STATISTICAL PROCESS CONTROL

Statistica1 quality control (SQC) is the term used to describe the set of statistical tools used by quality professionals. Statistical quality control can be divided into three broad categories:

1. **Descriptive statistics** are used to describe quality characteristics and relation-ships. Included are statistics such as the mean, standard deviation, the range, and a measure of the distribution of data.
2. **Statistical process control (SPC)** involves inspecting a random sample of the output from a process and deciding whether the process is producing products with characteristics that fall within a predetermined range. SPC answers the question of whether the process is functioning properly or not.
3. **Acceptance sampling** is the process of randomly inspecting a sample of goods and deciding whether to accept the entire lot based on the results. Acceptance sampling determines whether a batch of goods should be accepted or rejected.

The tools in each of these categories provide different types of information for use in analyzing quality. Descriptive statistics are used to describe certain quality characteristics, such as the central tendency and variability of observed data. Although descriptions of certain characteristics are helpful, they are not enough to help us evaluate whether there is a problem with quality. Acceptance sampling can help us do this. Acceptance sampling helps us decide whether desirable quality has been achieved for a batch of products, and whether to accept or reject the items produced. Although this information is helpful in making the quality acceptance decision *after* the product has been produced, it does not help us identify and catch a quality problem *during* the production process. For this we need tools in the statistical process control (SPC) category.

All three of these statistical quality control categories are helpful in measuring and evaluating the quality of products or services. However, statistical process control (SPC) tools are used most frequently because they identify quality problems during the production process. For this reason, we will devote most of the chapter to this category of tools. The quality control tools we will be learning about do not only measure the value of a quality characteristic. They also help us identify a *change* or variation in some quality characteristic of the product or process. We will first see what types of variation we can observe when measuring quality. Then we will be able to identify specific tools used for measuring this variation.

Variation in the production process leads to quality defects and a lack of product consistency. The Intel Corporation, the world's largest and most profitable manufacturer of microprocessors, understands this. Therefore, Intel has implemented a program it calls "copy-exactly" at all its manufacturing facilities. The idea is that regardless of whether the chips are made in Arizona, New Mexico, Ireland, or any of its other plants, they are made in exactly the same way. This means using the same equipment, the same exact materials, and workers performing the same tasks in the exact same order. The level of detail to which the "copy-exactly" concept goes is meticulous. For example, when a chip making machine was found to be a few feet long at one facility than another, Intel made them match.

When water quality was found to be different at one facility, Intel instituted a purification system to eliminate any differences. Even when a worker was found polishing equipment in one direction, he was asked to do it in the approved circular pattern. Why such attention to exactness of detail? The reason is to minimize all variations.

SOURCES OF VARIATION: COMMON AND ASSIGNABLE CAUSES

If you look at bottles of a soft drink in a grocery store, you will notice that no two bottles are filled to exactly the same level. Some are filled slightly higher and some slightly lower. Similarly, if you look at blueberry muffins in a bakery, you will notice that some are slightly larger than others and some have more blueberries than others. These types of differences are completely normal. No two products are exactly alike because of slight differences in materials, workers, machines, tools, and other factors.

These are called common, or random, causes of variation. Common causes of variation are based on random causes that we cannot identify. These types of variation are unavoidable and are due to slight differences in processing.

An important task in quality control is to find out the range of natural random variation in a process. For example, if the average bottle of a soft drink called Cocoa Fizz contains 16 ounces of liquid, we may determine that the amount of natural variation is between 15.8 and 16.2 ounces. If this were the case, we would monitor the production process to make sure that the amount stays within this range. If production goes out of this range — bottles are found to contain on average 15.6 ounces —this would lead us to believe that there is a problem with the process because the variation is greater than the natural random variation.

The second type of variation that can be observed involves variations where the causes can be precisely identified and eliminated. These are called assignable causes of variation. Examples of this type of variation are poor quality in raw materials, an employee who needs more training, or a machine in need of repair. In each of these examples the problem can be identified and corrected. Also, if the problem is allowed to persist, it will continue to create a problem in the quality of the product. In the example of the soft drink bottling operation, bottles filled with 15.6 ounces of liquid would signal a problem. The machine may need to be readjusted. This would be an assignable cause of variation. We can assign the variation to a particular cause (machine needs to be readjusted) and we can correct the problem (readjust the machine).

DESCRIPTIVE STATISTICS

Descriptive statistics can be helpful in describing certain characteristics of a product and a process. The most important descriptive statistics are measures of central tendency such as the mean, measures of variability such as the standard deviation and range, and measures of the distribution of data. We first review these descriptive statistics and then see how we can measure their changes.

The Mean

In the soft drink bottling example, we stated that the average bottle is filled with 16 ounces of liquid. The arithmetic average, or the mean, is a statistic that measures the central tendency of a set of data. Knowing the central point of a set of data is highly important. Just think how important that number is when you receive test scores!

To compute the mean we simply sum all the observations and divide by the total number of observations. The equation for computing the mean is

$$\bar{X} = \frac{\sum_{i=1}^{n} x}{n}$$

where $\overline{X}$= the mean
Xi = observation i, I = 1, . . . ,n
n = number of observations

The Range and Standard Deviation

In the bottling example we also stated that the amount of natural variation in the bottling process is between 15.8 and 16.2 ounces. This information provides us with the amount of variability of the data. It tells us how spread out the data is around the mean. There are two measures that can be used to determine the amount of variation in the data. The first measure is the range, which is the difference between the largest and smallest observations. In our example, the range for natural variation is 0.4 ounces. Another measure of variation is the standard deviation. The equation for computing the standard deviation is

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$$

σ = standard deviation of a sample
$\overline{x}$ = the mean
x_i = observation i, i = 1, . . . ,n
n = the number of observations in the sample

Small values of the range and standard deviation mean that the observations are closely clustered around the mean. Large values of the range and standard deviation mean that the observations are spread out around the mean. Figure 6-1 illustrates the differences between a small and a large standard deviation for our bottling operation. You can see that the figure shows two distributions, both with a mean of 16 ounces. However, in the first distribution the standard deviation is large and the data are spread out far around the mean. In the second distribution the standard deviation is small and the data are clustered close to the mean.

FIGURE 6-1 Normal distributions with varying standard deviations

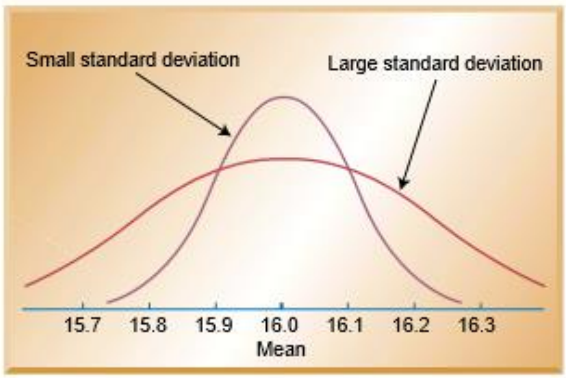
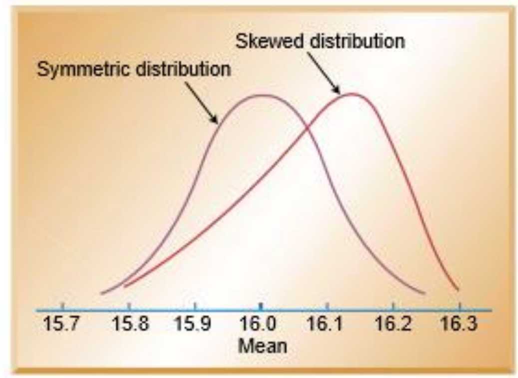


FIGURE 6-2 Differences between symmetric and skewed distributions



Distribution of Data

A third descriptive statistic used to measure quality characteristics is the shape of the distribution of the observed data. When a distribution is symmetric, there are the same number of observations below and above the mean. This is what we commonly find when only normal variation is present in the data. When a disproportionate number of observations are either above or below the mean, we say that the data has a skewed distribution. Figure 6-2 shows symmetric and skewed distributions for the bottling operation.

STATISTICAL PROCESS CONTROL METHODS

Statistical process control methods extend the use of descriptive statistics to monitor the quality of the product and process. As we have learned so far, there are common and assignable causes of variation in the production of every product. Using statistical process control we want to determine the amount of variation that is common or normal. Then we monitor the production process to make sure production stays within this normal range. That is, we want to make sure the process is in a state of control. The most commonly used tool for monitoring the production process is a control chart. Different types of control charts are used to monitor different aspects of the production process.

Developing Control Charts

A control chart (also called process chart or quality control chart) is a graph that shows whether a sample of data falls within the common or normal range of variation. A control chart has upper and lower control limits that separate common from assignable causes of variation. The common range of variation is defined by the use of control chart limits. We say that a process is out of control when a plot of data reveals that one or more samples fall outside the control limits. Figure 6-3 shows a control chart for the Cocoa Fizz bottling operation. The x axis represents samples (#1, #2, #3, etc.) taken from the process over time. The y axis represents the quality characteristic that is being monitored (ounces of liquid). The center line (CL) of the control chart is the mean, or average, of the quality characteristic that is being measured. In Figure 6-3 the mean is 16 ounces. The upper control limit (UCL) is the maximum acceptable variation from the mean for a process that is in a state of control. Similarly, the lower control limit (LCL) is the minimum acceptable variation from the mean for a process that is in a state of control. In our example, the upper and lower control limits are 16.2 and 15.8 ounces, respectively. You can see that if a sample of observations falls outside the control limits we need to look for assignable causes. The upper and lower control limits on a control chart are usually set at ±3 standard deviations from the mean. If we assume that the data exhibit a normal distribution, these control limits will capture 99.74 percent of the normal variation. Control limits can be set at ±2 standard deviations from the mean. In that case, control limits would capture 95.44 percent of the values. Figure 6-4 shows the percentage of values that fall within a particular range of standard deviation. Looking at Figure 6-4, we can conclude that observations that fall outside the set range represent assignable causes of variation. However, there is a small probability that a value that falls outside the limits is still due to normal variation. This is called Type I error, with the error being the chance of concluding that there are assignable causes of variation when only normal variation exists. Another name for this is alpha risk (α), where alpha refers to the sum of the probabilities in both tails of the distribution that falls outside the confidence limits. The chance of this happening is given by the percentage or probability represented by the shaded areas of Figure 6-5. For limits of ±3 standard deviations from the mean, the probability of a Type I error is .26% (100% - 99.74%), whereas for limits of ±2 standard deviations it is 4.56% (100% - 95.44%).

FIGURE 6-3

Quality control chart for Cocoa Fizz

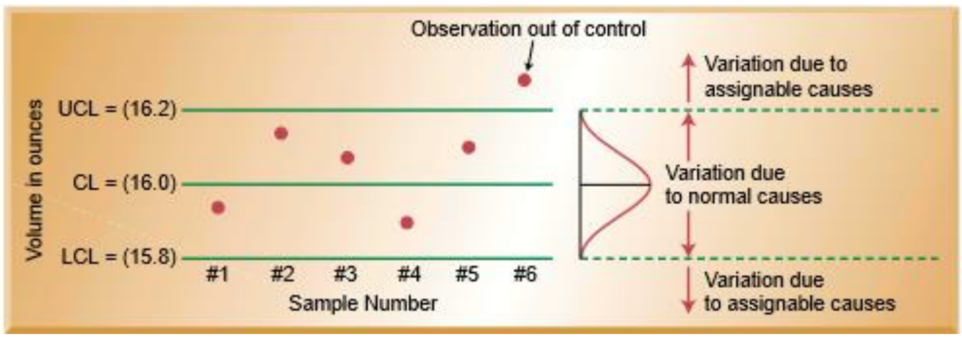


FIGURE 6-4 Percentage of values captured by different ranges of standard deviation

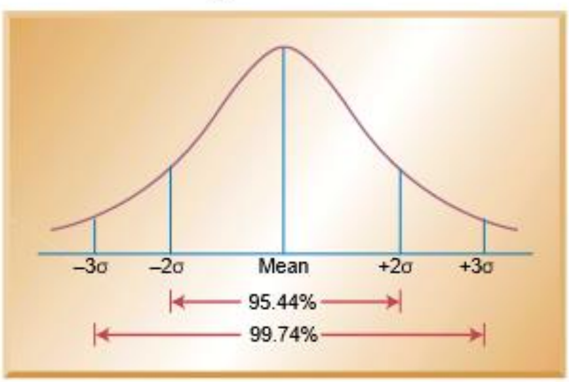
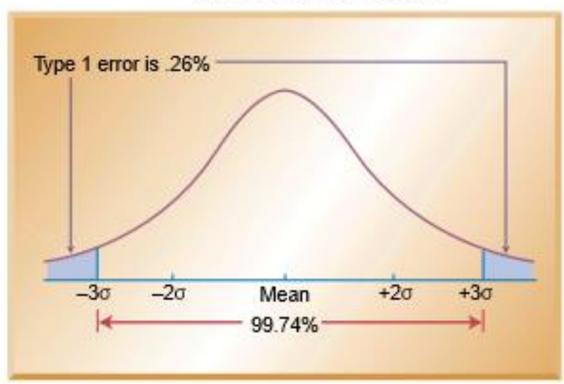


FIGURE 6-5 Chance of Type I error for $\pm 3\sigma$ (sigma-standard deviations)



Types of Control Charts

Control charts are one of the most commonly used tools in statistical process control. They can be used to measure any characteristic of a product, such as the weight of a cereal box, the number of chocolates in a box, or the volume of bottled water. The different characteristics that can be measured by control charts can be divided into two groups: variables and attributes. A control chart for variables is used to monitor characteristics that can be measured and have a continuum of values, such as height, weight, or volume. A soft drink bottling operation is an example of a variable measure, since the amount of liquid in the bottles is measured and can take on a number of different values. Other examples are the weight of a bag of sugar, the temperature of a baking oven, or the diameter of plastic tubing.

A control chart for attributes, on the other hand, is used to monitor characteristics that have discrete values and can be counted. Often they can be evaluated with a simple yes or no decision. Examples include color, taste, or smell. The monitoring of attributes usually takes less time than that of variables because a variable needs to be measured (e.g., the bottle of soft drink contains 15.9 ounces of liquid). An attribute requires only a single decision, such as yes or no, good or bad, acceptable or unacceptable (e.g., the apple is good or rotten, the meat is good or stale, the shoes have a defect or do not have a defect, the light bulb works or it does not work) or counting the number of defects (e.g., the number of broken cookies in the box, the number of dents in the car, the number of barnacles on the bottom of a boat). Statistical process control is used to monitor many different types of variables and attributes.

CONTROL CHARTS FOR VARIABLES

Control charts for variables monitor characteristics that can be measured and have a continuous scale, such as height, weight, volume, or width. When an item is inspected, the variable being monitored is measured and recorded. For example, if we were producing candles, height might be an important variable. We could take samples of candles and measure their heights. Two of the most commonly used control charts for variables monitor both the central tendency of the data (the mean) and the variability of the data (either the standard deviation or the range). Note that each chart monitors a different type of information. When observed values go outside the control limits, the process is assumed not to be in control. Production is stopped, and employees attempt to identify the cause of the problem and correct it.

A mean control chart is often referred to as an x-bar chart. It is used to monitor changes in the mean of a process. To construct a mean chart we first need to construct the center line of the chart. To do this we take multiple samples and compute their means. Usually these samples are small, with about four or five observations. Each sample has its own mean, $\bar{x}$. The center line of the chart is then computed as the mean of all K sample means, where K is the number of samples:

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots \bar{x}_K}{K}$$

To construct the upper and lower control limits of the chart, we use the following formulas:

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

where $\bar{\bar{x}}$ = the average of the sample means

z = standard normal variable (2 for 95.44% confidence, 3 for 99.74% confidence)

$\sigma_{\bar{x}}$ = standard deviation of the distribution of sample means, computed as $\sigma/\sqrt{n}$

σ = population (process) standard deviation

n = sample size (number of observations per sample)

Example 6.1 shows the construction of a mean (x-bar) chart.

A quality control inspector at the Cocoa Fizz soft drink company has taken twenty-five samples with four observations each of the volume of bottles filled. The data and the computed means are shown in the table. If the standard deviation of the bottling operation is 0.14 ounces, use this information to develop control limits of three standard deviations for the bottling operation.

Sample Number	Observations (bottle volume in ounces) 1	2	3	4	Average $\bar{x}$	Range R
1	15.85	16.02	15.83	15.93	15.91	0.19
2	16.12	16.00	15.85	16.01	15.99	0.27
3	16.00	15.91	15.94	15.83	15.92	0.17
4	16.20	15.85	15.74	15.93	15.93	0.46
5	15.74	15.86	16.21	16.10	15.98	0.47
6	15.94	16.01	16.14	16.03	16.03	0.20
7	15.75	16.21	16.01	15.86	15.96	0.46
8	15.82	15.94	16.02	15.94	15.93	0.20
9	16.04	15.98	15.83	15.98	15.96	0.21
10	15.64	15.86	15.94	15.89	15.83	0.30
11	16.11	16.00	16.01	15.82	15.99	0.29
12	15.72	15.85	16.12	16.15	15.96	0.43
13	15.85	15.76	15.74	15.98	15.83	0.24
14	15.73	15.84	15.96	16.10	15.91	0.37
15	16.20	16.01	16.10	15.89	16.05	0.31
16	16.12	16.08	15.83	15.94	15.99	0.29
17	16.01	15.93	15.81	15.68	15.86	0.33
18	15.78	16.04	16.11	16.12	16.01	0.34
19	15.84	15.92	16.05	16.12	15.98	0.28
20	15.92	16.09	16.12	15.93	16.02	0.20
21	16.11	16.02	16.00	15.88	16.00	0.23
22	15.98	15.82	15.89	15.89	15.90	0.16
23	16.05	15.73	15.73	15.93	15.86	0.32
24	16.01	16.01	15.89	15.86	15.94	0.15
25	16.08	15.78	15.92	15.98	15.94	0.30
Total					398.75	7.17

Solution

The center line of the control data is the average of the samples:

$$\bar{\bar{x}} = \frac{398.75}{25}$$

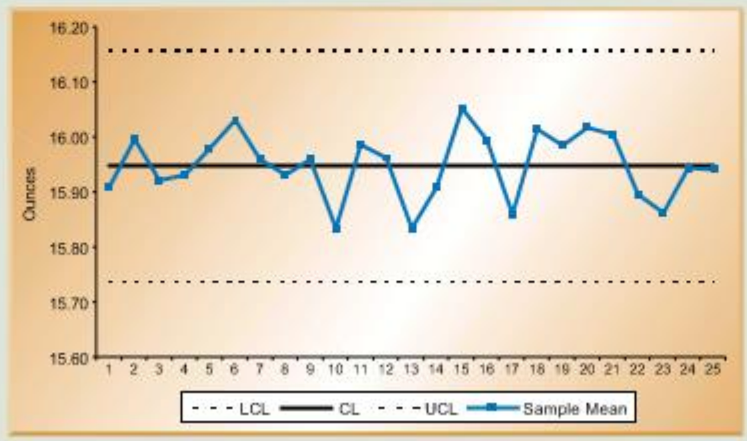
$$\bar{\bar{x}} = 15.95$$

The control limits are

$$\text{UCL} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 15.95 + 3\left(\frac{.14}{\sqrt{4}}\right) = 16.16$$

$$\text{LCL} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 15.95 - 3\left(\frac{.14}{\sqrt{4}}\right) = 15.74$$

The resulting control chart is:



Another way to construct the control limits is to use the sample range as an estimate of the variability of the process. Remember that the range is simply the difference between the largest and smallest values in the sample. The spread of the range can tell us about the variability of the data. In this case control limits would be constructed as follows:

$$\text{Upper control limit (UCL)} = \overline{\overline{x}} + A_2\overline{R}$$

$$\text{Lower control limit (LCL)} = \overline{\overline{x}} - A_2\overline{R}$$

where $\overline{\overline{x}}$ = average of the sample means
$\overline{R}$ = average range of the samples
A_2 = factor obtained from Table 6-1.

Notice that A_2 is a factor that includes three standard deviations of ranges and is dependent on the sample size being considered.

Range (R) Charts

Range (R) charts are another type of control chart for variables. Whereas x-bar charts measure shift in the central tendency of the process, range charts monitor the dispersion or variability of the process. The method for developing and using R-charts is the same as that for x-bar charts. The center line of the control chart is the average range, and the upper and lower control limits are computed as follows:

$$\text{CL} = \overline{R}$$

$$\text{UCL} = D_4\overline{R}$$

$$\text{LCL} = D_3\overline{R}$$

where values for D_4 and D_3 are obtained from Table 6-1.

TABLE 6-1

Factors for three-sigma control limits of $\bar{x}$ and R-charts

Source: Factors adapted from the *ASTM Manual on Quality Control of Materials.*

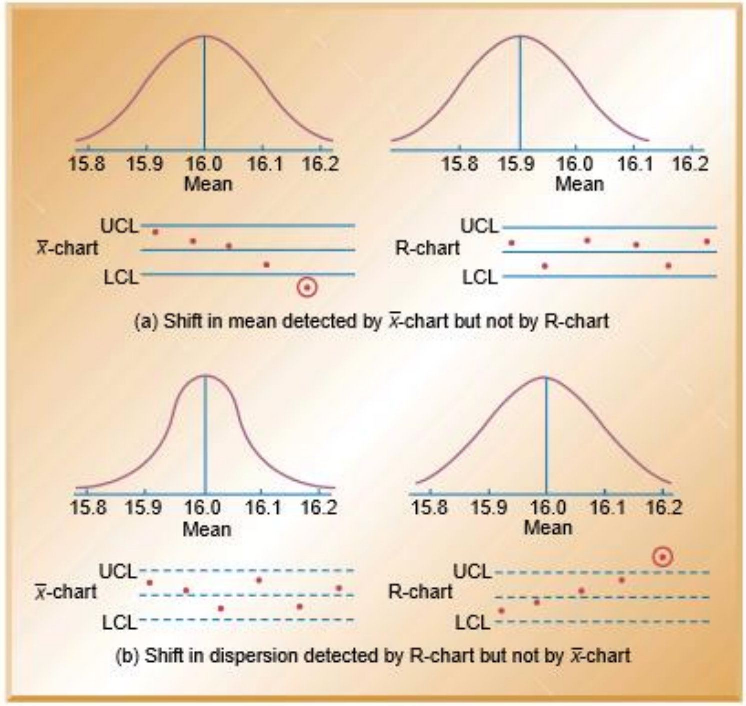
Sample Size n	Factor for $\bar{x}$-Chart A_2	Factors for R-Chart D_3	D_4
2	1.88	0	3.27
3	1.02	0	2.57
4	0.73	0	2.28
5	0.58	0	2.11
6	0.48	0	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65
16	0.21	0.36	1.64
17	0.20	0.38	1.62
18	0.19	0.39	1.61
19	0.19	0.40	1.60
20	0.18	0.41	1.59
21	0.17	0.43	1.58
22	0.17	0.43	1.57
23	0.16	0.44	1.56
24	0.16	0.45	1.55
25	0.15	0.46	1.54

Using Mean and Range Charts Together

The mean and range charts are used to monitor different variables. The mean or x-bar chart measures the central tendency of the process, whereas the range chart measures the dispersion or variance of the process. Since both variables are important, it makes sense to monitor a process using both mean a range charts. It is possible to have a shift in the mean of the product but not a change in the dispersion. For example, at the Cocoa Fizz bottling plant the machine setting can shift so that the average bottle filled contains not 16.0 ounces, but 15.9 ounces of liquid. The dispersion could be the same, and this shift would be detected by an x-bar chart but not by a range chart. This is shown in part (a) of Figure 6-6. On the other hand, there could be a shift in the dispersion of the product without a change in the mean. Cocoa Fizz may still be producing bottles with an average fill of 16.0 ounces. However, the dispersion of the product may have increased, as shown in part (b) of Figure 6-6. This condition would be detected by a range chart but not by an x-bar chart. Because a shift in either the mean or the range means that the process is out of control, it is important to use both charts to monitor the process.

FIGURE 6-6

Process shifts captured by $\bar{x}$-charts and R-charts



CONTROL CHARTS FOR ATTRIBUTES

Control charts for attributes are used to measure quality characteristics that are counted rather than measured. Attributes are discrete in nature and entail simple yes-or-no decisions. For example, this could be the number of nonfunctioning light bulbs, the proportion of broken eggs in a carton, the number of rotten apples, the number of scratches on a tile, or the number of complaints issued. Two of the most common types of control charts for attributes are p-charts and c-charts. P-charts are used to measure the proportion of items in a sample that are defective. Examples are the proportion of broken cookies in a batch and the proportion of cars produced with a misaligned fender. P-charts are appropriate when both the number of defectives measured and the size of the total sample can be counted. A proportion can then be computed and used as the statistic of measurement. C-charts count the actual number of defects. For example, we can count the number of complaints from customers in a month, the number of bacteria on a petri dish, or the number of barnacles on the bottom of a boat. However, we cannot compute the proportion of complaints from customers, the proportion of bacteria on a petri dish, or the proportion of barnacles on the bottom of a boat.

P-charts

P-charts are used to measure the proportion that is defective in a sample. The computation of the center line as well as the upper and lower control limits is similar to the computation for the other kinds of control charts. The center line is computed as the average proportion defective in the population, . This is obtained by taking a number of samples of observations at random and computing the average value of p across all samples. To construct the upper and lower control limits for a p-chart, we use the following formulas:

$$\text{UCL} = \bar{p} + z\sigma_p$$
$$\text{LCL} = \bar{p} - z\sigma_p$$

where z = standard normal variable
$\bar{p}$ = the sample proportion defective
σ_p = the standard deviation of the average proportion defective

As with the other charts, z is selected to be either 2 or 3 standard deviations, depending on the amount of data we wish to capture in our control limits. Usually, however, they are set at 3.

The sample standard deviation is computed as follows:

$$\sigma_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

where n is the sample size.

C-charts

A control chart used to monitor the number of defects per unit.
C-charts are used to monitor the number of defects per unit. Examples are the number of returned meals in a restaurant, the number of trucks that exceed their weight limit in a month, the number of discolorations on a square foot of carpet, and the number of bacteria in a milliliter of water. Note that the types of units of measurement we are considering are a period of time, a surface area, or a volume of liquid. The average number of defects is the center line of the control chart. The upper and lower control limits are computed as follows:

$$\text{UCL} = \bar{c} + z\sqrt{\bar{c}}$$
$$\text{LCL} = \bar{c} - z\sqrt{\bar{c}}$$

Acceptance Sampling

Acceptance sampling, the third branch of statistical quality control, refers to the process of randomly inspecting a certain number of items from a lot or batch in order to decide whether to accept or reject the entire batch. What makes acceptance sampling different from statistical process control is that acceptance sampling is performed either before or after the process, rather than during the process. Acceptance sampling before the process involves sampling materials received from a supplier, such as randomly inspecting crates of fruit that will be used in a restaurant, boxes of glass dishes that will be sold in a department store, or metal castings that will be used in a machine shop. Sampling after the process involves sampling finished items that are to be shipped either to a customer or to a distribution center. Examples include randomly testing a certain number of computers from a batch to make sure they meet operational requirements, and randomly inspecting snowboards to make sure that they are not defective. You may be wondering why we would only inspect some items in the lot and not the entire lot. Acceptance sampling is used when inspecting every item is not physically possible or would be overly expensive, or when inspecting a large number of items would lead to errors due to worker fatigue. This last concern is especially important when a large number of items are processed in a short period of time. Another example of when acceptance sampling would be used is in destructive testing, such as testing eggs for salmonella or vehicles for crash testing. Obviously, in these cases it would not be helpful to test every item! However, 100 percent inspection does make sense if the cost of inspecting an item is less than the cost of passing on a defective item. As you will see in this section, the goal of acceptance sampling is to determine the criteria for acceptance or rejection based on the size of the lot, the size of the sample, and the level of confidence we wish to attain. Acceptance sampling can be used for both attribute and variable measures, though it is most commonly used for attributes. In this section we will look at the different types of sampling plans and at ways to evaluate how well sampling plans discriminate between good and bad lots.

Sampling Plans

A sampling plan is a plan for acceptance sampling that precisely specifies the parameters of the sampling process and the acceptance/rejection criteria. The variables to be specified include the size of the lot (N), the size of the sample inspected from the lot (n), the number of defects above which a lot is rejected (c), and the number of samples that will be taken. There are different types of sampling plans. Some call for single sampling, in which a random sample is drawn from every lot. Each item in the sample is examined and is labeled as either "good" or "bad." Depending on the number of defects or "bad" items found, the entire lot is either accepted or rejected. For example, a lot size of 50 cookies is evaluated for acceptance by randomly inspecting 10 cookies from the lot. The cookies may be inspected to make sure they are not broken or burned. If 4 or more of the 10 cookies inspected are bad, the entire lot is rejected. In this example, the lot size N = 50, the sample size n = 10, and the maximum number of defects at which a lot is accepted is c = 4.

These parameters define the acceptance sampling plan. Another type of acceptance sampling is called double sampling. This provides an opportunity to sample the lot a second time if the results of the first sample are inconclusive.

In double sampling we first sample a lot of goods according to preset criteria for definite acceptance or rejection. However, if the results fall in the middle range, they are considered inconclusive and a second sample is taken. For example, a water treatment plant may sample the quality of the water ten times in random intervals throughout the day. Criteria may be set for acceptable or unacceptable water quality, such as .05 percent chlorine and .1 percent chlorine. However, a sample of water containing between .05 percent and .1 percent chlorine is inconclusive and calls for a second sample of water. In addition to single and double-sampling plans, there are multiple sampling plans. Multiple sampling plans are similar to double sampling plans except that criteria are set for more than two samples. The decision as to which sampling plan to select has a great deal to do with the cost involved in sampling, the time consumed by sampling, and the cost of passing on a defective item. In general, if the cost of collecting a sample is relatively high, single sampling is preferred. An extreme example is collecting a biopsy from a hospital patient. Because the actual cost of getting the sample is high, we want to get a large sample and sample only once. The opposite is true when the cost of collecting the sample is low but the actual cost of testing is high. This may be the case with a water treatment plant, where collecting the water is inexpensive but the chemical analysis is costly. In this section we focus primarily on single sampling plans.

Operating Characteristic (OC) Curves

Different sampling plans have different capabilities for discriminating between good and bad lots. At one extreme is 100 percent inspection, which has perfect discriminating power. However, as the size of the sample inspected decreases, so does the chance of accepting a defective lot. We can show the discriminating power of a sampling plan on a graph by means of an operating characteristic (OC) curve. This curve shows the probability or chance of accepting a lot given various proportions of defects in the lot. Figure 6-11 shows a typical OC curve. The x axis shows the percentage of items that are defective in a lot. This is called "lot quality." The y axis shows the probability or chance of accepting a lot. You can see that if we use 100 percent inspection we are certain of accepting only lots with zero defects. However, as the proportion of defects in the lot increases, our chance of accepting the lot decreases. For example, we have a 90 percent probability of accepting a lot with 5 percent defects and an 80 percent probability of accepting a lot with 8 percent defects. Regardless of which sampling plan we have selected, the plan is not perfect. That is, there is still a chance of accepting lots that are "bad" and rejecting "good" lots. The steeper the OC curve, the better our sampling plan is for discriminating between "good" and "bad." Figure 6-12 shows three different OC curves, A, B, and C. Curve A is the most discriminating and curve C the least. You can see that the steeper the slope of the curve, the more discriminating is the sampling plan. When 100 percent inspection is not possible, there is a certain amount of risk for consumers in accepting defective lots and a certain amount of risk for producers in rejecting good lots. There is a small percentage of defects that consumers are willing to accept. This is called the acceptable quality level (AQL) and is generally in the order of 1–2 percent. However, sometimes the percentage of defects that passes through is higher than the AQL. Consumers will usually tolerate a few more defects, but at some point the number of defects reaches a threshold level beyond which consumers will not tolerate them. This threshold level is called the lot tolerance percent defective (LTPD). The LTPD is the upper limit of the percentage of defective items consumers are willing to tolerate. Consumer's risk is the chance or probability that a lot will be accepted that contains a greater number of defects than the LTPD limit. This is the probability of making a Type II error—that is, accepting a lot that is truly "bad." Consumer's risk or Type II error is generally denoted by beta (β). The relationships among AQL, LTPD, and are shown in Figure 6-13. Producer's risk is the chance or probability that a lot containing an acceptable quality level will be rejected. This is the probability of making a Type I error—that is, rejecting a lot that is "good." It is generally denoted by alpha (α). Producer's risk is also shown in Figure 6-13. We can determine from an OC curve what the consumer's and producer's risks are. However, these values should not be left to chance. Rather, sampling plans are usually designed to meet specific levels of consumer's and producer's risk. For example, one common combination is to have a consumer's risk (β) of 10 percent and a producer's risk (α) of 5 percent, though many other combinations are possible.

Formula Review

1. Mean $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

2. Standard Deviation $\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}$

3. Control Limits for x-Bar Charts
   Upper control limit (UCL) $= \bar{\bar{x}} + z\sigma_{\bar{x}}$
   Lower control limit (LCL) $= \bar{\bar{x}} - z\sigma_{\bar{x}}$
   $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

4. Control Limits for x-Bar Charts Using Sample Range as an Estimate of Variability
   Upper control limit (UCL) $= \bar{\bar{x}} + A_2\bar{R}$
   Lower control limit (LCL) $= \bar{\bar{x}} - A_2\bar{R}$

5. Control Limits for R-Charts
   $UCL = D_4\bar{R}$
   $LCL = D_3\bar{R}$

6. Control Limits for p-Charts
   $UCL = \bar{p} + z(\sigma_p)$
   $LCL = \bar{p} - z(\sigma_p)$

7. Control Limits for c-Charts
   $UCL = \bar{c} + z\sqrt{\bar{c}}$
   $LCL = \bar{c} + z\sqrt{\bar{c}}$

8. Measures for Process Capability
   $$C_p = \frac{\text{specification width}}{\text{process width}} = \frac{USL - LSL}{6\sigma}$$
   $$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$

9. Average Outgoing Quality $AOQ = (P_{ac})p$