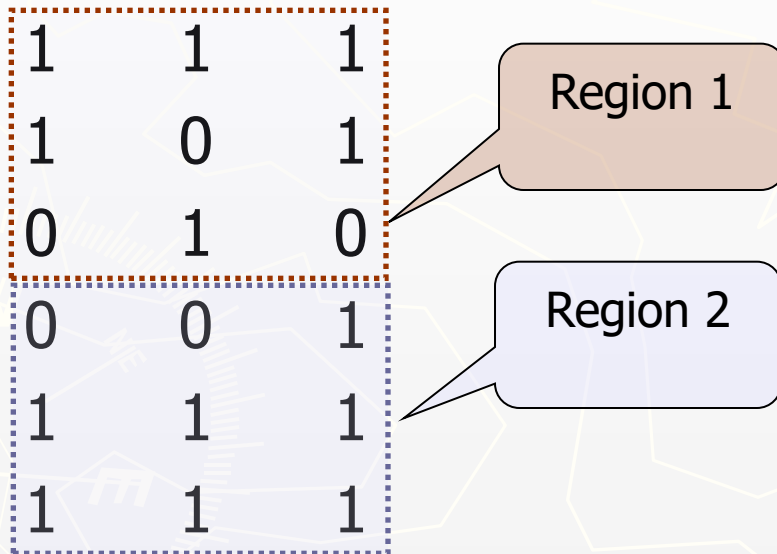
- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R .
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

► **Foreground and background**

- An image contains K disjoint regions, R_k , $k = 1, 2, \dots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement. All the points in R_u is called **foreground**;
All the points in $(R_u)^c$ is called **background**.

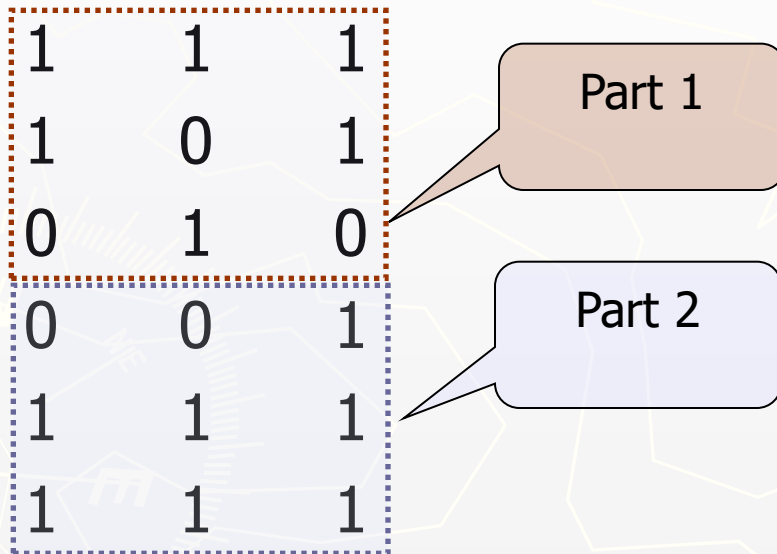
Question 1

- ▶ In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)

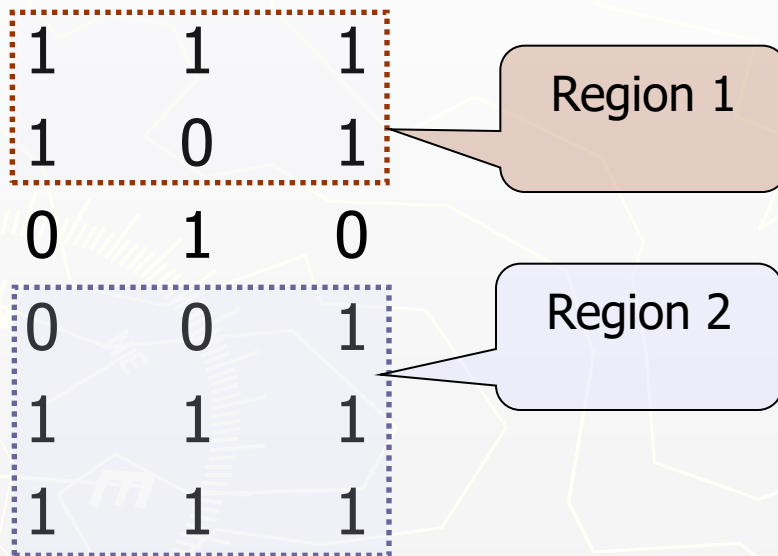


Question 2

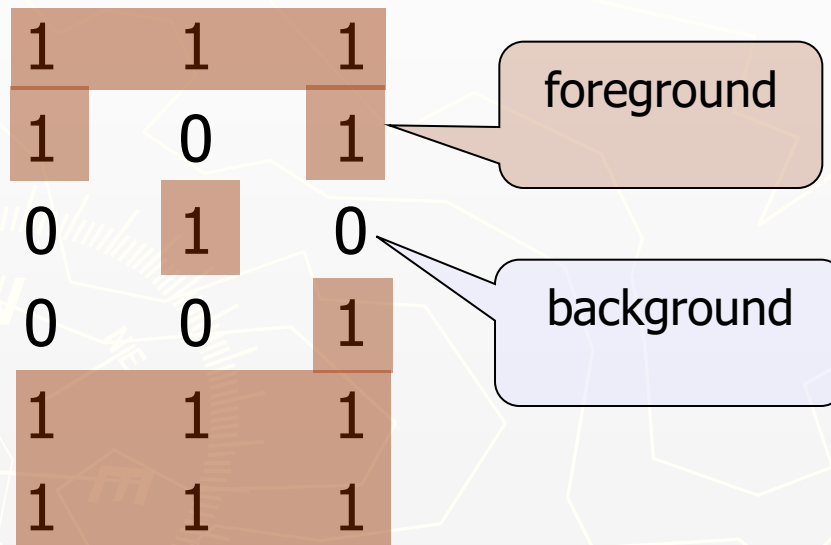
- ▶ In the following arrangement of pixels, are the two parts (of 1s) adjacent? (if 4-adjacency is used)



- ▶ In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)



- In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)



Question 3

- In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 4

- In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Distance Measures

- ▶ Given pixels p , q and z with coordinates (x, y) , (s, t) , (u, v) respectively, the distance function D has following properties:

a. $D(p, q) \geq 0$ $[D(p, q) = 0, \text{ iff } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

b. City Block Distance:

$$D_4(p, q) = |x-s| + |y-t|$$

c. Chess Board Distance:

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

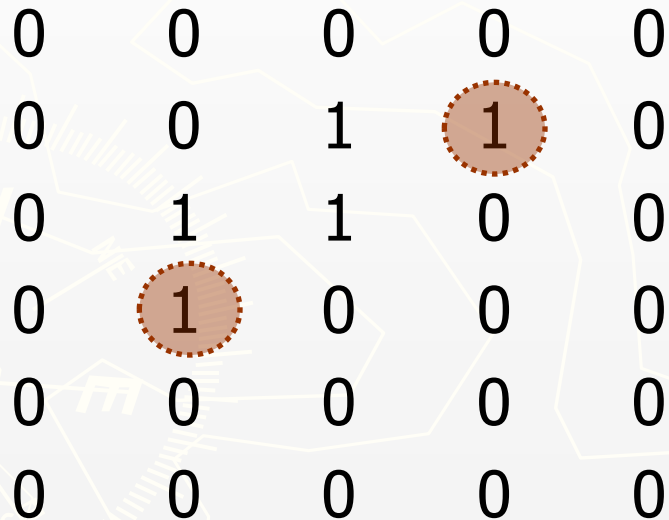
Question 5

- In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 6

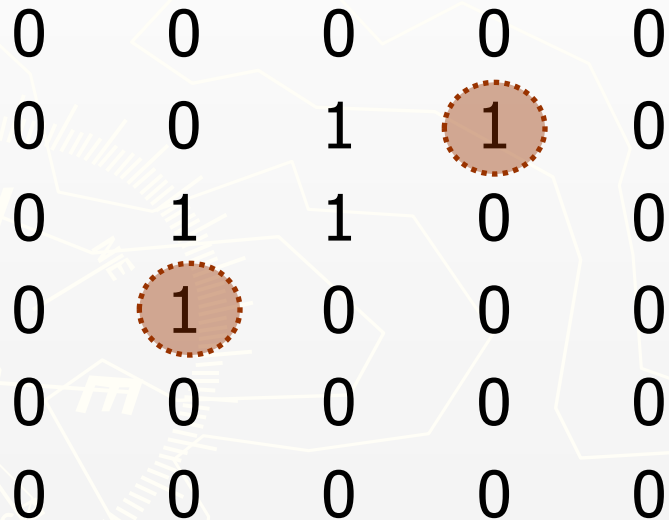
- In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?



0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 7

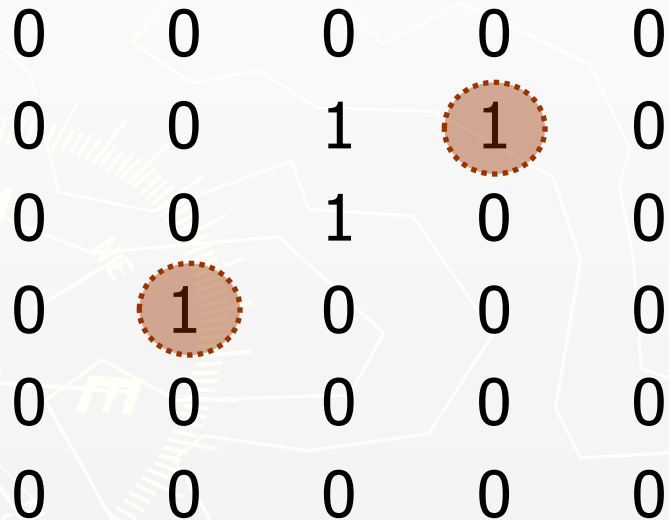
- In the following arrangement of pixels, what's the value of the length of the m-path between the circled two points?



0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 8

- In the following arrangement of pixels, what's the value of the length of the m-path between the circled two points?



0	0	0	0	0
0	0	1	1	0
0	0	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Introduction to Mathematical Operations in DIP

► Array vs. Matrix Operation

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array
product
operator

$$A .* B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix
product
operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

► Linear vs. Nonlinear Operation

$$H[f(x, y)] = g(x, y)$$

$$H[a_i f_i(x, y) + a_j f_j(x, y)]$$

$$= H[a_i f_i(x, y)] + H[a_j f_j(x, y)]$$

$$= a_i H[f_i(x, y)] + a_j H[f_j(x, y)]$$

$$= a_i g_i(x, y) + a_j g_j(x, y)$$

Additivity

Homogeneity

H is said to be a **linear operator**;

H is said to be a **nonlinear operator** if it does not meet the above qualification.

Arithmetic Operations

- ▶ Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$

$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: $f(x,y)$

Noise: $n(x,y)$ (at every pair of coordinates (x,y) , the noise is uncorrelated and has zero average value)

Corrupted image: $g(x,y)$

$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^K g_i(x, y)$$

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x, y) = \frac{1}{K} \sum_{i=1}^K g_i(x, y)$$

$$E\{\bar{g}(x, y)\} = E\left\{\frac{1}{K} \sum_{i=1}^K g_i(x, y)\right\}$$

$$= E\left\{\frac{1}{K} \sum_{i=1}^K [f(x, y) + n_i(x, y)]\right\}$$

$$= f(x, y) + E\left\{\frac{1}{K} \sum_{i=1}^K n_i(x, y)\right\}$$

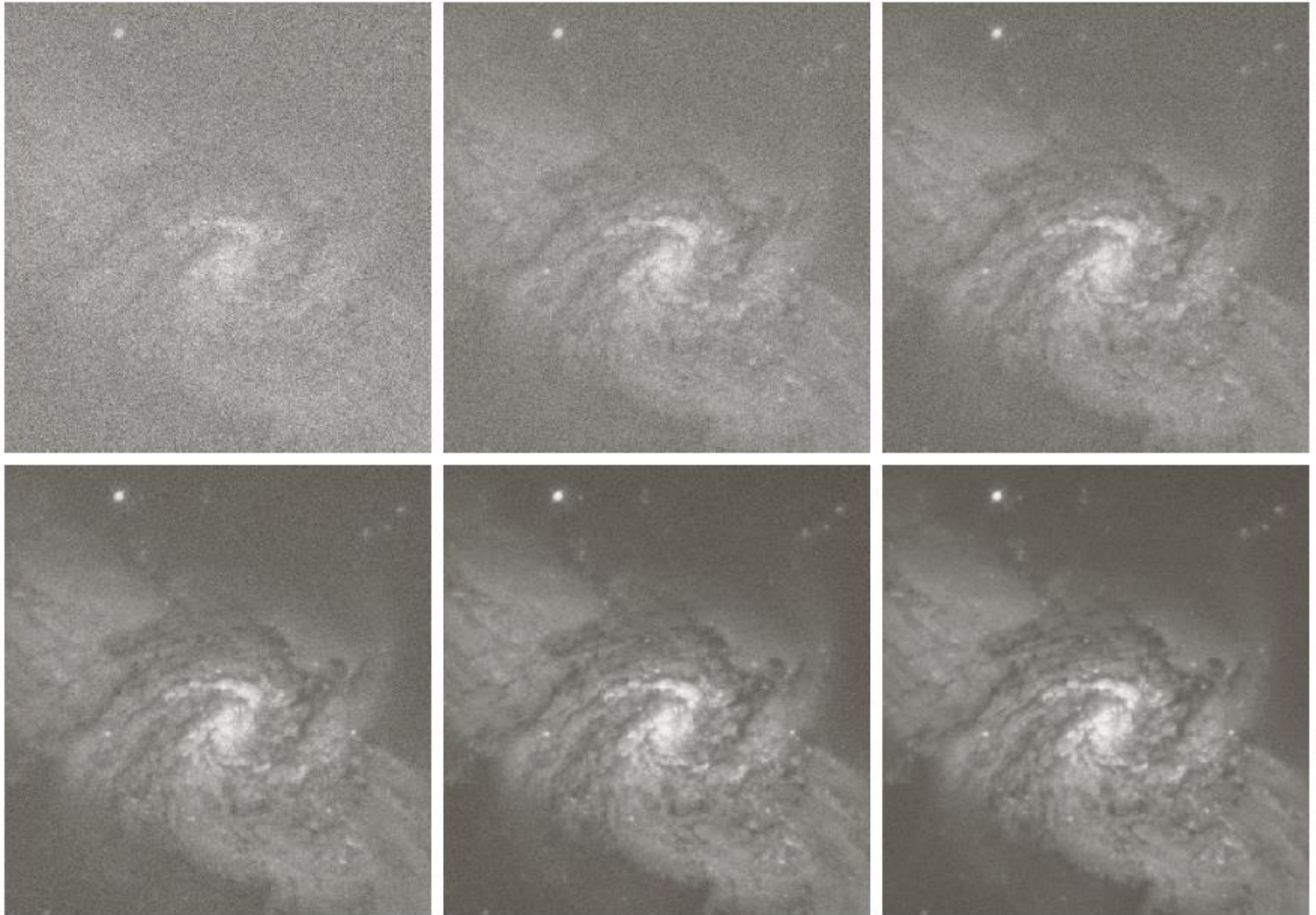
$$= f(x, y)$$

$$\sigma_{\bar{g}(x, y)}^2 = \sigma_{\frac{1}{K} \sum_{i=1}^K g_i(x, y)}^2$$

$$= \sigma_{\frac{1}{K} \sum_{i=1}^K n_i(x, y)}^2 = \frac{1}{K} \sigma_{n(x, y)}^2$$

Example: Addition of Noisy Images for Noise Reduction

- ▶ In astronomy, imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.
- ▶ In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

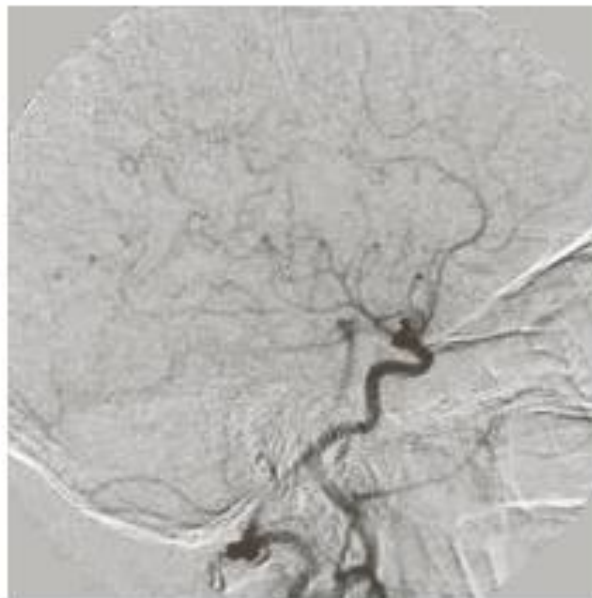
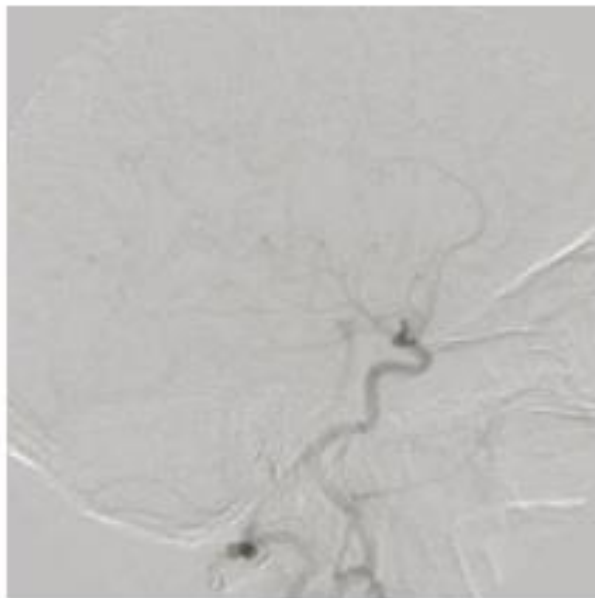
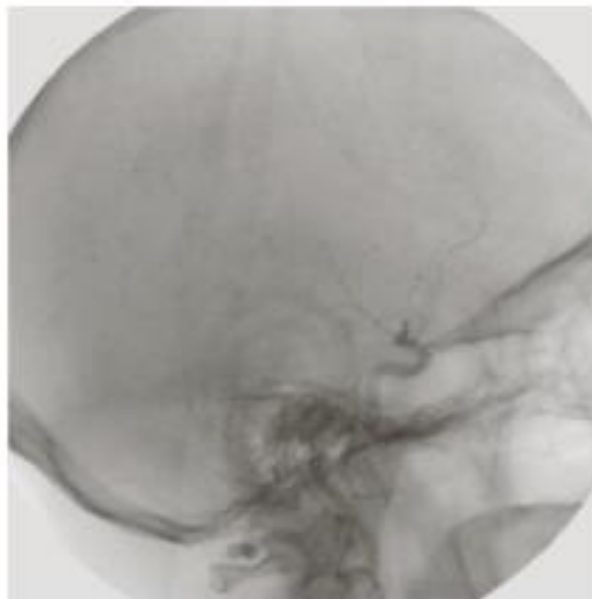
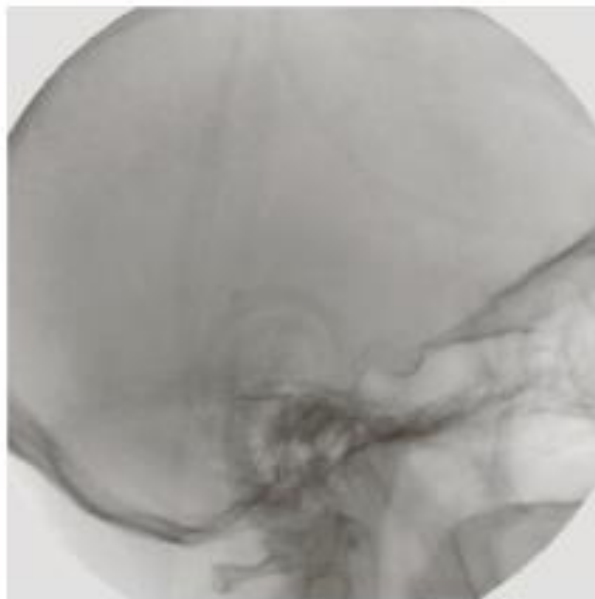
Mask $h(x,y)$: an X-ray image of a region of a patient's body

Live images $f(x,y)$: X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $g(x,y)$

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.



a	b
c	d

FIGURE 2.28

Digital subtraction angiography.

(a) Mask image.

(b) A live image.

(c) Difference

between (a) and

(b). (d) Enhanced

difference image.

(Figures (a) and

(b) courtesy of

The Image

Sciences Institute,

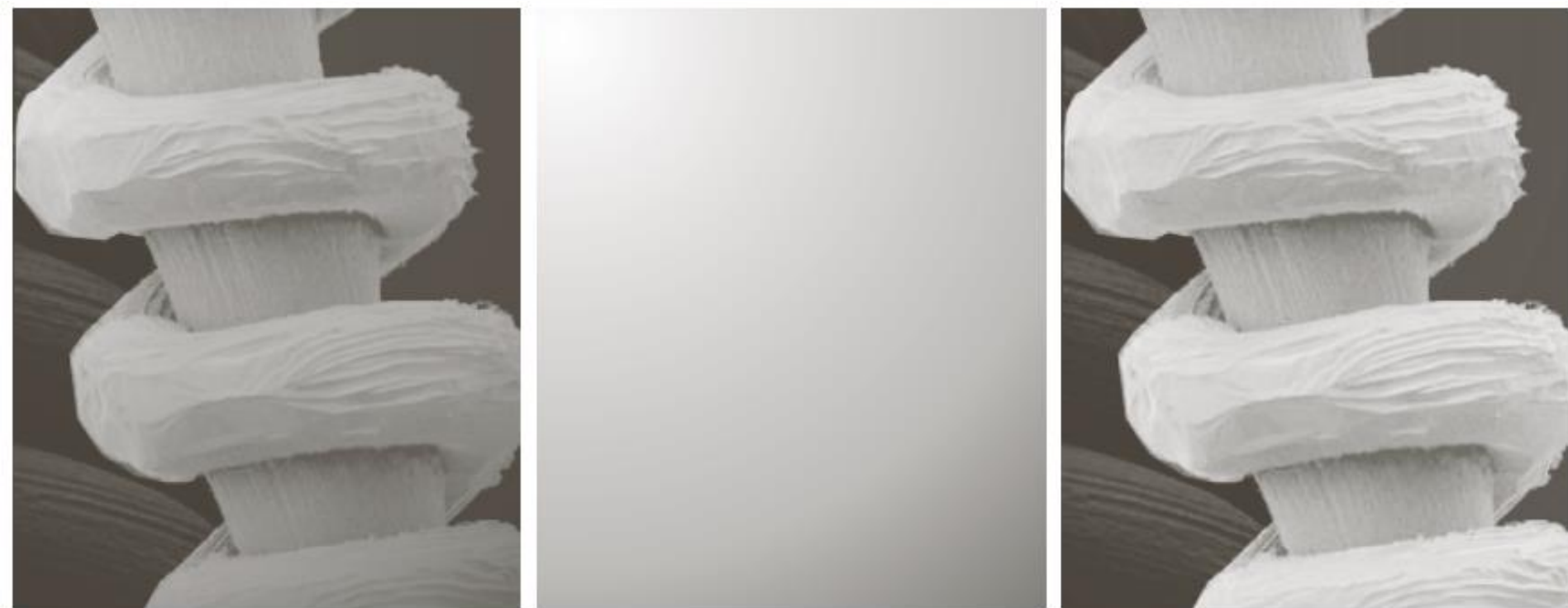
University

Medical Center,

Utrecht, The

Netherlands.)

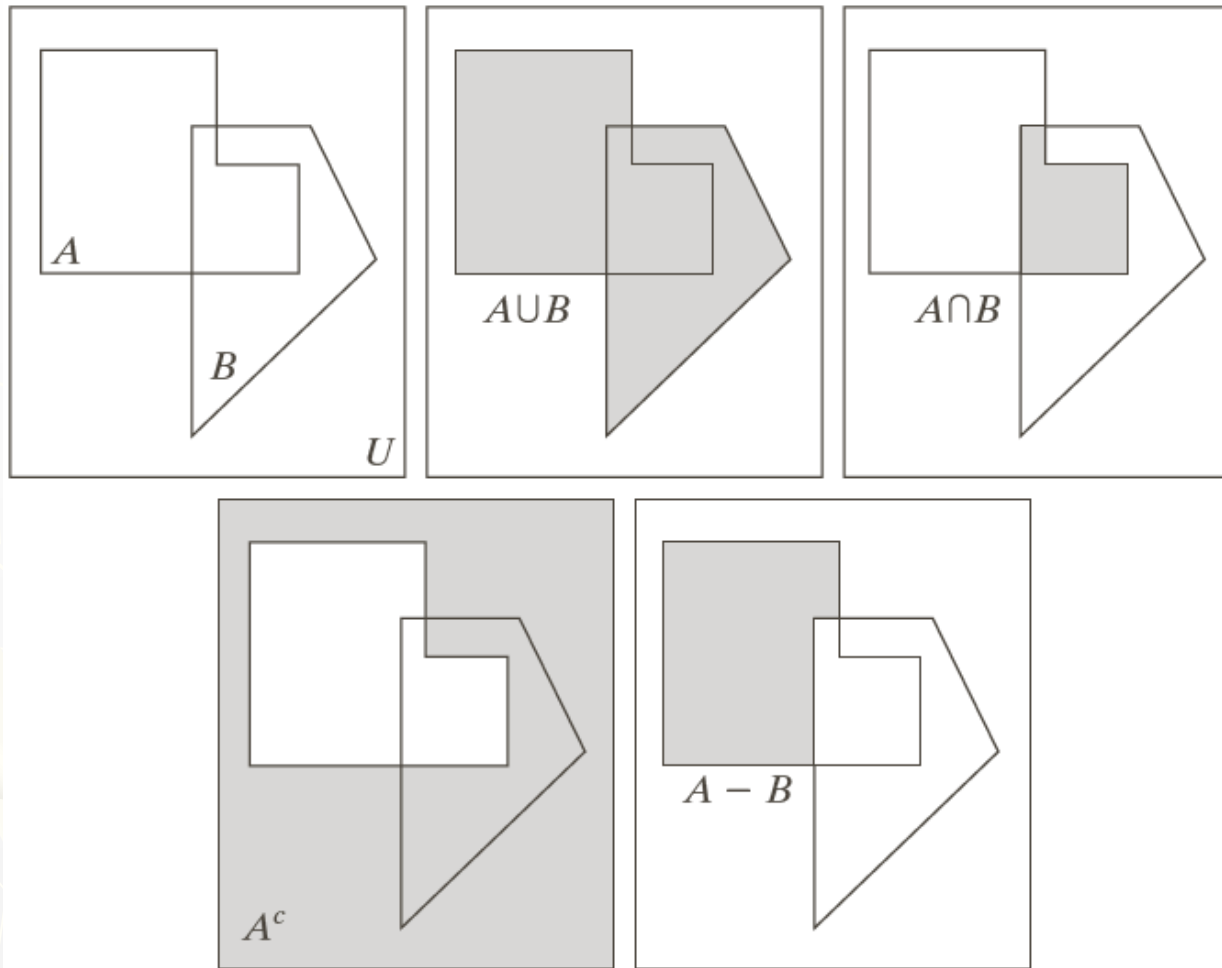
An Example of Image Multiplication



a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Set and Logical Operations



a	b	c
d	e	

FIGURE 2.31

(a) Two sets of coordinates, A and B , in 2-D space. (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B . In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set and Logical Operations

- ▶ Let A be the elements of a gray-scale image

The elements of A are triplets of the form (x, y, z) , where x and y are spatial coordinates and z denotes the intensity at the point (x, y) .

$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- ▶ The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$K = 2^k - 1$; k is the number of intensity bits used to represent z

Set and Logical Operations

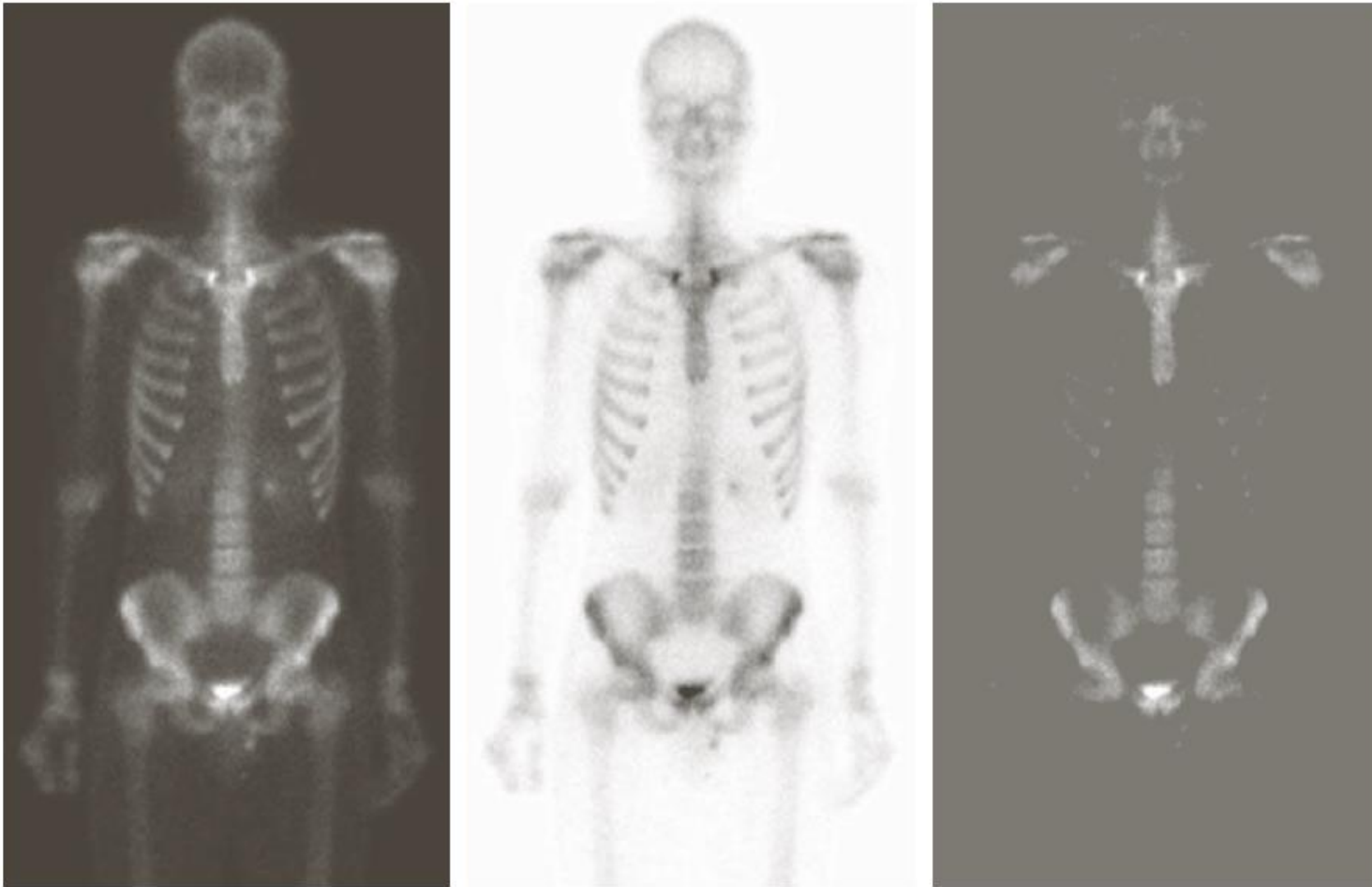
- ▶ The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{ \max_z(a, b) \mid a \in A, b \in B \}$$

Set and Logical Operations

a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)



Set and Logical Operations

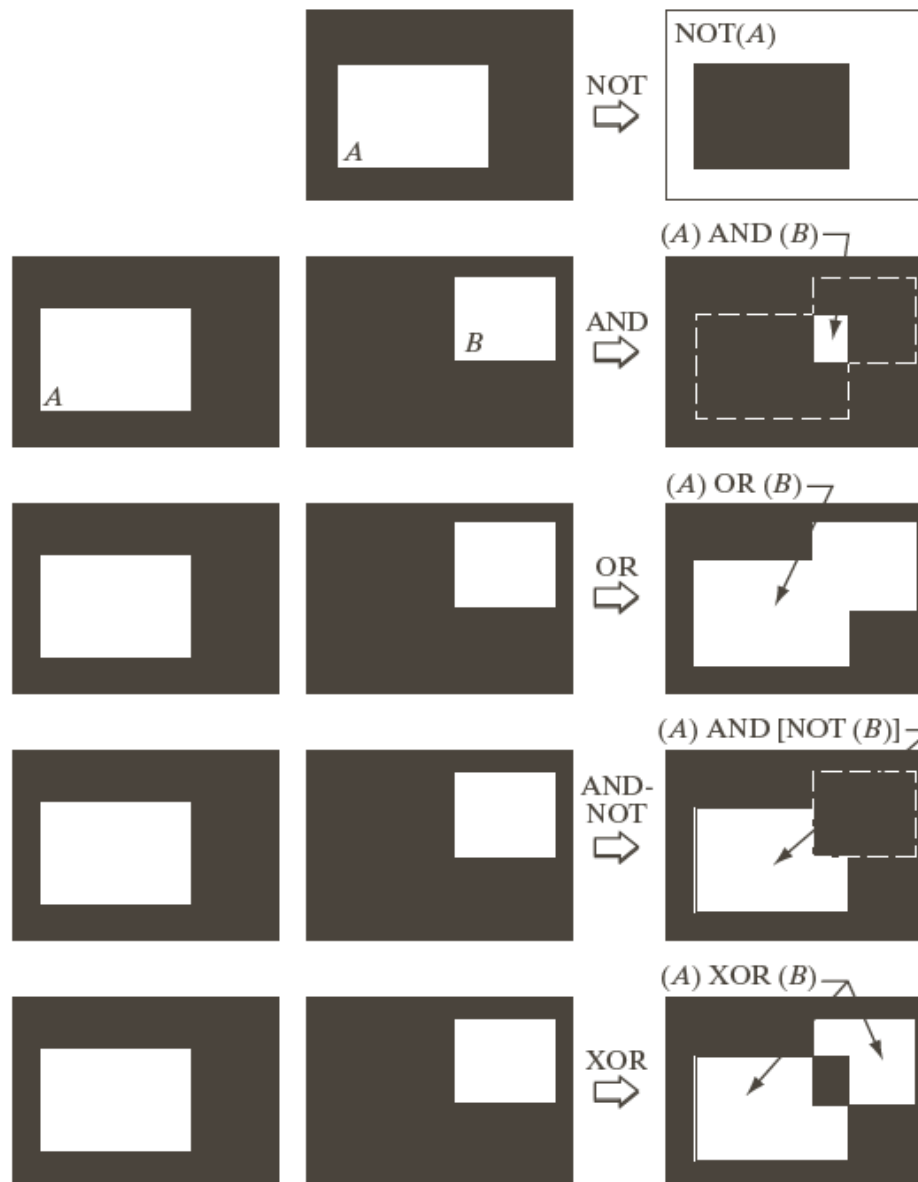


FIGURE 2.33

Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Spatial Operations

► Single-pixel operations

Alter the values of an image's pixels based on the intensity.

$$s = T(z)$$

e.g.,

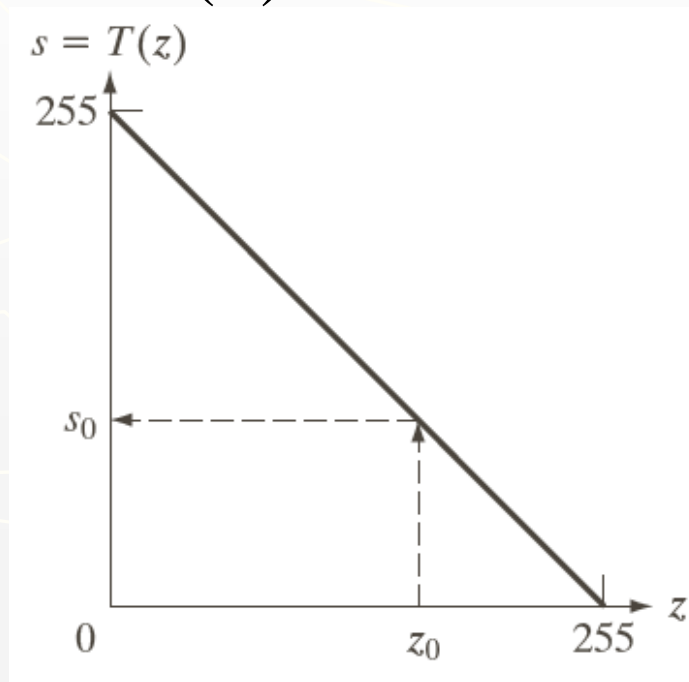
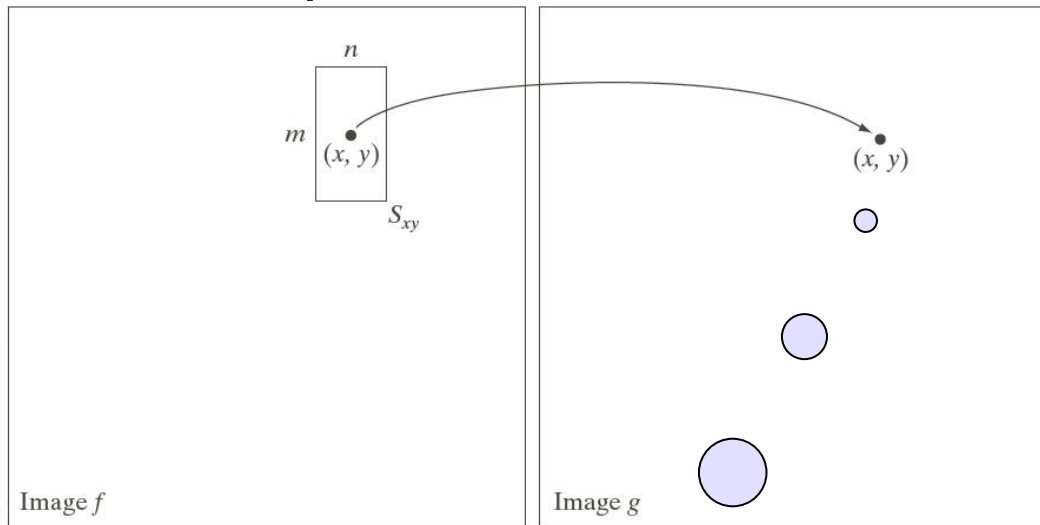


FIGURE 2.34 Intensity transformation function used to obtain the negative of an 8-bit image. The dashed arrows show transformation of an arbitrary input intensity value z_0 into its corresponding output value s_0 .

Spatial Operations

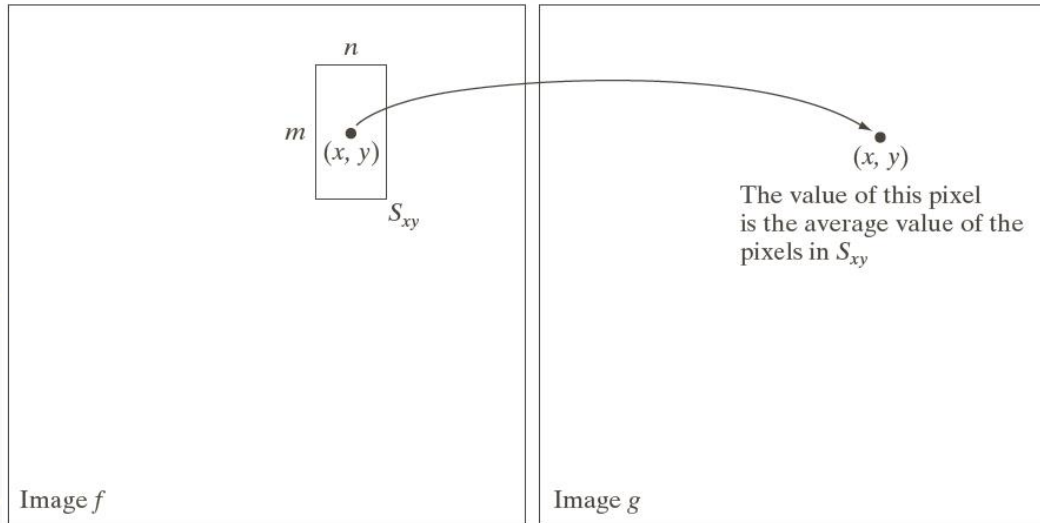
► Neighborhood operations



The value of this pixel is determined by a specified operation involving the pixels in the input image with coordinates in S_{xy}

Spatial Operations

► Neighborhood operations



Geometric Spatial Transformations

► Geometric transformation (rubber-sheet transformation)

— A spatial transformation of coordinates

$$(x, y) = T\{(v, w)\}$$

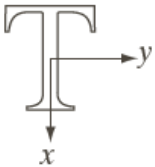
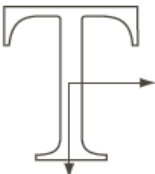


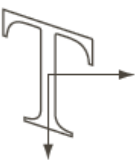

— intensity interpolation that assigns intensity values to the spatially transformed pixels.

► Affine transform

$$\begin{bmatrix} x & y & 1 \end{bmatrix} = \begin{bmatrix} v & w & 1 \end{bmatrix} \begin{bmatrix} t_{11} & t_{12} & 0 \\ t_{21} & t_{22} & 0 \\ t_{31} & t_{32} & 1 \end{bmatrix}$$

TABLE 2.2

Affine transformations based on Eq. (2.6.–23).

Transformation Name	Affine Matrix, T	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = w$	
Scaling	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = c_x v$ $y = c_y w$	
Rotation	$\begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v \cos \theta - w \sin \theta$ $y = v \sin \theta + w \cos \theta$	
Translation	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{bmatrix}$	$x = v + t_x$ $y = w + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & 0 & 0 \\ s_v & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v + s_v w$ $y = w$	
Shear (horizontal)	$\begin{bmatrix} 1 & s_h & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x = v$ $y = s_h v + w$	

Intensity Assignment

► Forward Mapping

$$(x, y) = T\{(v, w)\}$$

It's possible that two or more pixels can be transformed to the same location in the output image.

► Inverse Mapping

$$(v, w) = T^{-1}\{(x, y)\}$$

The nearest input pixels to determine the intensity of the output pixel value.

Inverse mappings are more efficient to implement than forward mappings.

Example: Image Rotation and Intensity Interpolation



FIGURE 2.36 (a) A 300 dpi image of the letter T. (b) Image rotated 21° clockwise using nearest neighbor interpolation to assign intensity values to the spatially transformed pixels. (c) Image rotated 21° using bilinear interpolation. (d) Image rotated 21° using bicubic interpolation. The enlarged sections show edge detail for the three interpolation approaches.

Image Registration

- ▶ Input and output images are available but the transformation function is unknown.

Goal: estimate the transformation function and use it to register the two images.

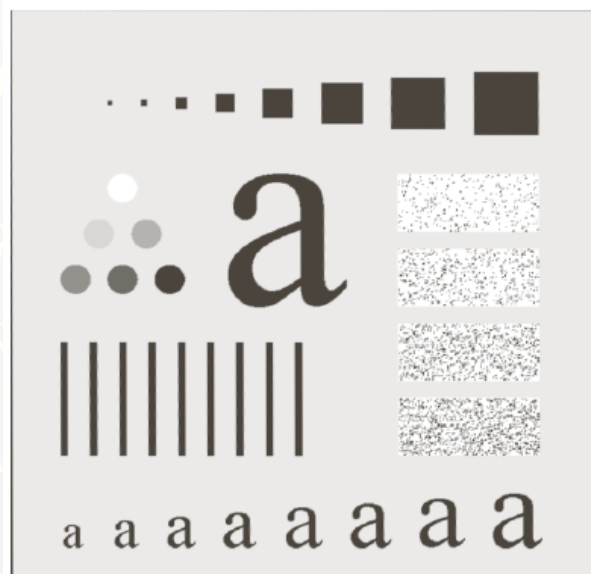
- ▶ One of the principal approaches for image registration is to use ***tie points*** (also called ***control points***)
 - ▶ The corresponding points are known precisely in the input and output (**reference**) images.

Image Registration

- ▶ A simple model based on bilinear approximation:

$$\begin{cases} x = c_1 v + c_2 w + c_3 vw + c_4 \\ y = c_5 v + c_6 w + c_7 vw + c_8 \end{cases}$$

Where (v, w) and (x, y) are the coordinates of tie points in the input and reference images.



a	b
c	d

FIGURE 2.37
Image registration.
(a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners.
(c) Registered image (note the errors in the borders).
(d) Difference between (a) and (c), showing more registration errors.

Image Transform

- ▶ A particularly important class of 2-D linear transforms, denoted $T(u, v)$

$$T(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) r(x, y, u, v)$$

where $f(x, y)$ is the input image,

$r(x, y, u, v)$ is the *forward transformation kernel*,

variables u and v are the transform variables,

$u = 0, 1, 2, \dots, M-1$ and $v = 0, 1, \dots, N-1$.

Image Transform

- ▶ Given $T(u, v)$, the original image $f(x, y)$ can be recovered using the inverse transformation of $T(u, v)$.

$$f(x, y) = \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u, v) s(x, y, u, v)$$

where $s(x, y, u, v)$ is the *inverse transformation kernel*,
 $x = 0, 1, 2, \dots, M-1$ and $y = 0, 1, \dots, N-1$.

Image Transform

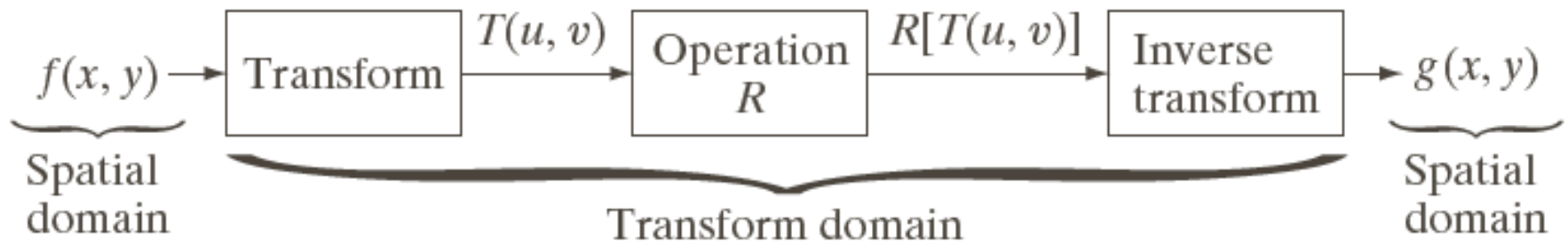
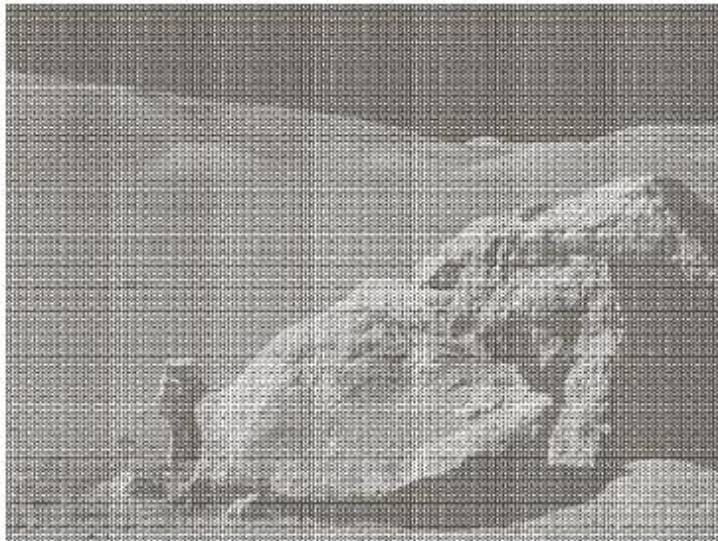


FIGURE 2.39
General approach
for operating in
the linear
transform
domain.

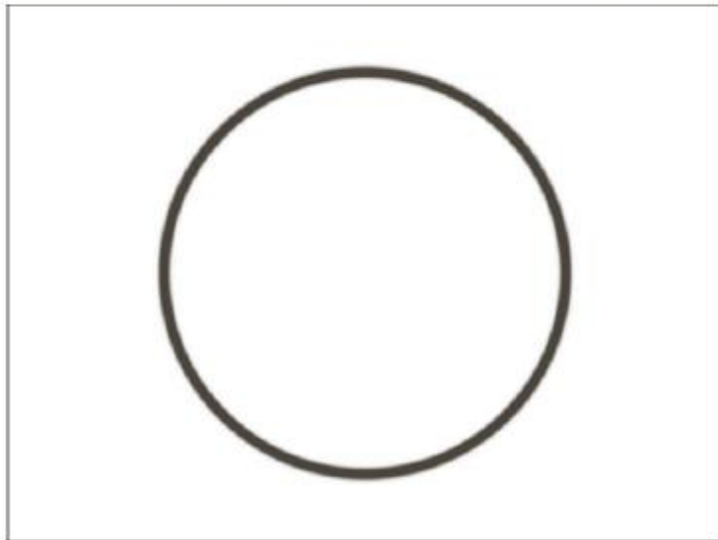
Example: Image Denoising by Using DCT Transform



a	b
c	d

FIGURE 2.40

(a) Image corrupted by sinusoidal interference. (b) Magnitude of the Fourier transform showing the bursts of energy responsible for the interference. (c) Mask used to eliminate the energy bursts. (d) Result of computing the inverse of the modified Fourier transform. (Original image courtesy of NASA.)



Forward Transform Kernel

$$T(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) r(x, y, u, v)$$

The kernel $r(x, y, u, v)$ is said to be SEPERABLE if

$$r(x, y, u, v) = r_1(x, u) r_2(y, v)$$

In addition, the kernel is said to be SYMMETRIC if

$r_1(x, u)$ is functionally equal to $r_2(y, v)$, so that

$$r(x, y, u, v) = r_1(x, u) r_1(y, u)$$

The Kernels for 2-D Fourier Transform

The *forward* kernel

$$r(x, y, u, v) = e^{-j2\pi(ux/M + vy/N)}$$

Where $j = \sqrt{-1}$

The *inverse* kernel

$$s(x, y, u, v) = \frac{1}{MN} e^{j2\pi(ux/M + vy/N)}$$

2-D Fourier Transform

$$T(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$$

$$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} T(u, v) e^{j2\pi(ux/M + vy/N)}$$

Probabilistic Methods

Let z_i , $i = 0, 1, 2, \dots, L-1$, denote the values of all possible intensities in an $M \times N$ digital image. The probability, $p(z_k)$, of intensity level z_k occurring in a given image is estimated as

$$p(z_k) = \frac{n_k}{MN},$$

where n_k is the number of times that intensity z_k occurs in the image.

$$\sum_{k=0}^{L-1} p(z_k) = 1$$

The mean (average) intensity is given by

$$m = \sum_{k=0}^{L-1} z_k p(z_k)$$

Probabilistic Methods

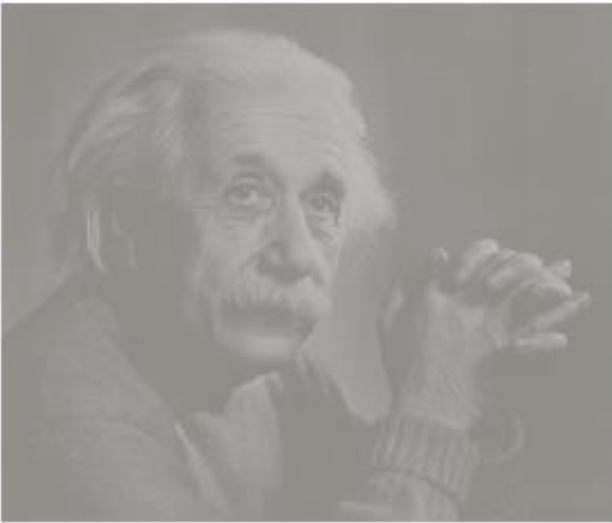
The variance of the intensities is given by

$$\sigma^2 = \sum_{k=0}^{L-1} (z_k - m)^2 p(z_k)$$

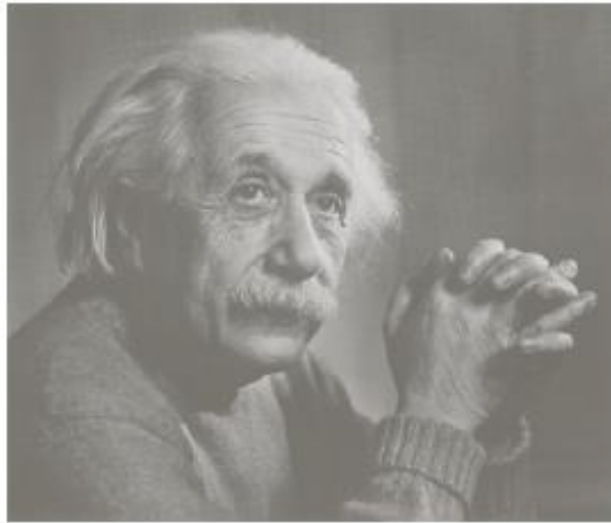
The n^{th} moment of the intensity variable z is

$$u_n(z) = \sum_{k=0}^{L-1} (z_k - m)^n p(z_k)$$

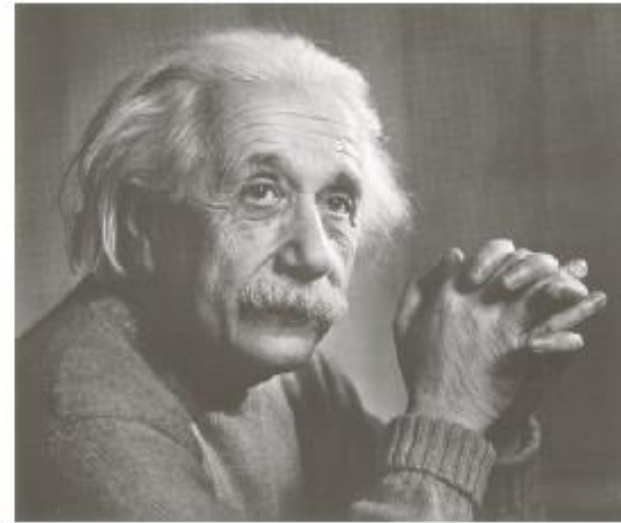
Example: Comparison of Standard Deviation Values



$$\sigma = 14.3$$



$$\sigma = 31.6$$



$$\sigma = 49.2$$



Courtesy:

Digital Image Processing (4th
Edition)

by Rafael C. Gonzalez , Richard E.
Woods