Lecture Notes on Nuclear Physics (PH:306 & PH:504)

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Chapitre 1

Nuclear Properties

Topics covered : Nuclear mass, charge, size, binding energy, spin and magnetic moment. Isobars, isotopes and isotones ; mass spectrometer (Bainbridge).

1.1 Nuclear mass

Though in nuclear physics we are much concerned with the masses of the nuclei, experimental determination using mass spectroscopes yield the atomic masses (M), which are usually shown in the periodic tables.Thus the nuclear mass (M_{nu}) is obtained by subtracting the masses of the Z orbital electrons (Zm_e) form the atomic mass. Hence

$$M_{nu} = M - Zm_e \tag{1.1}$$

The atomic masses are generally expressed in atomic unit which is defined as one-twelfth of the mass of ^{12}C atom taken to be exactly 12 unit and is designated by the symbol 'u' (unified atomic mass unit). Thus

$$1u = \frac{1}{12} \times \frac{12 \times 10^{-3}}{N_0} = 1.660566 \times 10^{-27} kg = 931.502 MeV \tag{1.2}$$

Particle	Rest mass (kg)	Rest mass (u)	Rest energy (MeV)
Electron (m_e)	9.10953×10^{-31}	5.48450×10^{-4}	0.511003
Proton (M_p)	1.67265×10^{-27}	1.0072765	938.280
Neutron (M_n)	1.67495×10^{-27}	1.0086650	939.573
$^1_1$H atom (M_H)	1.6736×10^{-27}	1.007825	938.79

1.2 Nuclear Charge

According to Rutherford's theory of α-particle scattering, number of α particles scattered by a nucleaus of charge +Ze (Z=atomic number of the element, e=charge of an electron) is proprtional to the nuclear charge number Z i.e. $N_S \propto Z^2$. James Chadwick, a scholar student of Rutherford first utilized this conclusion to deterimine the nuclear charge of several elements by his apparatus as shown bin Fig2x. Chadwick's working formula :

$$N_S = (\frac{2\pi N_D R^2}{A})(\frac{wnt}{a^3})(\frac{Ze^2}{4\pi\epsilon_0 M v^2})^2 cos\frac{\theta}{2} \tag{1.3}$$

where N_D= no of α particles falling directly on the detector screen D per second
R=distance between the α-source and the screen
A=area of the detector screen

w, t=width and thickness of the scattering foil RR'
θ=angle subtended by the scattering foil at the source S. Chadwick deterimed Z for a number of metallic elements using the above formula. Some of them are tabulated below.

Element	Z determined by Chadwick	Atomic number
Ag	46.3	47
Cu	29.3	29
Pt	77.4	78

1.3 Size

It was Ernest Rutherford who fast estimated the size of the nucleus by his famous α scattering experiment or Rutherford scattering experiment. Since Rutherford's time a variety of experiments have been performed to determine the nuclear dimensions, amongst them particle scattering is still a favored technique. Fast electrons and neutrons are ideal for this purpose. As electron interacts with the nucleus only through electric forces, it provides information on the charge distribution inside the nucleus. On the other hand, since neutron interacts with the nucleus only through nuclear forces, it provides information on the distribution of matters inside the nucleus. In both cases the de Broglie wavelength of the particle must be less than radius of the nucleus under investigation. The experiments suggested that the volume of a nucleus is directly proprtional to the number of nucleons it contains, which is its mass number A. Which in turn suggests that the density of the nucleons inside the nucleus is almost constant for all nuclei. Thus if R be the radius of a nucleus of mass number A, it's volume $\frac{4\pi R^3}{3}$ will be proprtional to A. Which results

$$R = R_0 A^{1/3}, R_0 \simeq 1.2 \times 10^{-15} m \simeq 1.2 fm \tag{1.4}$$

1.3.1 Tutorial

1. Find the radius and density of $^{12}_{6}$C nucleus. [Ans. $2.7fm$, $2.4\times10^{17}kg/m^3$]
2. Find the repulsive electric force on a proton whose center is 2.4fm from the center of another proton. Assume the protons are uniformly charged spheres of positive charge. (Note : Protons have internal quark

structure uud). Given $e = 1.6 \times 10^{-19}C$, $\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 Nm^2/C^2$. [Ans. 40N]

1.4 Binding energy

Minimum energy that must be supplied to a nucleus to break up it and to separate the constituent nucleons beyond the nuclear interaction range is called its binding energy. Conversely the energy that must be evolved in bringing nucleons together within the interaction range to form a nucleus is also called nuclear binding energy. For a nucleus of mass number A, atomic number Z, neutron number N=A-Z, proton mass $M_p = M_H - m_e$, neutron mass M_n and atomic mass M(A,Z), the binding energy is given by

$$\begin{aligned} E_B &= [ZM_H + NM_n - M(A,Z)]c^2 && (1.5)\\ &= [ZM_p + NM_n - M_{nuc}(A,Z)]c^2 && (1.6)\end{aligned}$$

In the case of formation of a nucleus the evolution of energy equal to the binding energy of the nucleus takes place due to the disappearence of a fraction of the total mass of the z protons and N neutrons, out of which the nucleus is formed. If the quantity of mass disappearing is ΔM, then the binding energy $E_B = \Delta Mc^2$. Further, the mass of the reulting nucleus must be less than the sum of the masses of constituent neutrons and protons.

1.5 Spin

Like electrons, protons and neutrons are fermions having intrinsic spin of spin quantum number s=1/2. This means that they have spin angular momenta **S** of magnitude

$$s = \sqrt{s(s+1)\hbar} = \sqrt{1/2(1/2+1)\hbar} = \frac{\sqrt{3}}{2}\hbar \qquad (1.7)$$

and spin magnetic quantum numbers of $m_s = \pm 1/2$.

1.6 Magnetic moment

Like the electron, proton and neutron posses intrinsic magnetic dipole moments. Experimental values of the magnetic moments of the proton and the neutron are

$$\mu_p = 2.7927\mu_N \tag{1.8}$$
$$\mu_n = -1.9131\mu_N \tag{1.9}$$
$$\mu_N = \frac{e\hbar}{2M_p} = 5.0571 \times 10^{27} J/T \tag{1.10}$$

where e and M_P are the charge and mass of the proton. μ_N is called the nuclear magneton in analogy with Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.2849 \times 10^{-24} J/T \tag{1.11}$$

It is very important to note that the magnetic moments of the proton and the neutron are intimately related to their intrinsic spin angular momenta. Their spin angular momenta are given by

$$p_p = s_p\hbar,\ p_n = s_n\hbar\ with\ s_p = s_n = \frac{1}{2} \tag{1.12}$$

The ratio of the magnetic moment μ_e to the spin angular momentum p_e (called the gyromagnetic ratio for the spin motion) of the electron is given by

$$\frac{\mu_e}{p_e} = g_e \frac{e}{2m_e} \tag{1.13}$$

where $p_e = s_e\hbar = \hbar/2$ and $g_e = 1 + \frac{j(j+1)+s(s+1)-l(l+1)}{2j(j+1)}$ is the Lande factor. $g_e = -2$ for electron assigned by S. Goudsmit and G. E. Uhlenbeck on ad hoc basis which later justified by Dirac electron theory. For proton and neutron

$$\frac{\mu_p}{p_p} = g_p \frac{e}{2M_p} \tag{1.14}$$
$$\frac{\mu_n}{p_n} = g_n \frac{e}{2M_p} \tag{1.15}$$

Substituting values of p_p and p_n, we get

$$\mu_p = g_p \frac{e}{2M_p}.s_p\hbar = g_p\mu_N/2 \tag{1.16}$$
$$\mu_n = g_n \frac{e}{2M_p}.s_n\hbar = g_n\mu_N/2 \tag{1.17}$$

Comparing Eq(1.7) and (1.8) with Eq(1.15) and (1.16) we get

$$g_p = 2 \times 2.7927; g_n = -2 \times 1.9131 \tag{1.18}$$

1.6.1 Tutorial Problems

1. Find the splitting factors for the levels $2p_{\frac{3}{2}}, 1d_{\frac{5}{2}}, 1g_{\frac{7}{2}}$.
2. Find the spin, parity and magnetic moment of ^{17}O.
3. Compare the magnetic potential energies (in eV) of an electron and of a proton in a magnetic field of 0.10T.

1.7 Isobar

Nuclides having equal mass number (A=Z+N) but different Z, N are called isobars. Examples are $(^6He,^6Li,^6Be)$, $(^{18}O,^{18}F)$ etc.

1.8 Isotope

Nuclides having equal proton number (Z) but different mass number (A=Z+N) are called isotopes. Examples are $^4He,^6He,^3He$ are isotopes of Hydrogen (Z=1), $^{16}O,^{18}O$ are isotopes of Oxygen (Z=8) etc.

1.9 Isotone

Nuclides having equal neutron number (N) but different mass number (A=Z+N) are called isotones. Examples are $^4He,^3H,^6Be$ are isotones with N=2, ^{17}O,^{18}F are isotones with (N=9) etc.

1.10 Mass spectrometer (Bainbridge)

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Chapitre 2

Structure of Nucleus

Topics cover here : Nature of forces between nucleons, nuclear stability and nuclear binding, the liquid drop model (descriptive) and the Bethe-Weizsacker mass formula, application to stability , extreme single particle shell model (qualitative discussion with emphasis on phenomenology with examples).

2.1 Nature of forces between nucleons

Lets begin the section with a tabular view of the forces of nature with their relative strengths, span of interaction etc.

Understanding the nature of nuclear forces has been a challenging problem ever since the discovery of the nucleus by Rutherford. The nucleus consists of positively charged protons and uncharged neutrons. Clearly, nuclear force is not just the Coulomb force, which in this case wosuld be repulsive and could not hold the nucleons together. The fact that the nucleus is stable and has a large binding energy (BE), (which is of the order of a few MeV, compared to a few eV for atomic systems bound by attractive Coulomb interaction) shows that the nuclear force is strong and attractive (at least part of the time). Since nuclear force cannot be "measured " directly by experiments, information must come indirectly from :

1. bound state properties of nuclei, e.g. BE, spin, angular momentum, parity, magnetic and quadrupole moments, excitation spectrum, transition rates, etc.
2. Scattering results- differential and total scattering cross sections for

TABLE 2.1 – Forces of nature, their relative strengths, associated particles and quanta.

Interaction	Relative strength	Range	Particles affected	Exchanged quanta
Electromagnetic	1	Long range	Charged particles	Photons
Strong	10	Short range ($10^{-15}m$)	Quarks	Gluons
Weak nuclear	10^{-12}	Very short range ($10^{-18}m$)	Quarks, leptons	$W^{\pm}, Z_0$ bosons
Gravitational	10^{-36}	Long range	Particles with mass	Graviton (undetected)

the scattering of a nucleon by another nucleon or a nucleus. The scattering cross section will depend on the force between the projectile and the target.

An assumption that the nuclear forces are derivable from a potential will be a farly good one, at least for low energy properties. We will examine this point more critically later. Since we experimentally measure only observable quantities, we can only compare them with the same observables calculated theoretically by assuming a particular potential and get indirect information about the potential. In this process, it is important that the theoretical calculations must be reliable -i.e., without approximations. Hence it is important to study few nucleon systems specifically two and three nucleon system-which can be solved exactly.

2.1.1 Gross qualitative features of nuclear forces

Gross nature of nuclear forces can be inferred from some of the general properties of nuclei :

— Saturation of nuclear density-The experimental fact that nuclear radius, $R \propto A^{1/3}$, for all but the lightest nuclei, implies that nuclear density is nearly constant.

— Saturation of binding fraction- For medium and heavy nuclei, the BE per nucleon becomes nearly constant (about 8.5 meV/nucleon). This is possible if the forces are short ranged (so that a given nucleon can interact with only those nucleons, which happen to be within the sphere of radius r_0, r_0 being the range of nuclear forces).

— The short range nature is also indicated by Rutherford's α- scattering experiment.

— However short range force alone cannot produce saturation. If the force is always attractive, one nucleon will pull all other nucleons within a small volume so that both BE/nucleon and density will increase, instead of reaching a saturation value. This can be prevented if
(1) two nucleons cannot come too close- i.e., nuclear force is strongly repulsive at very short distances. This will produce saturation in nuclear density.
(2) nuclear force is not *always* attractive and is *repulsive* at times, depending on the states of interacting nucleons. This will be the case, if *exchange forces* are present, which Heisenberg predicted.

— The exchange force is attractive or repulsive depending on the symmetry or antisymmetry of the two nucleon wave function (w. f.) under exchange of their variables (position, spin or isospin). Since in a nucleus, some of the nucleons will be in antisymmetric space (or spin or isospin) states, while others in symmetric space (or spin or isospin) states, so several of the nucleons will actually repel each other, while others will feel attraction. This will prevent a collapse of the nucleons into a tiny volume. This exchange force, together with a strong short range repulsion (at separations typically $\sim o.4fm$) can explain the saturation properties.

— We can summarize the qualitative features as follows. The nuclear force (1) is strong, short ranged ($\sim 1-2fm$) and ordinarily attractive. (2) is strongly repulsive at very short separations ($\sim 0.4fm$) (3) has exchange character, (4) conserves parity (since nuclear states have definite parity).
It is important to note that in case of non-interacting particles the projectile -target system undergoes scattering only if the get into physical touch. So the interaction distance must be less than or equal to the some of their radii. But for interacting particles scattering can occur even if the distance of the interacting projctile and target is greater than the sum of their radii.

2.1.2 Nature of two-nucleon interaction obtained from deuteron bound state

There is only one bound state of two nucleons, viz., the deuteron, which is a bound state of a proton (p) and a neutron (n). There are no bound diproton (pp) or dineutron (nn). The observed properties are :

1. BE of deuteron, B= 2.2246 MeV.
2. No excited states are known.
3. Ground state spin J (=total angular momentum) =1 (in units of $\hbar$).
4. Magnetic moment $\mu_d = 0.85735nm(\simeq \mu_p + \mu_n)$.
5. Quadrupole moment, $Q_d = 2.82 \times 10^{-3}b$ $(1b = 10^{-24}cm^2)$ (This is small compared to typical nuclear quadrupole moments, but certainly nonvanishing).

The BE/particle is only $\sim 1.1MeV$ and is much less than the average for heavier nuclei. Hence deuteron is weakly bound.

Since $\mu_d \simeq \mu_p + \mu_n$, but the difference is much larger than the experimental error shows that the contribution from the orbital motion is nonvanishing. This means that ground state (g.s.) has a large contribution from l=0, but $l \neq 0$ also contribute.

Since Q_d is small but nonvanishing, the g-s is nearly spherically symmetric, but not exactly. This once again shows that the g.s. is a mixture of $l = 0$ (large contribution) and $l \neq 0$. Hence l is not a good quantum number. This means that although nuclear force is dominantly central, but non central forces are also present.

Proton and neutron are both spin 1/2 particles. Hence the total spin of the np system (s) can either 0 (called singlet state, which is antisymmetric under spin exchange) or 1 (called spin triplet spin state, which is symmetric under spin exchange).

Now $\vec{J} = \vec{L} + \vec{S}$ and J is known to be 1. Then for S=0, L=1 $\rightarrow$ This is the ${}^1P_1$ state (in spectroscopic notation ${}^{2S+1}L_J$). For S=1, L can take the values 0, 1, or 2(by the rules of addition of angular momenta), corresponding to ${}^3S_1$, ${}^3P_1$ and ${}^3D_1$ respectively.

Now since parity is a good quantum number for nuclear states even and odd parity states cannot mix. On the other hand, since noncentral forces are present, different l states can mix. Hence for the g.s. of deuteron, the possible mixtures are :

$$(1)^3S_1 +^3 D_1 \, (even\, parity)$$

$$(2)^1P_1 +^3 P_1 \, (odd\, parity).$$

The fact that the g.s. has a large $l = 0$ contribution shows that it must be a mixture of ${}^3S_1$ and ${}^3D_1$. This mixing is possible, if the potential contains a tensor (S_{12}) term.

The nuclear force is dominantly central. Assuming a central short range potential of (say) square well shape and solving the two body Schrodinger equation, one can get an idea of the depth of the well from the known BE (=2.2246 MeV) and an idea of the range ($\sim 2fm$). The depth of the square well turns out to be $\sim 36MeV$.

2.1.3 Nature of nucler forces obtained from NN scattering at low energies

More information can be obtained from scattering experiments. Neutron targets are not available, but proton targets are readily available from materials containing hydrogen. Hence a beam of neutrons or protons is scattered by a proton target. Thus common scattering experiments are np and nn scattering. Consider a proton at rest in the lab and an incident neutron with speed v (magnitude of momentum p) approaching with an impact parameter b. The orbital angular momentum of the incident neutron

$$= L =\mid \vec{L} \mid = pb = 2\pi b\hbar/\lambda = \hbar\sqrt{l(l+1)} \, (semiclassically).$$

Hence

$$b = \frac{\lambda}{2\pi}\sqrt{l(l+1)} \tag{2.1}$$

Now since nuclear force has a short range (r_0), the incident neutron will not be influenced by the target (as in Fig 2(b)) if

$$b > r_0.$$

Hence the l^{th} partial wave will not be affected if

$$\frac{\lambda}{2\pi}\sqrt{l(l+1)} \quad > \quad r_0 \tag{2.2}$$

i.e. if

$$k \quad < \quad \frac{\sqrt{l(l+1)}}{r_0} \, (since\, k = 2\pi/\lambda = \sqrt{2ME/\hbar^2}) \tag{2.3}$$

Putting $l = 1$, we see that only $l = 0$ partial wave will be affected if

$$k \quad < \quad \frac{\sqrt{2}}{r_0} \tag{2.4}$$

Taking $r_0 \sim 2fm$, we see that only $l = 0$ partial wave is affected if $E_{lab} \leq 10MeB$. Thus at low energies only $l = 0$ is involved and the scattering is spherically symmetric. The partial wave analysis show that

$$Differential\ scattering\ crosssection \quad = \tfrac{d\sigma(\theta)}{d\Omega} = \tfrac{sin^2\delta_0}{k^2} \tag{2.5}$$

and

$$Total\ scattering\ crosssection \quad = \sigma = 4\pi\tfrac{sin^2\delta_0}{k^2} \tag{2.6}$$

where δ_0 is the phase shift of the $l = 0$ partial wave and θ is the centre of mass scattering angle. The meaning of δ_0 can be understood from Fig.3. The assymptotic form of the wave function $u(r)(= r\psi(r))$ in a short range (r_0) attractive potential V(r) will be $u(r) = sin(kr + \delta_0)$, since outside the range r_0, there is no potential . Compare this with a pure sine curve $u_0 = sin(kr)$ of same wave length and passing through the origin. [Note that u(r) also passes through the origin, due to the boundary condition that $\psi(r) = u(r)/r$ must be finite. The form of $u(r)$ is <u>not</u> sinusoida within the well]. Since the potential is attractive, the w.f. $u(r)$ will be <u>pulled</u> inside as compared to $u_0(r)$. Then the value of the w.f. at $r = r_1$ is $u = sin(kr_1 + \delta_0)$ and is the same as $u_0(r)$ at $r = r_1 + \delta_0/k$. For an attractive pot, $u(r)$ is pulled inside and $(r_1 + \delta_0/k)$ must be $>r_1$, so for an attractive potential $\delta_0 > 0$. Similarly for a repulsive potential (which will push $u(r)$ outwards), $\delta_0 < 0$. The phase shift δ_0 is a function of k and for $k \to 0$ (i.e. in the linit of zero energy of the projectile), $\delta_0 \to 0$. The scattering length (a) is defined as

$$a \quad = \quad \lim_{k\to 0}(-\frac{sin\delta_0}{k}) \tag{2.7}$$

Hence from eq(5)

$$(\frac{d\sigma(\theta)}{d\Omega})_{E=0} \quad = \quad a^2 \tag{2.8}$$

and

$$(\sigma)_{E=0} \quad = \quad 4\pi a^2 \tag{2.9}$$

Experimentally the zero energy n-p scattering cross-section can be measured with great precision. Hence the magnitude of the scattering length, but not its sign, is determined by direct experiment. At low energies, $\delta_0(k)$ is given by the effective range formula

$$kcot\delta_0 = -\frac{1}{a} + \frac{1}{2}k^2 r_{eff} \quad (small\ k) \tag{2.10}$$

where r_{eff} has the dimension of length and is called the effective range. For any given potential both a and r_{eff} can be calculated theoretically. From eq(6) and (10), total n-p scattering cross-section at a low energy (E) is

$$\begin{aligned} \sigma(E) &= \frac{4\pi}{k^2 cosec^2\delta_0} \\ &= \frac{4\pi}{k^2 + [-\frac{1}{a} + \frac{1}{2}k^2 r_{eff}]^2} \end{aligned} \tag{2.11}$$

This shows that all low energy scattering data can be fitted by adjusting only two parameters, namely a and r_{eff}. For a bound state, the effective range formula, eq(10), can be modified as (for np system)

$$\alpha = \frac{1}{a} + \frac{1}{2}\alpha^2 r_{eff}. \tag{2.12}$$

where $\alpha = \sqrt{MB/E}$, B being the binding energy (BE) of the system and M is the nucleon mass. Thus the BE is again given in terms of the same two parameters (a and r_{eff}). thus adjusting only two parameters, all low energy data (both scattering and bound state) can be fitted. Least square fitting all experimental results to eqns(11) and (12), one can obtain "experimental" values of a and r_{eff}. Now for *any* shape of the n-p potential, the depth (V_0) and range (r_0)can always be adjusted to reproduce these "experimental" values a and r_{eff}. Thus the shape of the n-p potential is not determinable from low energy data alone. This is the well known "Shape independence of nuclear potential".

Spin dependence of nuclear forces

The neutron and proton can be in relative spin singlet (S=0) or spin triplet (S=1) states. In an incoherent n-p scattering (scattering of an unpolarized n-beam by an unpolarized p-target) we have for zero energy

$$\sigma_{in}(0) = \frac{3}{4}.4\pi a_t^2 + \frac{1}{4}.4\pi a_s^2 = \pi(3a_t^2 + a_s^2) \tag{2.13}$$

This is measured accurately $[\sigma_{in}(0) = (20.442 \pm 0.23)b]$.here a_t and a_s are the triplet and singlet scattering lengths respectively. From this we cannot determine a_t and a_s separately. To determine a_t and a_s separately, we need to do coherent n-p scattering. This is possible if, (1) target contains more than one proton, having singular characteristics (eg H_2 molecule), (2) $\frac{\lambda}{2\pi}$ of incident neutron» inter proton separation, so that **same** neutron is scattered by both protons in the H_2 molecule. This requires very low energy neutrons. Analyzing the scattering data of very low energy neutrons from H_2 molecules para- hydrogen (with total spin of the two protons = $S_H = 0$) and orthohydrogen ($S_H = 1$), we get

$$\left.\begin{array}{rcl} a_s &=& (-23.710 \pm 0.030)\,fm \\ a_t &=& (5.432 \pm 0.005) fm \end{array}\right\} \tag{2.14}$$

Now a positive scattering length corresponds to a bound state and when scattering length is negative, there is no bound state. Thus for the n-p system, the bound state is a triplet one (deuteron ; we saw earlier that the g.s. of deuteron is $^3S_1 +^3 D_1$, i.e. a spin triplet state). There are no singlet bound n-p system. The n-p potential in the singlet state is not sufficiently strong to form a bound state.
Results (14) together with low energy incoherent n-p scattering cross sections fitted to appropriately weighted (for triplet and singlet weight factors are 3/4 and 1/4 respectively) eq.(11) gives

$$\left.\begin{array}{rcl} r_{eff,t} &=& (1.749 \pm 0.008)\,fm \\ r_{eff,s} &=& (2.73 \pm 0.03) fm \end{array}\right\} \tag{2.15}$$

2.1.4 Proton-proton (p-p) scattering at low energy

p-p scattering is more involved since protons are identical particles and Coulomb interaction interfares with nuclear interaction. The latter gives rise to a term linear in nuclear phase shift, which allows direct determination of the sign as well as the magnitude of the scattering length. Further more at low energy, only l=0 partial wave contributes for the nuclear interaction for which space part of w.f. is symmetric. For the identical particle p-p system, the total w.f. must be antisymmetric i.e. singlet. Thus only singlet phase shift and singlet scattering length appear in σ_{pp}. Experimentally, the proton is early detected by its ionization and leads to higher precision. Also since protons are charged, well collimated beams (by e.m. focussing) can be attained again giving better precision for measurement of θ. Analysis of experimental results give

$$\left.\begin{array}{rcl} a_p & = & (-7.821 \pm 0.004)\, fm \\ r_{eff,p} & = & (2.830 \pm 0.017) fm \end{array}\right\} \tag{2.16}$$

These results include the effects of nuclear as well as Coulomb interactions. If the Coulomb effect is theoretically substracted, we get

$$(a_p)_{nuclear\ only} = -(16.8 - 17.1) fm \tag{2.17}$$

This corresponds to singlet pp scattering length.

2.1.5 Neutron-neutron (n-n) scattering at low energy

Although it is not feasible experimentally to scatter neutrons by neutrons (since there are no convenient neutron targets available), one can extract information indirectly. One example is to scatter neutrons from deuteron (n-p) target and subtracting the already known n-p scattering part. Some corrections are needed due to the fact that deuteron is a bound n-p system. Analyzing such experimental data, one gets for n-n scattering

$$\left.\begin{array}{rcl} a_n & = & -(-17.4 \pm 1.8)\, fm \\ r_{eff,\,n} & = & (2.4 \pm 1.5) fm \end{array}\right\} \tag{2.18}$$

From eqns(16)-(18), we find that within experimental errors,

$$\left.\begin{array}{rcl} a_n & = & (a_p)_{nuclear\ only} \\ r_{eff,\,n} & = & r_{eff,\,p} \end{array}\right\} \tag{2.19}$$

This means that the nuclear part of p-p interaction, is, within experimental errors, the same as the n-n interaction. Thus the nuclear interaction is charge symmetric. Furthermore from eq.(14), (15) we see that $r_{eff,s}$ (for the singlet n-p interaction) is nearly equal to $r_{eff,n}$ and $r_{eff,p}$. Also, although a_s appears to be different from a_n or $(a_p)_{nuclear\ only}$, a large negative value means that the singlet state is *just unbound*. In *such a case*, the actual values of the parameters of the potential do not depend too strongly on the magnitude of $|a_s|$ (having a large negative value). Thus the potential parameters for nn, pp nuclear (both singlet interactions at low energy) and the singlet n-p interaction turn out to be the same (within experimental errors). Thus the nuclear interaction does not depend on the charge of nucleon. This is called

the **charge independence of nuclear forces**. However eq. (14), (15) show that the n-p singlet and triplet interaction parameters are quite different. This means that nuclear forces are **spin dependent**.
Analysis of high energy n-p and p-p scattering data show the following :
(1) Nuclear force has an exchange character.
(2) At very short separations ($\leq 0.5 fm$), the nuclear force is strongly repulsive.

2.1.6 Theoretical understanding of nucleon nucleon interaction

The simplest and the dominant mechanism at low energies by which a nucleon can interact with another, is by the exchange of a pion between them (Fig. 4) called one pion exchange proces. At the vertex 1, nucleon N_1 emits a pion, which is simultaneously absorbed by nucleon N_2 at the second vertex 2. At low energies, the K.E. of N_1 is not sufficient to produce a real pion (with rest mass $\simeq$ 140 MeV). Hence the emitted pion is a "virtual" one and must be reabsorbed (in order to have over all energy conservation) within a time Δt (in conformity with the uncertainty principle) where

$$\Delta E \Delta t \simeq \hbar \tag{2.20}$$

and $\Delta E \simeq 140 MeV$ is the energy fluctuation. The emitted pion can travel at most with speed of light (c) and hence can travel a distance

$$r_c = c\Delta t \simeq c\frac{\hbar}{\Delta E} \simeq \frac{2000 MeV fm}{140 MeV} \simeq 1.4 fm, \tag{2.21}$$

before it must be absorbed by N_2. Hence the range of nuclear interaction is about 1.4 fm. (If there are no other nucleons within this range, the pion will be absorbed by N_1, giving rise to its self energy). However this is an order of magnitude and the potential is not exactly zero for $r > 1.4 fm$, but has a much reduced magnitude. Calculating the contribution of the Feynmann diagram (Fig. 4), one has the one-pion exchange potential (OPEP) :

$$V_{OPEP}(\vec{r}) = \frac{g^2 m_\pi^3}{12M^2}(\vec{\tau_1}.\vec{\tau_2})[(\vec{\sigma_1}.\vec{\sigma_2}) + (1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2})S_{12}]\frac{e^{-\mu r}}{\mu r} \tag{2.22}$$

where $\mu = \frac{m_\pi c}{\hbar} = 0.7 fm^{-1}$, g is the pion nucleon coupling constant, m_π and M are pion and nucleon masses and $\vec{\tau_i}, \vec{\sigma_i} (i = 1, 2)$ are isospin and spin operators of the i^th nucleon. There is a strong tensor (S_{12}) term and spin dependence. The tensor operator has the form

$$S_{12} = 3(\vec{\sigma_1}.\hat{r})(\vec{\sigma_2}.\hat{r}) - (\vec{\sigma_1}.\vec{\sigma_2}). \tag{2.23}$$

However actual nucleon-nucleon force is not so simple. Exchanges of more than one pion or heavier mesons are also possible. For such processes, the mass of exchanged particles will be larger and so the range will be smaller. At very short separations ($\leq 0.5fm$) so many diagrams contribute that it is frustrating to calculate the potential theoretically. Such short separations correspond to very high momentum transfers and hence very high energy nucleon probes. For $E_{lab} \geq 300MeV$ (corresponding to $E_{cm} \geq m_\pi c^2$), "real" (as opposed to "virtual") pions can be produced. Then the very idea of a potential is questionable. Thus a complete theoretical derivation of the nuclear force is not possible. One then constructs a "realistic potential" by combining theoretical information with experimental inputs. There are several such standard nucleon-nucleon interactions, the Reid Soft Core, Reid Hard Core, Hamada-Johnston, Paris potential, Bonn potential etc. Parameters of such realistic potentials are obtained by least square fitting all two-nucleon data.

2.1.7 Three nucleon potential

If nuclear force consists only two body forces (2BF), one should be able to explain all nuclear data, using the realistic 2BF only. Although a many body Schrodinger equation cannot be solved exactly, the three body equation can be solved essentially exactly by momentum techniques. Solving the trinucleon systems (^3H and ^3He) with realistic 2BF only one finds that
(1) Calculated BE is 1MeV less than the experimental value.
(2) The calculated electric charge form factor differs considerably from the experimental one (Fig. 5). These indicate the existence of a three nucleon potential (3NP), in which all three nucleons interact simultaneously through an inseparable combination of their coordinates. The simplest mechanism which will give rise to a 3NP is a two-pion-exchange (2PE) process, shown in Fig6. Nucleon N_1 emits a virtual pion, which is absorbed by nucleon N_2, which then goes into the excited state Δ (1232MeV) of nucleon. This decays simultaneously by emitting another virtual pion, which is absorbed by the third nucleon N_3. The 3NP derived from this diagram, assuming nucleons to be point particles, has a very strong singularity (which goes as r^{-6}, r

being a measure of the size of the three body system) as $r \to 0$. Now 3NP is attractive for the equilateral triangle configuration of the three body system. Since the attractive singularity is worse than $1/r^2$, there cannot be any stable bound state in such a potential. unless an arbitrary cut off in smaller r is introduced. The falling is due to the diagonal of any structure of the nucleons and assuming them as point particles. Indeed, nucleons have finite structure and cannot come too close to one another. This structure effect is taken care of by the inclusion of pion-nucleon form factor in the derivation of 3NP. Such a procedure regularizes the 3NP. Theoretical calculations using a realistic 2BF and a regularized 3NP show marked improvement of the binding energy, which nearly agrees with the experimental value. but there is only marginal improvement of the charge form factor. this is puzzling. One reason may be that, at extremely short separations, the natures of 2BF and 3NP are uncertain (due to possible) contribution of higher order diagrams). The discrepancy in the charge form factor is mostly in the large momentum transfer region, which corresponds to inter nucleon separations $\leq 0.4fm$. In this region higher order diagrams contribute significantly. Moreover, meson exchange currents play an important role at high momentum transfers and explains most of the discrepancy of the charge form factor.

2.2 Isotopic spin or isospin of Nucleon

The charge symmetry and charge independence of nuclear forces together with the almost equality of masses of neutron and proton suggest that neutron and proton are the same particle in two different charge states and therefore the two may be treated in the same mathematical footing. This is conveniently done by introducing a 3D "charge space", called isotopic spin or isospin space (or isospace). The neutron and proton are described by an isotopic spinor (isospinor) field in the isospace. Since ordinary spin also has two states, the formal development of the isospinor algebra is exactly the same for the spin $\vec{s} = (\hbar/2)\sigma$ whose z-components are $s_z = \pm\hbar/2$. The common name given to neutron and proton is the nucleon which is a particle with isospin

$$t = \frac{1}{2}\tau$$

whose Z-componet in the isospin space

$$t_3 = \frac{1}{2}\tau_3 = +\frac{1}{2}$$

and

$$-\frac{1}{2}$$

for proton and neutron respectively. Three components of the matrix $\vec{\tau}$ are

$$\tau_3 = (0, 1; 1, 0), \tau_2 = (0 - i; i, 0), \tau_3 = (1, 0; 0 - 1).$$

The charge operator Q whose eigenvalues are $+1$ for the proton state and o for the neutron state is

$$Q = \frac{1}{2}(1 + \tau_3) = (1\,0; 0\,0)$$

with $Q\eta(p) = \eta p$ and $Q\eta(n) = 0$, $\eta(p)$ and $\eta(n)$ being respectively the proton and neutron wavefunction in the isospace. The total isospin $\vec{T}$ of two nucleons 1 and 2 is defined by

$$\vec{T} = t^1 + t^2 = \frac{1}{2}(\tau^1 + \tau^2)$$

with T=0 or 1. The pp and nn states are pure T=1 state , whereas the pn system is a mixture of T=1 and T=0 states. The isospin singlet state is antisymmetric in nucleons 1 and 2, and the isosoin triplet state is symmetric in nucleons 1 and 2.

2.3 Nuclear Isomerism

There is a large number of nuclei that are found in excited states with a half-life ranging from about a microsecond to many years, which are of extremely long duration on the nuclear time scale ($\sim 10^{-22}$sec). These nuclei are known as isomers.These nuclei de-excite by the emission of $\gamma-$rays or by emission of internal-pair conversion electrons $\gamma-$ rays are classified as electric (E) or magnetic (M) multipoles depending upon the change of spin and parity in the transition.

2.4 Question Answers

2.4.1 Can a photon transfer its entire energy to the electron in compton scattering ? Explain.

The kinetic energy of recoil of electron in Compton scattering is given by

$$E = \frac{h\nu\alpha(1 - cos\phi)}{1 + \alpha(1 - cos\phi)}$$

Which in terms of θ become

$$E = \frac{h\nu 2\alpha cos^2\theta}{(1+\alpha)^2 - \alpha^2 cos^2\theta}$$

where electron recoil angle θ is related to the scattering angle ϕ through $cot\frac{\phi}{2} = (1+\alpha)tan\theta$. The maximum recoil energy of the electron or the maximum photon energy that can be transfered to the electron is given by

$$E_{max} = h\nu\frac{2\alpha}{1 + 2\alpha}$$

where $\alpha = \frac{h\nu}{m_0c^2}$ is a positive quantity, so $\frac{2\alpha}{1+2\alpha}$ is less than unity. Hence

$$E_{max} < h\nu,$$

i.e. the maximum electron recoil energy is less than the photon energy. Thus we may conclude that in Compton scattering process incident photon cannot transfer its full energy tio the electron.

2.4.2 Show that a positron and an electron cannot annihilate into one photon. Conversely, show that a single photon cannot spontaneously get converted into an electron positron pair.

Let an electron and a positron collide together flying from opposite direction with equal and oppositelinear momentum as can be observed from their CM frame and annihilate into a photon. Since their net momentum is zero before collission that should also be zero after their annihilation due to collission according tothe principle of conservation of linear momentum, but the mometum of a photon can never be zero, hence electron-positron annihilation into a signle photon cannot occur. Again, let in the lab frame a sigle photon produces an electron-positron pair. In the CM of this electron-positron pair, the two particles will fly-off with equal and opposite momentum resulting in zero net momentum. But in no frame a photon can be at rest. Hence the violation of the momentum conservation principle shows that a photon cannot spontaneously get converted into an electron positron pair.

2.4.3 Estimate the density of the ${}^{12}_{6}C$ nucleus.

The atomic mass of ${}^{12}_{6}C$ is 12u. Neglecting masses and binding energies of 6 electrons, we have for the nuclear density

$$\rho = \frac{m}{4\pi R^3/3} = \frac{(12u)(1.66 \times 10^{-27} Kg/u)}{(4\pi/3)(2.7 \times 10^{-15} m)^3} = 2.4 \times 10^{17} kg/m^3$$

The above density value is essentially same for all nuclei. Can you guess how ? The density of a neutron star is comparable with the density of nuclear matter : a neutron star packs the mass of 1.4 to 3 suns into a sphere of about 10km in radius.

2.4.4 Calculate the magnitude of spin angular momentum and magnetic moment of a proton.

Protons, like neutrons or electrons are fermions with spin quantum numbers of s=1/2. This means that each of them has spin angular momentum **S** of magnitude $\sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}(\frac{1}{2}+1)}\hbar$ and spin magnetic quantum number $m_s = \pm 1/2$. Tthe spin magnetic moment of a proton or a neutron is expressed in nuclear magneton (μ_N), where

$$\mu_N = \frac{e\hbar}{2m_p} = 5.051 \times 10^{-27} J/T = 3.152 \times 10^{-8} eV/T$$

The spin magnetic moment of a proton is along its spin angular momentm **S** while that for the neutron is in opposition to each other. having magnitudes

$$\mu_{pz} = \pm 2.793 \mu_N$$

$$\mu_{nz} = \mp 1.913 \mu_N$$

2.5 Nuclear stability and nuclear binding

2.5.1 The liquid drop model and B-W mass formula

The fact that the density and the binding energy per nucleon are approximately the same for all (stable) nuclei was first noticed in the early 1930s, after a sufficient number of atomic masses had been measured. This led to the comparison of the nucleus with a liquid drop, which also has a constant density, independent of the number of molecules. The energy required to remove molecules from a liquid is the heat of vaporization. This is proportional to the mass or number of molecules in the liquid, just as the binding energy is proportional to the number of nucleons. Using this analogy, Weizsäcker in 1935 developed a formula for the mass of a nucleus (or the binding energy, since the two are related by Equation (?) as a function of A and Z, called the Weizsäcker semi-empirical mass formula. We shall write down one version of this formula and discuss the origin of the terms. The binding energy is written as

$$B = [+a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4 (A - 2Z)^2 A^{-1} \pm a_5 A^{-1/2}]c^2 \quad (2.24)$$

The first term in eq(13) accounts for the fact that the number of interactions is proportional to A and explains why the binding energy per nucleon is approximately constant. The second term is a correction to the first. The nucleons on the surface of the nucleus have fewer near neighbors, thus fewer interactions, than those in the interior of the nucleus. The effect is analogous to the surface tension of a liquid drop. The surface area is proportional to R^2, which is proportional to $A^{2/3}$. This term is negative because fewer interactions imply a smaller total binding energy. This is the term that accounts for the sharp decline in the binding energy per nucleon at low A values in Figure [Any reference books listed at the end] The third term accounts for the positive electrostatic energy of a charged drop. Because of the Coulomb repulsion of the protons, this effect equals the average electrostatic energy of a proton-proton pair, about $\frac{6ke^2}{5R}$ (see Problem ? ?) times the number of such pairs, which is Z (Z - 1)/2. Thus, the third term is

$$\frac{6e^2}{5R}\frac{1}{4\pi\epsilon_0}\frac{Z(Z-1)}{2} \simeq \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{(Ze)^2}{R_0 A^{1/3}} \simeq a_3 Z^2 A^{-1/3}$$

This positive energy of repulsion decreases the binding energy, so this term is negative. Although this effect exists for all nuclei with $Z > 1$, it is most important for high-Z nuclei and is primarily responsible for the slow decline in the binding energy per nucleon for large values of A. The fourth term has no analogy in the analysis of a liquid drop. It is a quantum mechanical term that accounts for the fact that if $N \neq Z$, the energy of the nucleus increases and the binding energy decreases because of the exclusion principle. The quantity $(A-2Z)^2/A = (N-Z)^2/A$ is the number of neutrons in excess of the number of protons. The expression $(A-2Z)^2/A = (N-Z)^2/A$ is an empirical term that is zero if N=Z and is independent of the sign of N-Z. It is referred to as the symmetry term. The last term is an empirical one to account for the pairing tendency of the nucleons that was mentioned earlier in connection with Table ? ?. The contribution to B is positive if Z and N are both even and negative for both Z and N odd. For the case of Z or N even and the other odd, the term is taken to be zero. (See Table ? ?) The results of many experiments have been used to fit Equation ? ?, or refinements of it, to the binding energies calculated from the measured masses. The solid curve in Figure ? ? is one such fit. Table ? ? lists the values of the coefficients a_1 through a_5 used to produce the curve in Figure ? ?. From Equations (? ?) and (? ?) and the preceding discussion, Weizsäcker's empirical formula for the mass M (Z, A) of a nucleus can then be written as

$$\begin{aligned} M(Z,A)c^2 &= Zm_pc^2 + Nm_nc^2 - B \\ &= Zm_pc^2 + Nm_nc^2 - [+a_1A - a_2A^{2/3} - a_3Z^2A^{-1/3} \\ &\quad -a_4(A-2Z)^2A^{-1} \pm a_5A^{-1/2}]c^2 \end{aligned} \tag{2.25}$$

Equation (? ?) is accurate to about $\pm 0.2 MeV$, which is quite good, all things considered. It has many useful applications. For example, a refined version of Equation (? ?) has been used by P. A. Seeger to compute and tabulate nearly 7500 atomic masses, including many that have obviously not yet been observed. It also provides some helpful panoramic views of nuclear properties. For example, setting $(\frac{\partial M}{\partial Z})_A = 0$ yields the value of Z for which a series of isobars has minimum mass. Determining the coefficients experimentally with R_0 as a parameter allows its calculation, yielding $R_0 = 1.237 fm$ in excellent agreement with the other methods. Plotting $M(Z,A)c^2$ values from Equation (? ?) as a third dimension on the N versus Z graph yields a contourlike graph in the rough shape of a valley whose floor lies along the line of stability. The resulting three-dimensional graph is very useful in discussing beta-decay radioactivity.

2.5.2 Application to stability considerations

Chapitre 3

Nuclear Decays

3.1 Discuss Fermi's Theory of Beta Decay.

In 1930, Wolfgang Pauli postulated the existence of the neutrino to explain the continuous distribution of energy of the electrons emitted in beta decay. Only with the emission of a third particle could momentum and energy be conserved. By 1934, Enrico Fermi had developed a theory of beta decay to include the neutrino, presumed to be massless as well as chargeless. Treating the beta decay as a transition that depended upon the strength of coupling between the initial and final states, Fermi developed a relationship which is now referred to as Fermi's Golden Rule :

$$\lambda_{if} = \frac{2\pi}{\hbar}|M_{if}|^2\rho_f, \tag{3.1}$$

where $\lambda_{if} \to$ transition probability, $M_{if} \to$ matrix element of the interactions and $\rho_f \to$ density of final states. Straightforward in concept, Fermi's Golden Rule says that the transition rate is proportional to the strength of the coupling between the initial and final states factored by the density of final states available to the system. But the nature of the interaction which led to beta decay was unknown in Fermi's time (the weak interaction). It took some 20 years to work out a detailed model which fit the observations. The nature of that model in terms of the distribution of electron momentum p is summarized in the relationship below. newpage

FIGURE 3.1 – Beta spectrum.

3.2 State the selection rules for γ and β transitions.

For $\gamma-$decay :

$$|J_i - J_f| \leq \lambda \leq (J_i + J_f)$$

$\pi_f \pi_i = (-1)^{\lambda}$ for $E\lambda$ transition
$\pi_f \pi_i = (-1)^{\lambda+1}$ for $M\lambda$ transition.

For $\beta-$-decay :
For the Fermi matrix element M_F, the operator connecting the initial and final nuclear states is the unit operator, which is scalar. Therefore no change in parity and spin is involved. While in Gammow-Teller nuclear matrix element, M_{GT}, we calculate the matrix element of the spin operator σ between the initial and final nuclear states.Since σ is pseudovector, which has three components, but is not changed under space reflection, the initial and final spins of the nucleus are coupled vectorially by a unit vector and the two states have the same parity. Thus we summarize the selection rules for allowed beta decay as follows :

$$\Delta J = 0;\ \Delta P = 0\ Fermiselectionrule$$

$$\Delta J = \pm 1, 0; 0 \to 0 forbidden;$$

$$\Delta P = 0\ Gammow - Teller\ selection\ rule$$

The allowed and forbidden nature of the transition is determined from the estimation of a quantity which is konown as the ft-value.

3.2.1 On which ideas Fermi's theory is based ?

Fermi's theory is based on the following ideas :
When a nucleus emits a $\beta-$particle its charge changes by one unit, while its mass is practically unchanged. When the ejected β particle is an electron, the number of protons in the nucleus is increased by one, and the number of neutrons is decreased by one. In positron emission the number of protons decreases by one and the number of neutrons increases by one. Beta-transformations may then be represented by the following processes :

$$\beta^- - emission:\ {}_0n^1 \rightarrow_1 H^1 +_{-1} e^0 + \overline{\nu}$$

$$\beta^+ - emission:\ {}_1H^1 \rightarrow_0 n^1 +_1 e^0 + \nu$$

$$orbital\ electron\ capture:\ {}_1H^1 +_- e^0 \rightarrow_0 n^1 + \nu$$

where ν represents the neutrions.

Define Helicity & indicate it's significance.

Helicity operator h is defined as

$$h = \frac{\sigma.\mathbf{p}}{|\sigma||p|}$$

where σ is the spin of the neutrino and p its momentum. From the above expression h can have two values i.e. $h = \pm 1$. h=+1 for anti-neutrino and h=-1 for neutrino. And this the only distinction between anti-neutriono and neutrino.

Give a short note on symmetry and corresponding conservation laws.

During the first haf of the 20th century it was recognized that, in general, a symmetry or invariance principle generates a conservation law. For example, i) the invariance of physical laws under displacement (or translation) in space has a consequence of conservation of linear momentum, ii) the invariance under a rotation in space has a consequence of the law of conservation of angular momentum. In the theory of relativity concept of invariance was extended to invariance under the Lorentz transformation. This new type of invariance is related to the impossibility of detecting absolute motion, to

the variation of mass with velocity, and to the relationship between mass and energy.The assimilation of relativity with quantum mechanics leads to an especially striking example of fundamental role of symmetry laws. The existence of positron was anticipated theoretically by the requirement that the quantum mechanical equation for the electron, the Dirac equation, must be invariant under Lorentz transformation. The existence of other antiparticles, e.g., antiproton, antineutron, antineutrino, has a similar theoretical interpretation, and their discovery testifies to the power and beauty of the invariance principles.
Another basic type of symmetry is that between right and left, or symmetry under a reflection. The principle of invariance involved may be stated in the following way : Any process which occurs in nature can also occur as it seen reflected in mirror ; the mirror image of any object is also a possible object in nature ; the motion in mirror is also a motion which would be permitted by the laws of nature ; an experiment made in a laboratory can also be made in the way it appears as seen in a mirror, and any resulting effect will be the mirror image of the actual effect. More precisely, we expect that the laws of nature are invariant under reflection, and experience seems to support this idea.
In quantum mechanics it has been established that the consequence of right-left symmetry (invariance under reflection) of the EM forces in the atom appears as the conservation of parity. Parity is a quantum number which takes value +1 or -1 depending on whether the Scrodinger wavefunction is unchanged or changes sign under a reflection through the origin of the spatial coordinates. Experimentally (in the study of atomic spectra) it has been found that in an atomic transition with the emission of a photon the parity of the initial state is equal to the total parity of the final state, i.e., to the product of the parities of the final atomic state and the photon emitted.

3.2.2 Experimental verification of parity violation in weak interaction

Lee and Yang proposed some experiments to test the invariance under reflection, one of which involved β-decay of ^{60}Co. The basic idea of this experiment is to build two sets of experimental arrangements which are mirror images of each other, and which contain weak interactions. The experiment was so designed to test whether the arrangements give same result or different in terms of their meters or (counters). The practical feasibility of the experiment depends on the facts that : i) the Co nucleus has a spin angular momentum ; ii) it rotates with a well defined angular momentum when it is in normal state. In a piece of Co^{60} under normal conditions, the nuclear spins are oriented in all possible directions. But at very low temperature, less than 0.1K since the thermal motions are suppressed, the external field can force the spins to a given direction and the nuclei are said to be polarized. The expt. proposed by Lee and Yang was simply to line up in the same axis, and then see whether the β- particles were emitted preferentially in one direction or the other along the axis. If there were preferential emission in one direction, the mirror image of the experiment would give preferential emission in the opposite direction ; if there is no preferential emission of β particles, the mirror image would give the same result. Possible emission patterns are shown in Fig (14-16 of Kaplan).

The circles represents sample of Co^{60} and the arrows represent electrons, with the arroheads showing the direction of emission. The nuclei in a given sample are aligned so that they spin in the sense indicated by the curved arrow. In case (a) there is no preferential emission ; electrons are emitted isotropically. In the image formed by the horizaontal mirror, the sense of rotation is the same and the electron distribution is again isotropic. In (a'), the direction in which the nuclei are aligned is rotaed through 180^o ; electron emission is again isotropic and the mirror image of the expt. is identical with the expt. itself.

The images in the vertical mirror looks just like the original turned upside down. Cases (a) and (a') show results expected with right-left symmetry and conservation of parity. In case (b), more electrons are emitted in the direction opposite to that in which the nuclei are aligned. The electron emission is anisotropic and the mirror image shows more electrons being emitted in the direction of the nuclear spins. This kind of result would be expected if parity is not conserved.

Wu expt

The expt suggested by Lee and Yang was performed by C. S. Wu and co-workers in 1957 at the National Bureau of Standards in Washington USA which gave a brilliant confirmation to Lee and Yang's suggestion. Co^{60} emits an electron of energy 0.312MeV in over 99% of its decays and two other electrons in the remaining decays, several gamma rays are also emitted in the process. To detect the β- particles, a thin anthracene crystal was placed inside the vcuum chamber near the cobalt source. The scintilation was transmitted to a PM tube located at the top of the cryostat. The degree of polarization of the Co-60 nuclei were measured by two NaI γ ray scintilation counters, one in the equatorial plane and the other near the polar position. The observed γ- ray anisotropy provided a measure of the polarization and hence of the temperature of the sample. Samples were made by good single crystals of cerium magnesium (CeMg) nitrate and growing on the upper surface of an additional crystalline layer about 0.002in thick, containing a few micro-Curie of Co^{60}. After the material in the cryostat was cooled to about 0.01K (at 10Tesla), the cooling magnet was turned off ; a vertical solenoid was raised around the lower part of the crystal within 20sec after the demagnetization, and the counting started. The result of the expt are shown in the Fig(14-18 of Kaplan) The time scale is actually a temperature scale measuring time from the instant when cooling is stoped. The curve labelled "$\gamma-$ anisotropy" is a measure of the degree (extent) of polarization ; after about 8min the nuclei have become sufficiently warm so that randomess removes γ anisotropy. The curve labelled "β assymetry" is the significant one ; it indicates the number of electrons emitted in the direction of the B field (in which the nuclei are aligned). In other words, the experimental result corresponds to case (b) of Fig (14-16 of Kaplan) ; the electrons are emitted in a preferred direction, the principle of right-left symmetry is violated, and parity is not conserved. The magnitude of the effect is remarkable ; the electron intensity in one direction along the axis of rotation was found to be 40% more than that in the opposite direction thereby leaving no doubt in the result.

Chapitre 4

Nuclear Models

4.1 Introduction

The study of the nucleus - to know its structure and the effective forces among the con- stituent particles - is one of the most challenging problems posed by the nature. The nucleon-nucleon (NN) force is not simple - nor it is uniquely and precisely known. Never- theless a lot of information about the NN force can be obtained from the study of two and three nucleon systems. The actual force among the nucleons in a fairly large nucleus may very well be different from the NN force between two isolated nucleons. Even if the NN force is assumed to be known, the quantum mechanical problem of a fairly large number (say 10 - 250) of nucleons is a very difficult problem and there are no exact solutions. However this is not unique for the nucleus alone. Any many body system - both classical (macroscopic) and quantum (microscopic)-has this difficulty. Examples are a drop of liq- uid, a volume of gas, a many electron atom, the planetary system, etc. However in many of these, the nature provides us with one form or another of simplification. For example, in the first two of these, the number of particles is so large that statistical methods can be used and give an excellent result. In the last two, there is a centre of force, which is such that the forces between it and each of the particles is much stronger than the forces among the particles. Thus each particle can be assumed to move in a common field of force and interparticle forces can be treated as a perturbation. In a nucleus, the number of particles is too large for an ab initio exact solution and too small for a statistical treat- ment. This makes the nuclear problem especially difficult and challenging.

On the other hand, there is no clear centre of force in a nucleus and the interaction between any two nucleons should, in principle, be the same. Thus the simplifications available in atomic or planetary systems are not expected to be valid in a nucleus. Thus we can summarize the situation as

1. A moderately heavy nucleus has too many nucleons for an exact solution and too few nucleons for a statistical treatment. Hence a theoretical solution is very difficult.
2. The internucleon force - especially the effective one inside the nucleus - is not accu- rately known.

Hence an exact and complete quantum mechanical treatment is beyond question. Even if it were possible, it would be desirable to look for a simplified description in terms of a f ew parameters, which will give us a grasp of the physical picture. At the same time, the parameters should be such that the physical picture is complete and incorporates the most important features. Hence we need conceptual or mathematical models of the nucleus. We assume the nucleus to be a physical system (the "model") with which we are familiar and which in some of its properties resembles a nucleus. The properties of this physical system (model) are investigated and compared with those of the nucleus obtained experimentally. If they agree, then the model is an acceptable one. However for such a complex system as the nu- cleus, each model (described by only a few parameters) must break down sooner or later. When that happens, the very cause of the breakdown often indicates in which way the model was defective and needs modification. Models are generally built on recognizable features of the nucleus. Models are built even by a stretch of imagination. An example is the **degenerate gas model**- in which the nucleons are assumed to be free particles constrained to be within the volume of the nucleus. Since the nucleons are forced to be within the nuclear volume, which is very small ($\sim fm^3 \sim 10^{-39}$ cc), the energy levels have spacings (which are inversely proportional to the linear dimensions) of several Mev. As the nucleons are fermions having spin 1/2, each momentum state can accommodate two nucleons of the same kind (proton or neutron). Since the spacing between the momentum states is large, almost all the lowest states are completely filled. This forms a degenerate gas. A simple calculation based on this model shows that the average kinetic energy of a nucleon in a nucleus is about 20 Mev and the maximum kinetic energy is about 33 Mev. Assuming that the binding energy per nucleon in a medium or heavy nucleus is about 8 Mev, the depth of the averaged potential well in which we assume the nucleus to move is about 41 Mev. This model also shows that the symmetric nucleus (in which N=Z=A/2, where N is the number of neutrons, Z is the number of protons and A is the total number of nucleons) is the most stable energetically. Consideration of Coulomb repulsion among 2the protons, which is proportional to Z(Z-1) shows that in heavy nuclei there will be more neutrons than protons. Since, according to this model , the symmetric nucleus is the most stable, it shows why a stable nucleus contains both protons and neutrons and not neutrons only. This is due to the Pauli exclusion principle - if we want to replace some protons by neutrons in the symmetric nucleus , these new neutrons must go to one of the unfilled higher energy levels and not the lowest energy levels previously occupied by the protons, thereby increasing the total energy of the nucleus. Thus as simple and unexpected a model as the degenerate gas model can explain some gross features of the nucleus. It brings out the quantum effects of a highly degenerate system, but it is not a suitable model for investigating the energetics or other observables of a nucleus.

4.2 The Shell Model

We know that in a many electron atom, there is a clear centre of force, namely the nucleus, so that each electron feels a dominantly attractive Coulomb force towards the nucleus. The mutual repulsion among the electrons is relatively weaker and can be treated as a perturbation. Then the common attractive field for all the electrons gives rise to a shell structure, which arises from the fact that each single particle level can be occupied by a maximum number of electrons, since the electrons obey Pauli exclusion principle. Even though there is no apparent common centre of attraction in a nuceus, there are overwhelming experimental evidences for a nuclear shell structure. The most important experimental evidence is the existence of "magic numbers". It is found that nuclei whose neutron number (N) or proton number (Z) or both are one of the following magic num- bers magic numbers→ 2, 8, 20, 28, 50, 82, 126 are particularly stable (have large binding energy) relative to their neighbours. Particularly spectacular are the " doubly magic nuclei" $\rightarrow^{4}He,^{16}O,^{40}Ca,^{48}Ca,^{208}Pb$ etc. These have very large BE and the first excited state is well above the ground state so that these are difficult to excite. Hence, in spite of the fact that there is no common centre of force in a nucleus, attempts were made in an ad hoc manner using simplest common potentials for each nucleon. The many body Hamiltonian, with two body interactions, has the form

$$H = \sum_{i=1}^{A} T_i + \sum_{i<j=2}^{A} V_{ij} \tag{4.1}$$

An exact solution of the many body Schrodinger equation

$$H\psi(\vec{r_1}, ..., \vec{r_A}) = E\psi(\vec{r_1}, ..., \vec{r_A}) \tag{4.2}$$

is not possible even if the two body potential V_{ij} is known. One can write

$$H_0 = \sum_{i=1}^{A} (T_i + \overline{U_i}) \tag{4.3}$$

and

$$H' = \sum_{i<j=2}^{A} V_{ij} + \sum_{i=1}^{A} \overline{U_i} \tag{4.4}$$

We can of course add and subtract a single particle term U_i, which has the same form for all nucleons in H, but then there is no a priori guarantee that H' will be small compared to H_0. Neverthless, we can contemplate the scenario that each nucleon moves in a random orbit, with respect to all other nucleons. If the orbital motions are fast, an individual nucleon will not "see" other nucleons individually, but only an average "nucleon cloud" produced by the fast motion of all the other nucleons. Thus the average effect of pairwise interactions of all the other orbiting nucleons will be as if that particular nucleon is placed in a constant (i.e. time independent) field of force. Now this will be true from the point of view of any particular nucleon. Hence each nucleon will experience the same " common " potential arising from the common field of force. This term is the average common potential $\overline{U}$, experienced by all nucleons. Since the origin of this average potential is indeed the net effect of all two body potentials $V(\vec{r_i} - \vec{r_j})$, we may expect H' to be small compared to H_0. Thus if we can approximate H by H_0, the Schrodinger equation becomes

$$H_0\psi(\vec{r_1}, ..., \vec{r_A}) = E\psi(\vec{r_1}, ..., \vec{r_A}) \tag{4.5}$$

Since H_0 is the sum of noninteracting single particle terms, ψ is a product w.f.

$$\psi(\vec{r_1}, ..., \vec{r_A}) = A[\phi_{\nu_1}(\vec{r_1}), \phi_{\nu_1}(\vec{r_1}), ..., \phi_{\nu_1}(\vec{r_A})], \tag{4.6}$$

where A is an antisymmetrization operator for the identical fermions. The single particle wave functions $\phi_\nu(\vec{r})$ satisfy

$$T(\vec{r}) + \overline{U}(\vec{r})\phi_\nu(\vec{r}) = \epsilon_\nu \phi_\nu(\vec{r}) \tag{4.7}$$

and

$$E = \sum_{\nu(occupied)} \epsilon_\nu \tag{4.8}$$

Thus in this case, the many body Schrodinger equation reduces to a one body Schrödinger equation [eqn.(9)], which has an infinite number of single particle levels $\epsilon_{\nu_1}, \epsilon_{\nu_2}, \ldots\ldots$. Then each such level is filled up with two nucleons of a given kind (corresponding to " spin up" and " spin down " states of a given nucleon), in accordance with the Pauli Principle.

4.2.1 Choice of average potential $\overline{U}(\vec{r})$

In the absence of a theoretical guidaence, one chooses simpliest forms of $\overline{U}(\vec{r})$, which allow analytic solutions. It is chosen as central, having one of the following shapes :

1. Infinite square well :

$$V(r) = \begin{cases} -V_0; \; for\, r < R \\ \infty; \; for\, r \geq R \end{cases} \tag{4.9}$$

2. Finite square well :

$$V(r) = \begin{cases} -V_0; \; for\, r < R \\ 0; \; for\, r \geq R \end{cases} \tag{4.10}$$

3. Harmonic oscilator well :

$$V(r) = -V_0 + 1/2M\omega^2 r^2 \tag{4.11}$$

We will discuss the third one (harmonic oscilator). The solutions can be obtained analytically

$$\epsilon_{nl} = \hbar\omega(\Lambda_{nl} + 3/2) \tag{4.12}$$

with

$$\Lambda_{nl} = 2(n-1) + l \tag{4.13}$$

All the possible (n, l) states giving the same Λ_{nl} are degenerate. Furthermore for a given l, there are $(2l+1)$ different values of $m(-l, -l+1, ..., +l)$. Each (nlm) state can accommodate two nucleons (corresponding to spin up and spin down) of one kind. Thus the total no. of nucleons of a given type that can be accommodated in a level with a fixed Λ is

$$N_\Lambda = \sum_l (2l+1), \tag{4.14}$$

where the sum over l includes all l values for a fixed Λ. The quantum nos and energy levels are shown bellow :
Thus the calculated magic numbers are 2, 8, 20, 40, 70, 112, ...Only the first three agree with the observed ones.
The infinite square well can also be solved analytically. In this case, the degeneracy among different n, l states no more exist. The predicted magic numbers are 2, 8, 20, 40, 58, ...etc and again only the first three are correctly reproduced.

It was found that a purely central $\overline{U}(r)$ cannot produce the observed magic numbers, no matter what shape we choose for $\overline{U}(r)$.
In 1949, Mayer and Haxel, Jensen and Suess independently suggested the inclusion of a spin orbit term

$$v_{ls}(r) = -f(r)\mathbf{l.s} \tag{4.15}$$

to the average potential $\overline{U}$. This will split the $j = l \pm 1/2$ levels. The sign is chosen such that the $j = l + 1/2$ level is depressed relative to the $j = l - 1/2$ level. Since

$$\begin{aligned} < v_{ls} > &= \epsilon_{ls} \\ &= - < (l.s) >< f(r) > \\ &= -l/2 < f(r) > \;\; for\; j = l + 1/2 \\ &= (l+1)/2 < f(r) > \;\; for\; j = l - 1/2 \end{aligned} \tag{4.16}$$

The new basis is $\mid nljm >$ and we have

$$V(r) = -V_0 + \frac{1}{2} M\omega^2 r^2 - f(r)\mathbf{l.s} \tag{4.17}$$

The form and strength of f(r) is chosen so as to reproduce the observed magic numbers. The calculated magic numbers then agree completely with the observed ones. The level scheme is shown in Fig 1.

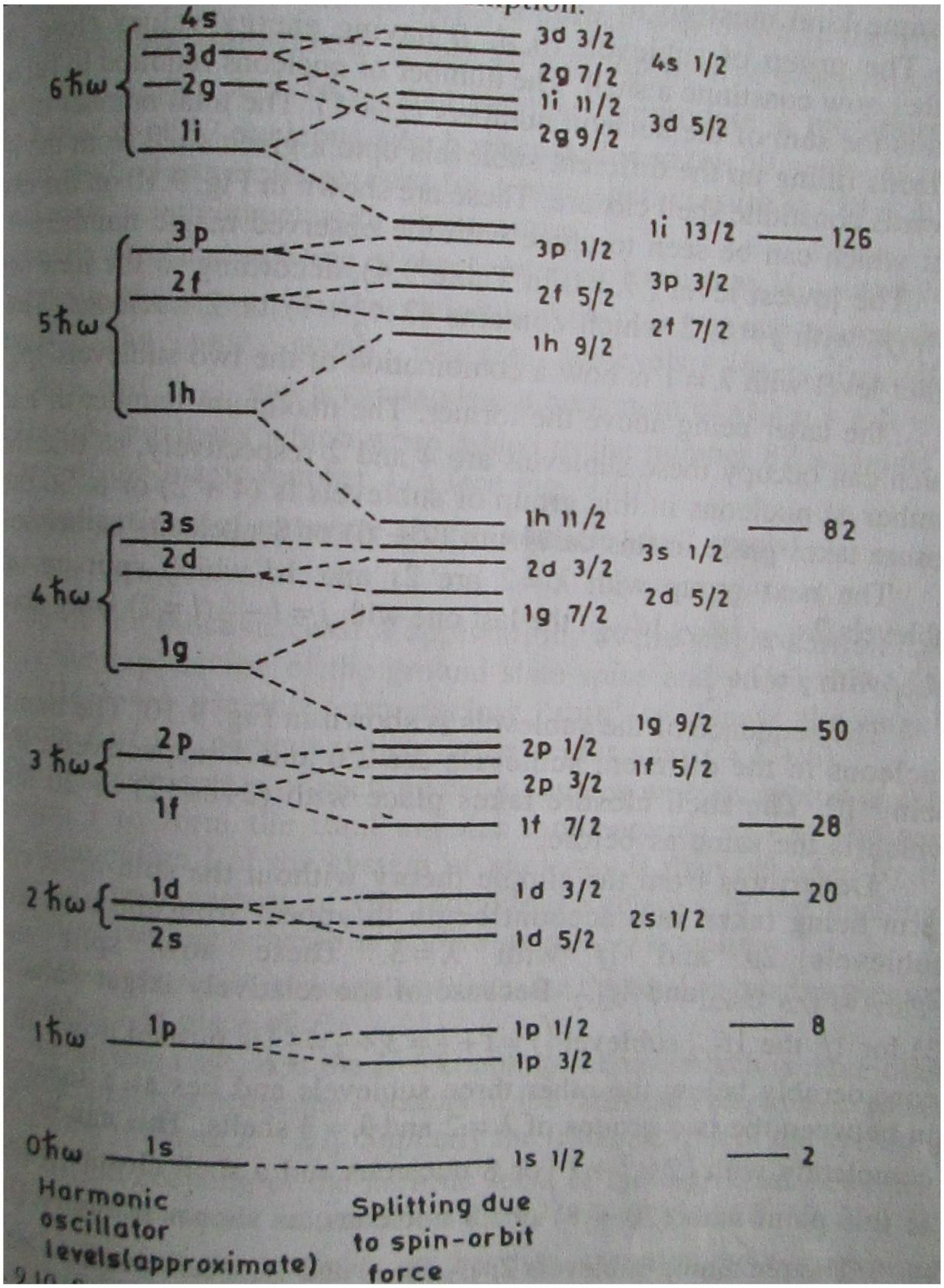


FIGURE 4.1 – Energy level diagrams for HO Potential Well

4.2.2 Prediction and success ESPSM

We have succeeded in explaining the magic numbers, which was the first goal. How- ever, with a few other assumptions, we can see that this model has a lot of successes. To proceed further, we now make another assumption. It is an experimental fact that all even-even nuclei have zero angular momentum in the ground state - the ground state is a 0+ state. We therefore postulate that each pair of like nucleons in the same (nlj) level of a nucleus pair off to a J=0, T=1 pair. The angular momentum and parity of the ground state of an odd A nucleus is then the angular momentum and parity of the last unpaired nucleon. With this assumption we have the extreme single particle shell model (ESPSM), which explains the spin and parity of all even-even nuclei correctly, many odd A nuclei correctly and the low lying spectrum of odd A nuclei with a fair amount of success.

4.2.3 Predictions about spin, parity and low lying spectrum

As an example consider ${}_8O_9^{17}$ - it has one neutron outside the doubly magic nucleus O^{16}. The last neutron must be in the $1d_{5/2}$ level according to Fig 1. Thus the ground state spin parity of O^{17} is $\frac{5}{2}^+$. This agrees with the observed value. Since lifting the unpaired neutron for $1d_{5/2}$ to $2s_{1/2}$ or $1d_{3/2}$ levels costs less energy than breaking a pair from the core and lifting one nucleon out of it, the low lying spectrum of O^{17} is expected to be as shown in Fig 2. - the excitation energies are just the corresponding level spacings in Fig 1. Little can be said about the odd-odd nuclei from this model alone.

4.2.4 Prediction of magnetic moments

In the ESPSM, only the last unpaired nucleon contributes to the magnetic moment. Thus the magnetic moment of the nucleus is

$$\vec{\mu} = (g_l a_l + g_s a_s)\vec{j} \tag{4.18}$$

where g_l, g_s are respectively the gyromagnetic ratios for the orbital and spin motions of the nucleons (neutrons and protons). $g_l = 0$ for neutron (since neutron is an uncharged particle its orbital motion does not produce any magnetic moment) and $g_l = 1$ for proton. And $g_s = g_n = -2 \times 1.9131$ for neutron and $g_s = g_p = 2 \times 2.7927$ for The coefficients $a_l j$ and $a_s j$ are the projections of $\vec{l}$ and $\vec{s}$ on $\vec{j}$, i.e.,

$$a_l j = \mu_N \frac{j(j+1) + l(l+1) - s(s+1)}{2\sqrt{j(j+1)}} \tag{4.19}$$

$$a_s j = \mu_N \frac{j(j+1) + s(s+1) - l(l+1)}{2\sqrt{j(j+1)}} \tag{4.20}$$

Since s = 1/2 and l = j + 1/2 or j-1/2, we get

$$for \quad l = j - 1/2 \quad \mu_z = \{g_l \frac{j-1/2}{j} + g_s \frac{1}{2j}\} j \mu_N \tag{4.21}$$

$$l = j + 1/2 \quad \mu_z = \{g_l \frac{j+3/2}{j+1} - g_s \frac{1}{2(j+1)}\} j \mu_N \tag{4.22}$$

In the case of odd-A nuclei, either the proton number is odd (odd-even nuclei) or the neutron number is odd (in the even-odd nuclei). So we have the following possibilities.
Odd neutron $(g_l = 0, g_s = g_n)$:

$$l = j - 1/2 \qquad \mu_z = \frac{g_n \mu_N}{2} \tag{4.23}$$

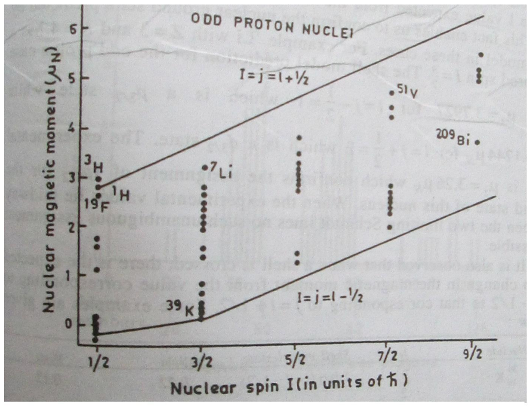
$$l = j + 1/2 \quad \mu_z = -\frac{j}{j+1} \frac{g_n \mu_N}{2} \tag{4.24}$$

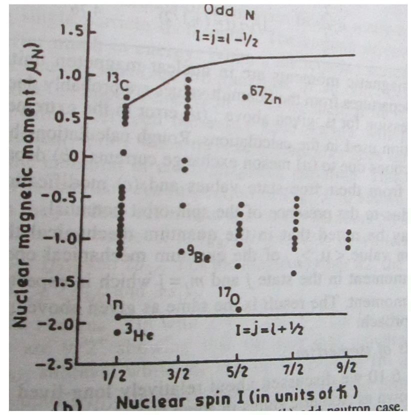
Odd proton $(g_l = 1, g_s = g_p)$:

$$l = j - 1/2 \qquad \mu_z = \{j - 1/2 + \frac{g_p}{2}\} \mu_N \tag{4.25}$$

$$l = j + 1/2 \quad \mu_z = \frac{j}{j+1} \{j + 3/2 - \frac{g_p}{2}\} \mu_N \tag{4.26}$$

The above values of nuclear magnetic moments are known as the Schmidt values and are plotted against j in Fig 3 and 4.





We see that the measured μ lies almost without exception between the two Schmidt lines - in general they are nearer to one line than the other. Thus there is a fair degree of success in explaining the magnetic moments.

4.2.5 Quadrapole Moment.

Once again in the ESPSM, the quadrapole moment of the nucleus is due to the last unpaired proton only. A straight forward calculation of

$$Q_{sp} = -\frac{2j-1}{2j+2} < r^2 > \tag{4.27}$$

shows that the calculated Q is of the same order as the measured Q for light nuclei but for A>100, the observed Q is nearly an order of magnitude larger. Thus the theoretical prediction is not quite successful. We see that in spite of rather sweeping assumptions, the ESPSM is quite successful in explaining not only the magic numbers, but also the g.s. spin, parity, and magnetic moment and the low lying spectra of quite a few nuclei. *The important failure is in the prediction of the Quadrapole moment.*
A better approach (called the shell model calculation) is to use the ESPSM states as the basis and diagonalizing the neglected part H' (called residual interaction) for one or more active particles outside an inert core. Agreement in general is better, but large nuclear quadrapole moments are not reproduced.

4.3 Single particle shell model

Unlike in the ESPSM, one has to consider the coupling of the angular momenta of all the odd nucleons in the last occupied sublevel outside the core of closed shells. We assume that the residual interaction between these loose nucleons in the outermost sublevel does not cause any perturbation to the single particle levels determined by (n, l, j). if there are k particles in the given outermost sublevel (n, l, j), then the different possible states that can be formed by them are degenerate in the extreme single particle model, which ignores the residual interaction between these particles. When the latter is taken into account, the degeneracy is removed and the levels differ slightly in energy. It is usually assumed that the residual interaction is weak compared to the spin-orbit force so that j-still remains a good quantum number i.e., the levels are still characterized by definite j values. If this is not the case, then the particle states may be regarded as the superposition of the different (n, l, j) states with energies close to one another. This is known as configuration mixing.

4.3.1 Summary of SPSM

1. For even Z even N nuclei, spin I=0
2. For odd A nuclei, I is determined by the j value of the last odd nucleon
3. For odd Z, odd N nuclei, the value of I is determined by the Nordheim rules.
   Nordheim number $N = (j_p - l_p) + (j_n - l_n)$. $N = 0$ for ($j_n = l_n \pm 1/2$ and $j_p = l_p \mp 1/2$). $N = \pm 1$ for ($j_n = l_n \pm 1/2$ and $j_p = l_p \pm 1/2$). Strong Rule : $N = 0, I =\mid j_n - j_p \mid$; Weak Rule : $N = \pm 1, I =\mid j_n - j_p \mid$ *or* $(j_n + j_p)$.

4.4 Individual (or independent) particle model

In this model all the A nucleons are assumed to move independent of one another in a common potential field. This model can be used for accurate determination of the wave function subject to the approximate validity of the shell model to some extent. As an example, let us consider the ^{7}Li (Z=3) nucleus with a core of 2 protons and 2 neutrons in $1s_{1/2}$ proton and neutron shells. In the extreme single particle shell model (ESPSM), we assume that the remaining 2 neutrons in the $1p_{3/2}$ neutron shell couple to give 0 spin, while the odd proton in the $1p_{3/2}$ proton shell determines the resultant spin

$I = J = 3/2$ for the nucleus[1]. In the independent particle model (IPM), on the other hand, no assumption is made in the pairing of the nucleons. So ^{7}Li is regarded as a closed shell plus 3 loose nucleons all in the same state $1p_{3/2}$ so that the configuration is $(1p_{3/2})^3$.

4.4.1 Basic idea of an actual calculation

4.4.2 Seniority scheme

4.4.3 Qualitative discussion of cfp

4.4.4 Diagonalization

4.5 The Collective Model

We saw that the extreme single particle shell model (ESPSM) or modifications of these (generally called independent particle model or simply "the shell model") can explain many nuclear properties, but fail to reproduce large nuclear quadrapole moments and spheroidal shapes, which many nuclei have. It is clear that such effects cannot arise from any model which takes into account only the single particle aspects, assuming that the nucleons are filled in pairs and only the unpaired nucleon contributes to the overall properties. Such large effects can only arise from the coordinated motion of many nucleons. The nucleons move as individual particles, but there can be coordination in the motion, such that many nucleons move collectively- giving the conglomerate of nucleons (consti- tuting the nucleus) a particular shape, which changes slowly (compared to the individual orbital motion of the nucleons) and in the extreme case may even remain stationary. This type of correlation is a long range one and is very different from the short range correlations in nuclear matter. If the "permanent" or "semi permanent" shape is non spherical, then the average po- tential felt by an individual nucleon will not be spherically symmetric and so the nucleons will readjust their orbits in accordance with the non-spherical average potential. This in turn tend to preserve the nuclear "shape". We can visualize this scenerio by a crude macroscopic analogy . We are familiar with the conglomerate of mosquitos above the head of a man in an open space in the evening. A single mosquito

1. In nuclear physics total angular momemntum quantum number is generally denoted by I instead of J

is an individual entity and moves independently. But the affinity to be together forces an individual to be a part of the conglomerate. Even though each mosquito moves independently, the overall shape is only slowly changing. If the man moves slowly, the conglomerate of mosquitos, also moves in phase, keeping the shape of the conglomerate nearly unchanged. This is because, as the man moves, the mosquitos nearest to the head (dark hair) move, trying to be just over the head. Then the mosquitos just above the bottom layer, try to be together and so "feels" a social force, which is not isotropic. The same thing happens to a flock birds flying in a conglomerate. Thus the motion of the individual nucleons is expected to be highly correlated. We can then imagine the nucleus to be a liquid drop, which can take spherical or nonspherical shapes. Since the forces on the individual nucleons depend on the shape of the surface, there will be coupling between the particles and surface motions. The surface can vibrate or rotate slowly (compared to nucleon periods). Such a model, which envisages the nu- cleus to be a liquid drop having a spherical or deformed shape and capable of executing slow vibrations or rotations is called a “collective model”. -the name derives from the collective motion of a large number of nucleons. The surface of a liquid drop of arbitary shape can be written as

$$R = R_0[1 + \sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} Y_\lambda^\mu(\theta, \phi] \tag{4.28}$$

$R(\theta, \phi)$ is the distance of the surface in the direction (θ, ϕ) from the origin. Now experimental information shows that the quadrupole deformation is the most im- portant. So we take $\lambda = 2$ only. [$\lambda = 0$ corresponds to volume change (which is not likely since nuclear matter is incompressible), λ= 1 corresponds to a shift of the centre of mass, and is not interesting]. Then

$$R = R_0[1 + \sum_{\mu=-2}^{2} \alpha_{2\mu} Y_2^\mu(\theta, \phi] \tag{4.29}$$

where (θ', ϕ') refer to the body fixed frame (axes are 1, 2 and 3). Referred to space fixed frame (axes are x, y, z), we have equation (21) with $\lambda = 2$ only. Hence

$$sum_{\mu=-2}^{2} \alpha_{2\mu} Y_2^\mu(\theta, \phi) = \sum_{\nu=-2}^{2} a_{2\nu} Y_2^\nu(\theta', \phi') \tag{4.30}$$

Suppose the orientation of the space fixed axes with respect to the body fixed axes is described by the Euler angles (Θ, Φ, Ψ). Then from the properties of the transformation of spherical harmonics under rotation of axes, we have

$$Y_{\lambda\mu}(\theta, \phi) = \sum_{\rho} Y_{\lambda\rho}(\theta', \phi') D^{(\lambda)}_{\mu\rho}(\Theta, \Phi, \Psi) \tag{4.31}$$

Hence from (23)

$$a_{2\nu} = \sum_{\mu} \alpha_{2\mu} D^{(2)}_{\mu\nu}(\Theta, \Phi, \Psi) \tag{4.32}$$

and

$$\alpha_{2\mu} = \sum_{\nu} a_{2\nu} D^{(2)}_{\mu\nu}(\Theta, \Phi, \Psi) \tag{4.33}$$

Let us take the body fixed axes as the principal axes. Hence the products of inertia are zero, which implies

$$a_{2,1} = a_{2,-1} = 0 a_{2,2} = a_{2,-2} \tag{4.34}$$

Thus a_{20}, a_{22} and the Euler angles would completely describe the system, replacing five $\alpha_{2\mu}$. Define

$$a_{20} = \beta cos\gamma; \;\; a_{22} = (\beta/\sqrt{2}) sin\gamma \tag{4.35}$$

Then

$$\sum_{\mu} \mid \alpha_{2\mu} \mid^2 = \sum_{\mu} \mid a_{2\mu} \mid^2 = \beta^2 \tag{4.36}$$

and β is a measure of the total deformation. The potential energy is

$$V = 1/2 C \sum_{\mu} \mid \alpha_{2\mu} \mid^2 \tag{4.37}$$

where C is a constant. We can verify that $\gamma = 0$ produces a prolate spheroid (cigar shape) with 3-axis as the symmetry axis. For $\gamma = \pi$, the shape is oblate spheroid (pancake) with 3-axis as the symmetry axis [See Fig. 1x] We can show that the total kinetic energy of the system is

$$T = 1/2 B(\dot{\beta}^2 + \beta^2 \dot{\gamma}^2) + 1/2 \sum_{k=1}^{3} I_k \omega_k^2 \tag{4.38}$$

where B is a constant, a dot denotes time derivative and

$$I_k = 4B\beta^2 sin^2(\gamma - k\frac{2\pi}{3}) \tag{4.39}$$

is the moment of inertia about the body fixed k-axis. If $\gamma = 0$ or π, $I_3 = 0$. Experimental evidence shows that there are no rotations about a symmetry axis. The last term in equation (1.54) indicates a rotation of the nucleus while the $\beta - \gamma$ term together with the potential energy term indicates collective vibrations. We quantize the classical expres- sions by replacing velocities (time derivatives) by the conjugate momenta and requiring that the variable and its conjugate momentum do not commute, as in the case of ordinary harmonic oscilator. Finally one can introduce creation and destruction operators to solve the Hamiltonian

$$H = H_\beta + H_\gamma + \sum_{k=1}^{3} \frac{L_k^2}{2I_k} + 1/2C\beta^2 \tag{4.40}$$

If the nuclear shape is "rigid", i.e. no change of shape, then only the rotational part of the Hamiltonian is active. In this case if the nucleus is axially symmetric about the 3-axis then $I_1 = I_2 = I$. We can specify the angular momentum J together with its component M along the z-axis and K along the 3-axis, and the states are denoted by |JMK>. Then for the rotational energy,

$$\begin{aligned} E_r = \sum_k \frac{\hbar^2}{2I_k} J_k^2 = \frac{\hbar^2}{2I}(J_1^2 + J_2^2) + \frac{\hbar^2}{2I_3} J_3^2 = \frac{\hbar^2}{2I}(J - J_3^2) + \frac{\hbar^2}{2I_3} J_3^2 \\ = \frac{\hbar^2}{2I}[J(J+1) - K^2] + \frac{\hbar^2}{2I_3} K^2 \end{aligned} \tag{4.41}$$

as the eigen values of J^2 and J_3 are J(J+1) and K respectively. Since the hydrodynamical model as well as the experimental facts show that there is no rotation about the symmetry axis, the component of angular momentum about the 3-axis is zero i.e., K=0. Hence

$$E_J = (\frac{\hbar^2}{2I})J(J+1) \tag{4.42}$$

For nonzero K, J takes the values K, K+1, K+2, ... Eqn. (1.58) agrees very well with observed rotational spectrum. The collective model also predicts collective β and γ vibrations (see eq. (1.56)), which have been observed in near spherical nuclei. Such energy levels are approximately equally spaced. Usually nuclei near a closed shell nucleus are near spherical in shape and exhibit vibrational excited states. Nuclei which are far removed from magic nuclei are strongly deformed and exhibit rotational excitation spectra according to eq (1.58). Thus the predictions of collective models are in agreement with observations for medium and heavy nuclei.

4.5.1 Coupling of particle and collective motions

For odd A, in the deformed nuclei region, coupling of the motion of the odd particle with the collective motion must not be disregarded since the particles are no longer in the same average potential and the motions of the nucleons will be altered. Coupling may be both weak and strong. For sphercal nuclei, the coupling is weak and may be treated as a perturbation. On the other hand, when the particle motion and the collective parameters strongly influence each other, we have a strong coupling limit. We assume that the single odd nucleon contributes $\Omega = K$ as the projection of the angular momentum on the axis of symmetry, the loose nucleons pairing off to $\sum \Omega_j = 0$ Hence K is odd half integral. The level energies of the axially symmtric nuclei are given by

$$E_{JK} = \epsilon_K + \frac{\hbar^2}{2I}[J(J+1) - K^2 + \delta_{K,1/2} a(-1)^{J+1/2}(J+1/2)] \qquad (4.43)$$

where $\delta_{K,1/2} = 0$ for $K \neq 1/2$ and $\delta_{K,1/2} = 1$ for $K = 1/2$. ϵ_K is the contribution of the energy due to the particle Hamiltonian, a is the decoupling parameter given by

$$a = \sum_j (-1)^{j+1/2}(j+1/2) \mid C_j \mid^2 \qquad (4.44)$$

where $\mid C_j \mid^2$ is the probability that the odd particle has the angular momentum j. The term involving a is absent for $K \neq 1/2$. The rotational bands are built on the particle states of different K which is entirely due to the particle angular momentum and is constant throughout the band. All higher values of $J > K$ are permitted. For $K = 1/2$, the lowest state has $K = J$ and the successive states have j=K+1, K+2 etc. The parity is the same as for the odd particle configuration. For $K = 1/2$, the level order is determined by the values of a. Eg. for $-3 < a < -2$, the level order is 3/2, 1/2, 7/2, 5/2, 11/2, 9/2 etc.

4.6 Nilsson model

The generalization of the phenomenological shell model to deformed nuclear shapes was first given by S. G. Nilsson in 1955, which is referred as the Nilsson model and also as a unified model. The purpose of the Nilsson model [2] is to produce a basic single-particle model applicable to nearly all deformed nuclei. It accounts for most of the observed features of single-particle levels in hundreds of deformed nuclei. The principal idea is to make the oscillator constants different in the different spatial directions. The form potential considered was an anisotropic harmonic potential :

$$v(r) = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \tag{4.45}$$

For the associated nuclear shape we may define a geometric nuclear surface consisting of all the points (x,y,z)

$$\frac{1}{m}\overline{\omega}_0^2 R^2 = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \tag{4.46}$$

where $\hbar\overline{\omega}_0 = 41MeV \times A^{-1/3}$ is the oscillator constant for the equivalent spherical nucleus. This describes an ellipsoid with axes X, Y and Z and given by

$$\overline{\omega}_0 R = \omega_x X = \omega_y Y = \omega_z Z \tag{4.47}$$

The condition of incompressibility of nuclear matter requires that the volume of the ellipsoid should be the same as that of the sphere, implying $R^3 = XYZ$, and this imposes a condition on the oscillator frequencies :

$$\overline{\omega}_0^3 = \omega_x \omega_y \omega_z \tag{4.48}$$

The single-particle Hamiltonian used originally by Nilsson, for a nucleus with symmetry axis z, is :

$$H = \frac{p^2}{2m} + \frac{1}{2}m[\omega_x^2(x^2 + y^2) + \omega_z^2 z^2] + C\mathbf{l}.\mathbf{s} + Dl^2 \tag{4.49}$$

where the first term of the potential is the kinetic energy of the single-particle, the second term is the anisotropic harmonic oscillator which is used as an average field, with $\omega_x^2 = \omega_0^2(1 + 2\delta_2/3) = \omega_y^2$ and $\omega_z^2 = \omega_0^2(1 - 4\delta_2/3)$ for the case of symmetry axis z. δ_2 is the parameter of deformation introduced by Nilsson and is related to β_2, to first order, by $\beta_2 \simeq 1.05\delta_2$ [2]. The third term is the spin-orbit interaction which has to be added to reproduce the correct magic numbers, with $C = -2\hbar\omega_0\kappa$ giving the strength of the spin orbit force. The fourth term, with $D = -\hbar\omega_0\kappa\mu$, accounts for the fact that, at large distances from the centre of the nucleus, the nucleons experience a deeper potential in the realistic case as compared to the harmonic oscillator, thus shifting the levels with higher l-values to lower energy. Different values of κ and μ are used for different shells by fitting the experimental data [3]. To study the observed single-particle levels, diagrams such as those in figures Fig.x ? and Figy ? are used. Positive parity orbits are indicated by solid lines, negative parity orbits by dashed lines. These diagrams show the single-particle energy levels for neutrons between the closed shells at 50 and 82 as a function of δ_2 deformation. The difference in energy between both proton and neutron single-particle levels is due to the Coulomb repulsion of the protons which is considered with an appropriate choice of the onstants κ and μ [3]. A typical Nilsson orbit is labelled as follows :

$$\Omega^{\pi}(Nn_z\Lambda)$$

where the first quantum number,Ω, gives the projection of the single-particle angular momentum, j, onto the symmetry axis and π is its parity. Inside the brackets the three quantum numbers are N, the principal quantum number of the major shell (or the number of oscillator quanta ($1\hbar\omega$, $2\hbar\omega$,...)) ; n_z, the number of nodes in the wave function along the z axis ; and Λ the projection of the orbital angular momentum l_i on the symmetry axis, l_{iz}. The reflection symmetry in nuclei means that the components $+\Omega$ and - Ω will have the same energy, giving the levels a degeneracy of two, as compared as (2j+1) for single particle states in spherical nuclei. Ω is the only good quantum number (apart from the parity) and the others (N, n_z and Λ) are not good quantum numbers for low deformations. These three quantum numbers become good quantum numbers at large deformations and therefore are called asymptotic quantum numbers [5].

For the spherical shape, the levels are grouped according to the principal quantum number N (with the splitting by the spin-orbit force determined through the total angular momentum j), but the behavior with deformation, depends on, how much of the excitation is in the z direction.

For prolate deformation, the potential becomes small in z direction, and the energy contributed by n_z excitations decreases. The cylindrical quantum numbers are thus helpful in understanding the splitting for small deformations. For very large deformations the influence of the spin-orbit and l^2 terms becomes less important and one may classify the levels according to the cylindrical quantum numbers.

4.7 Liquid Drop Model

One of the most important observed properties of the nucleus is the saturation property - which is manifested by the constancy of binding fraction (BE/A) of medium and heavy nuclei, the constancy of nuclear density (as evident from the formula for nucear radius, $r_0 A^{1/3}$, where r_0 is a constant), etc. The saturation of nuclear forces is very similar to the saturation of intermolecular forces in a liquid. Hence a formula for the total energy of a nucleus can be developed in which the most important terms are obtained from the analogy with a liquid drop (called the liquid drop model) , and finer detail, due to quantum nature of the nucleons, from the gas model. From the liquid drop model, we see that the total energy of the nucleus (besides the total mass energy of the protons and neutrons) will have a term proportional to the total number of nucleons ($\sim$ A), which is the heat of condensation of the liquid drop. In addition, there will be a surface tension term proportional to the surface area ($\sim A^{2/3}$) and an electrostatic repulsion term (due to the positively charged drop) proportional to $Z^2 A^{-1/3}$.

Two other terms are contributed by the gas model to this semi empirical mass formula. 3The gas model predicts the nucleus with N = Z to be most stable. So there is a contribu- tion to the energy due to neutron or proton excess and is proportional to $(A/2 - Z)^2/A$. Finally, according to the gas model, each energy level can accommodate two nucleons of a given kind. This means that the energy of the nucleus does not increase smoothly with the number of nucleons of a given kind. This contributes a step-like increase $\delta(A, Z)$.

$$\delta(A, Z) = \begin{cases} +f(A);\ A\, even,\ Z\, odd \\ 0;\ A\, odd \\ -f(A);\ A\, even,\ Z\, even \end{cases} \tag{4.50}$$

Thus we have the semi-empirical mass formula

$$M(A, Z) = M_p Z + M_n (A - Z) - a_1 A + a_2 A^{2/3} + a_3 \frac{Z^2}{A^{1/3}} + a_4 \frac{(A/2 - Z)^2}{A} + \delta(A, Z) \tag{4.51}$$

M_p = proton mass, M_n = neutron mass. a_1 =0.0169177, a_2 =0.019120, a_3 = 0.0007628, a_4 =0.10178, and f(A) is given as

$$f(A) = 0.010A^{-1/2} \tag{4.52}$$

The numerical values of the constants, as well as the form of f(A) are obtained from fitting experimental masses. The liquid drop model is the precursor of the collective model and is useful in the understanding of nuclear fission.

4.8 The Fermi Gas Model

Follow : Class notes

Chapitre 5

Nuclei far away from the stability valley

5.1 Exotic nuclei

In recent years revolutionary change in the scenario of theoretical and experimental research in nuclear physics has been observed with the discovery of exotic nuclei. The physics related to these exotic nuclei have been attracting increasing interest in recent years. A journey to this new branch of nuclear physics may reveal facts which are of particular interest to both theoretical and experimental nuclear physicists.

5.1.1 Definition of exotic nuclei

Exotic nuclei are those nuclei, which are not of the ordinary, commonly occurring type. Such nuclei are relatively rare. Exotic nuclei can be broadly divided into two classes :

1. The first class, usually called "halo nuclei", consists of nuclei near the neutron drip line or proton drip line with large neutron or proton excess respectively. Such nuclei are very unstable and short-lived. These were experimentally observed only in the mid-eighties after the development of radioactive ion beam facilities. So far, mainly neutron drip line nuclei have been observed and scant evidences exist for proton drip line nuclei. The first and by far the most well studied neutron drip line nucleus is ^{11}Li [1]. Other such nuclei observed so far are ^{6}He,

^{8}He, ^{11}Be, ^{12}Be, ^{14}Be, ^{17}B, etc. In such nuclei, one or two of the outermost neutrons are particularly loosely bound (with single or two neutron separation energies typically less than 1 MeV) and the density extends far out spatially, producing a low-density tail. Thus such nuclei appear to have a low-density neutron halo around a more stable core, and hence are called "neutron halo" nuclei. Existence of "proton halo" has only been inferred for the first excited state of ^{17}F.
Revolutionary changes in the picture of nuclear physics have taken place with the discovery of the drip line nuclei.

2. The second class consists of nuclei with one or more nucleon replaced by one or more hyperon. These are called "hypernuclei" and they were first experimentally discovered in 1952. Some of the experimentally known single-hyperon hypernuclei are $^{5}_{\Lambda}$He, $^{9}_{\Lambda}$Be, $^{10}_{\Lambda}$B, $^{12}_{\Lambda}$C, $^{8}_{\Xi}$He, $^{11}_{\Xi}$B, $^{4}_{\Sigma}$He, etc. Examples of double-hyperon hypernuclei are $^{6}_{\Lambda\Lambda}$He, $^{10}_{\Lambda\Lambda}$Be, $^{13}_{\Lambda\Lambda}$B etc. These nuclei are of special interest, since they are the only source of information about hyperon-nucleon and hyperon-hyperon interactions.

5.1.2 Drip line nuclei

In a plot of Z versus N, the broad line obtained for the stable nuclei is called the nuclear stability line. Limiting lines on either side of the broad region enclosing all existing stable nuclei is known as "drip lines". Along the proton drip line which is situated above the nuclear stability line, the single proton separation energy (S_p) is zero and along the neutron drip line, situated below the nuclear stability line, the single neutron separation energy (S_n) is also zero. According to conventional nuclear physics, no bound nucleus can survive beyond these drip lines. High-energy radioactive ion beam facility has made it possible to reach very short lived nuclei near the drip lines. Experimentally observed drip line nuclei are mostly near the neutron drip line. Strong evidence for the existence of proton drip line nuclei is not yet available. The most striking fact in the neutron drip line region has been the discovery of "halo" nuclei. There is experimental evidence that for nuclei in the vicinity of the neutron drip line, an extended tail with quite a low nucleon density could be formed around a relatively stable core by the quantum mechanical tunnelling of the last neutron(s). This is called a 'neutron halo'. Thus a halo nucleus may simply be pictured as a stable and relatively heavy core plus one or two very weakly bound extra nucleons, the latter moving around the core with a large r.m.s. radius. The density distribution of the valence nucleon around the core appears like the well-known halo around the head of saints. Halo nuclei are characterized by unusually large r.m.s. radius (compared to the famous $r_0 A^{1/3}$ formula) and very low one or two neutron separation energy. The halo is basically a threshold phenomenon resulting from the presence of a bound state very near to the continuum. The existence and main features of neutron halo are well studied for two cases, namely ^{11}Li and ^{6}He. In recent years several other nuclei like ^{14}Be, ^{19}C and ^{17}B have also been found to have neutron halo. On the other hand existence of proton halo is hindered by the additional Coulomb repulsion. The only known candidate for the single proton halo is the first excited state of ^{17}F, while other likely candidates are the nuclei ^{6}B and ^{17}Ne.

5.1.3 Hypernuclei

Hypernuclei are bound systems of neutrons, protons and one or more strange baryons, such as the Λ or Σ hyperons. Understanding the behavior of hypernuclei (how they are produced, their structure, spectroscopy and decay mechanisms) has been the subject of intense investigations during the last four decades. As defined earlier a hypernucleus must have at least one hyperon as its constituent. A lot of data have been accumulated for the nucleon-nucleon (NN) interaction. However, a better understanding of the interbaryon interaction can only be achieved by considering hyperons as well as nucleon. The hyperon-nucleon (YN) and hyperon-hyperon (YY) scattering data are very scarce. Although the available hyperon-nucleon scattering data are scarce, some attempt has been made by Nijmegen, Jülich and Tübingen groups to determine realistic hyperon-nucleon and also some pieces of the hyperon-hyperon interactions. It is nearly sixty years since the discovery of first hypernucleus in 1952. During the first two decades after the discovery, nuclear emulsion experiments provided a unique source of information on hypernuclei. These emulsion experiments were limited to measurements of binding energies of Λ particle in mostly light hypernuclei and some weak decay rate (lifetime) measurements. At present it is still not possible to pin down the realistic potentials, which can be used in *ab initio* hypernuclear structure calculations. In view of this situation, hypernuclear structure study will be aided by modeling effective ΛN and $\Lambda\Lambda$ potentials. One may determine the potentials phenomenologically so as to reproduce the binding energies of light Λ- and $\Lambda\Lambda$-hypernuclei before theoretically sound, realistic potentials become available.

5.1.4 Importance of studying exotic nuclei

The study of halo nuclei is of particular interest since they may yield important information about the nucleon-nucleon and nucleon-nucleus effective interactions under unusual circumstances viz., in the low density nuclear medium. From the theoretical viewpoint also, the study of halo nuclei is of high interest. **The empirical** mass formula is, so far, based on a fit to the observed properties of stable nuclei. We need information about halo nuclei to generalize the mass formula to regions near the drip line. Double-Λ hypernuclei, nuclear systems containing two Λ hyperons, have been of much interest since the seventies. As the only observed example of a multiply strange system, they give us unique opportunity to study $\Lambda\Lambda$ interaction in nuclear medium and test existing models of the baryon-baryon interaction. Hypernuclei are the *principal* source at present for studying the interaction between hyperons and nucleons. This together with nucleon-nucleon interaction may in turn provide insight into the fundamental quark-quark interaction.

5.1.5 Aim of the present study

The purpose of the present study is threefold.

1. We may determine phenomenologically the effective nucleon-nucleon and nucleon-nucleus interactions under unusual circumstances (i.e. in the low density nuclear medium) in the case of halo nuclei.
2. We may determine phenomenologically an effective ΛN as well as $\Lambda\Lambda$ potentials that reproduces the binding energies of experimentally observed single- and double-Λ hypernuclei.
3. We may explore the possibility whether hitherto unobserved single- and double-Λ hypernuclei with A_c=12,16,20,.. etc (A_c = mass number of the core) form bound state or not.
4. We may test the possibility of the existence of the excited states of these nuclei.
5. We may investigate the dependence of the effective ΛN interaction in a nuclear medium on the mass number.

5.1.6 Extremely neutron-rich nuclei

5.2 Super heavy nuclei

SN Ghosal PP-781 Limit of stability $Z^2/A \leq 50$. This limit is expected to be reached at Z=117 under classical considerations. Under shell structure considerations proton shell closure may occur at Z=114 and 126 while the neutron shell closure may occur at N=184. Theoretical considerations show that the nucleus with Z=114 and N=184 (A=298) may have a fairly long life. The mass number of super heavy nuclear isotopes may lie in the region $A \simeq 290 - 300$. However theoretical considerations also point towards relatively long lived SHN with $A \simeq 472$. Some attempts are being made to synthesize the Super Heavy Elements (SHE) through bombardment of heavy element targets with intense beams of heavy ions accelerated in special heavy ion accelerators.

Chapitre 6

Nuclear reactor theory

6.1 Introduction

Nuclear reactor is a devise in which self-sustained nuclear fission chain reaction can proceed in a controlled manner. An important factor which determines the characteristics of self-sustained chain reaction is the *multiplication factor* defined as $k = N/n$, where n is the number of neutrons absorbed in the fuel for production of nuclear fission and N denotes the number of fission generated neutrons that produce fission in the next generation. It is to be noted that all the fission generated neutrons do not produce fission in the subsequent stage. For uninterupted smooth operation of nuclear reactors at a constant rate, k must be unity. .

6.2 Fundamentals of nuclear fission

Nuclear fission is a special type of nuclear reaction in which an excited compund nucleus breaks up generally into fragments of comparable mass numbers and atomic numbers. Fission usually occurs amongst the isotopes of heaviest elements like isotopes of uranium, thorium etc.

When a heavy atom undergoes nuclear fission it breaks into two or more fission fragments. Also, several free neutrons, gamma rays, and neutrinos are emitted, and a large amount of energy is released. The sum of the rest masses of the fission fragments and ejected neutrons is less than the sum of the rest masses of the original atom and incident neutron (of course the fission fragments are not at rest). The mass difference is accounted for in the release of energy according to the equation $E = \Delta mc^2$:

$$\text{mass of released energy} = E/c^2 = m_{original} - m_{final}$$

Due to the extremely large value of the speed of light, c, a small decrease in mass is associated with a tremendous release of active energy (for example, the kinetic energy of the fission fragments). This energy (in the form of radiation and heat) carries the missing mass, when it leaves the reaction system (total mass, like total energy, is always conserved). While typical chemical reactions release energies on the order of a few eVs (e.g. the binding energy of the electron to hydrogen is 13.6 eV), nuclear fission reactions typically release energies on the order of hundreds of millions of eVs.

Two typical fission reactions are shown below with average values of energy released and number of neutrons ejected :

$$^{235}U + \textbf{neutron} \rightarrow \textbf{fission fragments} + 2.4neutrons + 192.9MeV$$

$$^{239}Pu + \textbf{neutron} \rightarrow \textbf{fission fragments} + 2.9neutrons + 198.5MeV$$

Note that these equations are for fissions caused by slow-moving (thermal) neutrons. The average energy released and number of neutrons ejected is a function of the incident neutron speed. Also, note that these equations exclude energy from neutrinos since these subatomic particles are extremely non-reactive and, therefore, rarely deposit their energy in the system.

6.2.1 Fission chain reaction

In a typical nuclear fission reaction a number of fresh neutrons are emited in addition to the release of a large quantity energy. These newly generated neutrons can be utilized to produce further fission in the fuel and the process can repeat itself producing even larger number of neutrons in the subsequent stages. Thus the fission reactions, under favourable condition, can be made to proceed in chain without any external interference as a self-sustaining process.

The chain reaction requires both the release of neutrons from fissile isotopes undergoing nuclear fission and the subsequent absorption of some of these neutrons in fissile isotopes. When an atom undergoes nuclear fission, a few neutrons (the exact number depends on several factors) are ejected from the reaction. These free neutrons will then interact with the surrounding medium, and if more fissile fuel is present, some may be absorbed and cause more fissions. Thus, the cycle repeats to give a reaction that is self-sustaining.

Nuclear power plants operate by precisely controlling the rate at which nuclear reactions occur, and that control is maintained through the use of several redundant layers of safety measures. Moreover, the materials in a nuclear reactor core and the uranium enrichment level make a nuclear explosion impossible, even if all safety measures failed. On the other hand, nuclear weapons are specifically engineered to produce a reaction that is so fast and intense it cannot be controlled after it has started. When properly designed, this uncontrolled reaction can lead to an explosive energy release. Nuclear fission fuel

6.3 Fission fuels

Nuclear weapons employ high quality, highly enriched fuel exceeding the critical size and geometry (critical mass) necessary in order to obtain an explosive chain reaction. The fuel for energy purposes, such as in a nuclear fission reactor, is very different, usually consisting of a low-enriched oxide material (e.g. UO2).

6.4 Neutron chain reaction

A nuclear chain reaction occurs when one single nuclear reaction causes an average of one or more subsequent nuclear reactions, thus leading to the possibility of a self-propagating series of these reactions. The specific nuclear reaction may be the fission of heavy isotopes (e.g., 235U). The nuclear chain reaction releases several million times more energy per reaction than any chemical reaction. The concept of a nuclear chain reaction was reportedly first hypothesized by Hungarian scientist Leó Szilárd on September 12, 1933 [1]. The neutron had been discovered in 1932, shortly before. Szilárd realized that if a nuclear reaction produced neutrons, which then caused further nuclear reactions, the process might be self-perpetuating. Szilárd, however, did not propose fission as the mechanism for his chain reaction, since the fission

1. Jogalekar, Ashutosh. "Leo Sziálrd, a traffic light and a slice of nuclear history". *Scientific American*. Retrieved 4 January 2016.

reaction was not yet discovered or even suspected. Instead, Szilárd proposed using mixtures of lighter known isotopes which produced neutrons in copious amounts. He filed a patent for his idea of a simple nuclear reactor the following year [2]

In 1936, Szilárd attempted to create a chain reaction using beryllium and indium, but was unsuccessful. After nuclear fission was discovered and proved by Otto Hahn and Fritz Strassmann in December 1938 [3] Szilárd and Enrico Fermi in 1939 searched for, and discovered, neutron multiplication in uranium, proving that a nuclear chain reaction by this mechanism was indeed possible [4]. This discovery prompted the letter from Szilárd[not in citation given] and signed by Albert Einstein to President Franklin D. Roosevelt warning of the possibility that Nazi Germany might be attempting to build an atomic bomb.[6][7]

On December 2, 1942, a team led by Enrico Fermi produced the first artificial self-sustaining nuclear chain reaction with the Chicago Pile-1 (CP-1) experimental reactor in a racquets court below the bleachers of Stagg Field at the University of Chicago. Fermi's experiments at the University of Chicago were part of Arthur H. Compton's Metallurgical Laboratory of the Manhattan Project ; the lab was later renamed Argonne National Laboratory, and tasked with conducting research in harnessing fission for nuclear energy.[8]

In 1956, Paul Kuroda of the University of Arkansas postulated that a natural fission reactor may have once existed. Since nuclear chain reactions only require natural materials (such as water and uranium), it is possible to have these chain reactions occur where there is the right combination of materials within the Earth's crust. Kuroda's prediction was verified with the discovery of evidence of natural self-sustaining nuclear chain reactions in the past at Oklo in Gabon, Africa in September 1972.[9]

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2. L. Szilárd, "Improvements in or relating to the transmutation of chemical elements," British patent number : GB630726 (filed : 28 June 1934 ; published : 30 March 1936).
3. Lise Meitner : Otto Hahn - the discoverer of nuclear fission. In : Forscher und Wissenschaftler im heutigen Europa. Stalling Verlag, Oldenburg/Hamburg 1955.
4. H. L. Anderson, E. Fermi, and Leo Szilárd, "Neutron production and absorption in uranium," The Physical Review, vol. 56, pages 284–286 (1 August 1939)

6.5 Multiplication factor

In connection with the nuclear chain reaction, the neutron multiplication factor, k, is defined as :

$$k = \frac{\textbf{number of neutrons in one generation}}{\textbf{number of neutrons in preceding generation}}$$

If k is greater than 1, the chain reaction is supercritical, and the neutron population will grow exponentially. If k is less than 1, the chain reaction is subcritical, and the neutron population will exponentially decay. If k = 1, the chain reaction is critical and the neutron population will remain constant.

In an infinite medium, neutrons cannot leak out of the system and the multiplication factor becomes the infinite multiplication factor, k = k_∞, which is approximated by the four-factor formula.

6.6 Condition for criticality

6.7 Breeding phenomena

6.8 Different types of reactors

6.9 Fusion

6.10 Nuclear fusion in stars

$$1u = \frac{1}{12} \times \frac{12 \times 10^{-3}}{N_0} = 1.660566 \times 10^{-27} kg = 931.502 MeV \tag{6.1}$$

6.11 Heavy ion collision

Nuclear reactions induced by Heavy ions (HI) (projectiles with A>4 or heavier than α-particles) needs specialized accelerators which can energise heavy ions up to ^{238}U with energy 1-2 MeV per nucleon. HI beams in the relativistic limit (2-5GeV) are also available. Experimental techniques have been developed to study HI reactions which are of very short half-lives $\simeq 10^{-15}s$ and fpor direct reaction 10^{-21} s. Identification of reaction products are made using E and ΔE counters where $E = mv^2/2$ (non-relativistic kinetic energy) since $dE/dx \propto Z^2M$. So identification is possible by simultaneous measurement of E and ΔE. Examples : Z^2M of two neighbouring isotopes ^{18}O and ^{19}F are 1152 and 1539 respectively which differes by 33.6%. For heavy isotopes the resolving power of the method is less. Z^2M for ^{42}Ca and ^{45}sC $\simeq$ 18% and that for ^{109}Ag and ^{113}Cd is 8%. Improvement in RP can be achieved by using two ΔE detectors in the telescope and measuring time of flight between them accurately, Combination of E-ΔE telescope with magnetic spectrometer is used to improve the method of identification.

6.11.1 Main features of HI induced reactions

i) Large mom. transfer ii) large ang. mom. transfer iii) exchange of large number of nucleons iv) spl. type of e.m. interaction such as multiple Coulomb excitations, high spin ionization, short range etc. Due to these spl features it is possible to study the properties of nuclei under unusual conditions which are not met with other kinds of reactions. i) nuclear matter with unusually high density (super dense) ii) rotationg at extremely high speed due to large ang. mom. iii) with extrremely short radiactive half-lives (due to being highly proton rich and hence away from the stability line) can be considered in these expts.
In addition the superheavy (in the trans uranic region) and super charged (nuclear molecules) nuclei can also be studied.
The schematic of the HI collision with diff impact parameter i'b' is sghown below.
For Coulomb scattering

$$E < \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 (R_{ion} + R_{nucl})}$$

where $r = R_{ion} + R_{nucl}$ and KE of projectile=Coulomb energy of interaction gives impact parameter. For b$\simeq R$ scattering due to nuclear force occur ii) for b<R compound nucleus formation occur and for bCoulomb scattering occur. The grazing ang. mom.

$$l_{gr} = (R/\hbar)[2M(E - \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 R})]^{1/2} = 0$$

for $E = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 R}$

Chapitre 7

Neutron Physics

Neutron was discovered by J Chadwick following the reaction

$$ {}^{9}_{4}Be +^{4}_{2} He \rightarrow^{13}_{6} C^{*} \rightarrow^{12}_{6} C +^{1}_{0} n $$

The mass of neutron was estimated following the reaction

$$ {}^{11}_{5}B +^{4}_{2} He \rightarrow^{15}_{7} N^{*} \rightarrow^{14}_{7} N +^{1}_{0} n $$

applying conservation of momentum and energy principles and assuming the collission an elastic one. The recoil velocity of the target in the L-system is

$$ v_2 = \frac{2m_1 u_1}{m_1 + m_2} cos\theta_2 $$

If the measurement is carried in the forward direction (i.e., $\theta_2 = 0^o$), then

$$ v_2 = \frac{2m_1 u_1}{m_1 + m_2} $$

For the protons ejected in the forward direction it was found that

$$ v_2 = v_p = \frac{2m_n u_n}{m_n + m_p} = \frac{2m_n u_n}{m_n + 1} = 3.3 \times 10^7 $$

While that for recoiled nitrogen in the forward direction it was found that

$$ v_2 = v_N = \frac{2m_n u_n}{m_n + m_N} = \frac{2m_n u_n}{m_n + 14} = 4.7 \times 10^6 $$

Thus

$$\frac{v_p}{v_N} = \frac{m_n + 14}{m_n + 1} = \frac{33}{4.7} \simeq 7$$

Solving which one could get

$$m_n = \frac{7}{6}u = 1.17u$$

For this great work Chadwick was awarded Nobel Prize in 1935. Mass of neutron was also estimated by Chadwick in collaboration with Goldhaber M using the disintegration of deuteron by γ-rays of known energy

$${}_1^2H + \gamma \rightarrow_1^1 H +_0^1 n$$

They used 2.62MeV γ-rays from thoriated carbon ThC^* which is the natural radioactive source of the most energetic γ-rays. The momentum of γ-rays is

$$p_\gamma = \frac{h\nu}{c} = 1.4 \times 10^{-21} kg.ms^{-1}$$

Conservation of energy, gives

$$M(^2H) + E_\gamma = M(^1H) + m_n + E_p + E_n$$

Kinetic energy of the proton emitted in the disintegration reaction as measured was $E_p = 0.225MeV$ which imparts proton the momentum

$$p_p = \sqrt{2m_pE_p} = \sqrt{(2)(1.66 \times 10^{-27})(0.225)(1.6 \times 10^{-13})} kg.ms^{-1}$$

$$= 1.09 \times 10^{20} kg.ms^{-1}$$

Which implies $p_\gamma << p_p$. Now momentum conservation gives

$$\vec{p_\gamma} = \vec{p_p} + \vec{p_n}$$

A very small value of the momentum of γ photon as compared to that for proton indicates that the neutron must have momentum nearly equal and opposite to that of proton. Hence, to a first approximation we can consider $p_p = p_n$ and $E_n = E_p = 0.225MeV = 0.0002415u$ and also $E_\gamma = 2.62MeV = 0.002813u$ Putting these in the energy equation we get mass of neutron in energy unit as

$$m_n = M(^2H) + E_\gamma - M(^1H) + -E_p - E_n$$

$$= (2.014102 - 1.007825 + 0.002813 - 0.0002415 \times 2)u$$
$$= 1.008607u$$

More precise value of neutron mass was obtained by R. E. Bell and L. G. Eliot (1950) by measuring energy of the γ-rays (=2.230MeV) emitted during capture of neutrons by protons,. They obtained $m_n = 1.0086613u$. The current accepted value of neutron mass is

$$m_n = 1.008685u$$

7.0.1 Classification of neutrons based on energies

1. Slow neutrons (0<E<$10^3 eV$) further classified as :
   i) Cold neutrons, 0<E $\leq$ 0.002eV having high penetrability through crystalline and polycrystalline materials
   ii) Thermal neutrons (E=kT) having low absorption cross-section in many materials and achieve thermal equilibrium within it with atoms and molecules and follow Maxwells velocity distribution formula. These neutrons find use in producing nuclear fission reactions which can further produce fast neutrons to initiate fission chain reactions.
   iii) Epithermal neutrons ($\geq 0.05eV$) : High energy neutrons on entering material medium achieve thermal equilibrium with its atoms and molecules at energies higher than that permitted by Maxwells distribution law.
   iv) Resonance neutrons (1<E<100eV) : These neutrons produces sharp resonance peaks in heavy nuclei in this energy range of neutrons.
2. Intermediate energy neutrons ($10^3 eV$<E<$10^5 eV$) : These neutrons mostly suffer elastic collision and are produced by slowing down of the fast neutrons by collisions with moderators.
3. Fast neutrons (10^5<E<10MeV) : These neutrons are widely used to produce various types of reactions as they undergoes in-elastic scattering which helps to investigate the excited states of target nuclei.
4. Very fast neutrons ($10MeV$<E<$50MeV$) : These neutrons are widely used to produce reactions of the type where two or more particles are emitted. Examples of such reactions are (n,2n), (n, pn) etc.
5. Ultra fast neutrons (E>50MeV) : These neutrons are widely used to produce spallation reactions in which the target nuclei are broken up

into large number of fragments by the impacts of ufn. Thes ufn's are produced in (p,n) reactions by very high energy protons accelerated in the ultra high energy accelerators. These neutrons are also avalable in cosmic rays.

7.0.2 Sources of neutrons

Nrutrons are produced in nuclear reactions which occur spontaneously or induced by high energy charged particles or intense radiations (X-rays or γ-rays). However accelerator based sources yield more intense neutron beams than those using natural radio-elements. Reactions which have larger reaction cross-sections are generally utilized fro neutron beam production. Sometimes special arrangements of materials are required for producing copious neutron flux in nuclear reactors.

Some of the neutron sources includes a) Radioactive (α, n) sources : The reaction ^{9}Be(α, n)^{12}C with Q value of 5.65 Mev and in which neutrons were discovered uses α from ^{210}Po (Z=84) producing α of energy 5.30MeV. The emitted neutrons have energy spread between 10.8MeV in the forward direction to 6.7 Mev in the backward direction. The lower limit of neutron energy could be 1Mev if ^{12}C is produced in its excited state. Other α sourrces include radium ^{226}Ra(1620y), plutonium ^{239}Pu(24400y) which have longer half lives compared to 140d of polonium ^{210}Po and can be utilized without any loss in neutron-intensity for years together. Radon ^{222}Rn having very short half-life (3.8d) has been abondonned due to technical reasons. Apart from beryllium ^{9}Be, other targets used in (α, n) reactions are ^{10}B, ^{11}B and ^{19}F.

Sources (Ref : S N Ghosal pp-623)	Yield (10^6 neutrons/Ci.s)
Ra-α-Be	17
Rn-α-Be	15
Ra-α-B	6.8
Po-α-Be	3
Ra-α-BeF_4	2.53

b) Photoneutron (γ, n) sources : These sources uses (γ, n) endoergic reaction on different targets like deuteron ^{2}H and beryylium ^{9}Be having reaction thresold energies 2.226MeV and 1.66MeV respectively using both natural and artificial sources of γ-rays. Other isotopes (like

^{7}Li) used as targets have threshold energies higher than 6MeV and for those high energy prompt γ-rays from various reactions and X-rays available from high energy electron accelerators are used.

c) Accelerated charged particle sources : These sources are preferred for the production of intense beams of mono-energetic neutrons using projectiles like protons, deuterons, tritons, α-particles and some heavier nuclei accelerated in a Cockroft-Walton generator, a Van de Graaff generator or a cyclotron.

Reaction (Ghosal pp-626)	Q-value	Yield
^{3}H(p,n)^{3}He	-0.764	
^{7}Li(p,n)^{7}Be	-1.644	11×10^6 per microCoulomb
^{2}H(d,n)^{3}He	+3.269	
^{3}H(d,n)^{4}He	+17.6	$\sim 10^8$ per microCoulomb
^{7}Li(d,n)^{8}Be	+15.03	
^{9}Be(d,n)^{10}B	+4.36	
Note	Q<0 :endoergic	Q>0 :exoergic reaction.

d) Ultrafast neutron sources : These are produced in different types of reactions induced by very high energy charged particles accelerated in ultra high energy accelerators. These neutrons are produced by evaporation process from the compound nucleus and have energy distribution characteristic of statistical theory. In case of deuteron projectiles, neutrons are produced mainly by stripping process having energy spread and angular spread.

e) Slow neutron sources : Slow neutrons are derived from intermediate or higher energy neutrons by allowing them to diffuse through low mass number medium called moderator. In such medium neutrons lose most of their energies by elastic collisions with moderator atoms until they reach thermal equilibrium in the medium. The velocity distribution approximately follows maxwells distribution law and have most probable speed $v_m = \sqrt{2kT/M}$ where k is the boltzmann constant and T absolute temperature. At room temperature (i.e., 300K) $v_m = 2.2 \times 10^3 kms^{-1}$

Chapitre 8

Nuclear Reactions

8.1 Introduction

When two nuclei collide there are 2 types of reactions :

1. Nuclei can coalesce to form highly excited Compound nucleus (CN) that lives for relatively long time. Long lifetime sufficient for excitation energy to be shared by all nucleons. If sufficient energy localised on one or more nucleons (usually neutrons) they can escape and CN decays. Independence hypothesis : CN lives long enough that it loses its memory of how it was formed. So probability of various decay modes independent of entrance channel.
2. Nuclei make 'glancing' contact and separate immediately, said to undergo Direct reactions(DI).

Projectile may lose some energy, or have one or more nucleons transferred to or from it. So, there are two types reactions (1) direct reactions and (2) compound nuclear reactions.

8.2 Direct reactions

1. Elastic scattering : A(a ; a)A zero Q-value. Internal states unchanged.
2. Inelastic scattering : $A(a;a')A^*$ or $A(a;a^*)A^*$. Projectile a gives up some of its energy to excite target nucleus A. If nucleus a also complex nucleus, it can also be excited. [If energy resolution in detection of a not small enough to resolve g.s. of target from low-lying excited states

then cross section will be sum of elastic and inelastic components. This is called quasi-elastic scattering].

3. Breakup reactions : Usually referring to breakup of projectile a into two or more fragments. This may be elastic breakup or inelastic breakup depending on whether target remains in ground state.
4. Transfer reactions : Stripping and Pickup reaction.
5. Charge exchange reactions : mass numbers remain the same and they can be elastic or inelastic.

8.3 Compound nuclear reactions

1. Fusion : Nuclei stick together
2. Fusion-evaporation : fusion followed by particle-evaporation and/or gamma emission
3. Fusion-fission : fusion followed by fission

8.4 Classification by outcome

Reactions can further be classified in terms of the outcomes as :

8.4.1 Elastic scattering

projectile and target stay in their g.s.

8.4.2 Inelastic scattering

projectile or target left in excited state

8.4.3 Transfer reaction

1 or more nucleons moved to the other nucleus

8.4.4 Fragmentation/Breakup/Knockout :

3 or more nuclei/nucleons in the final state

8.4.5 Charge Exchange :

A is constant but Z (charge) varies, e.g. by pion exchange

8.4.6 Multistep Processes :

intermediate steps can be any of the above ('virtual' rather than 'real')

8.4.7 Deep inelastic collisions :

Highly excited states produced

8.4.8 Fusion :

Nuclei stick together

8.4.9 Fusion-evaporation :

fusion followed by particle-evaporation and/or gamma emission

8.4.10 Fusion-fission :

fusion followed by fission The first 6 processes are Direct Reactions (DI) The last 3 processes give a Compound Nucleus (CN).

8.5 Comparission of Direct and Compound nuclear reactions

8.5.1 Locations :

1. CN reactions at small impact parameter,DI reactions at surface & large impact parameter.
2. CN reaction involves the whole nucleus while DI reaction usually occurs on the surface of the nucleus. This leads to diffraction effects : (i)Durations : A typical nucleon orbits within a nucleus with a period of 10^{-22} sec. [as K.E.$\sim$ 20 MeV]. If reaction complete within this time scale or less then no time for distribution of projectile energy around

target : DI reaction occurred. On the other hand CN reactions require » 10^{-22} sec.
(ii)Angular distributions : In DI reactions differential cross section strongly forward peaked as projectile continues to move in general forward direction. Diffrential cross sections for CN reactions do not vary much with angle (not complete isotropy as still slight dependence on direction of incident beam).

8.6 Some important terminologies :

Reaction channels : In nuclear reaction, each possible combination of nuclei is called a partition. Each partition further distinguished by state of excitation of each nucleus and each such pair of states is known as a reaction channel. The initial partition, a + A (both in their ground states) is known as the incident, or entrance channel. The various possible outcomes are the possible exit channels. In a particular reaction, if not enough energy for a particular exit channel then it is said to be closed.

8.7 Compound nucleus hypothesis

The compound nuclear hypothesis, propounded by Niels Bohr, is based on his observations on nuclear scattering and reaction experiments. According to the hypothesis when a nuclear projectile x enters a nuclear target X to initiate a nuclear reaction, an intermediate nucleus (C^*) is formed before the production of the final product nuclei y and Y or the emission of γ-rays. This intermediate nucleus having relatively longer lifetime ($\sim 10^{-15}s$) is called the compound nucleus and the process can be represented by

$$X + x \to C^* \to Y + y \; or \; X(x, y)Y$$

The incoming projectile x on entering the target nucleus X quickly dissipates its energy and merges with the closely packed nucleons thereby disturbing the random motion of the nucleons inside the nucleus. In the process none of the single nucleon is able to aquire sufficient energy from the projectile to get emitted from the nucleus. However after a long time when a very large number of collisions($\sim 10^7$) among the nucleons have taken place, one of the nucleons may accumulate suffiicient energy to escape from the nucleus thereby leading the residual nucleus to de-excite (cool off) to the ground state. This phenomenon can be compared to the evaporation mechanism of a heated liquid drop containing large number of molecules. The mean time interval between two successive collisions is about ($R/v = 2\times10^{-15}/5\times10^{7}$) $10^{-22}s$. So the lifetime of the compound nucleus is of the order of $10^7 \times 10^{-22}s \sim 10^{-15}s$. Compound nucleus being relatively longer lived, nuclear reaction proceeds in two stages : 1) formation of the CN by the absorption of the projectile (x) by the target (X), and 2) disintegration of CN in the reaction products y and Y, in a manner independent of its formation history (known as Bohr's independence hypothesis). Sometimes the residual nucleus (Y) is left in a highly excited state which "boils off" another particle y' leading to a two particle emission process X(x,yy')Y' and the process may continue further with the emission of y" and leaving excited Y". The two stages of reaction processes can be represented symbolically as

$$\begin{aligned} X + x \to C^* &\to Y + y \\ &\to C + \gamma \end{aligned}$$

The probability decay (measured in terms of the level width, Γ) being reciprocal of the mean-life τ of the compound nucleus, we have from uncertaity relation

$$\Gamma \sim \hbar/\tau$$

Now if Γ_y represent the partial level width for decay with the emission of y, then considering various particle emissions in the raction we may get the total width of levels as

$$\Gamma = \sum_y \Gamma_y + \Gamma_\gamma = (\Gamma_y + \gamma_{y'} + \Gamma_{y''} + ...) + \Gamma_\gamma \tag{8.1}$$

and the relative probabilities of different types of decay as

$$\eta_y = \frac{\Gamma_y}{\Gamma}, \eta_{y'} = \frac{\Gamma_{y'}}{\Gamma}, \eta_{y''} = \frac{\Gamma_{y''}}{\Gamma}, ..., \eta_\gamma = \frac{\Gamma_\gamma}{\Gamma} \tag{8.2}$$

Because of the independence hypothesis of CN decay, we may write the cross-section for the reaction process X(x,y)Y as a product of the cross-section σ_x for the formation of the compound nucleus and the probability of its decay

$$\sigma(x,y) = \sigma_x \eta_y = \sigma_x \frac{\Gamma_y}{\Gamma} \tag{8.3}$$

The above relation is written considering only one particular energy state of C^* or only one resonance. This is possible if the energy levels are well separated and are so sharp that they do not interfere themselves.

8.7.1 Derivation of Breit-Wigners one-level formula

The state of compund nucleus can be represented by a damped harmonic wave

$$\psi(t) = \psi_0 e^{-iE_r t/\hbar} e^{-\Gamma t/2\hbar} \tag{8.4}$$
$$= \psi_0 e^{-i(E_r - i\Gamma/2)t/hbar} \tag{8.5}$$

where $\Gamma/2$ is the half-width of the decaying level of life-time $\tau = \hbar/\Gamma$. Although the above function does not represenmt a stationary state, it may be assumed to be built up by the superposition of stationar states of different energies by Fourier integral method :

$$\psi(t) = \int_{-\infty}^{\infty} A_E e^{-iEt/\hbar} dE \tag{8.6}$$

By FT we have

$$\begin{aligned} A_E &= \frac{1}{2\pi}\int_0^{\infty} \psi(t')e^{iEt'/\hbar}dt' \\ &= \frac{1}{2\pi}\int_0^{\infty} \psi_0 e^{i(E-E_r+i\Gamma/2)t'/\hbar}dt' \\ &= \frac{\psi_0}{2\pi}\left[\frac{e^{i(E-E_r+i\Gamma/2)t'/\hbar}}{i(E-E_r+i\Gamma/2)t'/\hbar}\right]_0^{\infty} \end{aligned}$$

As the upper limit term vanishes due to damping term $e^{-\Gamma t/2\hbar}$, we get

$$A_E = \frac{\psi_0}{2\pi}\frac{i\hbar}{E-E_r+i\Gamma/2} \tag{8.7}$$
$$\mid A_E \mid^2 = \frac{\mid \psi_0 \mid^2}{4\pi^2}\frac{\hbar^2}{(E-E_r)^2+\Gamma^2/4} \tag{8.8}$$

But, the cross-section for the formation of compound nuclear state E_C in the process X+x is proportional to the squared amplitude. Hence, we may write

$$\sigma_x = \frac{C}{(E-E_r)^2+\Gamma^2/4} \tag{8.9}$$

where C is a constant. Applying reciprocity theorem the proprtionality constant C is found to be

$$C = \frac{\pi}{k^2}\Gamma_x\Gamma \tag{8.10}$$

Using eqs(9), (10) in (3) we get the cross-section for the reaction X(x,y)Y

$$\sigma(x,y) = \frac{\pi}{k^2}\frac{\Gamma_x\Gamma_y}{(E-E_r)^2+\Gamma^2/4} \tag{8.11}$$

The above eq(11) is known as Breit-Wigner one level formula for spinless nuclei at very low energies for which the relative angular momentum (l) of the particles in the entrance channel is zero i.e., l=0.

8.8 Differential & total Scattering cross-section

The number of particles scattered into unit solid angle per unit time, per unit incident flux, per target point, is called differential scattering cross-section. It is defined as

$$\frac{d\sigma}{d\Omega} = \mid f(\theta,\phi) \mid^2$$

If the incident plane wave e^{ikz} is so normalized that there is only one particle per unit volume, then

$$n_{inc} = \mid \psi_{inc} \mid^2 = \mid e^{ikz} \mid^2 = 1$$

So, the incident flux which is the number of particles incident per unit area is

$$v \mid \psi_{inc} \mid^2 = v,$$

v being the velocity of the incident particles. The probability of scattering by a single nuclear target into the solid angle $d\Omega$ is

$$\frac{d\sigma}{d\Omega} v \mid \psi_{inc} \mid^2 d\Omega = \frac{d\sigma}{d\Omega} v d\Omega$$

Now, the number of scattered particles per unit volume in direction (θ,ϕ) is $\mid \psi_{sc} \mid^2 = \mid f(\theta,\phi) \mid^2 /r^2$, so the scattered flux will be

$$v \mid \psi_{sc} \mid^2 = v \mid f(\theta,\phi) \mid^2 /r^2$$

Thus, the number of particles scattered into the soplid angle $d\Omega$ and passing through an elemental area dS at a distance r from the scattering centre is

$$(scattered\, flux) \times dS = v \mid f(\theta,\phi) \mid^2 /r^2 . r^2 d\Omega = v \mid f(\theta,\phi) \mid^2 d\Omega.$$

Comparing the preceeding relations we have the differential scattering cross-section

$$\frac{d\sigma}{d\Omega} = \mid f(\theta, \phi) \mid^2$$

[Note : In QM, flux means the probability current density given by

$$J = \frac{i\hbar}{2m}[\psi\nabla\psi^* - \psi^*\nabla\psi]$$

For incident wave,

$$\psi_{inc} = e^{ik.r}$$

,

$$J_{inc} = \frac{i\hbar}{2m}[e^{ik.r}\psi\nabla_r e^{-ik.r} - e^{-ik.r}\nabla_r e^{ik.r}] = \frac{\hbar k}{m}]$$

The total scattering cross-section is defined as

$$\sigma_s = \int_\Omega \frac{d\sigma}{d\Omega} d\Omega = \int_\Omega \mid f(\theta, \phi) \mid^2 sin\theta d\theta d\phi$$

8.8.1 Derivation of scattering cross-section (σ_{sc}) formula

A beam of mono-energetic incident particles proceeding along Z-direction can be represented by a plane wave

$$\psi_{inc} = e^{ikz} = e^{ikrcos\theta} = \sum_{l=0}^{\infty} i^l(2l+1)j_l(kr)P_l(cos\theta) \tag{8.12}$$

which in the assymptotic limit (ie. r$\rightarrow \infty$) can be expanded as

$$\begin{aligned}
\psi_{inc} &= \sum_{l=0}^{\infty} i^l(2l+1)\frac{sin(kr - l\pi/2)}{kr}P_l(cos\theta) \\
&= \frac{1}{2ikr}\sum_{l=0}^{\infty} i^l(2l+1)[e^{i(kr-l\pi/2)} - e^{-i(kr-l\pi/2)}]P_l(cos\theta) \qquad (8.13)
\end{aligned}$$

Thus the incident plane wave can be represented as superposition of a set of outgoing spherical wave e^{ikr}/r and a set of incoming spherical wave e^{-ikr}/r of equal amplitude. In the presence of the scattering nucleus, only the outgoing spherical wave will be affected in amplitude as well as in phase, and the total wavefunction can be expressed as

$$\psi(r, \theta) = \frac{1}{2ikr}\sum_{l=0}^{\infty} i^l(2l+1)[\eta_l e^{i(kr-l\pi/2)} - e^{-i(kr-l\pi/2)}]P_l(cos\theta) \tag{8.14}$$

Again, since the total wavefunction ψ in the presence of scatterer can also be represented as

$$\psi = \psi_{inc} + f(\theta, \phi)e^{ikr}/r \tag{8.15}$$

we have,

$$\begin{aligned}
f(\theta, \phi) &= re^{-ikr}[\psi - \psi_{inc}] \\
&= (re^{-ikr})\frac{1}{2ikr}\sum_{l=0}^{\infty} i^l(2l+1) \\
&\times [\eta_l e^{i(kr-l\pi/2)} - e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)} + e^{-i(kr-l\pi/2)}]P_l(cos\theta) \\
&= \frac{1}{2ik}\sum_{l=0}^{\infty} i^l(2l+1)[(\eta_l - 1)e^{-il\pi/2)}]P_l(cos\theta) \\
or,\ f(\theta, \phi) &= \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)[(\eta_l - 1)]P_l(cos\theta) \qquad (8.16)
\end{aligned}$$

Therefore, the differential elastic scattering cross-section for the l^{th} partial wave,

$$\frac{d\sigma_{sc}^l}{d\Omega} = \mid f_l(\theta, \phi) \mid^2 = \frac{(2l+1)^2}{4k^2} \mid \eta_l - 1 \mid^2 \{P_l(cos\theta)\}^2 \tag{8.17}$$

And the total elastic scattering cross-section,

$$\begin{aligned}
\sigma_{sc}^l &= \int_\Omega \frac{d\sigma^l}{d\Omega} d\Omega \\
&= \frac{1}{4k^2}(2l+1)^2 \mid \eta_l - 1 \mid^2 \int_\Omega \{P_l(cos\theta)\}^2 sin\theta d\theta d\phi \\
&= \frac{1}{4k^2}(2l+1)^2 \mid \eta_l - 1 \mid^2 \{2\pi.2/(2l+1)\} \\
&\quad [using\ orthogonality\ of\ Legendre\ polynomial] \\
or,\ \sigma_{sc}^l &= \frac{\pi}{k^2}(2l+1) \mid 1 - \eta_l \mid^2 \qquad (8.18)
\end{aligned}$$

Since, $\mid \eta_l \mid \leq 1$, we have

$$\sigma_{sc}^l \leq \frac{\pi}{k^2}(2l+1)$$

So, when there is no reaction, we can write

$$\eta_l = e^{2i\delta_l}$$

where δ_l is a real phase factor. Hence,

$$\begin{aligned} \sigma_{sc}^l &= \frac{\pi}{k^2}(2l+1) \mid 1-\eta_l \mid^2 \\ &= \frac{\pi}{k^2}(2l+1) \mid 1-e^{2i\delta_l} \mid^2 \\ &= \frac{\pi}{k^2}(2l+1).2(1-cos2\delta_l) \\ or,\ \sigma_{sc}^l &= \frac{4\pi}{k^2}(2l+1)sin^2\delta_l \end{aligned} \tag{8.19}$$

8.8.2 Derivation of Reaction cross-section (σ_{re}) formula

The total wavefunction in the presence of scatterer i.e. the target nucleu can be expressed as the sum of the sperical outgoing and speherical incoming waves of different angular momenta l, so,

$$\psi(\vec{r}) = \psi(r,\theta) = \sum_l \psi_{in}^l + \sum_l \psi_{out}^l \tag{8.20}$$

where

$$\psi_{in}^l = \frac{1}{2ikr}(2l+1)i^l P_l(cos\theta)e^{-i(kr-l\pi/2)} \tag{8.21}$$

$$\psi_{out}^l = \frac{1}{2ikr}(2l+1)i^l P_l(cos\theta)\eta_l e^{i(kr-l\pi/2)} \tag{8.22}$$

The radial part of the above waves are respectively

$$u_{in}^l = \frac{1}{2ikr}(2l+1)i^l e^{-i(kr-l\pi/2)} \tag{8.23}$$

$$u_{out}^l = \frac{1}{2ikr}(2l+1)i^l \eta_l e^{i(kr-l\pi/2)} \tag{8.24}$$

As the probability current density is defined as

$$J = \frac{i\hbar}{2m}[\psi\nabla\psi^* - \psi^*\nabla\psi] \tag{8.25}$$

We have

$$J_{in}^l = \frac{i\hbar}{2m}[u_{in}^l\frac{\partial u_{in}^{l*}}{\partial r} - u_{in}^{l*}\frac{\partial u_{in}^l}{\partial r}] = -\frac{\hbar(2l+1)^2}{4mkr^2} \tag{8.26}$$

$$J_{out}^l = \frac{i\hbar}{2m}[u_{out}^l\frac{\partial u_{out}^{l*}}{\partial r} - u_{out}^{l*}\frac{\partial u_{out}^l}{\partial r}] = -\frac{\hbar(2l+1)^2}{4mkr^2} \mid \eta_l \mid^2 \tag{8.27}$$

Hence, the net outgoing current density for the l^{th} partial wave will be

$$J^l = J^l_{in} - J^l_{out} = -\frac{\hbar(2l+1)^2}{4mkr^2}(1- \mid \eta_l \mid^2) \tag{8.28}$$

Now, if N be the flux of the incident particles then the reaction cross-section for the l^{th} partial wave will be

$$\sigma^l_{re} = N_{re}/N = N_a/N$$

where $N_{re} = N_a$ is the total number of particles absorbed in going through a spherical surface of radius r :

$$N_a = -\int\int J^l r^2 d\Omega \{P_l(cos\theta)\}^2$$

the minus sign is introduced to make the net number of particles positive. We have

$$\begin{aligned} N_a &= \frac{\hbar(2l+1)^2}{4mk}(1- \mid \eta_l \mid^2)\int\int\{P_l(cos\theta)\}^2 \frac{r^2 sin\theta d\theta d\phi}{r^2} \\ &= \frac{\pi\hbar(2l+1)}{mk}(1- \mid \eta_l \mid^2) \end{aligned} \tag{8.29}$$

The incident flux is

$$N = v \mid \psi_{inc} \mid^2 = v = p/m = \hbar k/m \tag{8.30}$$

Therefore from (43) & (44),

$$\sigma^l_{re} = N_a/N = \frac{\pi(2l+1)}{k^2}(1- \mid \eta_l \mid^2) \tag{8.31}$$

8.8.3 Total partial cross-section & its limiting values

The total scattering and reaction cross-section is

$$\begin{aligned} \sigma^l_t &= \sigma^l_{sc} + \sigma^l_{re} \\ &= \frac{\pi(2l+1)}{k^2} \mid 1 - \eta_l \mid^2 + \frac{\pi(2l+1)}{k^2}(1- \mid \eta_l \mid^2) \end{aligned} \tag{8.32}$$

i) For $\eta_l = 1$, both $\sigma^l_{sc} = 0$ and $\sigma^l_{re} = 0$, so $\sigma^l_t = 0$.
ii)For $\eta_l = 0$, both $\sigma^l_{sc} = \frac{\pi(2l+1)}{k^2}$ and $\sigma^l_{re} = \frac{\pi(2l+1)}{k^2}$, so $\sigma^l_t = \frac{2\pi(2l+1)}{k^2}$.
iii) For $\eta_l = -1$, $\sigma^l_{sc} = \frac{4\pi(2l+1)}{k^2}$ and $\sigma^l_{re} = 0$, so $\sigma^l_t = \sigma^l_{sc}(max) = \frac{4\pi(2l+1)}{k^2}$
Thus, we may conclude that there can be elastic scattering without reaction, but reaction cannot take place without scattering. For reaction to be possible $-1 < \eta_l < 1$.

8.8.4 Total cross-section & Optical theorem

The total cross-section for all the partial components is given by

$$\begin{aligned}
\sigma_t &= \sum_l (\sigma^l_{sc} + \sigma^l_{re}) = \frac{\pi}{k^2}\sum_l (2l+1)\{\mid 1-\eta_l \mid^2 + (1- \mid \eta_l \mid^2)\} \\
&= \frac{2\pi}{k^2}\sum_l (2l+1)(1-Re\eta_l) \qquad (8.33) \\
Again,\ f(\theta) &= \frac{1}{2ik}\sum_l (2l+1)P_l(cos\theta)(\eta_l - 1) \\
&= \frac{1}{2ik}\sum_l (2l+1)P_l(cos\theta)(Re\eta_l + iIm\eta_l - 1) \\
&= \frac{1}{2k}\sum_l (2l+1)P_l(cos\theta)\{Im\eta_l + i(1-Re\eta_l)\} \\
or,\ Imf(\theta) &= \frac{1}{2k}\sum_l (2l+1)P_l(cos\theta)(1-Re\eta_l) \\
or,\ Imf(0) &= \frac{1}{2k}\sum_l (2l+1)(1-Re\eta_l) \qquad (8.34) \\
&= \frac{1}{2k}\frac{k^2}{2\pi}\sigma_t \ \ [using\, eq(22)] \\
or,\ \sigma_t &= \frac{4\pi}{k} Imf(0) \qquad (8.35)
\end{aligned}$$

The relation (24) is known as the optical theorem. It gives a direct relationship between the total cross-section and the forward scattering amplitude, as a general consequence of the wave theory for destructive interference between incident and scattered wave.

8.9 Scattering matrix

We know that the scattering wave function is given by

$$f(\theta) = \sum_l (2l+1)T_l P_l(cos\theta) \qquad (8.36)$$

where

$$T_l = e^{i\delta_l} sin\delta_l = \frac{1}{2i}(S_l - 1) \qquad (8.37)$$

In the above relations T_l is called the partial wave T matrix and S_l is called the partial wave S matrix. Expressiing $f(\theta)$ in the integral form we have

$$f(\theta) \;=\; -\frac{m}{2\pi\hbar^2}\int d\vec{r}e^{-i\vec{k}'.\vec{r}}V(r)\psi_{\vec{k}}^{(+)}(\vec{r}) \tag{8.38}$$

Which in Dirac (bra-ket) notation can be put as

$$\begin{aligned} f(\theta) \;&=\; -\frac{m}{2\pi\hbar^2} < \vec{k}' \mid V \mid \psi_{\vec{k}}^{(+)} > && (8.39)\\ &=\; -\frac{m}{2\pi\hbar^2}T(\vec{k}',\vec{k}) && (8.40)\end{aligned}$$

$\vec{k} \rightarrow$ incident momentum, $\vec{k}' \rightarrow$ scattered momentum, $\psi_{\vec{k}}^{(+)} \rightarrow$ outgoing wave and $T(\vec{k}',\vec{k})$ is called the transition matrix element.

8.10 Overlapping levels :

8.10.1 Statistical model & Ghosal's experiment

Existence of resonances in the reaction cross-section versus beam energy plot is one of the features of compound nucleus model when the level width (Γ) is much larger than the separation (D) between two levels (i.e., $\Gamma >> D$). Further these resonances appear at very low incident energies (usually for neutrons) which indicates that isolated levels exist at excitation energies close to the separation energy of the neutron (S_n) from the compound nucleus. The total energy E ($E_n + S_n$) brought in by the incident particle is distributed in very large number of ways amongst the nucleons in the compound nucleus, untill enough energy is concentrated on a particular nucleon or group of nucleons which will enable it to escape the nucleus. However, even if it is energetically favoured, the particle may not be actually able to escape the nucleus due the existence of sharp discontinuity in the potential at the surface of the nucleus, which allows only a part of the incident wave to transmit throgh the surface boundary. The average period of transmission of a particle through the barrier repeat itself many times with an averge time interval T$\sim \frac{\lambda}{2\pi D}$ where D is the mean level spacing of the compound nucleus. After many such periods, the particle finally escapes after a time $\tau = \frac{\lambda}{2\pi\Gamma}$ where Γ is the level width (which occurs due to uncertainty). Since $\tau >> T, \Gamma << D$, which is the condition for formation of compound nucleus. As the energy

of the incident particle increases, the trasmission coefficient increses which interms reduces τ, so the level width increasess. When γ becomes larger than D, overlapping of levels occur thereby by giving rise to continuum of levels. At continuum any level may be supposed to be a superpostion of different states of different energies and different phases. However when there is complete overlapping due to very large number of exit channelds being available, there cannot be any definite phase relationshipo between different levels superposed and the independence hypothesis regains its validity. This is the so called continuum region. In this region, the mode of formation of compound state is forgotten at the time of its decay, not because of the time scales involved, but beacuse any effect of formation will be averaged out over many oerlapping states. This is known as the statistical assumption.
The validity of the statistical assumtion in the continuum region was conmfirmed experimentally by S. N. Ghosal in 1950. In his experiment the same compound nucleus ${}^{64}_{30}Zn^*$ was formed in the same state of excitation by the following two process :

$$ {}^{60}_{28}Ni + \alpha \rightarrow {}^{64}_{30} Zn^* $$

$$ {}^{63}_{29}Cu + p \rightarrow {}^{64}_{30} Zn^* $$

In the first process α particles with energies raging from 10MeV to 40MeV were used to obtained the excitation function (absorption cross-section α-energy curve) using ^{60}Ni as target. In the second process protons with energies raging from 3 MeV to 33 MeV were directed to ^{63}Cu target for the same purpose. In both the process same compound nucleus $^{64}Zn^*$ were formed in the same t to the condition

$$ E_\alpha = E_p + 7MeV $$

The absoption of a proton of energy E_p by the target ^{63}Cu produces the compound nucleus $^{64}Zn^*$ in a state of excitation energy

$$ E_c = E_p + S_p $$

where S_p is the separation energy of the proton from the ground state of ^{64}Zn. Similarly the absoption of an α particle of energy E_α by the target ^{60}Ni produces the same compound nucleus $^{64}Zn^*$ in the same state of excitation energy

$$ E_c = E_\alpha + S_\alpha $$

where S_α is the separation energy of the α particle from the ground state of ^{64}Zn. Comparing the above relation one may find

$$E_\alpha = E_p + (S_p - S_\alpha)$$

The currently accepted values of $(S_p - S_\alpha)$ is 3.7MeV which is smaller than that obtained in Ghosal's experiment which is due to the uncertainties in the defintion of energy by stacked foil method used in Ghosal's experiment. In both the process involved in Ghosal's experiment the following disintegration channels were studied :

$$^{64}_{30}Zn^* \rightarrow^{63}_{30} Zn + n$$
$$\rightarrow^{62}_{30} Zn + 2n$$
$$\rightarrow^{62}_{29} Cu + p + n$$

The excitation functions of the above processes were determined as functions of the proton and α- energies for the two reaction processes. And the following relationshiop was satisfied for the cases with all proton energies between 3-33MeV and corresponding α-energies between 10-40 MeV according to the relation $E_\alpha = E_p + 7MeV$:

$$\sigma(\alpha, n) : \sigma(\alpha, 2n) : \sigma(\alpha, pn) = \sigma(p, n) : \sigma(p, 2n) : \sigma(p, pn)$$

which gives a direct confirmation of the independence hypothesis under statistical assumptions.

For a reaction of the type

$$X + x \rightarrow C^* \rightarrow Y + y$$

we can write

$$\sigma(x, y) = \sigma_x(E_x)\eta_y$$

where $\sigma_x(E_x)$ is the cross-section for the formation of the compound nucleus C^* in a particular state of excitation due to absorption of x by the target X for the energy E_x and η_y is the probability of decay of the compound nucleus C^* into Y+y.

Again, for the reaction

$$A + a \rightarrow C^* \rightarrow Y + y$$

in which the same compound compound nucleus C^* is formed in the same state of excitation due to absorption of a different particle 'a' of energy E_a

by a different target 'A' producing the same decay products Y+y, we can write the cross-section

$$\sigma(a, y) = \sigma_a(E_a)\eta_y$$

Thus we have,

$$\frac{\sigma(x, y)}{\sigma(a, y)} = \frac{\sigma_x(E_x)}{\sigma_a(E_a)}$$

If the decay of C^* takes place through another exit channel

$$C^* \rightarrow B + b$$

in which two other particles B and b are produced in the final state, we can write the corresponding cross-sections in the two cases as

$$\sigma(x, b) = \sigma_x(E_x)\eta_b$$

$$\sigma(a, b) = \sigma_a(E_a)\eta_b$$

Taking the ratios again we get,

$$\frac{\sigma(x, b)}{\sigma(a, b)} = \frac{\sigma_x(E_x)}{\sigma_a(E_a)}$$

Thus we have

$$\frac{\sigma(x, y)}{\sigma(a, y)} = \frac{\sigma(x, b)}{\sigma(a, b)}$$

The above equation is the general form of the result obtained in Ghosal's experiment.

Chapitre 9

Assigments

Coming shortly

9.1 References

1. **Nuclear Astrophysics- A Course of Lectures by Dr Md Abdul Khan, Levant Books ; & CRC Press, NY**
2. Nuclear Physics- S. N. Ghosal
3. Theory of Nuclear Structure- M A Preston
4. Concepts of Nuclear Physoics- E. Cohen
5. Nuclear Physics- Liley

Most important : Follow my class notes.