These include glass mirrors, aluminium sheet and aluminium foil. The main disadvantage with these cookers is that although they can cook all types of food items, they require continuous attention, as a result of which the operator has to be in the sun most of the time. Another disadvantage is that except for glass, the reflectivity of all other surfaces decreases with the passage of time.

2.3 SOME OBSERVATIONS

It will be useful to conclude this chapter with some observations. Our survey of various solar thermal applications shows that water heating systems for domestic as well as industrial purposes have already made a small and encouraging impact on the Indian energy scene. The same is true of the box-type solar cooker. In fact, India now has the distinction of having the largest number of solar cookers amongst all the countries of the world.

Other applications using heat directly like space heating, crop and timber drying, and distillation have not yet made the same impact. This is a little surprising because all of them have a definite, important role to play in certain regions or for specific purposes. For example, space heating by passive methods if integrated into the initial design of a building can be very beneficial in the northern parts of India. Similarly solar distillation has an important role to play in meeting the needs of thousands of small communities across the country where drinking water is scarce. A concerted effort has to be made to make these technologies more cost effective and reliable, and to disseminate information regarding their use.

As far as power generation is concerned, it has to be recognized that solar thermal energy can make a real impact if it leads to large scale cost-effective electrical power generation. Our survey of the world scene shows that although this is far from being the case, impressive developments have taken place in the last decade. Medium temperature systems using line focussing parabolic collector technology have been commercialized to some extent. Also, the central receiver concept has been tested extensively on a pilot scale and the solar chimney concept has been proved. Both systems seem promising, but need a considerable amount of developmental work before their suitability and viability can be assessed.

Three

Solar Radiation

This chapter is concerned with the availability of solar radiation as an energy source. Extra-terrestrial radiation, its spectral distribution and the radiation at the earth's surface is discussed first. Instruments used for measuring solar radiation and methods used for presenting data are then described. The geometry of solar radiation and the various angles used are defined so that it is possible to convert the flux on one plane to an equivalent flux on another. Equations for predicting the availability of solar radiation at a location are also given.

3.1 SOLAR RADIATION OUTSIDE THE EARTH'S ATMOSPHERE

The characteristics of the sun's energy available outside the earth's atmosphere are first considered.

The sun is a large sphere of very hot gases, the heat being generated by various kinds of fusion reactions. Its diameter is 1.39×10^6 km, while that of the earth is 1.27×10^4 km. The mean distance between the two is 1.496×10^8 km. Although the sun is large, it subtends an angle of only 32 minutes at the earth's surface. This is because it is also at a very large distance. Thus, the beam radiation received from the sun on the earth is almost parallel. The brightness of the sun varies from its centre to its edge. However, for engineering calculations, it is customary to assume that the brightness all over the solar disc is uniform.

Measurements indicate that the energy flux received from the sun outside the earth's atmosphere is essentially constant. The *solar constant* I_{sc} is the rate at which energy is received from the sun on a unit area perpendicular to the rays of the sun, at the mean distance of the earth from the sun. The value of the solar constant has been the subject of many experimental investigations. Based on measurements made up to 1970, a standard value of 1353 W/m^2 was adopted in 1971.* However, based on subsequent measurements, a revised values of 1367 W/m^2 has been recommended.† The difference between the two values is only 1 per cent.

The earth revolves around the sun in an elliptical orbit having a very small eccentricity, and with the sun at one of the foci. Consequently, the distance between the earth and the sun varies a little through the year. Because of this variation, the extra-terrestrial flux also varies. The value on any day can be calculated from the equation

$$I'_{sc} = I_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right) \tag{3.1}$$

where n is the day of the year.

It is also useful to know the spectral distribution of extra-terrestrial solar radiation. Measurements corresponding to both the values of the solar constant are given in Table 3.1. It will be seen that the measurements are in close agreement with each other. For both cases, the spectral value first increases sharply with wavelength, passes through a maximum at a wavelength of 0.48 μm and then decreases asymptotically to zero. 99 per cent of the sun's radiation is obtained up to a wavelength of 4 μm. The percentage of radiation obtained up to a certain wavelength is also given in Table 3.1. This value is useful for calculating the solar flux emitted in any specified range of wavelengths. For easy reference and visualization, the spectral distribution data corresponding to the solar constant value of 1353 W/m^2 is plotted in Fig. 3.1.

The radiation coming from the sun is essentially equivalent to blackbody radiation. Using the Stefan-Boltzmann law, the equivalent backbody temperature can be shown to be 5762 K for a solar constant of 1353 W/m^2 and 5779 K for a solar constant of 1367 W/m^2

*M.P. Thekaekara and A.J. Drummond, "Standard Values for the Solar Constant and its Spectral Components", *Nat. Phys. Sci.*, **229**, 6 (1971).

†C. Frolich and R.W. Brusca, "Solar Radiation and its Variation in Time", *Solar Physics*, **74**, 209 (1981).

Table 3.1 Spectral Distribution of Extra-terrestrial Solar Radiation at Mean Sun-earth Distance

Solar constant I_{sc} W/m^2	1353*		1367†	
λ μm	e_λ W/m^2-μm	D_λ	e_λ W/m^2-μm	D_λ
0.2	10.7	0.0081	—	—
0.3	514	1.21	528	1.21
0.38	1120	7.00	1210	6.46
0.4	1429	8.73	1702	8.03
0.48	2074	19.68	2055	18.85
0.5	1942	22.60	1863	21.68
0.6	1666	35.68	1720	35.26
0.7	1369	46.88	1428	46.84
0.78	1159	54.35	1196	54.47
0.8	1109	56.02	1145	56.16
0.9	891	63.37	913	63.59
1.0	748	69.49	744	69.45
2.0	103	93.49	119	94.10
3.0	31	97.83	25	98.07
4.0	9.5	99.06	8.5	99.16
5.0	3.8	99.51	3.5	99.57
6.0	1.8	99.72	1.8	99.75
7.0	1.0	99.82	0.95	99.84
8.0	0.59	99.88	0.55	99.89
10.0	0.25	99.94	0.20	99.95

$$D_\lambda = \left\{\left[\int_0^\lambda e_\lambda\, d\lambda\right] \Big/ \left[\int_0^\infty e_\lambda\, d\lambda\right]\right\} \times 100 = \left\{\left[\int_0^\lambda e_\lambda\, d\lambda\right] \Big/ I_{sc}\right\} \times 100$$

(*Sources*: *M.P. Thekaekara, "Solar Energy Outside the Earth's Atmosphere", *Solar Energy*, 14, 109 (1973). Adapted with permission.
†Data from the World Radiation Centre, Davos, Switzerland (1981).)

3.2 SOLAR RADIATION AT THE EARTH'S SURFACE

Solar radiation is received at the earth's surface in an attenuated form because it is subjected to the mechanisms of absorption and scattering

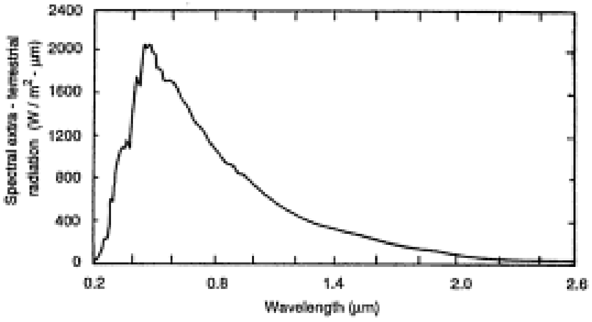

Fig. 3.1 Spectral Distribution of Extra-terrestrial Solar Radiation (Air Mass Zero)
[From National Aeronautics and Space Administration Report, NASA SP-8005 (1971).]

as it passes through the earth's atmosphere (Fig. 3.2). Absorption occurs primarily because of the presence of ozone and water vapour in the atmosphere, and to a lesser extent due to other gases (like CO_2, NO_2, CO, O_2 and CH_4) and particulate matter. It results in an increase in the internal energy of the atmosphere. On the other hand, scattering occurs due to all gaseous molecules as well as particulate matter in the atmosphere. The scattered radiation is redistributed in all directions, some going back into space and some reaching the earth's surface.

The atmosphere at any location on the earth's surface is often classified into two broad types—an atmosphere without clouds and an atmosphere with clouds. In the former case, the sky is cloudless everywhere, while in the latter, the sky is partly or fully covered by clouds. The mechanisms of absorption and scattering (described in the previous paragraph) are similar with both types of atmosphere. However it is obvious that less attenuation takes place in a cloudless sky. Consequently maximum radiation is received on the earth's surface under the conditions of a cloudless sky.

Solar radiation received at the earth's surface without change of direction, i.e. in line with the sun, is called *beam* or *direct radiation*. The radiation received at the earth's surface from all parts of the sky's hemisphere (after being subjected to scattering in the atmosphere) is called *diffuse radiation*. The sum of the beam and diffuse radiation is referred to as *total* or *global radiation*.

In general, the intensity of diffuse radiation coming from various

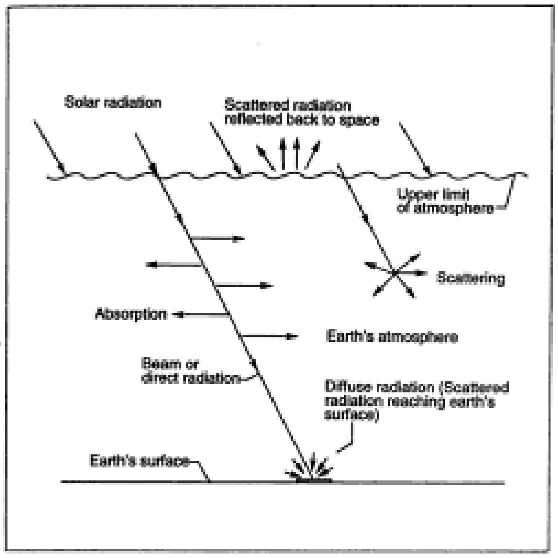

Fig. 3.2 Schematic Representation of (i) the Mechanisms of Absorption and Scattering, and (ii) Beam and Diffuse Radiation Received at the Earth's Surface

directions in the sky is not uniform. The diffuse radiation is therefore said to be anisotropic in nature. However in many situations (e.g. part or heavy cloud cover), the intensity from all directions tends to be reasonably uniform. It is then modelled as being perfectly uniform and is said to be isotropic in nature.

A term called the *air mass* (AM) is often used as a measure of the distance travelled by beam radiation through the atmosphere before it reaches a location on the earth's surface. It is defined as the ratio of the mass of the atmosphere through which the beam radiation passes to the mass it would pass through if the sun is directly overhead (i.e. at its zenith). The *zenith angle* θ_z is the angle made by the sun's rays with the normal to a horizontal surface. It can be shown approximately that for locations at sea level and zenith angles from 0 to 70°, the air mass is equal to the secant of the zenith angle. Thus air mass zero (AM0) corresponds to extra-terrestrial radiation, air mass one (AM1) corresponds to the case of the sun at its zenith, and air mass two (AM2) corresponds to the case of a zenith angle of 60°.

Extensive studies have been made on the mechanisms of absorption and scattering, and on the determination of attenuation coefficients for various substances. Nevertheless, it is in general not possible to predict, to reasonable degree of accuracy, the variation with time of the beam and diffuse radiation which might be expected at a specified location on the earth's surface. Thus the designer of solar process equipments has to resort to one of the following options

1. Make measurements over a period of time at the location in question where the solar equipment is to be installed.
2. Use measurements available for some other location where the climate is known to be reasonably similar to the location under consideration.
3. Use empirical predictive equations which link the values of solar radiation with other meteorological parameters whose values are known for the location under consideration. Examples of such parameters are the number of hours of sunshine received per day, the precipitation, the cloud cover, etc.

3.3 INSTRUMENTS FOR MEASURING SOLAR RADIATION AND SUNSHINE

The instruments used for measurement will now be described. Solar radiation flux is usually measured with the help of a *pyranometer* or a *pyrheliometer*.

A *pyranometer* is an instrument which measures either global or diffuse radiation over a hemispherical field of view (refer Photo No. 3). A sketch of one type of *pyranometer* as installed for the measurement of global radiation is shown in Fig. 3.3. Basically the pyranometer consists of a 'black' surface which heats up when exposed to solar radiation. Its temperature increases until the rate of heat gain by solar radiation equals the rate of heat loss by convection, conduction and reradiation. The hot junctions of a thermopile are attached to the black surface, while the cold junctions are located in such a way that they do not receive the radiation. As a result, an emf is generated. This emf which is usually in the range of 0 to 10 mV can be read, recorded or integrated over a period of time and is a measure of the global radiation.

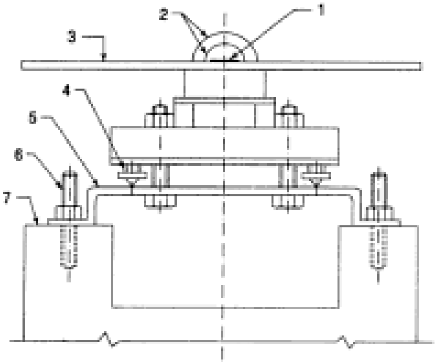

Fig. 3.3 Pyranometer for Measuring Global Radiation (1. Black Surface, 2. Glass Domes, 3. Guard Plate, 4. Levelling Screws, 5. Mounting Plate, 6. Grouted Bolts, 7. Platform)

The pyranometer shown in Fig. 3.3 is used commonly in India. It has its hot junctions arranged in the form of a circular disc of diameter 25 mm and is coated with a special black lacquer having a very high absorptivity in the solar wavelength region. Two concentric hemispheres, 30 and 50 mm in diameter respectively, made of optical glass having excellent transmission characteristics, are used to protect the disc surface from the weather. An accuracy of about ±2 per cent can be obtained with the instrument.

The pyranometer can also be used for the measurement of diffuse radiation. This is done by mounting it at the centre of a semicircular *shading ring*. The shading ring is fixed in such a way that its plane is parallel to the plane of the path of the sun's daily movement across the sky and it shades the thermopile element and the two glass domes of the pyranometer at all times from direct sunshine. Consequently, the pyranometer measures only the diffuse radiation received from the sky. The pyranometer is mounted at the centre of the shading ring.

The construction of one type of shading ring is shown in Fig. 3.4. *ABCD* is a horizontal rectangular frame 35 cm × 80 cm with its long sides in an east-west direction. To the sides *AB* and *CD* of the frame are pivoted two angle-iron arms *EF* and *GH*, 70 cm long with slots along their length, carrying sliders, *SS*, on which is mounted the semicircular shading ring *R*. The arms are pivoted about a horizontal

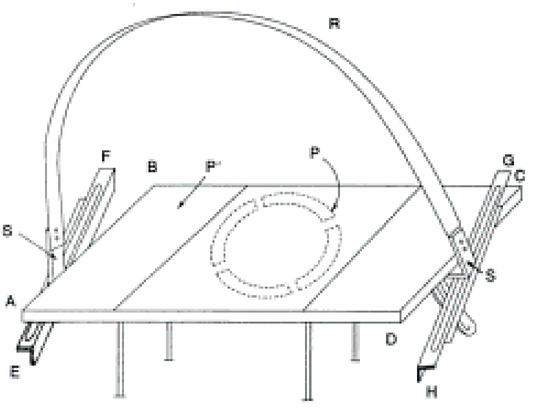

Fig. 3.4 Shading Ring Arrangement for the Measurement of Diffuse Radiation

axis which passes through the centre of the rectangular frame and can be adjusted at an angle to the horizontal equal to the latitude of the station. The movement of the ring up and down the arms allows for changes in the sun's declination. The shading ring is of aluminium, 50 mm broad, and is bent to a radius of 450 mm. The inner surface of the ring is painted dull black, while the rest of the shading ring arrangement is painted dull matt white. To the bottom of the frame *ABCD* is fixed a thick metal plate *P* with a circular slot so that the frame, when fixed on a masonry platform with nuts and bolts, can be adjusted in its proper position by rotation about a vertical axis. To the top of the frame is fitted another thick metal plate *P'* on which the pyranometer is mounted.

A *pyrheliometer* is an instrument which measures beam radiation. In contrast to a pyranometer, the black absorber plate (with the hot junctions of a thermopile attached to it) is located at the base of a tube (Fig. 3.5). The tube is aligned with the direction of the sun's rays with the help of a two-axis tracking mechanism and an alignment indicator. Thus the black plate receives only beam radiation and a small amount of diffuse radiation falling within the 'acceptance angle' of the instrument.

The duration of bright sunshine in a day is measured by means of

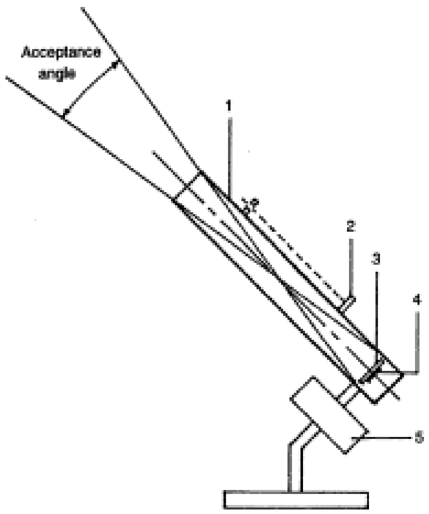

Fig. 3.5 Pyrheliometer for Measuring Beam Radiation (1. Tube Blackened on Inside Surface, 2. Alignment Indicator, 3. Black Absorber Plate, 4. Thermopile Junctions, 5. Two-axis Tracking Mechanism)

a *sunshine recorder* shown in Fig. 3.6 (refer Photo No. 4). The sun's rays are focussed by a glass sphere to a point on a card strip held in a groove in a spherical bowl mounted concentrically with the sphere. Whenever there is bright sunshine, the image formed is intense enough to burn a spot on the card strip. Through the day as the sun moves across the sky, the image moves along the strip. Thus, a burnt trace whose length is proportional to the duration of sunshine is obtained on the strip.

3.4 SOLAR RADIATION DATA

Most radiation data is measured for horizontal surfaces. A typical daily record of the global and diffuse radiation measured on a clear day is

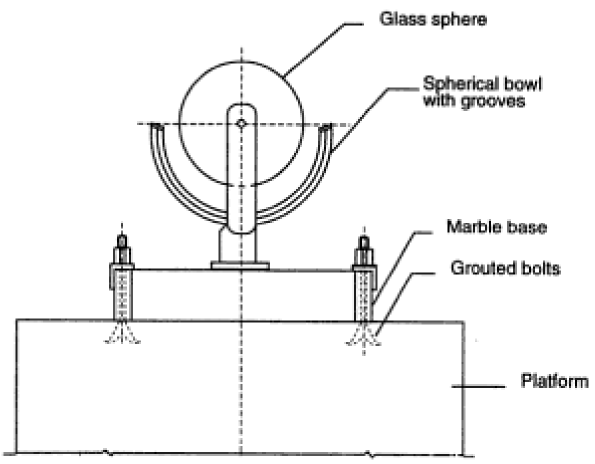

Fig. 3.6 Sunshine Recorder

shown in Fig. 3.7. It is seen that a fairly, smooth variation with the maximum occurring around noon is obtained on a clear day. In contrast, an irregular variation with many peaks and valleys may be obtained on cloudy day.

Solar radiation flux is often reported in langleys per hour or per day (1 langley = 1 cal/cm^2). The unit 'langley' has been adopted in honour of Samuel Langley who made the first measurement of the spectral distribution of the sun.

A solar designer is primarily interested in average values for a location. The averaging is usually done over a month, and tabulations showing the hourly variation of global and diffuse radiation, the amount received per day and the sunshine hours per day are prepared. Detailed tabulations for many locations in India have been published*. Data for a few locations is given in Appendix 3.

A general idea of the availability of solar radiation over different regions can be obtained by constructing solar radiation maps. Such maps have been drawn by Mani and Chacko† for India and are

*A. Mani, *Handbook of Solar Radiation Data for India*, Allied Publishers, New Delhi (1980).

†A. Mani and O. Chacko, "Solar Radiation Climate of India", *Solar Energy*, **14**, 139 (1973).

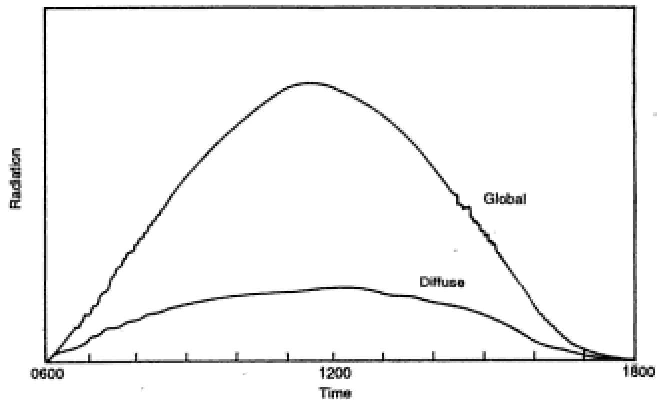

Fig. 3.7 Record of Global and Diffuse Radiation Measured on a Clear Day

reproduced in Fig. 3.8 and Fig. 3.9. Figure 3.8 shows the distribution of the average daily global radiation, while Fig. 3.9 shows the distribution of the average daily diffuse radiation.

From Fig. 3.8, it is seen that the annual average daily global radiation received over the whole country is around 450 langleys. Peak values are generally measured in April or May, with parts of Rajasthan and Gujarat receiving over 600 langleys. In contrast, during the monsoon and winter months, the daily global radiation decreases to about 300–400 langleys.

From Fig. 3.9, it is observed that the annual average daily diffuse radiation received over the whole country is around 175 langleys. The maximum values measured over the whole country are about 300 langleys in Gujarat in July, while minimum values, between 75 and 100 langleys, are measured over many parts of the country during November and December as winter sets in.

3.5 SOLAR RADIATION GEOMETRY

In order to find the beam energy falling on a surface having any orientation, it is necessary to convert the value of the beam flux coming from the direction of the sun to an equivalent value corresponding to

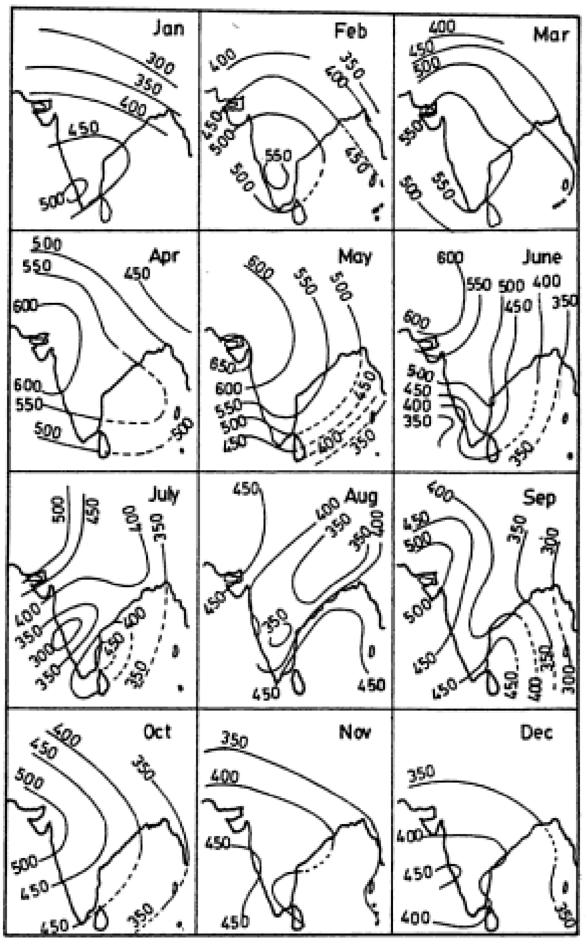

Fig. 3.8 Average Daily Global Radiation Over India in cal/cm²-day
(From Mani and Chacko.)

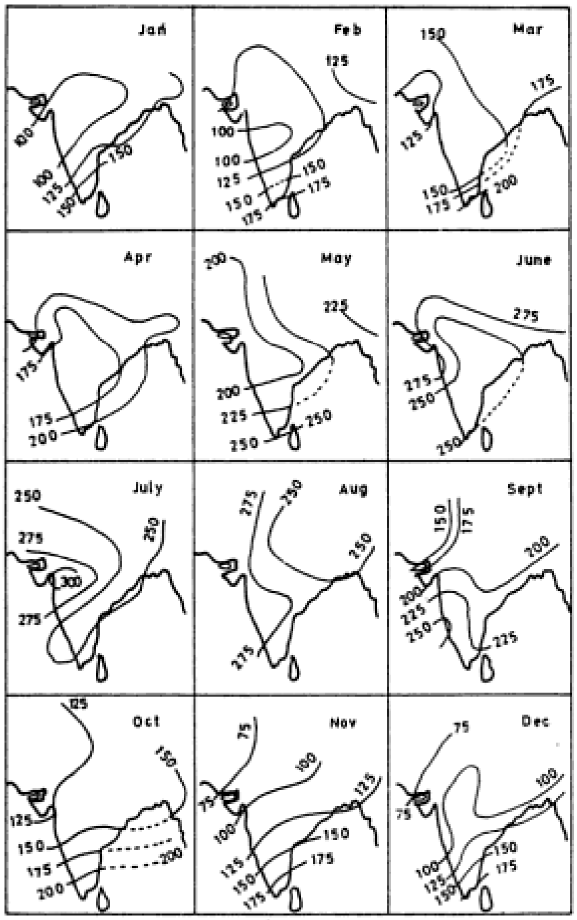

Fig. 3.9 Average Daily Diffuse Radiation Over India in cal/cm²-day
(From Mani and Chacko.)

the normal direction to the surface. Relationships for making this conversion will now be given.

If θ is the angle between an incident beam of flux I_{bn} and the normal to a plane surface, then the equivalent flux falling normal to the surface is given by $I_{bn}\cos\theta$. The angle θ can be related by a general equation to ϕ the latitude, δ the declination, γ the surface azimuth angle, ω the hour angle, and β the slope. Each of these will first be defined.

The *latitude* ϕ of a location is the angle made by the radial line joining the location to the centre of the earth with the projection of the line on the equatorial plane. By convention, the latitude is measured as positive for the northern hemisphere. It can vary from -90° + 90°.

The *declination* δ is the angle made by the line joining the centres of the sun and the earth with its projection on the equatorial plane. It arises by virtue of the fact that the earth rotates about an axis which makes an angle of approximately 66.5° with the plane of its rotation around the sun. The declination angle varies from a maximum value of +23.45° on June 21 to a minimum value of −23.45° on December 21. It is zero on the two equinox days of March 21 and September 22. Cooper* has given the following simple relation for calculating the declination

$$\delta \text{ (in degrees)} = 23.45 \sin\left[\frac{360}{365}(284+n)\right] \tag{3.2}$$

where n is the day of the year. Equation (3.2) is plotted in Fig. 3.10. Its accuracy of prediction is adequate for engineering purposes.

The *surface azimuth angle* γ is the angle made in the horizontal plane between the line due south and the projection of the normal to the surface on the horizontal plane. It can vary from −180° to +180°. We adopt the convention that the angle is positive if the normal is east of south and negative if west of south.

The *hour angle* ω is an angular measure of time and is equivalent to 15° per hour. It also varies from −180° to +180°. We adopt the convention of measuring it from noon based on local apparent time (LAT), being positive in the morning and negative in the afternoon. (The term 'local apparent time' will be defined later.)

The *slope* β is the angle made by the plane surface with the horizontal. It can vary from 0 to 180°.

It can be shown that

$$\begin{aligned}\cos\theta &= \sin\phi\,(\sin\delta\cos\beta + \cos\delta\cos\gamma\cos\omega\sin\beta)\\ &\quad + \cos\phi\,(\cos\delta\cos\omega\cos\beta - \sin\delta\cos\gamma\sin\beta)\\ &\quad + \cos\delta\sin\gamma\sin\omega\sin\beta\end{aligned} \tag{3.3}$$

*P.I. Cooper, "The Absorption of Solar Radiation in Solar Stills", *Solar Energy*, **12**, 3 (1969).

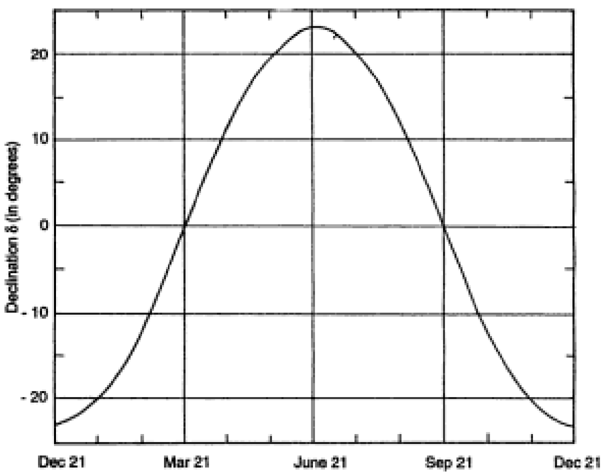

Fig. 3.10 Variation of Declination Over the Year—Eq. (3.2)

Simpler versions of Eq. (3.3) are normally required. Some of these are as follows

Vertical surface $\beta = 90°$,

$$\cos\theta = \sin\phi\cos\delta\cos\gamma\cos\omega - \cos\phi\sin\delta\cos\gamma + \cos\delta\sin\gamma\sin\omega \tag{3.4}$$

Horizontal surface $\beta = 0°$,

$$\cos\theta = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega \tag{3.5}$$

The angle θ in this case is the *zenith angle* θ_z (defined earlier in Sec. 3.2). The complement of the zenith angle is also used quite often in calculations. It is called the *solar altitude angle* and will be denoted by the symbol α_a. Some of the above angles are shown in Fig. 3.11.

Surface facing due south $\gamma = 0°$,

$$\begin{aligned}\cos\theta &= \sin\phi\,(\sin\delta\cos\beta + \cos\delta\cos\omega\sin\beta)\\ &\quad + \cos\phi\,(\cos\delta\cos\omega\cos\beta - \sin\delta\sin\beta)\\ &= \sin\delta\sin(\phi-\beta) + \cos\delta\cos\omega\cos(\phi-\beta)\end{aligned} \tag{3.6}$$

Vertical surface facing due south $\beta = 90°$, $\gamma = 0°$,

$$\cos\theta = \sin\phi\cos\delta\cos\omega - \cos\phi\sin\delta. \tag{3.7}$$

The angle of incidence θ can also be expressed in terms of θ_z the

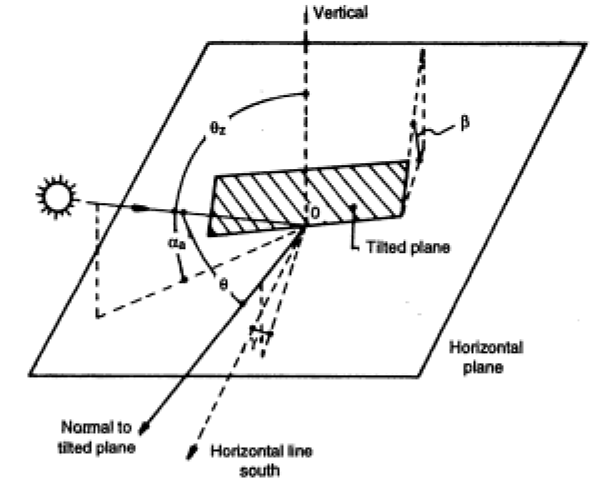

Fig. 3.11 Diagram Illustrating the Angle of Incidence θ, the Zenith Angle θ_z, the Solar Altitude Angle α_a, the Slope β and the Surface Azimuth Angle γ

zenith angle, β the slope, γ the surface azimuth angle and γ_s the solar azimuth angle. Braun and Mitchell* have shown that

$$\cos\theta = \cos\theta_z \cos\beta + \sin\theta_z \sin\beta \cos(\gamma_s - \gamma) \tag{3.8}$$

The *solar azimuth angle* γ_s is the angle made in the horizontal plane between the line due south and the projection of the line of sight of the sun on the horizontal plane. Thus it gives the direction of the shadow cast in the horizontal plane by a vertical rod. By convention, the solar azimuth angle is taken to be positive if the projection of the line of sight is east of south and negative if west of south. In order to use Eq. (3.8), it is necessary to first calculate θ_z and γ_s. θ_z is obtained from Eq. (3.5), while γ_s is obtained from the expression†

$$\cos\gamma_s = (\cos\theta_z \sin\phi - \sin\delta)/\sin\theta_z \cos\phi \tag{3.9}$$

*J.E. Braun and J.C. Mitchell, "Solar Geometry for Fixed and Tracking Surfaces", *Solar Energy*, 31, 439 (1983).

†M. Iqbal, *An Introduction to Solar Radiation*, Academic Press, Canada (1983).

Example 3.1

Calculate the angle made by beam radiation with the normal to a flat-plate collector on December 1 at 0900 h (local apparent time). The collector is located in New Delhi (28°35′ N, 77°12′ E), and is tilted at an angle of 36° with the horizontal and is pointing due south.

For this case, $\gamma = 0°$ and Eq. (3.6) is applicable. On December 1, $n = 335$.

From Eq. (3.2),

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + 335)\right]$$
$$= -22.11°$$

At 0900 h (local apparent time), $\omega = 45°$. Substituting in Eq. (3.6), we have

$$\cos\theta = \sin(-22.11°)\sin(28.58° - 36°)$$
$$+ \cos(-22.11°)\cos 45° \cos(28.58° - 36°)$$
$$= 0.6982$$
$$\theta = 45.7°$$

The angle θ can also be calculated using Eq. (3.8) in conjunction with Eqs (3.5) and (3.9). From Eq. (3.5)

$$\cos\theta_z = \sin 28.58° \sin(-22.11°) + \cos 28.58° \cos(-22.11°)\cos 45°$$
$$= 0.39522$$
$$\theta_z = 66.72°$$

From Eq. (3.9)

$$\cos\gamma_s = [\cos 66.72° \sin 28.58° - \sin(-22.11°)]/\sin 66.72° \cos 28.58°$$
$$= 0.70103$$
$$\gamma_s = 45.49°$$

Substituting into Eq. (3.8), we get

$$\cos\theta = \cos 66.72° \cos 36° + \sin 66.72° \sin 36° \cos(45.49° - 0°)$$
$$= 0.6982.$$
$$\theta = 45.7°$$

Apparent Motion of the Sun

It will be useful at this stage to describe the apparent motion of the sun as seen from the earth. To an observer on the earth, on any given day, the sun rises in the east, moves in a plane tilted at an angle of $(90° - \phi)$ with the horizontal, and finally sets in the west. Thus, the apparent plane in which the sun moves intersects the horizontal in a line pointing east-west. However because of the declination angle this E-W line of intersection does not coincide with the E-W line passing through the observer *O* (Fig. 3.12). For a location in the northern hemisphere, this line is to the south in winter (declination negative), to the north in summer (declination positive) and coincides with the E-W line passing through the observer on the two equinox days of March 21 and September 22 (declination zero).

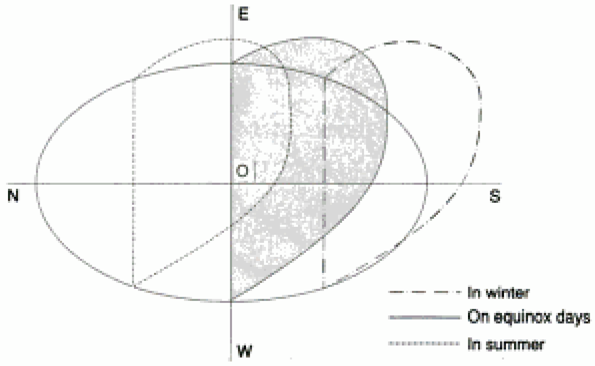

Fig. 3.12 Apparent Plane of Motion of the Sun for a Location in the Northern Hemisphere

Sunrise, Sunset and Day Length

The hour angle corresponding to sunrise or sunset (ω_s) on a horizontal surface can be found from Eq. (3.5) if one substitutes the value of 90° for the zenith angle. We obtain

$$\cos \omega_s = -\tan \phi \tan \delta$$

$$\omega_s = \cos^{-1} (-\tan \phi \tan \delta) \tag{3.10}$$

Equation (3.10) yields a positive and a negative value for ω_s, the positive value corresponding to sunrise and the negative to sunset. Since 15° of the hour angle is equivalent to 1 hour, the corresponding day length (in hours)

$$S_{max} = \frac{2}{15} \cos^{-1} (-\tan \phi \tan \delta) \tag{3.11}$$

The hour angle at sunrise or sunset as seen by an observer on an

inclined surface facing south ($\gamma = 0°$) will also be given by Eq. (3.10) if the day under consideration lies between September 22 and March 21, and the location is in the northern hemisphere. This is because during this period, the declination is negative and the apparent plane of motion of the sun intersects the horizontal plane in an E-W line which lies to the south of the E-W line passing through an observer on the inclined plane. However, if the day under consideration lies between March 21 and September 22, the hour angle at sunrise or sunset (ω_{st}) would be smaller in magnitude than the value given by Eq. (3.10) and would be obtained by substituting $\theta = 90°$ in Eq. (3.6). This yields

$$\omega_{st} = \cos^{-1} [-\tan (\phi - \beta) \tan \delta] \tag{3.12}$$

Thus the magnitude of ω_{st} for an inclined surface facing south ($\gamma = 0°$) in the northern hemisphere is the smaller of the magnitudes of the values given by Eqs (3.10) and (3.12).

In general, for a plane surface not symmetrically oriented, hour angles at sunrise and sunset would be unequal in magnitude apart from having opposite signs. The general procedure would be to calculate ω_{st} by substituting $\theta = 90°$ in Eq. (3.3) and by using Eq. (3.10). Depending upon the day of the year and the orientation of the surface, proper judgement would need to be exercised in selecting the correct values from the solutions thus obtained.

Example 3.2

Calculate the hour angle at sunrise and sunset on June 21 and December 21 for a surface inclined at an angle of 10° and facing due south ($\gamma = 0°$). The surface is located in Bombay (19°07′ N, 72°51′ E).

For June 21, we use Eq. (3.12),

$$\omega_{st} = \cos^{-1} [-\tan (19.12° - 10°) \tan 23.45°] = \pm 94.0°$$

For December 21, we use Eq. (3.10),

$$\omega_{st} = \cos^{-1} [-\tan 19.12° \tan (-23.45°)] = \pm 81.4°$$

Local Apparent Time

The time used for calculating the hour angle ω is the *local apparent time*. This can be obtained from the standard time observed on a clock by applying two corrections. The first correction arises because of the difference between the longitude of a location and the meridian on which the standard time is based. The correction has a magnitude of 4 minutes for every degree difference in longitude. The second correction called the *equation of time* correction is due to the fact that the earth's orbit and rate of rotation are subject to small fluctuations. This correction is based on experimental observations and is plotted in Fig. 3.13.

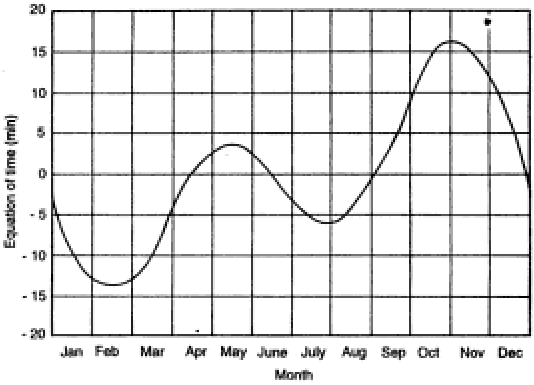

Fig. 3.13 Equation of Time Correction

Thus, Local apparent time = Standard time
± 4(Standard time longitude
− longitude of location)
+ (Equation of time correction)

The negative sign in the first correction is applicable for the eastern hemisphere, while the positive sign is applicable for the western hemisphere.

Example 3.3

Determine local apparent time (LAT) corresponding to 1430 h (IST) at Bombay (19°07′ N, 72°51′ E) on July 1. In India, standard time is based on 82.50° E.

From Fig. 3.13, the equation of time correction on July 1 is (−4) minutes. Therefore,

Local apparent time = 1430 h − 4(82.50 − 72.85) minutes
+ (−4 minutes)
= 1430 h − 38.6 minutes − 4 minutes
= 1347 h

3.6 EMPIRICAL EQUATIONS FOR PREDICTING THE AVAILABILITY OF SOLAR RADIATION

Since measurements of solar radiation are often not available, attempts have been made by many investigators to establish relationships linking the values of radiation (global or diffuse) with meteorological parameters like number of sunshine hours, cloud cover, and precipitation. Some of these equations will be given in Sections 3.6.1–3.6.5 and their use illustrated with the help of a few numerical examples. These equations are generally valid for cloudy skies.

3.6.1 Monthly Average Daily Global Radiation

The first attempt at estimating solar radiation was due to Angstrom who suggested that it could be related to the amount of sunshine by a simple linear relation of the form

$$\frac{\overline{H}_g}{\overline{H}_c} = a + b\left(\frac{\overline{S}}{\overline{S}_{max}}\right) \tag{3.14}$$

where, $\overline{H}_g$ = monthly average of the daily global radiation on a horizontal surface at a location (kJ/m^2-day),

$\overline{H}_c$ = monthly average of the daily global radiation on a horizontal surface at the same location on a clear day (kJ/m^2-day),

$\overline{S}$ = monthly average of the sunshine hours per day at the location (h),

$\overline{S}_{max}$ = monthly average of the maximum possible sunshine hours per day at the location, i.e. the day length on a horizontal surface (h),

a, b = constants obtained by fitting data.

Because of difficulties in deciding what constitutes a clear day, Page* suggested that $\overline{H}_c$ in Eq. (3.14) be replaced by $\overline{H}_0$, the monthly average of the daily extra-terrestrial radiation which would fall on a horizonatl surface at the location under consideration.

$$\frac{\overline{H}_g}{\overline{H}_0} = a + b\left(\frac{\overline{S}}{\overline{S}_{max}}\right) \tag{3.15}$$

Values of a and b have been obtained for many cities in the world by Lof *et al*. They are given in Table 3.2. A similar set of values have also

*J.K. Page, *Proc. U.N. Conf. New Sources of Energy, Rome*, **4**, 378 (1961).

been obtained for 17 Indian cities by carrying out a citywise regression analysis. These values are given in Table 3.3.

Table 3.2 Constants *a* and *b* in Eq. (3.15) for Many Cities of the World*

Location	Range of $\overline{S}/\overline{S}_{max}$	*a*	*b*
Albuquerque, New Mexico, USA	0.68–0.85	0.41	0.37
Atlanta, Georgia, USA	0.45–0.71	0.38	0.26
Blue Hill, Mass., USA	0.42–0.60	0.22	0.50
Brownsville, Texas, USA	0.47–0.80	0.35	0.31
Buenos Aires, Argentina	0.47–0.68	0.26	0.50
Charleston, S.C., USA	0.60–0.75	0.48	0.09
Dairen, Manchuria	0.55–0.81	0.36	0.23
El Paso, Texas, USA	0.78–0.88	0.54	0.20
Ely, Nevada, USA	0.61–0.89	0.54	0.18
Hamburg, Germany	0.11–0.49	0.22	0.57
Honolulu, Hawaii, USA	0.57–0.77	0.14	0.73
Madison, Wisconsin, USA	0.40–0.72	0.30	0.34
Malange, Angola	0.41–0.84	0.34	0.34
Miami, Florida, USA	0.56–0.71	0.42	0.22
Nice, France	0.49–0.76	0.17	0.63
Pune, India	0.25–0.49	0.30	0.51
	0.65–0.89	0.41	0.34
Stanleyville, Congo	0.34–0.56	0.28	0.39
Tamanrasset, Algeria	0.76–0.88	0.30	0.43

*G.O.G. Lof, J.A. Duffie and C.O. Smith, "World Distribution of Solar Radiation", *Solar Energy*, **10**, 27 (1966). Adapted with permission.

In the above computations, the quantity $\overline{H}_0$ is the mean of the value (H_0) for each day of the month. H_0 is obtained by integrating over the day length as follows:

$$H_0 = I_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right) \int (\sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega)\, dt \tag{3.16}$$

Now, $$t = \frac{180\ \omega}{15\ \pi}$$

where t is in hours and ω is in radians.

Hence, $$dt = \frac{180}{15\pi}\, d\omega$$

Substituting in Eq. (3.16),

$$H_0 = \frac{12}{\pi} I_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right) \int_{-\omega_s}^{+\omega_s} (\sin \phi \sin \delta + \cos \phi \cos \delta \cos \omega)\, d\omega$$

$$= \frac{24}{\pi} I_{sc}\left(1 + 0.033 \cos \frac{360n}{365}\right)(\omega_s \sin \phi \sin \delta + \cos \phi \cos \delta \sin \omega_s) \tag{3.17}$$

Table 3.3 Constants *a* and *b* in Eq. (3.15) for Indian Cities*

Location	*a*	*b*	Mean error (per cent)
Ahmedabad	0.28	0.48	3.0
Bangalore	0.18	0.64	3.9
Bhavnagar	0.28	0.47	2.8
Calcutta	0.28	0.42	1.3
Goa	0.30	0.48	2.1
Jodhpur	0.33	0.46	2.0
Kodaikanal	0.32	0.55	2.9
Madras	0.30	0.44	3.5
Mangalore	0.27	0.43	4.2
Minicoy	0.26	0.39	1.4
Nagpur	0.27	0.50	1.6
New Delhi	0.25	0.57	3.0
Pune	0.31	0.43	1.9
Shillong	0.22	0.57	3.0
Srinagar	0.35	0.40	4.7
Thiruvananthapuram	0.37	0.39	2.5
Vishakhapatnam	0.28	0.47	1.2

*Vijay Modi and S.P. Sukhatme, "Estimation of Daily Total and Diffuse Insolation in India from Weather Data", *Solar Energy*, **22**, 407 (1979). Used with permission.

The calculation of $\overline{H}_0$ has been simplified by Klein*, who has determined the particular day in each month on which the extra-terrestrial radiation is nearly equal to the monthly mean value. The dates on which the value of H_0 is equal to $\overline{H}_0$ are as follows: January 17, February 16, March 16, April 15, May 15, June 11, July 17, August 16, September 15, October 15, November 14 and December 10. As expected, these dates are close to the middle of the month.

Equation (3.15) can be used for calculating the average daily global radiation at a location when data on sunshine hours is available, and

*S.A. Klein, "Calculation of Monthly Average Insolation on Tilted Surfaces", *Solar Energy*, **19**, 325 (1977).

values of a and b are known for a nearby location with a similar climate. The procedure is illustrated in the following example.

Example 3.4

Estimate the monthly average daily global radiation on a horizontal surface at Baroda (22°00′ N, 73°10′ E) during the month of March if the average sunshine hours per day is 9.5.

We will assume that the values of $a = 0.28$ and $b = 0.48$ obtained for Ahmedabad (Table 3.3) are valid for Baroda. This is a good assumption since the two cities are fairly close to each other and their climate is similar. Based on Klein's recommendation, we will take $\overline{H}_0$ to be equal to the value of H_0 on March 16. Further, we will assume that this recommendation is also valid for the calculation of $\overline{S}_{max}$.

On March 16, $n = 75$. From Eq. (3.2),

$$\delta = 23.45 \sin \frac{360}{365}(284 + 75)$$

$$= -2.42°$$

From Eq. (3.10), the sunrise hour angle

$$\omega_s = \cos^{-1}\{-\tan 22° \tan(-2.42°)\}$$

$$= 89.02° = 1.554 \text{ radians}$$

Day length, $S_{max} = \dfrac{2}{15} \times 89.02 = 11.87$ h

From Eq. (3.17),

$$H_0 = \frac{24}{\pi} \times 1.367 \times 3600\left(1 + 0.033 \cos\frac{360}{365} \times 75\right)\Big\{1.554 \sin 22° \sin(-2.42°) + \cos 22° \cos(-2.42°) \sin 89.02°\Big\}$$

$= 34\,206$ kJ/m^2-day

Therefore, from Eq. (3.15),

$$\overline{H}_g = \left\{0.28 + 0.48\left(\frac{9.5}{11.87}\right)\right\} \times 34\,206$$

$$= 22\,718 \text{ kJ/m}^2\text{-day}$$

Other meteorological parameters have also been used for predicting solar radiation. These include cloud cover (the amount of sky dome covered by clouds) and precipitation (the number of days in the month with precipitation greater than 0.3 mm). However, in general, the sunshine ratio parameter ($\overline{S}/\overline{S}_{max}$) has been found to be the most reliable predictor.

3.6.2 Monthly Average Daily Diffuse Radiation

Based on a study of data for a few countries, Liu and Jordan* showed that the daily diffuse-to-global radiation ratio could be correlated against the daily global-to-extra-terrestrial radiation ratio. The correlation was expressed by the following cubic equation

$$\frac{\overline{H}_d}{\overline{H}_g} = 1.390 - 4.027\left[\frac{\overline{H}_g}{\overline{H}_0}\right] + 5.531\left[\frac{\overline{H}_g}{\overline{H}_0}\right]^2 - 3.108\left[\frac{\overline{H}_g}{\overline{H}_0}\right]^3 \quad (3.18)$$

where $\overline{H}_d$ = monthly average of the daily diffuse radiation on a horizontal surface (kJ/m^2-day).

The other symbols have the same meaning as given earlier. The ratio ($\overline{H}_g/\overline{H}_0$) is often denoted by the symbol $\overline{K}_T$ and is called the monthly average clearness index. Kreith† has pointed out that Eq. (3.18) has been obtained with a value of 1394 W/m^2 for the solar constant.

When available Indian data was analysed,‡ the following linear equation was obtained

$$\frac{\overline{H}_d}{\overline{H}_g} = 1.411 - 1.696\left[\frac{\overline{H}_g}{\overline{H}_0}\right] \quad (3.19)$$

In a separate study, Gupta *et al.*** have obtained the equation

$$\frac{\overline{H}_d}{\overline{H}_g} = 1.354 - 1.570\left[\frac{\overline{H}_g}{\overline{H}_0}\right] \quad (3.20)$$

Equations (3.19) and (3.20) are valid for $0.3 < (\overline{H}_g/\overline{H}_0) < 0.7$. They agree well with each other. However, when compared with Liu and Jordan's Eq. (3.18), it is observed that there are significant differences. These are due to the fact that the diffuse component is much larger in India.

Example 3.5

Estimate the monthly average daily diffuse radiation on a horizontal surface at Baroda during the month of March.

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*B.Y.H. Liu and R.C. Jordan, "The Interrelationship and Characteristic Distribution of Direct, Diffuse and Total Solar Radiation", *Solar Energy*, 4(3), 1 (1960).

†F. Kreith and J.F. Kreider, *Principles of Solar Engineering*, Chapter 2, McGraw-Hill, New York (1978).

‡Vijay Modi and S.P. Sukhatme, "Estimation of Daily, Total and Diffuse Insolation in India from Weather Data", *Solar Energy*, 22, 407 (1979).

**C.L. Gupta, K. Usha Rao and T.A. Reddy, "Radiation Design Data for Solar Energy Applications", *Energy Management*, 3, 299 (1979).

We will use Eqs (3.19) and (3.20) for making the estimation. From Example 3.4, $\overline{H}_g = 22\,718$ kJ/m^2-day and $\overline{H}_0 = 34\,206$ kJ/m^2-day. Monthly average clearness index $\overline{K}_T = 0.6642$.

From Eq. (3.19),

$$\overline{H}_d = 22\,718(1.411 - 1.696 \times 0.6642) = 6465 \text{ kJ/m}^2\text{-day}$$

From Eq. (3.20),

$$\overline{H}_d = 22\,718(1.354 - 1.570 \times 0.6642) = 7072 \text{ kJ/m}^2\text{-day}$$

Thus we see that the predictions made by Eqs (3.19) and (3.20) are within 10 per cent of each other.

3.6.3 Monthly Average Hourly Global Radiation

A number of studies have also been conducted with the objective of obtaining relations for predicting the diurnal variation of the monthly average hourly global radiation at a location. Collares-Pereira and Rabl* have developed the following relation

$$\frac{\overline{I}_g}{\overline{H}_g} = \frac{\overline{I}_o}{\overline{H}_0}(a + b \cos \omega) \tag{3.21}$$

where $a = 0.409 + 0.5016 \sin(\omega_s - 60°)$,

$b = 0.6609 - 0.4767 \sin(\omega_s - 60°)$,

$\overline{I}_g$ = monthly average of the hourly global radiation on a horizontal surface (kJ/m^2-h),

$\overline{I}_0$ = monthly average of the hourly extra-terrestrial radiation on a horizontal surface (kJ/m^2-h).

The other symbols in Eq. (3.21) have been defined earlier. Eq. (3.21) has been developed on the basis of the following facts

(i) Measured data generally show a similarity between the diurnal variation of $\overline{I}_g$ and $\overline{I}_0$.

(ii) There is a close correlation between the values of the ratios $(\overline{I}_g/\overline{H}_g)$ and $(\overline{I}_0/\overline{H}_0)$.

It will be noted from Eq. (3.21) that the symbol I has been used for denoting an hourly value (kJ/m^2-h), while earlier it was used for denoting an instantaneous value i.e. a flux (kW/m^2). For most situations involving solar radiation, transient processes occur at a slow pace. It is therefore not necessary to make a distinction between the

*M. Collares-Pereira and A. Rabl, "The Average Distribution of Solar Radiation Correlations between Diffuse and Hemispherical and between Daily and Hourly Insolation Values", *Solar Energy*, **22**, 155 (1979).

two quantities. If necessary, an hourly value (kJ/m^2-h) can be calculated by obtaining the instantaneous value (kW/m^2) at the mid-point of the hour and multiplying by 3600.

Satyamurty and Lahiri* have tested the predictions of Eq. (3.21) against the measured data of fourteen locations in India and shown that there is a good correlation between the two sets of values. The rms difference between the predicted and measured values of $(\overline{I}_g/\overline{H}_g)$ ranged only between 2.9 and 5.5 per cent for thirteen of the fourteen locations. A significant difference of 10.4 per cent was obtained only at one location.

3.6.4 Monthly Average Hourly Diffuse Radiation

In a manner similar to that adopted for developing Eq. (3.21), Liu and Jordan† have suggested the following relation for estimating the monthly average hourly diffuse radiation

$$\frac{\overline{I}_d}{\overline{H}_d} = \frac{\overline{I}_0}{\overline{H}_0} \tag{3.22}$$

Satyamurty and Lahiri‡ have also tested the predictions of Eq. (3.22) against the measured data of fourteen locations in India. In this case, relatively poor agreement is obtained, the rms difference between the predicted and measured values of $(\overline{I}_d/\overline{H}_d)$ ranging between 5.7 and 13.4 per cent. Satyamurty and Lahiri have therefore suggested the following improved relation

$$\frac{\overline{I}_d}{\overline{H}_d} = (a' + b' \cos \omega)\frac{\overline{I}_0}{\overline{H}_0} \tag{3.23}$$

where $a' = 0.4922 + \{0.27/(\overline{H}_d/\overline{H}_g)\}$ for $0.1 \le (\overline{H}_d/\overline{H}_g) \le 0.7$

or $a' = 0.76 + \{0.113/(\overline{H}_d/\overline{H}_g)\}$ for $0.7 < (\overline{H}_d/\overline{H}_g) \le 0.9$

and $b' = 2(1 - a')(\sin \omega_s - \omega_s \cos \omega_s)/(\omega_s - 0.5 \sin 2\omega_s)$

Example 3.6

Use the prediction equations, to calculate the monthly average hourly global and hourly diffuse radiation during the month of April on a horizontal surface at New Delhi (28°35′ N, 77°12′ E). Time: 0900 to 1000 h (LAT). The average number of sunshine hours per day is 8.6. Compare the predicted values with the available measurements.

*V.V. Satyamurty and P.K. Lahiri, "Estimation of Symmetric and Asymmetric Hourly Global and Diffuse Radiation from Daily Values", *Solar Energy*, **48**, 7 (1992).

†B.Y.H. Liu and R.C. Jordan, op. cit.

‡V.V. Satyamurty and P.K. Lahiri, op. cit.

Daily Radiation

The representative day in April is the 15th. On April 15, $n = 105$.

From Eq. (3.2),

$$\delta = 23.45 \sin\left[\frac{360}{365}(284 + 105)\right] = 9.42°$$

From Eq. (3.10),

$$\omega_s = \cos^{-1}(-\tan 28.58° \times \tan 9.42°)$$
$$= 95.18° = 1.661 \text{ radians}$$
$$S_{max} = \frac{2}{15} \times 95.18 = 12.69 \text{ h}$$

Substituting in Eq. (3.17), we get

$$H_0 = \frac{24}{\pi} \times 1.367 \times 3600\left(1 + 0.033 \cos \frac{360}{365} \times 105\right)$$
$$\times (1.661 \sin 28.58° \sin 9.42° + \cos 28.58° \cos 9.42° \cos 95.18°)$$
$$= 37\,034 \text{ kJ/m}^2\text{-day}$$

From Table 3.3, $a = 0.25$ and $b = 0.57$.

Putting $\overline{H}_0$ equal to H_0 on April 15, and substituting in Eq. (3.15), we get

$$\overline{H}_g = 37\,034[0.25 + 0.57 \times (8.6/12.69)]$$
$$= 23\,564 \text{ kJ/m}^2\text{-day}$$

From Eq. (3.19),

$$\overline{H}_d = 23\,564[1.411 - 1.696 \times (23\,564/37\,034)]$$
$$= 7820 \text{ kJ/m}^2\text{-day}$$

Hourly Radiation

The hourly extra-terrestrial radiation on a horizontal surface on April 15 between 0900 and 1000 h is obtained by calculating the instantaneous value at 0930 h and multiplying by 3600 s.

From Eq. (3.5),

$$I_0 = 1.367\left(1 + 0.033 \cos \frac{360}{365} \times 105\right)$$
$$\times (\sin 28.58° \sin 9.42° + \cos 28.58° \cos 9.42° \cos 37.5°)$$
$$= 1.0384 \text{ kW/m}^2 = 3738 \text{ kJ/m}^2\text{-h}$$

Substituting in Eq. (3.21),

$$a = 0.409 + 0.5016 \sin(95.18° - 60°) = 0.69800$$
$$b = 0.6609 - 0.4767 \sin(95.18° - 60°) = 0.38625$$

Once again putting $\overline{I}_0$ equal to I_0 on April 15, we get

$$\frac{\overline{I}_g}{23\,564} = \frac{3738}{37\,034}(0.69800 + 0.38625 \cos 37.5°)$$
$$\overline{I}_g = 2389 \text{ kJ/m}^2\text{-h}$$

Finally, from Eq. (3.22),

$$\overline{I}_d = 7820 \times (3738/37034)$$
$$= 789 \text{ kJ/m}^2\text{-h}$$

Instead of Eq. (3.22), if we use Eq. (3.23), a slightly different value is obtained. We get

$$\overline{I}_d = 780 \text{ kJ/m}^2\text{-h}$$

Comparison Between Predicted and Measured Values

Measured values of $\overline{H}_g, \overline{H}_d, \overline{I}_g$ and $\overline{I}_d$ for New Delhi are given in Tables A 3.1 and A 3.2. Table 3.4 gives a comparison between the predicted and measured values. It is seen that in all the cases the predicted values are in reasonably good agreement with the measured values.

Table 3.4 Comparison between the Predicted and Measured Values of Solar Radiation

Location : New Delhi
Month : April
Time : 0900–1000 h (LAT)

	Predicted	Measured	Difference (%)
$\overline{H}_g$ (kJ/m^2-day)	23 564	24 966	5.6
$\overline{H}_d$ (kJ/m^2-day)	7820	8902	12.2
$\overline{I}_g$ (kJ/m^2-h)	2389	2617	8.7
$\overline{I}_d$ (kJ/m^2-h)	789	850	7.2

3.6.5 Daily and Hourly Diffuse Radiation

A number of correlations have been suggested for predicting the value of the daily diffuse radiation H_d from a given value of the daily global radiation H_g on an individual day. These correlations are useful because the daily global radiation is recorded at many stations, whereas the daily diffuse radiation is not. Collares-Pereira and Rabl* have suggested the following correlation based on an analysis of data recorded at a number of stations in the United States.

*M. Collares-Pereira and A. Rabl, op. cit.

$$\frac{H_d}{H_g} = 0.99 \quad \text{for} \quad K_T \le 0.17$$
$$= 1.188 - 2.272\,K_T + 9.473\,K_T^2 - 21.856\,K_T^3 + 14.648\,K_T^4$$
$$\text{for} \quad 0.17 < K_T \le 0.8 \qquad (3.24)$$

where $K_T = H_g/H_0$.

In a similar manner, correlations have also been developed for estimating the hourly diffuse radiation I_d from measured values of I_g. Spencer* has suggested the following correlation

$$\frac{I_d}{I_g} = a - b\left(\frac{I_g}{I_0}\right) \qquad (3.25)$$

where $a = 0.949 + 0.0118|\phi|$

$b = 1.185 + 0.0135|\phi|$

and ϕ = latitude in degrees.

Eq. (3.25) is valid for $0.35 \le (I_g/I_0) \le 0.75$.

3.6.6 Hourly Global, Beam and Diffuse Radiation Under Cloudless Skies

We now turn our attention to the prediction of radiation under cloudless skies (clear days). Based on an analysis of US data, ASHRAE† has given a method for estimating the hourly global and diffuse solar radiation falling on a horizontal surface under such conditions. The equations are based on an exponential decay model in which the beam radiation decreases with increase in the distance traversed through the atmosphere. The global radiation I_g reaching a horizontal surface on the earth is given by

$$I_g = I_b + I_d$$

where I_g = hourly global radiation,

I_b = hourly beam radiation,

I_d = hourly diffuse radiation.

Now, $I_b = I_{bn} \cos\theta_z \qquad (3.26)$

where I_{bn} = beam radiation in the direction of the rays,

and θ_z = angle of incidence on a horizontal surface, i.e. the zenith angle.

*J.W. Spencer, "A Comparison of Methods for Estimating Hourly Diffuse Solar Radiation from Global Solar Radiation", *Solar Energy*, **29**, 19 (1982).

†American Society of Heating, Refrigerating and Air-conditioning Engineers (ASHRAE), *Handbook of Fundamentals*, pp. 385–443 (1972).

Thus, $$I_g = I_{bn} \cos\theta_z + I_d \qquad (3.27)$$

In the ASHRAE model, it is postulated that

$$I_{bn} = A \exp\left[-B/\cos\theta_z\right] \qquad (3.28)$$

and $$I_d = C\,I_{bn} \qquad (3.29)$$

where A, B and C are constants whose values have been determined on a monthwise basis. These constants change during the year because of seasonal changes in the dust and water vapour content of the atmosphere, and also because of the changing earth-sun distance. The values of A, B and C were initially given by Threlkeld and Jordan* and subsequently revised by Iqbal†. The revised values are given in Table 3.5 and are recommended for use.

Table 3.5 Values of the Constants A, B and C Used for Predicting Hourly Solar Radiation on Clear Days

	A (W/m^2)	B	C
January 21	1202	0.141	0.103
February 21	1187	0.142	0.104
March 21	1164	0.149	0.109
April 21	1130	0.164	0.120
May 21	1106	0.177	0.130
June 21	1092	0.185	0.137
July 21	1093	0.186	0.138
August 21	1107	0.182	0.134
September 21	1136	0.165	0.121
October 21	1136	0.152	0.111
November 21	1190	0.144	0.106
December 21	1204	0.141	0.103

Example 3.7

The following hourly values are measured around mid-day on a clear day at Nagpur (21°06′ N, 79°03′ E).

*J.L. Threlkeld and R.C. Jordan, "Direct Solar Radiation Available on Clear Days", *ASHRAE Transactions*, **64**, 45 (1958).

†M. Iqbal, *An Introduction to Solar Radiation*, Academic Press, Canada (1983).

Date: June 19, 1968.

Local apparent time	0900–1000	1000–1100	1100–1200	1200–1300	1300–1400	1400–1500
Global Radiation (langleys/h)	67.6	77.0	79.3	84.5	79.5	64.0
Diffuse Radiation (langleys/h)	17.1	15.2	17.6	18.6	16.9	17.6

Compare these values with values which would be predicted by the ASHRAE method.

On June 19, 1968, $n = 171$

From Eq. (3.2),

$$\delta = 23.45 \sin\left\{\frac{360}{365}(284 + 171)\right\} = 23.44^\circ$$

Therefore from Eq. (3.5),

$$\cos\theta_z = \sin 21.10^\circ \sin 23.44^\circ + \cos 21.10^\circ \cos 23.44^\circ \cos\omega$$
$$= 0.1432 + 0.8560 \cos\omega$$

From Table 3.5, the values of A, B and C are

$$A = 1092 \text{ W/m}^2 = 3931.2 \text{ kJ/m}^2\text{-h}$$
$$B = 0.185$$
$$C = 0.137$$

The remaining calculation is conveniently done in tabular form. The values obtained for I_b, I_d and I_g from Eqs (3.26)–(3.29) are given in Table 3.6 and are compared with the measured data. It is seen that the values of A, B, C obtained on the basis of US data predict high values of the beam radiation and low values of the diffuse radiation, when used to predict radiation under Indian conditions. The two effects tend to balance each other to some extent. As a result, the values of global radiation are predicted to a reasonable degree of accuracy.

Table 3.6 Comparison of Predicted and Measured Values of Hourly Radiation

LAT	ω (deg)	I_b (kJ/m²-h)		I_d (kJ/m²-h)		I_g (kJ/m²-h)	
		Predicted	Measured	Predicted	Measured	Predicted	Measured
0900–1000	37.5	2581	2114	430	716	3011	2830
1000–1100	22.5	3012	2588	442	636	3454	3224
1100–1200	7.5	3236	2583	447	737	3683	3320
1200–1300	−7.5	3236	2759	447	779	3683	3538
1300–1400	−22.5	3012	2621	442	708	3454	3329

3.7 SOLAR RADIATION ON TILTED SURFACES

From the preceding sections, it is seen that very often measuring instruments give the values of solar radiation falling on a horizontal surface. The same is also true of the empirical equations given in Sec. 3.6. However, most solar equipments (e.g. flat-plate collectors), for absorbing radiation, are tilted at an angle to the horizontal. It therefore becomes necessary to calculate the flux which falls on a tilted surface. This flux is the sum of the beam and diffuse radiation falling directly on the surface and the radiation reflected onto the surface from the surroundings.

Beam Radiation

The ratio of the beam radiation flux falling on a tilted surface to that falling on a horizontal surface is called the *tilt factor* for beam radiation. It is denoted by the symbol r_b. For the case of a tilted surface facing south (i.e. $\gamma = 0^\circ$),

$$\cos\theta = \sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)$$

while for a horizontal surface

$$\cos\theta_z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega$$

Hence
$$r_b = \frac{\cos\theta}{\cos\theta_z} = \frac{\sin\delta \sin(\phi - \beta) + \cos\delta \cos\omega \cos(\phi - \beta)}{\sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega} \quad (3.30)$$

Similarly, expressions for r_b can be derived for other situations in which the tilted surface is oriented in a different direction with $\gamma \neq 0^\circ$

Diffuse Radiation

The tilt factor r_d for diffuse radiation is the ratio of the diffuse radiation flux falling on the tilted surface to that falling on a horizontal surface. The value of this tilt factor depends upon the distribution of diffuse radiation over the sky and on the portion of the sky dome seen by the tilted surface. Assuming that the sky is an isotropic source of diffuse radiation, we have for a tilted surface with a slope β,

$$r_d = \frac{1 + \cos\beta}{2} \quad (3.31)$$

since $(1 + \cos\beta)/2$ is the radiation shape factor for a tilted surface with respect to the sky.

Reflected Radiation

Since $(1 + \cos\beta)/2$ is the radiation shape factor for a tilted surface with respect to the sky, it follows that $(1-\cos\beta)/2$ is the radiation shape factor for the surface with respect to the surrounding ground. Assuming that the reflection of the beam and diffuse radiations falling on the ground is diffuse and isotropic, and that the reflectivity is ρ, the tilt factor for reflected radiation is given by

$$r_r = \rho\left(\frac{1-\cos\beta}{2}\right) \tag{3.32}$$

Flux on Tilted Surface

The flux I_T falling on a tilted surface at any instant is thus given by

$$I_T = I_b r_b + I_d r_d + (I_b + I_d) r_r \tag{3.33}$$

where the values of r_b, r_d and r_r are as given in Eqs (3.30), (3.31) and (3.32). It should be noted that Eq. (3.30) is valid only for a tilted surface with $\gamma = 0°$, whereas Eqs (3.31) and (3.32) are valid for any tilted surface with a slope β. Dividing both sides of Eq. (3.33) by I_g, we obtain the ratio of the flux falling on a tilted surface at any instant to that on a horizontal surface.

$$\frac{I_T}{I_g} = \left(1 - \frac{I_d}{I_g}\right) r_b + \frac{I_d}{I_g} r_d + r_r \tag{3.34}$$

One difficulty associated with using Eq. (3.33) is that the value of the diffuse reflectivity ρ is not known for most situations. A value around 0.2, generally expected with surfaces of concrete or grass, can be used. Fortunately, the reflected radiation term does not very often contribute much to the total.

Equation (3.34) can be used for calculating the hourly radiation falling on a tilted surface if the value of ω is taken at the mid-point of the hour. It can also be used for calculating the monthly average hourly value ($\bar{I}_T$) if the calculations are done for the representative day of the month as specified in Sec. 3.6.1. Eq. (3.34) is then used in the modified form

$$\frac{\bar{I}_T}{\bar{I}_g} = \left(1 - \frac{\bar{I}_d}{\bar{I}_g}\right) \bar{r}_b + \frac{\bar{I}_d}{\bar{I}_g} \bar{r}_d + \bar{r}_r \tag{3.35}$$

where $\bar{r}_b = r_b$ on the representative day,

$\bar{r}_d = r_d = (1+\cos\beta)/2$,

$\bar{r}_r = r_r = \rho(1-\cos\beta)/2$

The daily radiation falling on a tilted surface is also of interest in many applications. Liu and Jordan* have proposed that the ratio of

*B.Y.H. Liu and R.C. Jordan, "Daily Insolation on Surfaces Tilted Toward the Equator", *Trans. ASHRAE*, 67, 526 (1961).

the daily radiation falling on such a surface (H_T) to the daily global radiation on a horizontal surface (H_g) is given by an equation having a form similar to Eq. (3.34). Thus

$$\frac{H_T}{H_g} = \left(1 - \frac{H_d}{H_g}\right) R_b + \frac{H_d}{H_g} R_d + R_r \tag{3.36}$$

For a south-facing surface ($\gamma = 0°$), Liu and Jordan show that

$$R_b = \frac{\omega_{st}\sin\delta\sin(\phi-\beta) + \cos\delta\sin\omega_{st}\cos(\phi-\beta)}{\omega_s\sin\phi\sin\delta + \cos\phi\cos\delta\sin\omega_s} \tag{3.37}$$

$$R_d = r_d = \frac{1+\cos\beta}{2} \tag{3.38}$$

$$R_r = r_r = \rho\left(\frac{1-\cos\beta}{2}\right) \tag{3.39}$$

In Equation (3.37), ω_{st} and ω_s are the sunrise or sunset hour angles (expressed in radians) for the tilted surface and a horizontal surface respectively.

Equation (3.36) can also be used for calculating the monthly average daily radiation falling on a tilted surface if the values required are calculated for the representative day of the month. Eq. (3.36) is then used in the modified form

$$\frac{\bar{H}_T}{\bar{H}_g} = \left(1 - \frac{\bar{H}_d}{\bar{H}_g}\right) \bar{R}_b + \frac{\bar{H}_d}{\bar{H}_g} \bar{R}_d + \bar{R}_r \tag{3.40}$$

where $\bar{R}_b = R_b$ on the representative day,

$\bar{R}_d = R_d = (1+\cos\beta)/2$

$\bar{R}_r = R_r = \rho(1-\cos\beta)/2$

Example 3.8

Calculate the monthly average hourly radiation falling on a flat-plate collector facing south ($\gamma = 0°$) with a slope of 15°, given the following data

Location	—	Madras (13°00′ N)
Month	—	October
Time	—	1100 – 1200 h (LAT)
$\bar{I}_g$	—	2408 kJ/m^2-h
$\bar{I}_d$	—	1073 kJ/m^2-h

Assume ground reflectivity to be 0.2.

The calculations will be done for the representative day in October, viz. October 15.

On this day, $n = 288$.

From Eq. (3.2), $\delta = -9.599°$

Substituting in Eq. (3.30),

$$r_b = \frac{\sin(-9.599°)\sin(13.00° - 15°) + \cos(-9.599°)\cos 7.5°\cos(13.00° - 15°)}{\sin 13.00°\sin(-9.599°) + \cos 13.00°\cos(-9.599°)\cos 7.5°}$$

$= 1.1200$

From Eqs (3.31) and (3.32),

$$r_d = (1 + \cos 15°)/2 = 0.9830$$

$$r_r = 0.2(1 - \cos 15°)/2 = 0.0034$$

Putting $\bar{r}_b$ equal to r_b on October 15, $\bar{r}_d = r_d$ and $\bar{r}_r = r_r$, and substituting in Eq. (3.35), we have

$$\frac{\bar{I}_T}{\bar{I}_g} = \left(1 - \frac{1073}{2408}\right) \times 1.1200 + \frac{1073}{2408} \times 0.9830 + 0.0034$$

$= 1.0624$

$\bar{I}_T = 2558$ kJ/m^2-h

PROBLEMS

1. (a) Find the temperature of the sun's surface if it is assumed to be black and the value of the solar constant is 1367 W/m^2.
   (b) Using Planck's law, calculate the spectral distribution for the temperature calculated in (a). Compare this distribution with measured values of the distribution given in Table 3.1.
2. Estimate the rate at which the sun emits energy. What fraction of this energy is intercepted by the earth and what is the amount intercepted?
3. Calculate the angle of incidence of beam radiation on a flat-plate collector for the following situation:

Location	Nagpur (21°06′ N, 79°03′ E)
Slope of collector	31°
Surface azimuth angle	15°
Date	December 1, 1979
Time	0900 h (IST)

4. Calculate and plot the variation of the angle of incidence of beam radiation on May 1st on a flat surface located in Bombay (19°07′ N, 72°51′E). The surface is tilted at an angle of 10° with the horizontal and is pointing due south. Would you prefer to plot the variation against IST or LAT?
5. Find the days of the year on which the sun is directly overhead at 1200 h (LAT) at Pune (18°32′ N).
6. An array of flat-plate collectors is facing due south and is inclined at an angle β to the horizontal. Reflecting mirrors of the same length L as the collectors are used to boost the output (Fig. 3.14). Using the criterion that the sun's rays striking the top of the mirrors (line AA) at 1200 h (LAT) should be reflected to the top of

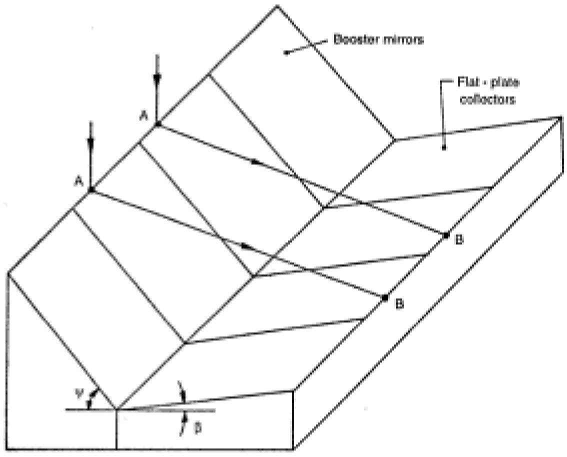

Fig. 3.14 Problem 6

the collectors (line BB), derive an expression for the correct inclination ψ of the booster mirrors in terms of β, δ and ϕ.

7. Plot the variation of the day length on a horizontal surface through the year for the following locations:

Srinagar	(34°06′ N, 74°51′ E),
Calcutta	(22°39′ N, 88°27′ E)
Vishakhapatnam	(17°43′ N, 83°14′ E)
Thiruvananthapuram	(8°29′ N, 76°57′ E)

8. Calculate the day length on May 1 and December 1 for a south-facing surface tilted at an angle of 40° and located at New Delhi (28°35′ N, 77°12′ E).
9. Calculate the hour angle at sunrise and sunset on a plane surface tilted at an angle of 40°. Given $\phi = 28°$ N, $\delta = -21°$ and $\gamma = 48°$.
10. Calculate the length and direction of the shadow cast on the ground by a 1 m long vertical stick for the following situation

Location	Bombay (19°07′ N, 72°51′ E)
Date	February 13, 1980
Time	1000 h (IST)

11. A horizontal stick, 1 m. long, is fixed at right angles to a vertical south facing wall ($\gamma = 0°$). Calculate the length and direction of the shadow cast by the stick on the wall for the following situation

Location	Jodhpur (26°18′ N, 73°01′ E)
Date	November 5
Time	0900 h (LAT)

12. Test the validity of the recommendation that the value of H_0 for July can be taken equal to the value of H_0 on the seventeenth of the month by doing calculations for Shillong (25°34′ N, 91°56′ E).