Aliah University

Department of Mathematics and Statistics

Syllabus for

2 Year M.Sc. in Mathematics (CBCS)

2021

2 Year M.Sc. in Mathematics

Programme Outcomes (PO)

At the end of the programme, the students will be able to do the following:

PO1: Apply knowledge of mathematics in different field of science, engineering and technology.

PO2: To formulate and develop mathematical arguments.

PO3: To acquire knowledge and understanding in advanced areas of mathematics, chosen by the student from the courses available.

PO4: Understand the fundamental axioms in mathematics and grow capability of developing ideas based on them.

PO5: Understand, formulate and use mathematical models arising in science, technology and other related areas.

Programme Specific Outcomes (PSO)

PSO1: Prepare and motivate students for research studies in mathematics and allied disciplines.

PSO2: Provide knowledge of a wide range of mathematical techniques and application of mathematical methods/ tools in other scientific and engineering fields.

PSO3: Provide advanced knowledge on topics in mathematics, empowering the students to pursue higher studies at academic institutions of national and international importance.

PSO4: Strong foundation on different areas of mathematics which have strong links and applications in real life.

PSO5: Nurture problem solving skills, thinking, and creativity through assignments, project and research works.

PSO6: Assist students to make them competent for competitive exams, e.g., JAM, NET, GATE, SET, etc.

Semester – I

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATPGCCT01	Groups, Rings and Modules	Core	4	4	50	In Page 4
2	MATPGCCT02	Topology	Core	4	4	50	In Page 5
3	MATPGCCT03	Measure and Probability	Core	4	4	50	In Page 6
4	MATPGCCT04	ODE & Integral Equations	Core	4	4	50	In Page 7
5	MATPGCCT05	Partial Differential Equations	Core	4	4	50	In Page 8
6	PGAUC01	Elementary Arabic and Islamic Studies	AUC	0	4	50	–
Total				20	24	300	

Semester – II

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATPGCCT06	Linear Algebra and Field Extension	Core	4	4	50	In Page 9
2	MATPGCCT07	Complex Analysis	Core	4	4	50	In Page 11
3	MATPGCCT08	Functional Analysis	Core	4	4	50	In Page 12
4	MATPGCCT09	Classical Mechanics and Calculus of Variations	Core	4	4	50	In Page 13
5	MATPGCCT10	Continuum Mechanics & Control Theory or Graph Theory & Geometry of Curves and Surfaces	Core	4	4	50	In Page 15
6	PGAEC01	Disaster Management/ Human Rights & Value Education/ Yoga & Life Skills	AEC	0	4	50	–
Total				20	24	300	

Semester – III

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATPGCCT11	Numerical Analysis	Core	4	4	50	In Page 20
2	MATPGCCT12	Optimization Techniques & Integral Transform	Core	4	4	50	In Page 21
3	MATPGDET01	Any Course from List A1 or List B1	DSE	4	4	50	–
4	MATPGDET02	Another Course from List A1 or List B1†	DSE	4	4	50	–
5	.....PGGEC01	To be chosen from other Master's Programme or from Courses offered by other Departments	GE	4	4	50	–
Total				20	20	250	

† Both Courses to be chosen from the same list.

Semester – IV

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATPGCCP01	Computational Lab	Core	4	4	50	In Page 23
2	MATPGDET03	Any Course from List A2 or List B2	DSE	4	4	50	–
3	MATPGDET04	Another Course from List A2 or List B2‡	DSE	4	4	50	–
4	.....PGGEC02	To be chosen from other Master's Programme or from Courses offered by other Departments	GE	4	4	50	–
5	MATPGPRJ01	Project & Dissertation	Project	4	4	50	–
Total				20	20	250	

‡ Both Courses to be chosen from the same list.

List of DSE Courses

List A1

Sl. No.	Course Title	Contents
1	Operator Algebra – I	In Page 24
2	Commutative Algebra – I	In Page 25
3	Operator Theory – I	In Page 26
4	Advanced Topology – I	In Page 27
5	Advanced Complex Analysis – I	In Page 28
6	Advanced Differential Geometry – I	In Page 30
7	Algebraic Graph Theory – I	In Page 31

List B1

Sl. No.	Course Title	Contents
1	Dynamical Systems – I	In Page 32
2	Solid Mechanics – I	In Page 33
3	Mathematical Biology – I	In Page 35
4	Fluid Mechanics – I	In Page 36
5	Operations Research – I	In Page 38
6	Financial Mathematics – I	In Page 38
7	Difference Equations in Ecology and Eco-epidemiology	In Page 39

List A2

Sl. No.	Course Title	Contents
1	Operator Algebra – II	In Page 41
2	Commutative Algebra – II	In Page 42
3	Operator Theory – II	In Page 43
4	Advanced Topology – II	In Page 43
5	Advanced Complex Analysis – II	In Page 44
6	Advanced Differential Geometry – II	In Page 46
7	Algebraic Graph Theory – II	In Page 46

List B2

Sl. No.	Course Title	Contents
1	Dynamical Systems – II	In Page 47
2	Solid Mechanics – II	In Page 49
3	Mathematical Biology – II	In Page 50
4	Fluid Mechanics – II	In Page 52
5	Operations Research – II	In Page 53
6	Financial Mathematics – II	In Page 54
7	Lie Theory of Ordinary and Partial Differential Equations	In Page 55

MATPGCCT01 (Groups, Rings and Modules)

Learning Objectives

The student will –

- Revise their knowledge on group actions and Sylow's theorems in Group Theory; and polynomial rings, EDs and PIDs in Ring Theory.
- Study solvable and nilpotent groups, semidirect products and free groups.
- Gain knowledge on the basic theory for noetherian and artinian rings, rings of fractions and the Jacobson radical.
- Understand and investigate basic results in module theory, know constructions like tensor product.

Course Outcomes

Upon successful completion of this course, the students will –

- Develop a good understanding of groups and rings, and to be able to prove important theorems and solve problems.
- Have an insight of module theory as vector spaces over rings having a well developed theory.

Course Details

Unit I: Group Theory

Review of Group actions, p-groups, Sylow's theorems and consequences, Cauchy's theorem, Simplicity of A_n for $n \geq 5$, non-simplicity tests. Structure theorem for finite Abelian groups, structure theorem for finitely generated Abelian groups. Normal and subnormal series, composition series, Jordan–Hölder theorem, solvable groups and nilpotent groups. Semi direct products. Free groups. [14 – 16 Lectures]

Unit II: Ring Theory

Review of Polynomial rings, factorization of polynomials, irreducibility tests. Ring embeddings. Euclidean domains, Principal ideal domains, Unique factorization domains. Noetherian and Artinian rings, Hilbert Basis Theorem, Primary decomposition in Noetherian rings. Ring of fractions, Extension and contraction of ideals. The Jacobson radical, Jacobson semisimple ring. [20 – 24 Lectures]

Unit III: Module Theory

Modules: Basic definitions and examples, quotient modules and module homomorphisms.
Generation of modules, direct sum and free modules.
Tensor product of modules.
Exact sequences, Projective, injective and flat modules. [16 – 20 Lectures]

References

[1] D. S. Malik, J. N. Mordeson, M. K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill International Editions, 1997.

[2] D.S. Dummit, R. M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 1999.

[3] N.S. Gopalakrishnan, University Algebra, New Age International Pvt. Ltd., 2018.

[4] T. W. Hungerford, Algebra, Springer, 1980.

[5] I. N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.

[6] J.A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

[7] M. Artin, Algebra, Prentice Hall of India,1994.

[8] S. Lang, Algebra I, 3rd Ed., Addison Wesley, 2005.

[9] G. Birkhoff, S. Mac Lane, A Survey of Modern Algebra, Taylor and Francis, 2008.

[10] J. Rotman, An Introduction to the Theory of Groups, Springer-Verlag, 1990.

MATPGCCT02 (Topology)

Learning Objectives

- This course is an introduction to topological spaces.
- It deals with constructions like subspaces, product spaces, and quotient spaces, and properties like completeness, compactness and connectedness.

Course Outcomes

Upon successful completion of this course, the students –

- can work with sets and functions, images and preimages, and can distinguish between finite, countable, and uncountable sets.
- know how the topology on a space is determined by the collection of open sets, by the collection of closed sets, or by a basis of neighbourhoods at each point, and know what it means for a function to be continuous.
- know the definition and basic properties of connected spaces, path connected spaces, compact spaces, and locally compact spaces.
- know what it means for a metric space to be complete, and can characterize compact metric spaces.
- become familiar with the Urysohn lemma and the Tietze extension theorem, and can characterize metrizable spaces.

Course Details

Definition and examples of topological spaces. Basis and subbasis for a topology. Interior, closure and boundary of a set. The order Topology. Subspace and relative topology. Closed sets and Limit points. Hausdorff Spaces. Continuous Functions and topological properties. Construction of continuous functions. The Product topology. Metric topology and Uniform limit theorem. The quotient topology and quotient maps. Homeomorphisms. [24 – 30 Lectures]

Connectedness and Compactness: Connected spaces and its properties. Connected subspaces of the Real line .Components and local Connectedness. Compact spaces and their properties. Compact subspaces of the Real line. Limit point compactness. Sequentially compact spaces. Local compactness and properties. Countability and Separation Axioms. Ascoli-Arzela theorem, equicontinuity. [24 – 30 Lectures]

References

[1] C. Adams and R. Franzosa, Introduction to Topology: Pure and Applied, Pearson, 2009.

[2] J.R. Munkres, A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

[3] J. Dugundji, Topology, Allyn and Bacon, 1966.

[4] G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.

[5] J.L. Kelley,General Topology, Van Nostrand Reinhold Co., New York, 1995.

[6] J. Hocking, G. Young, Topology, Addison-Wesley Reading, 1961.

[7] L. Steen, J. Seebach, Counterexamples in Topology, Holt, Reinhart and Winston, New York, 1970.

Note: This course is based on the Book [1], Chapters 1 – 7.

MATPGCCT03 (Measure and Probability)

Learning Objectives

In this course students are expected to be able to:

- know about the measurable set and the measurable function.
- understand Lebesgue Integral and compare it with Riemann Integral.
- learn Probability Theory from the viewpoint of Measure Theory.

Course Outcomes

Upon successful completion of the course, a student will –

- acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- demonstrate capacity to solve problems and communicate mathematical ideas efficiently.
- be able to define measures in a probability space and to apply those to solve problems in probability.

Course Details

Unit I: Fundamentals of Measure and Integration Theory
Fields, σ-Fields and Measures, Extension of Measures, Lebesgue-Stieltjes Measures and Distribution Functions, Measurable Functions and Integration (Basic Integration Theorems). [14 – 18 Lectures]

Unit II: Results on Measure and Integration Theory
Comparison of Lebesgue and Riemann Integrals, Radon-Nikodym Theorem and related results, L^p Spaces, Convergence of sequences of measurable functions, Product measures and Fubini's Theorem, Measures on Infinite Product Spaces, Weak Convergence of measures. [16 – 20 Lectures]

Unit III: Review of Probability Theory
Discrete Probability Spaces, Independence, Bernoulli Trials, Conditional Probability, Random variables and Random vector, Expectation. Application of Conditional probability and expectation, Conditional Expectation given a σ-Field, Properties of Conditional expectation and regular, conditional probability. [10 – 12 Lectures]

Unit IV: Martingle Theory and Central Limit Theorem
Strong laws of Large numbers and Martingle Theory (Concepts only upto Uniform integrability and optional sampling theory), Central limit Theorem. [8 – 10 Lectures]

References

[1] Robert B. Ash, Catherine A. Doleans-Dade, Probability and Measure Theory, 2nd Edition, Academic Press, 1999.

[2] P. Billingsley, Probability and Measure, 3rd Edition, Wiley, 2008.

[3] W. Feller, Introduction to Probability Theory and Its Applications: Vol. 1 and 2, 2nd Edition, Wiley, 1957.

[4] G. De Barra, Measure Theory and Integration, 2nd Edition, New Age International Publishers, 2013.

[5] P.R. Halmos, Measure Theory, 2nd Edition, Springer, 2008.

[6] H.L. Royden, P. Fitzpatrick, Real Analysis, 4th Edition, Pearson India, 2009.

[7] A. Gut, Probability: A Graduate Course, 2nd Edition, Springer, 2007.

[8] R.G. Laha and V.K. Rohatgi, Probability Theory, 1st Edition, Dover Publications Inc, 2020.

[9] W. Rudin, Principles of Mathematical Analysis, 3rd Edition, McGraw-Hill Education, 1976.

Note: This course is based on the Book [1], Chapters 1 – 7.

MATPGCCT04 (ODE & Integral Equations)

Learning Objectives

- The objective of the course is to give students a good basic understanding of the theory and methods of finding solutions for differential equations and integral equations and to introduce them to applications in modelling physical phenomena and mechanical systems.
- The course will provide both a comprehensive foundation of basic theory of differential and integral equations and practice for a broad range of subjects in Science & Technology.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- find solutions to certain classes of linear differential equations.
- determine solutions of a systems oflinear differential equations by using eigenvalues and eigenvectors.
- find the solutions of a certain classes of nonlinear differential equations by applying the linearization technique.
- classify different kind of integral equations and to solve them.

Course Details

First order system of equations: The fundamental existence and uniqueness theory, simple illustrations. Dependence of solution on initial conditions. Local stability analysis, Peano's and Picard's theorems. [6 – 8 Lectures]

Sturm's theory: Some basic results on Sturm's theory. Sturm's separation theorem, Sturm's comparison theorem. [2 – 4 Lectures]

Ordinary point and singularity of a second order linear differential equation in the complex plane. Classification of singularities of a second order differential equation. Solution about an ordinary point, solution of Hermite equation as an example. Regular singularity, Frobenius' method – solution about a regular singularity of second order differential equations. [10 – 12 Lectures]

Legendre polynomial: Generating function; Rodrigue's formula, recurrence relations and differential equations satisfied by it; Its orthogonality, expansion of a function in a series of Legendre Polynomials. Bessel's functions and its orthogonality. Legendre and Bessel Fourier series. [8 – 10 Lectures]

Green's function. [4 – 6 Lectures]

Integral Equations: Introductory concepts: Definition, Fredholm and Volterra integral equations. Non-linear Equations, Singular equations, Integro-differential equations. Relations between differential and integral equations. [4 – 6 Lectures]

Fredholm integral equation: Equations of the first and second Kind. Solution with separable kernels. [6 Lectures]

Volterra integral equation: First and second type. Solution with separable kernels. [4 Lectures]
Characteristic numbers and eigenfunctions. Resolvent kernel. [4 Lectures]

References

[1] J.C. Burkill, The Theory of Ordinary Differential Equations, 3rd edition, Prentice Hall Press, 1975.

[2] E.A. Codington, N. Levinson, Theory of Ordinary Differential Equations, 3rd edition, Krieger Publishing Company, 1984.

[3] G.F. Simmons, Steven G. Krantz, Differential Equations, Indian edition, McGraw Hill Education, 2017.

[4] G. Birkhoff, G-C. Rota, Ordinary Differential Equations, 4th edition, Willey India, 2016.

[5] N.N. Lebedev, Special Functions and Their Applications, 2nd edition, Dover Publications Inc., 2003.

[6] S.M. Jemyan, The Classical Theory of Integral Equations, 12th edition, Birkhauser, 2012.

[7] W.V. Lovitt, Linear Integral Equations, 2nd edition, Dover Publications Inc., 2005.

[8] F.G. Tricomi, Integral Equations, 2nd edition, Dover Publications Inc., 1985.

[9] A.M. Wazwaz: A First Course in Integral Equations, 2nd edition, World Scientific, 2015.

MATPGCCT05 (Partial Differential Equations)

Learning Objectives

- The main objective of this course is to introduce concepts and techniques for the solution of partial differential equations (PDEs).
- In particular, the course aims to present the method of characteristics for first-order PDEs.
- The theory of separation of variables for Laplace's and the linear wave equation in cylindrical and spherical polar coordinates.
- Green's function methods for the one-dimensional heat equation and two- and three-dimensional waves.

Course Outcomes

- Formulate and solve the Monge equations for a first order quasi-linear PDE in 2D. Calculate the earliest time at which solutions become multi-valued.
- Construct shock solutions and study their behaviour, in particular calculate shock speeds. Apply the theory to model specific problems in traffic flow.
- Apply separation of variables to construct solutions to Laplace's equation and the wave equation in cylindrical and spherical polar coordinates in terms of Bessel functions and Legendre polynomials.
- Formulate and solve initial value problems for the wave equation in 1D, 2D and 3D.

Course Details

Unit I: Classification of second order Partial Differential Equations (PDEs)

The Canonical forms for Hyperbolic, Parabolic and Elliptic equations. Existence, uniqueness and continuous dependence of the solution to the initial and boundary conditions. Dirichlets, Neumann's and mixed problems. Derivation of Laplace, Wave and Diffusion equation. Adjoint operator, Green's Divergence theorem, Lagrange's and Green's identities. [12 – 16 Lectures]

Unit II: Laplace equation

Fundamental solution. Mean value properties of solutions. Maximum-minimum principle and its consequences. Laplace equation in polar, Spherical polar and cylindrical polar coordinates. Standard methods of solution: Separation of variables (Fourier method). Stability theory. Method of solution: theory of Green's function method. [10 – 12 Lectures]

Unit III: Wave equation

Occurrence of wave equation in physics. Elementary solution of one-dimensional equation, Riemann method of solution, vibrating membrane: application of calculus of variation. Uniqueness of the solution. Non-homogeneous wave equation. Duhamel's principle. [8 – 10 Lectures]

Unit IV: Diffusion equation

Fundamental solution. Occurrence in physics. Elementary solution of diffusion equation. General solution, Cauchy Method of Characteristics. Maximum–minimum principle and consequences. Green's function method. Application in ecology. [8 – 10 Lectures]

Unit V: Boundary value problems of Dirichlet and Neumann

Dirichlet's principle, Dirichlet problem for a rectangle, The Neumann problem for a rectangle, Interior Dirichlet problem for a circle, Exterior Dirichlet problem for a circle, Interior Neumann problem for a circle, Green's function solution of Dirichlet's and Neumann's problem for sphere. [10 – 12 Lectures]

References

[1] I.N. Sneddon, Elements of Partial Differential Equations, Mcgraw Hill, 1957.

[2] F. John, Partial Differential Equations, Springer, New York, 2012.

[3] T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Pub. House, 2003.

[4] K.S. Rao, Partial differential equations. PHI publishers. 2011.

[5] Tyn Myint-U, L. Debnath. Linear Partial Differential Equations for Scientists and Engineers, Birkhäuser, Boston, 2007.

[6] P. Prasad, R. Ravindran, Partial Differential Equations, New Age International Publishers, 2011.

[7] L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, American Mathematical Society, 2010.

[8] W.E. Williams, Partial Differential Equations, Clarendon Press, 1980.

[9] F.H. Miller, Partial Differential Equations, John Wiley, 1941.

[10] I.G. Petrovsky, Lectures on Partial Differential Equations, Dover Publications, 2012.

[11] G.F.D. Duff, D. Naylor, Partial Differential Equations of Applied Mathematics. Willey, New York, 1966.

MATPGCCT06 (Linear Algebra and Field Extension)

Learning Objectives

The student will –

- understand the relationships among field extensions, polynomial rings and roots of polynomials.
- apply the above in problems involving straightedge and compass constructions.
- gain knowledge on splitting fields, separable extensions and cyclotomic polynomials.
- understand the basics of Galois theory.
- gain knowledge about important characteristics of matrices, its four fundamental subspaces.
- study rational canonical form and Jordan canonical form in perspective of modules over principal ideal domains.

Course Outcomes

Upon successful completion of this course, the students will –

- demonstrate proper use of advanced algebraic techniques in solving problems.
- develop a deep understanding of the concepts and theorems in modules over PID and field theory.
- apply knowledge acquired to solve problems and analyze examples.

Course Details

Unit I: Linear Algebra

Review of Inner Product Spaces, Quadratic forms and operators. [2 – 4 Lectures]

LU and LDV factorization of matrices. [2 – 4 Lectures]

Four Fundamental subspaces associated with a matrix, Fundamental Theorem of Linear Algebra, Its impact on system of equations and its solution; Hermitian, Self-Adjoint, Unitary and Orthogonal transformation for complex and real spaces. Bilinear and Quadratic forms. The spectral Theorem (for normal matrix/ normal operator). The structure of orthogonal transformations in real Euclidean spaces. [8 – 10 Lectures]

Finitely generated modules over PID. [2 – 4 Lectures]

Existence and Uniqueness theorems on Invariant factors, elementary divisors (proof may be omitted).

Companion matrix,Primary decomposition theorem, Rational Canonical form and Jordan Canonical form (through the R-module approach). Computational algorithm for these forms (upto 4×4 case) through similarity. [8 – 10 Lectures]

Unit II: Field Theory

Basic Theory of Field Extensions. [2 – 4 Lectures]

Algebraic Extensions. [4 – 6 Lectures]

Classical Straightedge and Compass Constructions. [2 – 4 Lectures]

Splitting Fields and Algebraic Closures. [4 – 6 Lectures]

Separable and Inseparable Extensions. [2 – 4 Lectures]

Cyclotomic Polynomials and Extensions. [1 – 2 Lectures]

Galois Theory: Fundamental Theorem of Galois Theory, Finite fields, Galois Groups of polynomials, Solvability of polynomials by radicals, Insolvability of a Quintic. [8 – 10 Lectures]

References

[1] K. Hoffman, R. Kunze, Linear Algebra, 2nd Ed., Prentice Hall of India, 1999.

[2] G. Strang, Introduction to Linear Algebra, 5th Ed., Wellesley-Cambridge Press and SIAM, 2016.

[3] A. Ramachandra Rao, P. Bhimasankaran, Linear Algebra, Hindustan Book Agency, 2000.

[4] V. Sahai, V. Bist, Linear Algebra, Alpha Science International Ltd., 2002.

[5] J.H. Kwak, S. Hong, Linear Algebra, 2nd Ed., Birkhuser, 2004.

[6] S.H. Friedberg, A.J. Insel, L.E. Spence, Linear Algebra, 4th Ed., Prentice Hall of India, 2003.

[7] D.S. Dummit, R.M. Foote, Abstract Algebra, 2nd Ed., John Wiley, 1999.

[8] D.S. Malik, J.N. Mordeson, M.K. Sen, Fundamentals of Abstract Algebra, McGraw-Hill International Editions, 1997.

[9] T.W. Hungerford, Algebra, Springer, 1980.

[10] J.A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa, 1999.

[11] I.N. Herstein, Topics in Algebra, Wiley Eastern Ltd. New Delhi, 1975.

[12] J. Rotman, Galois Theory, Springer, 2012.

[13] N.S. Gopalakrishnan, University Algebra, New Age International Pvt. Ltd., 2018.

MATPGCCT07 (Complex Analysis)

Learning Objectives

In this course students are expected to be able to:

- determine continuity/differentiability/analyticity of a function.
- evaluate a contour integral using parameterization, fundamental theorem of calculus and Cauchy's integral formula.
- find the Taylor's series of a function and determine its circle or annulus of convergence.
- compute the residue of a function and use the residue theorem to evaluate a contour integral or an integral over the real line.
- determine the number of zeros of an analytic function using Rouche's theorem.
- get some idea of normal families, Montel's theorem, Riemann mapping theorem.

Course Outcomes

Upon successful completion of this course, the students will –

- acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- demonstrate capacity to solve problems and communicate mathematical ideas efficiently.

Course Details

Unit I: Basics of Complex Valued Function

Functions of a complex variable, limit and continuity; differentiability, analytic functions, Cauchy-Riemann equations, sufficient condition for differentiability of a complex function; harmonic functions, conjugate harmonic functions, relation between analytic function and harmonic function; bilinear transformation; convergence of sequence and series of function and power series. multi-valued function, branch point, branch cut, conformal mapping, idea of analytic continuation. [10 – 12 Lectures]

Unit II: Complex Integration and Cauchy's Theorem

Complex integration; Cauchy's fundamental theorem (statement only) and its consequences; index of a closed curve, homotopy version of Cauchy's theorem; Cauchy's integral formula, derivative of an analytic function, Taylor's theorem, Morera's theorem, Cauchy's inequality, Liouville's theorem, Little Picard's theorem; zeros of an analytic function, fundamental theorem of classical algebra, uniqueness theorem, identity theorem; maximum modulus theorem, minimum modulus theorem, Schwarz's lemma. [20 – 24 Lectures]

Unit III: Classification of Singularities and Residue Theorem

Laurent series, classification of singularities; Riemann's theorem, Casorati-Weierstrass theorem; residue theorem, evaluation of some real integrals; argument principle, Rouche's theorem, open mapping theorem.

[16 – 18 Lectures]

Unit IV: Space of Continuous Functions

Space of continuous functions, normal families, Arzela-Ascoli theorem (statement only), compactness and convergence in the space of analytic functions, Montel's theorem (statement only), space of meromorphic functions, Riemann mapping theorem (statement only). [4 – 6 Lectures]

References

[1] S. Ponnusamy, H. Silverman, Complex Variable with Applications, 1st Edition, Birkhauser, 2006.

[2] J.W. Brown, R.V. Churchill, Complex Variables and Applications, 8th Ed., McGraw-Hill International Edition, 2009.

[3] T.W. Gamelin, Complex Analysis, 2nd Edition, Springer, 2013.

[4] J.B. Conway, Functions of one Complex Variable, 2nd Edition, Springer, 1995.

[5] L.V. Ahlfors, Complex Analysis, 3rd Edition, McGraw-Hill Education, 2017.

[6] S. Ponnusamy, Foundation of Complex Analysis, 2nd Edition, Narosa Publications, 2011.

[7] H.S. Kasana, Complex Variables: Theory and Application, 2nd Edition, Prentice Hall India Learning Private Ltd., 2005.

[8] E.T. Copson, An Introduction to the Theory of Functions of a Complex Variable, 1st Edition, Oxford University Press, 1970.

[9] E.M. Stein, R. Shakarchi, Complex Analysis, 1st Edition, Princeton University Press, 2013.

[10] S.G. Krantz, Complex Analysis: The Geometric Viewpoint, 2nd Edition, The Mathematical Association of America, 2004.

MATPGCCT08 (Functional Analysis)

Learning Objectives

Objective of the course is

- to introduce students to the ideas and some of the fundamental theorems of functional analysis.
- to show students the use of abstract algebraic/topological structures in studying spaces of functions.
- to allow students to taste the subject with a view to further work as a postgraduate.
- to give students a working knowledge of the basic properties of Banach spaces, Hilbert spaces and bounded linear operators.
- to show students the idea of duals and adjoints.
- to show students the value of looking at the spectrum of a bounded linear operator.
- to demonstrate significant applications of the theory of functional analysis.

Course Outcomes

Upon successful completion of this course, the students will –

- appreciate how functional analysis uses and unifies ideas from the vector spaces, theory of metrics, and complex analysis.
- understand and apply fundamental theorems from the theory of normed and Banach spaces, including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem, and the Stone-Weierstrass theorem.
- understand and apply ideas from the theory of Hilbert spaces to other areas, including Fourier series, Fredholm operators and wavelet analysis.

Course Details

Banach Spaces

Normed Linear Spaces, Banach Spaces, Equivalent Norms, Finite dimensional normed linear spaces and local compactness, Quotient Space of normed linear spaces and its completeness, Riesz Lemma, Fixed Point Theorems and its applications. Bounded Linear Transformations, Normed linear spaces of bounded linear transformations, Uniform Boundedness Theorem, Principle of Condensation of Singularities, Open Mapping Theorem, Closed Graph Theorem, Linear Functionals, Hahn-Banach Theorem, Dual Space, Reflexivity of Banach Spaces. [24 – 30 Lectures]

Hilbert Spaces

Real Inner Product Spaces and its Complexification, Cauchy-Schwarz Inequality, Parallelogram law, Pythagorean Theorem, Bessel's Inequality, Gram-Schmidt Orthogonalization Process, Hilbert Spaces, Orthonormal Sets, Complete Orthonormal Sets and Parseval's Identity, Structure of Hilbert Spaces, Orthogonal Complement and Projection Theorem. Riesz Representation Theorem, Adjoint of an Operator on a Hilbert Space, Reflexivity of Hilbert Spaces, Self-adjoint Operators, Positive Operators, Projection Operators, Normal Operators, Unitary Operators. Introduction to Spectral Properties of Bounded Linear Operators.

[24 – 30 Lectures]

References

[1] B.V. Limaye, Functional Analysis, New Age International Limited, Publishers, 2014.

[2] E. Kreyszig, Introductory Functional Analysis with Applications, Wiley, 2014.

[3] C.D. Aliprantis, O. Burkinshaw, Principles of Real Analysis, 3rd Edition, Harcourt Asia Pte Ltd., 1998.

[4] C. Goffman, G. Pedrick, First Course in Functional Analysis, Prentice Hall of India, New Delhi, 1987.

[5] G. Bachman, L. Narici, Functional Analysis, Academic Press, 1966.

[6] A.E. Taylor, Introduction to Functional Analysis, John Wiley and Sons, New York, 1958.

[7] G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.

[8] J.B. Conway, A Course in Functional Analysis, Springer Verlag, New York, 1990.

[9] Y. Eidelman, V. Milman, and A. Tsolomitis, Functional Analysis: An Introduction, American Mathematical Society, 2004.

Note: This course is based on the Book [1] and [2].

MATPGCCT09 (Classical Mechanics and Calculus of Variations)

Learning Objectives

The objective of the course is to enable the students

- to distinguish between 'inertial frame of reference' and 'non-inertial frame of reference'.
- to know how to impose constraints on a system in order to simplify the methods to be used in solving physics problems.
- to know the importance of concepts such as generalized coordinates and constrained motion.
- understand Poisson brackets, understand canonical transformations.
- to find the linear approximation to any dynamical system near equilibrium and also know how to derive and solve the wave equation for small oscillations.

Course Outcomes

Upon successful completion of this course, the students will –

- learn about Lagrangian and Hamiltonian formulation of Classical Mechanics.
- be able to state the conservation principles involving momentum, angular momentum and energy and understand that they follow from the fundamental equations of motion.
- have a deep understanding of Newton's laws.

Course Details

Fundamental of mechanics

Mechanics of a system of particles: Constraints; Generalized coordinates; Degrees of freedom; Virtual displacement and principle of virtual work; D'Alembert's principle; Generalized forces; Generalized momentum. [4 – 6 Lectures]

Rotating frames of reference

Frames of reference rotating with constant angular velocity; Coriolis forces; Virial Theorem. Two- and Three-Body Motions; Foucault's pendulum. [4 – 6 Lectures]

Lagrange's equation

Lagrangian; Lagrange's equations of motion; cyclic coordinates; Routh's process for the ignoration of co-ordinates; Conservative system; Natural System; Liouville's system; velocity dependent potential; Principle of energy; Rayleigh's dissipation function.

[10 – 12 Lectures]

Hamilton's equations

Calculus of variations and its applications in shortest distance, minimum surface of revolution, Euler-Lagrange equation, Brachistochrone problem, geodesic. Hamilton's principle. Lagrange's undetermined multipliers. Derivation of Hamilton's Equations; Hamiltonian; The form of the Hamiltonian function; Legendre transformation; Hamilton's principle from D'Alembert's principle; Lagrange's equations of motion from Hamilton's principle; Hamilton's equations of motion from Hamilton's principle. Principle of least action. Symmetry properties and conservation laws; Noether's theorem. [6 – 8 Lectures]

Canonical Transformations

Canonical coordinates and canonical transformations. Principal forms of Generating function; Lagrange's and Poisson's brackets and their relation under canonical transformations, Hamilton's equations of motion in Poisson's bracket. Canonical transformations in Poisson's bracket; constant of motion. [4 – 6 Lectures]

Hamilton-Jacobi equation

Hamilton's principal function; Jacobi's Theorem. Hamilton's Principal Function. Hamilton's Characteristic Function Action-angle variables; Adiabatic Invariance. [4 – 6 Lectures]

Theory of Small Oscillations (Conservative System)

Normal Coordinates. Oscillations under constraints. Stationary Character of Normal Modes. Elements of Non-linear Oscillations. [4 – 6 Lectures]

An Introduction to Special Theory of Relativity [4 – 6 Lectures]

References

[1] F.A. Chorlton, Text Book of Dynamics, CBS, 2002.

[2] L.D. Landau, E.M. Lifshitz, Mechanics: Course of Theoretical Physics: Vol. 1, 3rd Edition, Elsevier India, 2010.

[3] H. Goldstein, Classical Mechanics, Narosa Publ., New Delhi, 1998.

[4] N.C. Rana, P.S. Joag, Classical Mechanics, Tata McGraw Hill, New Delhi, 2002.

[5] N.H. Louis, J.D. Finch, Analytical Mechanics, CUP, 1998.

[6] L. Elsgolts, Differential Equations and the Calculus of Variations, MIR Publishers. Moscow, 1973.

[7] J.L. Synge and B.A. Griffith, Principles of Mechanics. McGraw-Hill, NewYork, 1970.

[8] E.C.G. Sudarshan, N. Mukunda, Classical Dynamics: A Modern Perspective, John Wiley & Sons, 1974.

[9] V.A. Ugarov, Special Theory of Relativity, Mir Publishers, Moscow, 1979.

[10] E.T. Whittaker, A Treatise of Analytical Dynamics of Particles and Rigid Bodies, Cambridge Univ. Press, Cambridge, 1977.

[11] F. Gantmacher, Lectures in Analytical Mechanics, Mir Publ., 1975.

[12] V.I. Arnold, Mathematical Methods of Classical Mechanics, 2nd ed., Springer-Verlag, 1997.

[13] N.G. Chetaev, Theoretical Mechanics, Springer-Verlag, 1990.

[14] M. Calkin, Lagrangian and Hamiltonian Mechanics, World Sci. Publ., Singapore, 1996.

[15] J.R. Taylor, Classical Mechanics, University Science Books, California, 2004.

MATPGCCT10 (Continuum Mechanics & Control Theory)

Module – I (Continuum Mechanics) (25 Marks)

Learning Objectives

The objective of the course is –

- to expose the students to the foundation in Continuum Mechanics.
- to learn the conservation principles and derive the equations governing the mechanics of continuum.
- to learn the constitutive equations for solid and fluid.
- to develop practical skills in working with tensors.
- to develop problem solving skills, applying the conservation principles and the constitutive equations to solve practical engineering problems.

Course Outcomes

Upon successful completion of this course, the students will –

- be able to appreciate a wide variety of advanced courses in solid and fluid mechanics.

Course Details

Analysis of Stress

The Continuum Concept, Homogeneity. Isotropy. Mass-Density, Body Forces. Surface Forces, Cauchy's Stress Principle. The Stress Vector, State of Stress at a Point. Stress Tensor, The Stress Tensor – Stress Vector Relationship, Force and Moment. Equilibrium. Stress Tensor Symmetry, Stress Transformation Laws , Stress Quadric of Cauchy, Principal Stresses. Stress Invariants. Stress Ellipsoid, Maximum and Minimum Shear Stress Values, Mohr's Circles for Stress, Plane Stress, Deviator and Spherical Stress Tensors. [8 Lectures]

Deformation and Strain

Particles and Points, Continuum Configuration. Deformation and Flow Concepts, Position Vector. Displacement Vector, Lagrangian and Eulerian Descriptions, Deformation Gradients. Displacement Gradients, Deformation Tensors. Finite Strain Tensors, Small Deformation Theory. Infinitesimal Strain Tensors, Relative Displacements. Linear Rotation Tensor. Rotation Vector, Interpretation of the Linear Strain Tensors, Stretch Ratio. Finite Strain Interpretation, Stretch Tensors. Rotation Tensor, Transformation Properties of Strain Tensors, Principal Strains. Strain Invariants. Cubical Dilatation, Spherical and Deviator Strain Tensors, Plane Strain. Mohr's Circles for Strain, Compatibility Equations for Linear Strains. [8 Lectures]

Motion and Flow

Motion. Flow. Material Derivative, Velocity. Acceleration. Instantaneous Velocity Field, Path Lines. Stream Lines. Steady Motion, Rate of Deformation. Vorticity. Natural Strain, Physical Interpretation of Rate of Deformation and Vorticity Tensors, Material Derivatives of Volume, Area and Line Elements, Material Derivatives of Volume, Surface and Line Integrals.

[8 Lectures]

Fundamental Laws of Continuum Mechanics

Conservation of Mass. Continuity Equation, Linear Momentum Principle. Equations of Motion. Equilibrium Equations, Moment of Momentum (Angular Momentum) Principle, Conservation of Energy. First Law of Thermodynamics. Energy Equation, Equations of State. Entropy. Second Law of Thermodynamics, The Clausius-Duhem Inequality. Dissipation Function, Constitutive Equations. Thermomechanical and Mechanical Continua. [6 Lectures]

References

[1] G.E. Mase, Theory and Problems of Continuum Mechanics, Schaums Outline Series, Mcgraw Hill Book Company, 1970.

[2] R.N. Chatterjee, Mathematical Theory of Continuum Mechanics, Narosa, 2002.

[3] J.N. Reddy, Principles of Continuum Mechanics, Cambridge University Press, 2010.

[4] Y.C. Fung, A First Course in Continuum Mechanics, Prentice Hall, 1977.

[5] R.C. Batra, Elements of Continuum Mechanics, AIAA, 2005.

[6] W.M. Lai, D. Rubin, E. Krempl, Continuum Mechanics, Butterworth Heinemann, 1999.

[7] S. Nair, Introduction to Continuum Mechanics, Cambridge University Press, 2009.

[8] J. L. Wegner, J. B. Haddow, Elements of Continuum Mechanics and Thermodynamics, Cambridge University Press, 2009.

[9] D.S. Chandrasekharaiah, L. Debnath, Continuum Mechanics, Academic Press, 1994.

[10] T. J. Chung, Applied Continuum Mechanics, Cambridge University Press, 1996.

[11] A.C. Eringen, Mechanics of continua, Robert E. Krieger Publishing Company, 1980.

[12] L.E. Malvern, Introduction to the Mechanics of a Continuous Medium, Prentice-Hall, Inc, 1977.

[13] L.I. Sedov, Introduction to the Mechanics of a Continuous Medium, Addison Wesley Publishing Company, 1965.

Module – II (Control Theory) (25 Marks)

Learning Objectives

- To provide fundamental knowledge in analysis and design of dynamical system.
- To learn about the dynamical system and control method.
- To implement the feedback control system with performance specifications.

Course Outcomes

Upon successful completion of this course, the students will –

- understand fundamental limits of control and estimation.
- be able to use advanced mathematical techniques to formulate and solve control problems.
- appreciate issues of robustness, optimality, architecture and uncertainty in control problems.
- identify practical challenges in posing control problems.

Course Details

Optimal Control Theory

Introduction, Optimal control problems, Mathematical methods for optimal control design: Calculus of variation, Pontryagin's optimum policy, Bang-Bang Control, Hamilton-Jacobi Principle, Maximum principle I, Mangasarian's theorem I; Maximum principle II, Mangasarian's theorem II. Application in ecology [8 Lectures]

Discrete-time Control

Introduction, Discretization of continuous time state space equations, Lyapunov stability analysis, Controllability and Observability. [6 Lectures]

Sufficient condition for control

Sufficiency theorem in the absence of state space constraints, The Arrow sufficiency theorem, mixed restrictions on the state and control variables, An Arrow type sufficiency theorem in case of mixed constraints. [8 Lectures]

Adaptive control

Parametric models of dynamical systems, Adaptive control problem. Design of Minimum variance and Moving average controllers, Stochastic self tuning regulators, Unification of direct self tuning regulations, Linear quadratic STR, adaptive predictive control. [8 Lectures]

References

[1] J-J E. Slotine and W. Li, Applied Nonlinear Control, Englewood Cliffs, NJ: Prentice-Hall, 1991.

[2] M.N. Bandyopadhyay, Control Engineering: Theory and Practice, Prentice-Hall of India Private Limited, 2003.

[3] K. Ogata, Discrete-time Control Systems, Pearson Education, 2005.

[4] W.L. Brogan, Modern Control Theory, Pearson, 3rd edition, 2011.

[5] S.H. Zak, Systems and Control, 1st edition, Oxford University Press, 2002.

[6] M. Gopal, Modern Control System Theory, New Age International(P) Ltd., 2nd edition, 2014.

[7] M. Gopal, Digital Control & State Variable Methods, Tata McGraw Hill Education, 2008.

[8] T. Kailath, Linear Systems by , Prentice-Hall Inc.,1980.

MATPGCCT10 (Graph Theory & Geometry of Curves and Surfaces)

Module – I (Graph Theory) (25 Marks)

Learning Objectives

- This is an introductory course on Graph Theory.
- This covers basic elements of graphs such as degree, walk, path, cycle, distance, eccentricity, radius, diameter, etc.
- Different types of graphs such as simple graph, multigraph, directed graph, Eulerian graph, Hamiltonian graph, bipartite graph, trees, etc. will be discussed.
- Concepts of connectivity, colouring, covering, matching and planarity will be introduced.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- write precise and accurate mathematical definitions of objects in graph theory.
- use mathematical definitions to identify and construct examples and counter-examples.
- use a combination of theoretical knowledge and independent mathematical thinking in creative investigation of questions in graph theory.
- apply graph theoretical knowledge in solving real life problems.

Course Details

Introduction to Graphs
Definition of a graph, digraphs and relations, simple graph, weighted graph, degree of a vertex, matrix representation of graphs: adjacency matrix and incidence matrix, regular graph, bipartite graph, sub-graph, complete graph, complement of a graph, graph operations, isomorphism. [4 Lectures]

Connectivity
Walks, trails, paths and cycles, connectedness of a graph, disconnected graphs and their components, distance, centre, radius, cut-vertices, cut-edges, blocks, vertex connectivity, edge connectivity, shortest path problem: Dijkstra's algorithm. [3 Lectures]

Eulerian and Hamiltonian Graphs
Motivation and origin, necessary conditions and sufficient conditions for existence of Eulerian trail and Hamiltonian cycle.
[2 Lectures]

Trees
Definition and characterizations, rooted and binary trees, spanning trees, counting of trees: Cayley's formula, breadth first and depth first search, minimum spanning tree problem: Prim's algorithm and Kruskal's algorithm. [6 Lectures]

Independent Sets, Coverings and Matchings
Independent sets and coverings, matchings in bipartite graphs, perfect matchings in graphs. [4 Lectures]

Graph Colouring
Vertex colouring, cliques and chromatic number, chromatic polynomial. [3 Lectures]

Planarity
Planar graphs, Euler's formula, Kuratowski's graphs, detection of planarity, geometric dual, combinatorial dual. [4 Lectures]

Directed Graphs
Underlying graph of a digraph, out-degrees and in-degrees, tournaments. [4 Lectures]

References

[1] Robin J. Wilson, Introduction to Graph Theory, 4th Edition, Pearson, 2007.

[2] S. Pirzada, An Introduction to Graph Theory, Universities Press, 2009.

[3] D.B. West, Introduction to Graph Theory, 2nd Edition, Pearson, 2015.

[4] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications, Springer, 2008.

Module – II (Geometry of Curves and Surfaces) (25 Marks)

Learning Objectives

- The course is designed with a target to introduce the students with the geometric properties of curves and surfaces in space using techniques of Differential Calculus.
- Students will come to know the notions of curvature, torsion; the scalars studying which one can feel the visual representations of curves and surfaces.
- The prerequisite for this course is working knowledge of Calculus (Ordinary and Partial Derivatives, Chain Rule, Integration etc.) and Linear Algebra (Basis of a linear space, Matrices, Determinants, Norm, Scalar Product).

Course Outcomes

Upon successful completion of this course, the students will –

- come to know regarding various notions governing the shape of curves and surfaces.
- be able to calculate the signed curvature of a plane curve; curvature and torsion of a space curve; first and second fundamental forms, normal, geodesic, Gaussian and mean curvatures of a surface.
- feel the shapes of geometric objects and the beauty of isometry and conformal mappings, above all.

Course Details

Curves
Curves: Curves, Arc Length Parameter, Reparametrisation of a Curve, Unit Speed Reparametrisation. [4 Lectures]

Curvature
Serret Frenet Formulae, Notions of Curvature and Torsion. [4 Lectures]

Surface
Surfaces, Smooth Surfaces, Smooth Maps between Surfaces, Tangents and Derivatives, Normals and Orientability. [8 Lectures]

The First Fundamental Form
Length of Curves on Surfaces, Isometries of Surfaces, Conformal Mappings of Surfaces. [6 Lectures]

Curvature of Surfaces
The Second Fundamental Form, The Gauss and Weingarten Maps, Normal and Geodesic Curvatures, Gaussian and Mean Curvatures. [8 Lectures]

References

[1] Andrew Pressley, Elementary Differential Geometry, Revised 2e, Springer Verlag London, 2012. (This is the Text Book. The syllabus almost blindly follows selected topics from the first eight chapters of this book).

[2] John Macleary, Geometry from a Differentiable Viewpoint, Cambridge University Press, 1994. (This book contains many interesting historical notes).

[3] Manfredo P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, 1976. (This book is a neatly written exposition of the subject but at some advanced level).

[4] Barrett O' Neill, Elementary Differential Geometry, Revised 2e, Academic Press, 2006. (This is an excellent one for those having further interest in the subject).

[5] J. A. Thorpe, Elementary Topics in Differential Geometry, Springer, 2004. (A nice book, at some points approach is different from our text book).

MATPGCCT11 (Numerical Analysis)

Learning Objectives

The objective of the course is –

- to provide suitable and effective methods called Numerical Methods, for obtaining approximate representative numerical results of the problems.
- to solve problems in the field of Applied Mathematics, Theoretical Physics and Engineering which requires computing of numerical results using certain raw data.
- to solve complex mathematical problems using only simple arithmetic operations. The approach involves formulation of mathematical models of physical situations that can be solved with arithmetic operations.
- to deal with various topics like finding roots of equations, solving systems of linear algebraic equations, interpolation, numerical integration and differentiation, solution of differential equation, boundary value problems, solution of matrix problems.
- to facilitate numerical computing.

Course Outcomes

At the end of this course the student will –

- be familiar with numerical methods for different types of interpolation, solution of nonlinear and polynomial equations, function approximation, differentiation and integration, the solution of ordinary differential equations, the solution of system of linear equations and matrix inversion, and the calculation of eigenvalues and eigenvectors.
- understand the theory behind these methods.
- be able to apply these methods to find numerical approximations and error estimates in a range of problems.

Course Details

Computer Number System

Control of round-off errors, Instabilities – Inherent and Induced, Hazards in approximate computations, Well-posed computations, Well-posed and Ill-posed problems, Direct and inverse and identification problems of computation. [2 Lectures]

Solution of System of Linear Equations

Successive over relaxation (SOR) method and convergence condition. Rate of convergence of methods. Operational counts in Gauss elimination method. Matrix factorization method: LU decomposition method: Crout's and Doolittle method. Tridiagonal form. [6 – 8 Lectures]

Solution of Non-linear Equations

Iteration methods: Tchebyshev method, Multipoint method, Modified Newton-Raphson method (for real roots: simple or repeated), Newton's Method, Quasi-Newton's method (System of equations). Rate of convergence of all iteration methods. Solution for polynomial equations. [8 – 10 Lectures]

Eigenvalues and Eigenvectors of Real Matrix

Jacobi's method for symmetric matrix (algorithm only) [4 Lectures]

Polynomial interpolation: Hermite interpolation, Error term, Cubic spline interpolation, Convergence properties (statement only), Inverse interpolation. [4 – 6 Lectures]

Approximation of Functions

Least squares polynomial approximation, Approximation with orthogonal polynomials, Chebyshev polynomials. [4 – 6 Lectures]

Numerical Integration

Gauss-Legendre and Gauss-Chebyshev quadratures, Richardson extrapolation, Romberg integration. Double integrals – Cubature formula of Simpson Type, Improper integrals. [6 – 8 Lectures]

Numerical Solution of Initial Value Problems for ODE

Multistep predictor-corrector methods – Adams-Bashforth method, Adams-Moulton method, Milne's method, Convergence and stability. Two Point Boundary Value Problem for ODE: Finite difference method, Shooting Method. [6 – 8 Lectures]

Solution of PDE by Finite Difference Methods

Parabolic equation in one dimension, Explicit finite difference method, Implicit Crank-Nicolson method, One-dimensional Hyperbolic Equation – Finite difference method, Method of characteristics (Consistency, Convergence and Stability), Solution of Laplace Equation. [6 – 8 Lectures]

References

[1] M.K. Jain, S.R.K. Iyenger, R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International, 2012.

[2] A. Ralston, A First Course in Numerical Analysis, McGraw Hill, N.Y., 1965.

[3] K.E. Atkinson, An Introduction to Numerical Analysis, John Wiley & Sons, 1989.

[4] G.D. Smith, Numerical Solution of Partial Differential Equations, 1985.

[5] I.S. Berezin, N.P. Zhidkov, Computing Method, Academic Press, 1979.

[6] E. Isacson, H. Keller, Analysis of Numerical Methods, Dover Publications, 2012.

[7] R. Zurmuhl, Numerical Analysis for Engineers and Physicists, Springer-Verlag, 1977.

[8] A. Ralston, P. Rabinowitz, A First Course in Numerical Analysis, McGraw Hill, N.Y., 1978.

[9] M.K. Jain, Numerical Solution of Differential Equations, New Age International, 2018.

[10] L. Fox, Numerical Solution of Ordinary and Partial Differential Equations, Oxford, 1962.

[11] W.F. Ames, Numerical Methods for PDEs, Academic Press, N.Y., 1977.

[12] J.H. Ahlberg, E.N. Nilson, J.L. Walsh, The Theory of Splines and Their Applications, Academic Press, N.Y., 1967.

[13] J.H. Wilkinson, Rounding Errors in Algebraic Processes, Prentice Hall, Englewood Cliffs, N.J., 1963.

[14] C.E. Fröberg, Introduction to Numerical Analysis, Addison-Wesley Publishing Company, 1969.

[15] C. Pozrikidis, Numerical Computation in Science and Engineering, Oxford University Press, Inc., N.Y., 1998.

MATPGCCT12 (Optimization Techniques & Integral Transform)

Learning Objectives

The student will be able to –

- know the basics of different evolutionary search algorithms.
- enumerate the fundamental knowledge of Linear Programming and Dynamic Programming problems.
- learn classical optimization techniques and numerical methods of optimization.
- explain Integer programming techniques and apply different optimization techniques to solve various models arising from engineering areas.
- equip with the methods of finding Laplace transform and Fourier Transforms as well as inverse transform of different functions.
- make them familiar with the methods of solving ordinary differential equations, partial differential equations, IVP and BVP, integral equations using Laplace transforms and Fourier transforms.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- use search methods in different evolutionary algorithms.
- use the fundamental knowledge of Linear Programming and Dynamic Programming problems.
- use classical optimization techniques and numerical methods of optimization.
- use the programming technique and apply different techniques to solve various optimization problems arising from engineering areas.
- recognize the different methods of finding Laplace transforms and Fourier transforms of different functions.
- apply the knowledge of Laplace transform, Fourier transform in finding the solutions of differential equations, initial value problems, boundary value problems.

Course Details

Module A: Optimization Techniques **(25 Marks)**

Unconstrained Optimization
One-Dimensional Search Methods: Golden section search, Fibonacci search, Newton's method. [4 – 6 Lectures]

Integer Programming
Gomory's cutting plane method, branch and bound method. [2 – 4 Lectures]

Linear Constrained Optimization
Revised Simplex method, dual simplex method, sensitivity analysis. [4 – 6 Lectures]

Non-Linear Constrained Optimization
Problems with equality constraints: Problem formulation, Lagrange conditions, second order conditions, minimizing quadratics subject to linear constraints. [6 – 8 Lectures]

Problems with inequality constraints
Karush-Kuhn-Tucker conditions, second order conditions. [4 – 6 Lectures]

Inventory and Queuing Models
Elementary queuing and inventory models. Basic EOQ and EPQ models (with and without shortage). Steady-state solutions of Markovian queuing models: M/M/1, M/M/C with limited waiting space. [4 – 6 Lectures]

Module B: Integral Transform **(25 Marks)**

Integral Transforms
Fourier Transforms: Fourier integral Theorem. Definition and properties. Fourier transform of the derivative. Derivative of Fourier transform. Fourier transforms of some useful functions. Fourier cosine and sine transforms. Inverse of Fourier transforms. Convolution. Properties of convolution function. Parseval relation. Applications. [12 – 14 Lectures]

Laplace Transforms: Definition and properties. Sufficient conditions for the existence of Laplace Transform. Laplace Transform of some elementary functions. Laplace transform of the derivatives. Inverse of Laplace transform. Bromwich Integral Theorem. Evaluation of inverse transforms by residue. Initial and final value theorems. Convolution theorem. Applications.

[14 –16 Lectures]

References

[1] S.S. Rao, Engineering Optimization: Theory and Practice, 3rd Ediiton, New Age International, 2010.

[2] Edwin K.P. Chong, Stanislaw H. Zak, An Introduction to Optimization, 2nd Wiley, 2005.

[3] M.S. Bazaraa, H.D. Sherali, C.M. Shetty, Nonlinear Programming: Theory and Algorithms, Wiley-Interscience, 2006.

[4] M. Aokie, Introduction to Optimization Techniques: Fundamentals and Applications of Nonlinear Programming, The Macmillan Company, 1971.

[5] I.N. Sneddon, Fourier Transform, McGraw Hill, 1951.

[6] L. Debnath, D. Bhatta, Integral Transforms and Their Applications, Chapman and Hall/ CRC, 2006.

[7] B. Davies, Integral Transforms and Their Applications, Springer, 2002.

[8] O.L. Mangasarian, Nonlinear Programming, SIAM (Classics in Applied Mathematics Series), 1994.

[9] R. Fletcher, Practical Methods of Optimization, Wiley India, 2009.

[10] F.C. Titchmash, Introduction to the theory of Fourier Integrals, Oxford Press, 1937.

[11] Peter K.F. Kuhfittig, Introduction to the Laplace Transform, Plenum Press, N.Y., 1980.

[12] E.J. Watson, Laplace Transforms and Application, Van Nostland Reinhold Co. Ltd., 1981.

[13] C. J. Tranter, Integral Transforms in Mathematical Physics, Methuen & Co., 1962.

[14] A. Pinkus, S. Zafrany, Fourier Series and Integral Transforms, Cambridge University Press, 1997.

MATPGCCP01 (Computational Lab)

Learning Objectives

The student will –

- be familiar with mathematical software like MATLAB and SageMath.
- be able to carry out mathematical computations using softwares.
- visualize mathematical concepts with the help of numerical examples.

Course Outcomes

Upon successful completion of this course, the students will –

- be a able to solve algebraic and trancendental equations using MATLAB or SageMath.
- find out the determinant, eigenvalues and eigenvectors of matrices.
- plot given curves and surfaces.
- compute graph parameters and plot simple graphs.
- solve ODE and PDEs.

Course Details

(Students can choose **Module 1** and **Module 3** OR **Module 2, Module 3** and **Module 4** OR **Module 2, Module 3** and **Module 5**)

Module 1: Computational Mathematics Using SageMath

Introduction: Exploring Integers, Solving Equations, 2d Plotting, 3d Plotting. [6 Hours]

Calculus: Calculus of One Variable, Applications of Derivatives, Integrals, Applications of Integrals, Partial Derivatives and Gradients, Jacobians, Local Maximum-Minimum. [18 Hours]

Linear Algebra: Working with Vectors, Solving System of Linear Equations, Vector Spaces, Linear Transformations, Eigenvalues and Eigenvectors. [18 Hours]

Graph Theory: Common Graphs, Basic Graph Operations, Graph Products, Paths and Cycles, Graph Properties (whether Eulerian, Hamiltonian, Regular, Planar, Bipartite, etc.), Spanning Trees, Graph Colouring, Independent Sets, Plotting of

Graphs.

[24 Hours]

Module 2: Introduction to MATLAB

General Information, Data Types and Variables, Operators, Flow Control, Functions, Input/Output, Array Manipulation, Writing and Running Programs, Plotting. [21 hours]

Module 3: Numerical Analysis

Numerical Solution of Algebraic and transcendental Equations, Numerical Solutions of System of Linear Equations, Interpolations and curve fitting, Numerical Integration, Numerical Eigenvalues and eigenvectors, Numerical Solution of ODE: Initial Value Problems and Boundary Value Problems, Numerical Solution of Partial Differential Equations. [24 Hours]

Module 4: Numerical Simulation in Dynamical Systems

Determination of equilibrium positions by plotting nullclines of planner dynamical systems in a bounded region $E \subset \mathbb{R}^2$. Direction field and stability of limit cycles associated with Lienard systems. Phase diagrams of the chaotic dynamics of the Rössler system, Lorenz system and Chua's Circuit. [21 Hours]

Module 5: Numerical Inversion of the Laplace Transform

Bellman Method, Gaver-Stehfest method, Talbot's Method. [21 Hours]

References

[1] Jaan Kiusalaas, Numerical Methods in engineering with MATLAB, Cambridge University Press, New York, 2005.

[2] Won Young Yang, Wenwu Cao, Tae-Sang Chung, John Morris, Applied Numerical Methods Using MATLAB, John Wiley & Sons, Inc., Hoboken, New Jersey, 2005.

[3] John H Mathews, Kurtis D Fink, Numerical Methods using MATLAB, Prentice Hall, Upper Saddle River, 1999.

[4] Steven T. Karris, Numerical Analysis Using MATLAB and Spreadsheets, Orchard Publications, Fremont, California.

[5] Dean G. Duffy, Advanced Engineering Mathematics with MATLAB, CRC Press, Boca Raton, 2017.

[6] Stormy Attaway, MATLAB: A Practical Introduction to Programming and Problem Solving, Butterworth-Heinemann, New York, 2009.

[7] S.R. Otto, J.P. Denier, An Introduction to Programming and Numerical Methods in MATLAB, Springer-Verlag London Limited, 2005.

[8] Stephen Lynch, Dynamical Systems with Applications Using MATLAB, Birkhausar, 2014.

[9] Paul Zimmermann, Mathematical Computation with Sage, available from http://www.sagemath.org.

[10] Razvan A Mezei, An Introduction to SAGE Programming: With Applications to SAGE Interacts for Numerical Methods, Wiley, 2016.

MATPGDET01/ MATPGDET02 (Operator Algebra – I)

Learning Objectives

- Students will be introduced to the fundamentals of Banach Algebra and C^*-Algebra.
- They will learn some important theorems in this area.

Course Outcomes

- Upon successful completion of the course, students will be able use the concepts gathered here effectively.
- They will appreciate the subject as a junction between Functional Analysis and Algebra, and handle some concepts of algebra and analysis together.

Course Details

Banach Algebra

Definition of Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an elements, The spectral radius, Gelfand formula, Multiplicative linear functionals and the maximal ideal space, Gleason-Kahane-Zelazko Theorem, The Gelfand Transforms, The Spectral mapping theorem, Isometric Gelfand Transform. [26 – 30 Lectures]

C^*-Algebras

Definition of C^*-Algebras and examples, Self-adjoint, Unitary, Normal, Positive and Projection elements in C^*-Algebras, Commutative C^*-Algebras, C^*-Homomorphisms, Representation of commutative C^*-Algebras, subalgebras and the spectrum, The Spectral Theorem, Positive linear functionals in C^*-algebras, States and the GNS construction. [26 – 30 Lectures]

References

[1] F.F. Bonsall, J. Duncan, Complete Normed algebras, Springer-Verlag, 2011.

[2] R.V. Kadison, J.R. Ringrose, Fundamentals of the theory of operator algebras, Vol. I - III, AMS, 2013.

[3] C.E. Rickart, General theory of Banach Algebras: Van Nostrand, 1960.

[4] W. Arveson, An invitation to C^*-Algebras, Springer-Verlag, 1976.

[5] T.W. Palmer, Banach Algebras and the General Theory of C^*-Algebras, Cambridge University Press, 2001.

MATPGDET01/ MATPGDET02 (Commutative Algebra – I)

Learning Objectives

The main objective is to

- give an introduction to modern commutative algebra with a special regard to commutative ring theory.
- learn proofs of fundamental theorems of commutative algebra.
- understand relations between commutative algebra and other areas of mathematics, such as algebraic geometry and algebraic number theory.
- apply theorems to provide proofs of statements about commutative rings and modules over them (problem solving).

Course Outcomes

At the end of the course students are expected to be able to

- state fundamental theorems of commutative algebra, provide proofs.
- apply the theorems to solve problems and analyze examples.
- apply knowledge of commutative algebra to analyze examples of particular rings and modules, for example given in terms of generators and relations.
- develop insight regarding abstract theory development also applicable to other parts of mathematics, like number theory and algebraic geometry.

Course Details

Nilradical and Jacobson radical, Nakayama's Lemma, Operations on Ideals, Prime Avoidance, Chinese Remainder Theorem, Extension and Contraction of ideals. [10 – 14 Lectures]

Free modules, projective modules, tensor product of modules, flat modules. [6 – 8 Lectures]

Rings and Modules of Fractions, Localisation and local rings. [6 – 8 Lectures]

Noetherian Rings, Primary Decomposition in Noetherian Rings. [6 – 8 Lectures]

Artininan modules, modules of finite length. [6 – 8 Lectures]

Integral Dependence, Going-Up Theorem, Integrally Closed Domains, Going-Down Theorem, Hilbert Nullstellensatz. [10 – 14 Lectures]

References

[1] D.S. Dummit, R.M. Foote, Abstract Algebra, Second Edition, John Wiley & Sons Inc., 1999.

[2] M.F. Atiyah, I.G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley, 1969.

[3] M. Reid, Undergraduate Commutative Algebra, Cambridge University Press, 1995.

[4] N.S. Gopalakrishnan, Commutative Algebra, Universities Press, 2016.

[5] H. Matsumara, Commutative Ring Theory, Cambridge Studies in Advanced Mathematics, 1989.

MATPGDET01/ MATPGDET02 (Operator Theory – I)

Learning Objectives

- In Operator Theory I, students will be introduced to some topics of operator theory (with an emphasis on spectral theory) and to the fundamentals of Banach algebra theory.
- It will introduce the student to terms, concepts and results for bounded linear operators which are commonly used in this particular area of mathematics.
- It will also introduce the students which are relevant to current research and prepare the student to pursue such a career.

Course Outcomes

Upon successful completion of this course, the students will –

- prove the continuity of concrete linear operators between topological vector spaces.
- understand whether a given linear operator is compact or not.
- find the essential spectra of linear operators.
- find the maximal spectra of concrete commutative Banach algebras.
- describe the functional calculi and the spectral decompositions of concrete self-adjoint operators and Compact Operators.
- be able to apply spectral theorems of bounded selfadjoint linear operators, spectral theorem of normal operators.
- be able to apply Spectral theorem for unitary and selfadjoint operators, multiplication operator and differentiation operators.
- be efficient to calculate the numerical range of some special operators.

Course Details

Bounded linear Operators

Resolvent set, Spectrum, Point spectrum, Continuous spectrum, Residual spectrum, Approximate point spectrum, Spectral radius, Spectral properties of a bounded linear operator, Spectral mapping theorem for polynomials. Numerical range, Numerical radius, Convexity of numerical range, Closure of numerical range contains the spectrum, Relation between the numerical radius and norm of a bounded linear operator. [26 – 30 Lectures]

Banach Algebra

Definition of normed and Banach Algebra and examples, Singular and Non-singular elements, The spectrum of an element, The spectral radius. [8 – 12 Lectures]

Compact linear operators

Spectral properties of compact linear operators on a normed linear space, Operator equations involving compact linear operators, Fredholm alternative theorem, Fredholm alternative for integral equations. Spectral theorem for compact normal operators. [14 – 18 Lectures]

References

[1] Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and sons, 2014.

[2] G. Bachman, L. Narici, Functional Analysis, Dover Publications, 2012.

[3] A . Taylor, D. Lay, Introduction to Functional Analysis, John Wiley and Sons, 1980.

[4] N. Dunford, J.T. Schwarts, Linear Operators – Part III, Wiley, 1988.

[5] Y. Eidelman, V. Milman, A. Tsolomitis,, Functional Analysis An Introduction, American Mathematical Society, 2004.

MATPGDET01/ MATPGDET02 (Advanced Topology – I)

Learning Objectives

- This is a two semester course. The first semester deals with deeper aspects of Point Set Topology.
- It consists of concepts like Separability, Metrizability, Compactification, Nets and Filters etc.
- Prerequisite to this course is a strong concept in Real analysis; and familiarity with basic notions of point set topology.
- In this course, the student comes to know the difference between metric spaces and spaces which does not 'admit' a metric; they even come to know under what conditions a topological space can be endowed with a metric.
- Once studied sequences, students can enjoy its generalization nets and filters.
- Students will be delighted to observe how basic geometric structures may be studied by transforming them into algebraic questions.
- Studying geometric objects by associating algebraic invariants to them is a powerful idea which influenced many areas of mathematics. As for example, deciding about the existence of a map between spaces (often a difficult task) may be translated into deciding whether an algebraic equation has a solution (which is often quite simple).

Course Outcomes

- This course naturally develops students' geometric maturity.
- Apart from gaining knowledge in topological concepts, students learn analyzing logic while trying to prove a theorem.

Course Details

Countability and Separation Axioms

Countability Axioms, The Separation Axioms, Equation spaces, Lindelöf spaces, Regular spaces, Normal spaces, Urysohn Lemma, Tietze Extension Theorem. [10 – 12 Lectures]

Nets and Filters

Directed Sets, Nets and Subnets, Convergence of a net, Ultranets, Partially Ordered Sets and Filters, Convergence of a filter, Ultrafilters, Basis and Subbase of a filter, Nets and Filters in Topology. [8 – 10Lectures]

Tychonoff Theorem and Compactification

Tychonoff Theorem, Completely Regular spaces, Local Compactness, One-point compactification, Stone-Čech Compactification. [8 – 10 Lectures]

Metrization

Urysohn Metrization Theorem, Topological Imbedding, Imbedding Theorem of a regular space with countable base in $\mathbb{R}^n$, Partitions of Unity, Topological m-Manifolds, Imbedding Theorem of a compact m-manifold in $\mathbb{R}^n$. Local Finiteness, Nagata-Smirnov Metrization Theorem, Para-compactness, Stone's Theorem, Local Metrizability, Smirnov Metrization Theorem. Uniform Spaces. [10 – 14 Lectures]

Complete Metric Spaces and Function Spaces

Complete Metric Spaces, The Peano Space-Filling Curve, Hahn-Mazurkiewicz Theorem (statement only). Compactness in Metric Spaces, Equicontinuity, Pointwise and Compact Convergence, The Compact-Open Topology, Stone-Weierstrass Theorem, Ascoli's Theorem, Baire Spaces, A Nowhere Differentiable Function. [10 – 14 Lectures]

References

[1] J.R. Munkres, Topology: A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

[2] J. Dugundji, Topology, Allyn and Bacon, 1966.

[3] G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.

[4] J.L. Kelley, General Topology, Van Nostrand Reinhold Co., New York, 1995.

[5] N. Bourbaki, Topologie Générale, Springer-Verlag, 1989.

[6] J. Hocking, G. Young, Topology, Addison-Wesley Reading, 1961.

[7] L. Steen, J. Seebach, Counter Examples in Topology, Springer-Verlag, 1970.

MATPGDET01/ MATPGDET02 (Advanced Complex Analysis – I)

Learning Objectives

The student will –

- understand normal families and related theorems.
- understand Dirichlet problem for harmonic functions, Poisson's Integral formula and its consequences.
- gain knowledge on Doubly periodic function, Weierstrass elliptic function and Weierstrass sigma function.
- study entire functions and meromorphic functions.
- study the Nevanlinna's Theory.

Course Outcomes

Upon successful completion of this course, the students will –

- acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- develop a deep understanding of the concepts and theorems in Complex Analysis.
- acquire necessary skills to research on various topic of Complex Analysis such as Nevanlinna's Theory, Order and Growth properties of meromorphic functions, Uniqueness theory of meromorphic functions etc.

Course Details

Unit I

Equicontinuity, Arzela-Ascoli theorem, Schwarz's lemma, Montel's theorem, normal families, Riemann mapping theorem.

[10 Lectures]

Unit II

Dirichlet problem for harmonic functions, Poisson integral functions, Jensen's formula, infinite products, Hadamard factorization theorem. [10 Lectures]

Unit III

Doubly periodic functions. Weierstrass Elliptic function, Weierstrass σ-function and their properties. [10 Lectures]

Unit IV

Entire functions, $M(r,f)$ and its properties. [4 Lectures]

Unit V

Meromorphic functions, Definition of the functions $m(r,a), N(r,a)$ and $T(r)$. Nevanlinna's first fundamental theorem. Cartan's identity and convexity theorems. Orders of growth. Order of a meromorphic function. Comparative growth of $\log M(r)$ and $T(r)$. Nevanlinna's second fundamental theorem. Estimation of $S(r)$. Nevanlinna's theorem on deficient functions. Nevanlinna's five-point uniqueness theorem. Milloux theorem. Milloux basic result. [22 Lectures]

References

[1] L.V. Ahlfors, Complex Analysis, 3rd Edition, McGrawHill Education, 2017.

[2] J.B.Conway, Functions of one Complex Variable, 2nd Edition, Springer, 1995.

[3] E.C. Titchmarsh, The Theory of Functions, 2nd Edition, Oxford University Press, 1976.

[4] E.T. Copson, An Introduction to the Theory of Functions of a Complex Variable, 1st Edition, Oxford University Press, 1970.

[5] A.I. Markushevich, Theory of Functions of a Complex Variable, (Vol. I, II, III), 2nd Edition, AMS, 2001.

[6] W. Kaplan, An Introduction to Analytic Functions, 1st Edition, Addison-Wesley Educational Publishers, 1967.

[7] H. Cartan, Elementary Theory of Analytic Functions of one or several complex variables, New Edition, Dover Publications Inc., 1995.

[8] W.K. Hayman, Meromorphic Functions, New Edition, Oxford University Press, 1968.

[9] L. Yang, Value Distribution Theory, 1st Edition, Springer, 1993.

[10] C.C. Yang, Y.X. Yi, Uniqueness theory of meromorphic functions, 3rd Edition, Kluwer Academic Publishers, 2003.

MATPGDET01/ MATPGDET02 (Advanced Differential Geometry – I)

Learning Objectives

- This is a two semester course. The syllabus is designed to discuss the concepts of Differential Geometry on smooth manifolds.
- As a prerequisite the reader is supposed to have done a course in Differential Geometry of Curves and Surfaces; and a course in Several Variable Calculus. A smooth manifold is an object which locally looks like n- dimensional Euclidean space and over which we can do Calculus.
- First objective of this course is to introduce the student with the concepts of manifolds and submanifolds with several examples. The syllabus focus on extension of calculus from curves and surfaces to its higher dimensional analogues, i.e. manifolds. The notion of vector fields is discussed in several ways. The concept of differential forms is also discussed. Notions of Lie Groups is also to be discussed so that a student can interlink the topological and algebraic structures on Manifolds.

Course Outcomes

- After going through the course, a student will come to know what is meant by manifolds, submanifolds and Lie groups.
- They will also be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate free approach.
- In short, they will be matured enough to enter into the realm of Riemannian Geometry.

Course Details

A Brief Review of Calculus of Several Real Variables
Partial derivatives and Directional Derivatives, Differentiability and Chain Rule, Real Analytic Functions, Taylor's Theorem with Remainder. [6 – 8 Lectures]

Tangent Vectors in Euclidean Spaces as Derivations
Germs of Functions, Derivations at a Point, Vector Fields, Vector Fields as Derivations. [6 – 8 Lectures]

The Exterior Algebra of Multi-covectors
Dual Space, Multilinear Functions, Permutation Action on Multilinear Functions, Symmetrizing and Alternating Operators, Tensor Product and Wedge Product, Anti-commutativity and Associativity of Wedge Product, Basis for k-vectors. [12 – 14 Lectures]

Differential Forms on Euclidean Spaces
Differential 1-forms, Differential of a function, Differential k-forms, Differential Forms as Multilinear Functions on Vector Fields, The Exterior Derivative, Closed Forms and Exact Forms, Convention on Subscripts and Superscripts. [12 – 14 Lectures]

Manifolds
Topological Manifolds, Compatible Charts, Smooth Manifolds, Examples of Smooth Manifolds. [8 Lectures]

Smooth Maps on a Manifold
Smooth Functions on a Manifold, Smooth Maps between Manifolds, Diffeomorphisms, Smoothness in terms of Components, Partial Derivatives, The Inverse Function Theorem. [8 Lectures]

References

[1] Loring W. Tu, An Introduction to Manifolds, Springer, 2011.

[2] S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindustan Book Agency (Text and Readings in Mathematics 22), 2002.

[3] John M. Lee, Introduction to Manifolds, 2nd edition, Springer, 2013.

MATPGDET01/ MATPGDET02 (Algebraic Graph Theory – I)

Learning Objectives

- To make the students aware about linear algebraic treatment to Graph Theory is the main objective of this course.
- Properties of the incidence matrix, eigenvalues of the adjacency matrix and the Laplacian matrix of graphs will be discussed in general.
- Some bounds involving the eigenvalues and other graph parameters will also be taught.
- Concepts of distance matrix in general graph, and particularly that in trees will be introduced.
- Strongly regular graphs and their spectra will be covered.
- Paley graphs will be introduced.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- find the spectrum of some simple graphs like complete graph, path graph, cycle graph and star graph.
- find the spectrum of graphs obtained through graph operations like Cartesian product.
- find the number of spanning trees of any given graph using Laplacian matrix.
- understand the algebraic connectivity of graphs.
- analyze a given tree in terms of the distance matrix and distance eigenvalues.
- identify a strongly regular graph and to find it's spectrum.
- develop a solid knowledge about finite fields and their connection with graphs.
- analyze a real world/ synthetic network in terms of spectral radius and to rank the nodes using Perron eigenvector.

Course Details

Unit I

Incidence matrix of a graph, Matchings in bipartite graphs. [8 – 10 Lectures]

Unit II

Adjacency matrix of a graph and its eigenvalues – bounds, energy of a graph, Spectral radius of graphs. [10 – 12 Lectures]

Unit III

Regular graphs and Line graphs, Strongly regular graphs, Cycles and Cuts. [8 – 10 Lectures]

Unit IV

Laplacian matrix of a graph, Algebraic connectivity, Laplacian spectral radius of graphs. [12 – 14 Lectures]

Unit V

Distance matrix of a graph, distance regular graph, distance spectral radius of graphs. [12 – 14 Lectures]

References

[1] N. Biggs, Algebraic Graph Theory, Cambridge University Press, 1993.

[2] C. Godsil, G. Royle, Algebraic Graph Theory, Graduate Texts in Mathematics 207, Springer-Verlag, 2001.

[3] R.B. Bapat, Graphs and Matrices, Universitext, Springer, Hindustan Book Agency, New Delhi, 2010.

[4] A.E. Brouwer, W.H. Haemers, Spectra of graphs, Springer, 2011.

MATPGDET01/ MATPGDET02 (Dynamical Systems – I)

Learning Objectives

- Systems of nonlinear ordinary differential equations, depending on parameters, have equilibrium solutions that may be classified as stable or unstable.
- The stable solutions determine the long term behavior of the model and as the parameters change, this stability may change giving rise to bifurcations.

Course Outcomes

- For one and two-dimensional systems of ODE, find the fixed points.
- Determine the linearization of the system about such solutions and discuss their stability.
- Understand the distinction between hyperbolic and non-hyperbolic fixed points.
- Sketch a phase portrait of linear and nonlinear one- and two-dimensional systems of ODEs.

Course Details

Unit I

The fundamental existence and uniqueness theorem; Dependence on initial conditions and system parameters. The maximal interval of existence. The flow defined by a system of nonlinear ordinary differential equations. Linearization and its limitations. Equilibria of a nonlinear system and its linearization about each equilibrium point. Hyperbolic and non-hyperbolic critical points. Eigenvalues and eigenspaces. Invariant spaces. The Hartman-Grobman theorem. Stability: asymptotic stability; Lyapunov functions; Lyapunov's theorems. [12 – 14 Lectures]

Unit II

The stable manifold theorem; saddles, nodes, foci and Centers of planar dynamical system. Nonhyperbolic critical points; Center Manifold Theory. Normal Form Theory. Gradient and Hamiltonian Systems. [10 – 12 Lectures]

Unit III

Poincare-Bendixon's theorem: Closed orbits. Bendixon's negative criterion. Dulac's criterion. Limit cycles. Theorems on the number of limit cycles in $\mathbb{R}^2$. Lienard System; van der Pol oscillator, Poincaré map; connection between stability of Poincare maps and corresponding flows. Invariant set; attractor, basins of attraction, boundary or separatrix. [10 – 12 Lectures]

Unit IV

Limit sets and attractors, Periodic and eventually periodic orbits and their stability; Limit cycles and separatrix; The Poincare map; Stable manifold Theory for periodic orbits; The Poincare sphere and behavior at infinity; Index theory. [10 – 12 Lectures]

Unit V

Graphical analysis of orbits of one dimensional maps. Logistic map; some properties of logistic map. Tent map; Periodic orbits of Tent map. Skinny Baker's map; Hyperbolicity of maps. Conjugacy of maps. Topological conjugacy and semi-conjugacy of maps. Conjugacy between Tent and Logistic maps. Sensitive dependence on initial conditions (SDIC). Topological transitivity (TT). [8 – 10 Lectures]

References

[1] S.H. Strogatz, Nonlinear Dynamics and Chaos, Addison Wesley, 1994.

[2] L. Perko, Differential Equation and Dynamical Systems, Springer, 3rd Eds, 2014.

[3] S.N. Rasband, Chaotic Dynamics and Nonlinear Systems, John Wiley & Sons, N.Y., 1989.

[4] J. Guckenheimer, P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer-Verlag, 1983.

[5] J. Marsden, M. McCracken, The Hopf Bifurcation and Its Applications, Springer-Verlag, 1976.

[6] G. Iooss, D.D. Joseph, Elementary Stability and Bifurcation Theory, Springer Verlag, 1997.

[7] B.D. Hassard, N.D. Kazarinoff and Y.H. Wan, Theory and Applications of Hopf Bifurcations, Cambridge University Press, 1981.

[8] V.I. Arnold, Dynamical Systems III, Springer, 1993.

[9] P.M. Morse , H. Feshbach, Methods of Theoretical Physics, Part I, McGraw Hill, Kogaknsha, 1953.

[10] J.W. Cain, A.M. Reynolds, Ordinary and Partial Differential Equations: An introduction to Dynamical Systems, Virginia Commonwealth University, 2010.

MATPGDET01/ MATPGDET02 (Solid Mechanics – I)

Learning Objectives

- To make students understand the principles of elasticity.
- To familiarize students with basic equations of elasticity in Cartesian as well as Curvilinear coordinates.
- To expose students to two dimensional problems in Cartesian and polar coordinates.
- To expose students to extension and bending of beams.
- To expose students to bending of plates.
- To expose students to three dimensional Problems of elasticity.
- To provide students with exposure to the systematic methods for solving problems in elasticity.
- To provide to students the foundation for pursuing other advanced solid mechanics courses.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- learn about the elastic behavior of material and evaluate stress invariants, principal stresses and their directions.
- determine strain invariants, principal strains and their directions.
- develop constitutive relationships between stress and strain for linearly elastic solid.
- apply linear elasticity in the design and analysis of structures such as beams, plates, shells.
- analyze the structural sections subjected to torsion, bending and extension.
- learn about formulations and solution strategies of various boundary value problems.
- use Airy stress functions for solution of 2-D plane stress and plane strain problems.

Course Details

Equations of Elasticity

Hooke's law. Generalized Hooke's law. Various cases of Elastic symmetry of a body. Homogeneous isotropic media. Elastic modulii for isotropic media. Equilibrium and dynamical equations for an isotropic elastic solid. Beltrami-Michell compatibility equations. The strain energy function and its connection with Hooke's law. Betti's identity. Clapeyrons formula and Clapeyrons theorem. Fundamental boundary value problems of elasticity. Uniqueness and existence of solutions. Saint Venant's principle.

[10 – 12 Lectures]

Saint-Venant's semi-inverse method of solution (Statement). Application to some simple problems of Theory of Elasticity. Extension of Beams by longitudinal forces, Beam stretched by its own weight, Bending of beams by terminal Couples.

[6 Lectures]

Orthogonal curvilinear coordinates

Equations of equilibrium, Strain and rotation tensors, Hooke's law. Applications in the solution of some problems of elasticity theory. [10 – 12 Lectures]

Plane problems

Plane strain and plane stress. Generalized plane stress. Airy's stress function. Solution of plane problems by means of polynomials. General Equations of the plane problems in polar coordinates. Complex variable method for plane problems and its applications. [10 – 12 Lectures]

Theory of thin plates

Approximate theory of bending of thin plates by transverse loads (Small deflection of thin plates). Deflection of circular, elliptic and rectangular plates under different edge conditions. [8 Lectures]

Solution of Problem in Elasticity by Potentials

The scalar and vector potentials for the displacement vector field. Equations of motion in terms of displacement potentials. Strain potential. Galerkin vector. Vertical load on the horizontal surface of a semi-infinite solid. Neuber-Papkovich solution.

[10 Lectures]

References

[1] I.S. Sokolnikoff, Mathematical Theory of Elasticity. McGraw Hill, 1956.

[2] A.E.H. Love, A treatise on mathematical theory of elasticity, Dover, 1954.

[3] S.P. Timoshenko, J.N. Goodier, Theory of Elasticity, McGraw Hill, 1970.

[4] Y.A. Amenzade, Theory of Elasticity, Mir Publishers, Moscow, 1971.

[5] N.I . Muskhelisvili, Some basic problems on the theory of elasticity. Nordhoff, 1953.

[6] Y.C. Fung, Foundation of solid mechanics. Prentice Hall, 1965.

[7] Y.C. Fung, P. Tong, Classical and Computational Solid Mechanics, World Scientific, 2005.

[8] M.H. Sadd, Elasticity: Theory, Applications and Numerics, Academic Press, 2014.

[9] L.D. Landau, E. M. Lifshitz: Theory of Elasticity, Pergamon Press, 1989.

[10] A.C. Eringen, E.S. Suhubi, Elastodynamics, Vol.I and II, Academic Press, 1974.

[11] V.Z. Parton, P.I. Perlin, Mathematical Methods of the theory of elasticity, vol. I, II, Mir Publishers, 1984.

[12] A.E. Green, W. Zerna, Theoretical Elasticity, Dover, 2012.

[13] S.G. Lekhnitskii, Theory of elasticity of an anisotropic body, Mir Publisher, Moscow, 1981.

[14] J.D. Achenbach, Wave Propagation in Elastic Solids, North Holland Publishing Company, 1984.

[15] K.F. Graff, Wave Motion in Elastic Solids, Dover Publications Inc., 2003.

[16] W. Nowacki, Thermoelasticity, Addison-Wesley Publishing Co., 1962.

[17] W. Nowacki, Dynamic Problems of Thermoelasticity, Springer, 1975.

[18] J.L. Nowinski, Theory of thermoelasticity with applications. Sijthoff and Noardhaoff International Publishers, 1978.

[19] R.B. Hetnarski, M.R. Eslami, Thermal Stresses – Advanced Theory and Applications, Springer, 2008.

[20] M.R. Eslami, R.B. Hetnarski, J. Ignaczak, N. Noda, N. Sumi, Y. Tanigawa, Theory of Elasticity and Thermal Stresses, Springer, 2015.

[21] R.S. Dhaliwal, A. Singh, Dynamic Coupled Thermoelasticity, Hindustan Publishing Corporation, 1986.

MATPGDET01/ MATPGDET02 (Mathematical Biology – I)

Learning Objectives

- To understand the qualitative properties of dynamical system like stability, bifurcation and chaos.
- To know the linear, nonlinear differential equations and difference equations and their solutions.
- To know basics of ecosystem and different types of community.
- To formulate simple growth problems and study of population models.
- To study the continuous population models for single species and interacting population.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- formulate many biological and ecological systems by mathematical modelling.
- study several qualitative properties of the systems.
- making of predictions about the biological systems.
- simulate real world problems in biological sciences and health care industry.

Course Details

System of linear and nonlinear differential equations

Preliminary concept on autonomous and non- autonomous system of differential equations. The flow defined by a differential equation, linearization of dynamical systems (two, three and higher dimension), Stability: (i) asymptotic stability (Hartman's theorem), (ii) global stability (Liapunov's second method). Bifurcations: saddle-node, transcritical and pitchfork bifurcations, Hopf- bifurcation. [10 – 12 Lectures]

Linear difference equations

Difference equations, existence and uniqueness of solutions, linear difference equations with constant coefficients, systems of linear difference equations, qualitative behavior of solutions to linear difference equations. [6 – 8 Lectures]

Basic concept of ecosystem and mathematical models

Individual, Population, Community, Ecosystem, Components: Abiotic, Biotic; Different types of ecosystem. Deterministic and Random and Stochastic model [6 – 8 Lectures]

Continuous time models of single species

Basic postulates, Growth model equations: Malthus, Logistic with Allee effect, Gompertz growth model. Effects on growth: Density Dependence, variable growth coefficient, simple harmonic growth coefficient. Leslie growth model. [8 – 10 Lectures]

Models for interacting population

Host-parasites interactions; phase-plane analysis, (Lotka- Volterra system), Alternate form, Effect of migration, some derivations. Competition model: Gauss exclusion principle. [8 – 10 Lectures]

Nonlinear discrete equations and discrete models (Map)

Maps, Maps vs. Differential equations, Fixed points, Stability and Cobweb diagram, the logistic difference equation, systems of nonlinear difference equations, stability criteria for second order equations, stability criteria for higher order system. Nicholson-Bailey, plant-herbivore interaction model, Stability, Existence of periodic solution, bifurcation and chaotic behavior.

[10 – 12 Lectures]

References

[1] D.W. Jordan, P. Smith, Nonlinear Ordinary Equations - An Introduction to Dynamical Systems (Third Edition), Oxford Univ. Press, 1998.

[2] L. Perko, Differential Equations and Dynamical Systems, Springer Verlag, 1991.

[3] F. Verhulust, Nonlinear Differential Equations and Dynamical Systems, Springer Verlag, 1996.

[4] W.G. Kelley, A.C. Peterson, Difference Equations- An Introduction with Applications, Academic Press, 1991.

[5] L.A. Segel, Modelling Dynamical Phenomena in Molecular Biology, Cambridge University Press, 1984.

[6] J.D. Murray, Mathematical Biology, Springer and Verlag, 2001.

[7] H.I. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel Dekker, Inc., New York, 1980.

[8] M. Kot, Elements of Mathematical Ecology, Cambridge Univ. Press., 2001.

[9] T.C. Gard, Stochastic differential equation with application in population dynamics: Marcel-Dekker, New York, 1987.

[10] C.W. Gardiner, Stochastic Methods for Physics, Chemistry and Natural Sciences, Springer Verlag, 1983.

[11] L. Arnold, Stochastic Differential Equations: Theory and Applications, Wiley, 1974.

MATPGDET01/ MATPGDET02 (Fluid Mechanics – I)

Learning Objectives

- To give the students knowledge of basic principles of fluid mechanics.
- To expose the students to basic laws of fluid motion.
- To learn the complex variable method and its applications in potential flows.
- To expose the students to the knowledge of irrotational motion and vortex motion.
- To learn about different kinds of waves.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- determine streamlines, path of particles, velocity potentials.
- derive equation of motion and equation of continuity in orthogonal curvilinear coordinates.
- determine circulation about circular and elliptic cylinder.
- understand Kelvin's minimum energy theorem, uniqueness theorem.

- understand theorem of Blasius, theorem of Kutta and Juokowski, Schwartz-chirstoffel theorem.
- define source, sinks, doublets, images and Stoke's stream function.
- determine vortex line, vortex filament in case of steady motion.
- understand vortex pair, vortex doublet and image of a vortex with regards to plane a circular cylinder.
- understand energy of waves and group velocity.

Course Details

Unit I

Lagrange's and Euler's methods in fluid motion. Equation of motion and equation of continuity, Boundary conditions and boundary surface stream lines and paths of particles. Irrotational and rotational flows, velocity potential. Bernoullis equation. Impulsive action equations of motion and equation of continuity in orthogonal curvilinear co- ordinate. Euler's momentum theorem and D'Alembert's paradox. [10 – 12 Lectures]

Unit II

Theory of irrotational motion flow and circulation.Permanence irrotational motion. Connectivity of regions of space. Cyclic constant and acyclic and cyclic motion.Kinetic energy. Kelvin's minimum. Energy theorem. Uniqueness theorem. Two Dimensional irrotational motion. [10 – 12 Lectures]

Unit III

Complex potential, sources. sinks, doublets and their images circle theorem. Theorem of Blasius. Motion of circular and elliptic cylinders. Circulation about circular and elliptic cylinder. Steady streaming with circulation. Rotation of elliptic cylinder. Theorem of Kutta and Juokowski. Conformal transformation. Juokowski transformation. Schwarz Christoffel theorem.

[10 – 12 Lectures]

Unit IV

Motion of a sphere. Stoke's stream function. Source, sinks, doublets and their images with regards to a plane and sphere.

[4 Lectures]

Unit V

Vortex motion. Vortex line and filament equation of surface formed by stream lines and vortex lines in case of steady motion. Strength of a filament.Velocity field and kinetic energy of a vortex system. Uniqueness theorem rectilinear vortices. Vortex pair. Vortex doublet. Images of a vortex with regards to plane and a circular cylinder. Angle infinite row of vortices. Karman's vortex sheet. [10 – 12 Lectures]

Unit VI

Waves: Surface waves. Paths of particles. Energy of waves. Group velocity. Energy of a long wave. [8 Lectures]

References

[1] A.S. Ramsay, Hydrodynamics, Bell & Sons Ltd, 1913.

[2] H. Lamb, Hydrodynamics, Cambridge University Press, 1932.

[3] L.D. Landou, E.M. Lifchiz, Fluid Mechanics, Pergamon Press, London, 1959.

[4] L.M. Thomson, Theoretical hydrodynamics, Dover Publication, 1938.

[5] L.M. Thomson, Theoretical Aerodynamics, Dover Publication, 1958.

[6] Shih-I. Pai, Introduction to the theory of compressible flow, Van Nostrand, 1959.

[7] P. Niyogi, Inviscid Gas Dynamics, McMillan, 1975.

[8] K. Oswatitsch, Gas Dynamics, Academic Press, 1956.

MATPGDET01/ MATPGDET02 (Operations Research – I)

Learning Objectives

- Main objective this course is to let the students understand the basic concepts, strategies and techniques to analyze a variety of inventory systems and make optimal decisions for the improvement of these systems.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- comprehend the basic principles, concepts, and techniques of production and inventory management as they relate to the entire supply chain (customer demand, distribution, and product transformation processes).
- understand the methods used by organizations to obtain the right quantities of stock or inventory.
- familiarize themselves with inventory management practices.
- deal with lot size systems with different cost properties, such as quantity discounts, price-change anticipation, perishable goods system.
- have knowledge of multi-item inventory models with single linear restriction, more than one linear restriction, and non-linear restrictions.

Course Details

Production and inventory management
Historical origin and background of the subject. Nature of inventory problems. Structure of inventory systems. Definition of inventory problem. Important parameters associated with inventory problems. Variables in inventory problems. Controlled and uncontrolled variables. Types of inventory systems and inventory policies. [12 – 16 Lectures]

Statistical and dynamical inventory problems
Deterministic inventory models/ systems. Harris-Wilson model. Economic lot size systems. Sensitivity of the lot size systems.

[10 – 14 Lectures]

Order level systems and their sensitivity analysis
Order level lot size and their sensitivity studies. Non-constant demand models under $(s,q),(t,s_i)$ and (t_i,s_i) policies. Power law and linear travel demand situations.

[10 – 14 Lectures]

Lot size systems with different cost properties
(i) Quantity discounts, (ii) Price-change anticipation, (iii) Perishable goods system. Multi-item inventory models with (i) single linear restriction, (ii) More than one linear restriction, (iii) non-linear restrictions. [12 – 16 Lectures]

References

[1] E. Nadder, Inventory Systems, John Wiley, 1966.

[2] G. Hadley, T. M. Whitin, Analysis of Inventory Systems, Prentice Hall, 1963.

[3] R.J. Tersine, M. Hays, Principles of Inventory and Material Management, Prentice Hall, 1994.

MATPGDET01/ MATPGDET02 (Financial Mathematics – I)

Learning Objectives

The student will learn –

- The Time Value of Money, Cash flow, Annuities, Loans, Bond, Portfolio Immunization.

Course Outcomes

- Upon successful completion of this course, the students will develop the idea of describing and using the financial instruments like bonds, loans, cash flow, differences of different assets.

Course Details

Interest Theory

Time Value of Money: interest rate (rate of interest), simple interest, compound interest, accumulation function, future value, current value, present value, net present value, discount factor, discount rate (rate of discount), convertible m-thly, nominal rate, effective rate, inflation and real rate of interest, force of interest, equation of value. [8 – 10 Lectures]

Annuities/ Cash Flows

Annuity-immediate, annuity due, perpetuity, payable m-thly or payable continuously, level payment annuity, arithmetic increasing/decreasing annuity, geometric increasing/ decreasing annuity, term of annuity. immediate or due, present value, future value, current value, interest rate, payment amount, and term of annuity; Level annuity, a. finite term, b. Level perpetuity, c. Non-level annuities/ cash flows. [8 – 10 Lectures]

Loans

Principal, interest, term of loan, outstanding balance, final payment (drop payment, balloon payment), amortization, sinking fund. term of loan, interest rate, payment amount, payment period, principal. [8 – 10 Lectures]

Bonds

Price, book value, amortization of premium, accumulation of discount, redemption value, par value/ face value, yield rate, coupon, coupon rate, term of bond, callable/ noncallable. [8 – 10 Lectures]

General Cash Flows and Portfolios

Yield rate/ rate of return, dollar-weighted rate of return, time- weighted rate of return, current value, duration (Macaulay and modified), convexity (Macaulay and modified), portfolio, spot rate, forward rate, yield curve, stock price, stock dividend.

[8 – 10 Lectures]

Immunization

Cash flow matching, immunization (including full immunization), Redington immunization. [8 – 10 Lectures]

References

[1] S.A. Broverman, Mathematics of Investment and Credit (Fifth Edition), ACTEX Publications, 2010.

[2] J.W. Daniel, L.J.F. Vaaler, Mathematical Interest Theory (Second Edition), The Mathematical Association of America, 2009.

[3] S.G. Kellison, The Theory of Interest (Third Edition), McGraw-Hill, 2009.

[4] C. Ruckman, J. Francis, Financial Mathematics: A Practical Guide for Actuaries and other Business Professionals (Second Edition), BPP Professional Education, 2005.

[5] Wai-Sum. Chan, Yiu-Kuen. Tse, Financial Mathematics for Actuaries, Updated Edition, McGraw- Hill Education (Asia), 2013.

[6] R.L. McDonald, Derivatives Markets (Third Edition), Pearson, 2013.

MATPGDET01/ MATPGDET02 (Difference Equations in Ecology and Eco-epidemiology)

Learning Objectives

- Develop an appreciation of the modern scope of scientific inquiry in the field of Ecology and Eco-epidemiology.
- Become familiar with the variety of ways that organisms interact with both the physical and the biological environment.

- Develop an understanding of the differences in the structure and function of different types of functions.
- Learn techniques of data analysis as well as methods of presenting scientific information in figures and tables.
- Develop an appreciation of the natural world through direct experience with local systems.
- Learn techniques for gathering data in the field.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- learn basic concepts of Ecology and Eco-epidemiology.
- learn dynamics of population.
- learn ecology of communities.
- learn functional and structural features of communities.
- learn ecological cycles and biological accumulations.
- learn ecological problems of humanity, protection of the nature and biological variability.
- learn economics of environment and planning, sustainable development.

Course Details

Basic concepts of discrete-time dynamical systems

Local and global bifurcation with special emphasis on Neimar-sacker bifurcation and flip bifurcation. Centre manifold theorem. Direction of Hopf-bifurcation. L Chaos. Different route to chaos. Lyapunov Exponent. Introduction to Mathematical Modeling. Non-dimensionalisation techniques. Stability criteria of a system. Jury's condition. Boundedness of a system. Positiveness of a solution. Persistence of a system. Kolmogorov Analysis. Single species difference equation models-Malthus model, Logistic model, Model with harvesting, Model with Allee effect. Insect outbreak model. [10 – 12 Lectures]

Modeling predator functional response

Holling type-I, type-II, type-III, type-IV, ratio-dependent, Beddington – DeAngelis. [2 – 4 Lectures]

Two and three species difference equation models

Predator-prey model. Food chain model. Model with one prey-two predators. Model with one predator-two preys. Model with generalist predator. Model with two preys-two predators. [6 – 8 Lectures]

Modeling two and three species discrete time models

Incorporating different phenomena like imposition of a population floor, addition of refugia, omnivory, intraspecific density dependence, toxic inhibition, spatial effect, disease in prey, dispersal, predator switching, Allee effect , additional predator, additional food , harvesting of predator, disease in predator, mutualism, commensalism, parasitism, fear factor, seasonal variation, cooperation, group defense, cross predation, anti-predator, etc. [8 – 10 Lectures]

Modeling of excitable systems

Chemostat Model. Tumour-growth model. Cancer model. HIVmodel. Model of divorce prediction and marriage repair. Metapopulation and patch Model. [4 – 6 Lectures]

Epidemic Models (SI,SIR, SIS, SIRS, SEIR etc). Models with time delay (single and double delay). Derivation of a critical delay for stability of a system. Periodic solutions. [4 – 6 Lectures]

Model with prey switching/ generalist predator/ Allee effect (Boukal-Sabelis-Berec model). (ii) Local and Global Stability (iii) Food Web (iv) Age-dependent populations [4 – 6 Lectures]

Eco-epidemic systems

Prey-diseased-predator interactions, Predator-diseased-prey interactions, Diseased competing species models, Eco-epidemics models of symbiotic communities (Disease effects on the symbiotic system, Disease control by use of a symbiotic species)

[4 – 6 Lectures]

References

[1] A.D. Bazykin, A.I. Khibnik, B. Krauskopf, Nonlinear dynamics of interacting populations, 2nd edition, World Scientific, 1998.

[2] J.D. Murray, Mathematical Biology, 2nd edition, Springer-Verlag,1989.

[3] D.W. Jordan, P. Smith, Nonlinear Ordinary Differential Equations, 4th edition, Oxford University Press, 2007.

[4] S.H. Strogatz, Nonlinear dynamics and chaos with applications to Physics, Biology, Chemistry, and Engineering, 2nd edition, Westview Press, 2001.

[5] J. Pastor, Mathematical Ecology of Populations and Ecosystems, 2nd edition, Wiley-Blackwell, 2008.

[6] J. Guckenheimer, P.J. Holme, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer, 2002.

[7] B.D. Hassard, N.D. Kazarinoff and Y.H. Wan, Theory and Applications of Hopf Bifurcation, Cambridge University Press, 1981.

[8] M. Kot, Elements of Mathematical Ecology, Cambridge University Press, 2001.

MATPGDET03/ MATPGDET04 (Operator Algebra – II)

Learning Objectives

- Students will be introduced to the fundamentals of Von Neumann Algebras.
- They will learn different types of algebras and their representations.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- prove the theorems taught and effectively use the concepts presented here.
- appreciate the subject being in the junction of functional analysis and algebra, and they will be able to handle some concepts of algebra and analysis together.

Course Details

Von Neumann algebras, Monotone sequence of operators, Range Projections, The Commutant, The Double Commutant theorem, The Kaplansky Density theorem, L^∞ as Von Neumann Algebra, Maximal Abelian Algebras, Abelian Von Neumann Algebras, Cyclic and separating vectors, Representation of Abelian Von Neumann Algebras, The L^∞ functional calculus, Connectedness of the unitary group, The projection lattice, Kaplansky's formula, The centre of a Von Neumann Algebra, Various types of projections. [48 – 60 Lectures]

References

[1] F.F. Bonsall, J. Duncan, Complete Normed algebras, Springer-Verlag. 2011.

[2] R.V. Kadison, J.R. Ringrose, Fundamentals of the theory of operator algebras, Vol. I - III, AMS, 2013.

[3] C.E. Rickart, General theory of Banach Algebras, Van Nostrand, 1960.

[4] W. Arveson, An invitation to C^*-Algebras, Springer-Verlag, 1976.

[5] T.W. Palmer, Banach Algebras and the general theory of C^*-algebras, Cambridge University Press, 2001.

MATPGDET03/ MATPGDET04 (Commutative Algebra – II)

Learning Objectives

- The main objective is to give an introduction to modern commutative algebra with a special regard to commutative ring theory.
- Learn proofs of fundamental theorems of commutative algebra.
- Understand relations between commutative algebra and other areas of mathematics, such as algebraic geometry and algebraic number theory.
- Apply theorems to provide proofs of statements about commutative rings and modules over them (problem solving).

Course Outcomes

Upon successful completion of this course, the students will be able to –

- state fundamental theorems of commutative algebra, provide proofs.
- apply the theorems to solve problems and analyze examples.
- apply knowledge of commutative algebra to analyze examples of particular rings and modules, for example given in terms of generators and relations.
- develop insight regarding abstract theory development also applicable to other parts of mathematics, like number theory and algebraic geometry.

Course Details

Unit I
Valuation Rings, Discrete Valuation Rings, Dedekind Domains, Fractional and Invertible Ideals. [8 – 10 Lectures]

Unit II
Valuations, Places and valuations, the rank of a valuation, Krull Domains, Prufer Domains, Bezout Domains. [10 – 12 Lectures]

Unit III
Topologies and Completions, Filtrations. [10 – 12 Lectures]

Unit IV
Graded Rings and Modules, the Associated Graded Ring. [10 – 12 Lectures]

Unit V
Hilbert Functions, Dimension Theory of Noetherian Local Rings, Krull's Principal Ideal Theorem, Regular Local Rings, Transcendental Dimension. [10 – 14 Lectures]

References

[1] D.S. Dummit, R.M. Foote, Abstract Algebra, Second Edition, John Wiley & Sons Inc., 1999.

[2] M.F. Atiyah, I.G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley, 1969.

[3] M. Reid, Undergraduate Commutative Algebra, Cambridge University Press, 1995.

[4] N.S. Gopalakrishnan, Commutative Algebra, Universities Press, 2016.

[5] H. Matsumara, Commutative Ring Theory, Cambridge Studies in Advanced Mathematics, 1989.

MATPGDET03/ MATPGDET04 (Operator Theory – II)

Learning Objectives

- In Operator Theory II, Students will learn Self adjoint operators (with an emphasis on spectral representation theorems), specially, positive operators and projection operators. They will also study normal operators and unbonded linear operators in Hilbert spaces.
- This course will introduce the students special classes of bounded linear operator and study why each of them is important and significant.
- Describe the functional calculi and the spectral decompositions of concrete selfadjoint operators and Compact Operators.

Course Outcomes

Upon successful completion of this course, the students will be –

- able to apply spectral theorems of bounded selfadjoint linear operators, Spectral theorem of normal operators.
- able to apply Spectral theorem for unitary and selfadjoint operators, Multiplication operator and differentiation operators.

Course Details

Selfadjoint operators
Spectral properties of bounded selfadjoint linear operators on a complex Hilbert space, Positive operators, Square root of a positive operator, Projection operators, Spectral family of a bounded selfadjoint linear operator and its properties, Spectral theorem for a bounded selfadjoint linear operator. [16 – 20 Lectures]

Normal Operators
Spectral properties for bounded normal operators, Spectral theorem for bounded normal operators. [8 – 10 Lectures]

Unbounded linear operators in Hilbert space
Hellinger-Toeplitz theorem, Symmetric and selfadjoint operators, Closed linear operators, Spectrum of an unbounded selfadjoint linear operator, Cayley Transformation $U = (T - iI)(T + iI)^{-1}$ of an operator T, Spectral theorem for unitary and selfadjoint operators, Multiplication operator and differentiation operator, Application to Quantum Mechanics. [26 – 30 Lectures]

References

[1] Erwin Kreyszig, Introductory Functional Analysis with Applications, John Wiley and sons, 2014.

[2] G. Bachman, L. Narici, Functional Analysis, Dover Publications, 2012.

[3] A . Taylor, D. Lay, Introduction to Functional Analysis, John Wiley and Sons, 1980.

[4] N. Dunford, J.T. Schwarts, Linear Operators – Part III, Wiley, 1988.

[5] Y. Eidelman, V. Milman, A. Tsolomitis, Functional Analysis An Introduction, American Mathematical Society, 2004.

MATPGDET03/ MATPGDET04 (Advanced Topology – II)

Learning Objectives

- This course is continuation of the course Advanced Topology – I in previous semester.
- This gives an introduction to homotopy and homology.
- A course in group theory is a must as a prerequisite.

Course Outcomes

- This course also develops students' geometric maturity.
- Apart from gaining knowledge in topological concepts, students learn analyzing logic while trying to prove a theorem.

Course Details

The Fundamental Group and Covering Spaces

Homotopy of paths, Fundamental Group, Covering Spaces, Fundamental Group of the Circle, Fundamental Group of the Punctured Plane, Special Van Kampen Theorem, Fundamental Group of S^n, Seifert - Van Kampen Theorem (statement and applications), Fundamental Group of surfaces. Essential and Inessential Maps, Borsuk – Ulam Theorem for S^2 , Fundam ental Theorem of Algebra, Vector Fields and Fixed Points, Brouwer's Fixed-Point Theorem for the disc, Homotopy Type, Deformation Retract, Strong Deform ation Retract. Jordan Separation Theorem, Jordan Curve Theorem (statement only). Classification of Covering Spaces, General Lifting Lemma, Existence of Coverings, Semilocally Simply Connectivity, Deck Transformations.

[24 – 30 Lectures]

Simplicial Homology

Geometric Complexes and Polyhedra, Orientation of Geometric Complexes. Chains, Cycles, Boundaries and Homology Groups, Examples of Homology Groups, The Structure of Homology Groups, Euler - Poincaré Theorem, Pseudomanifolds and the Homology Groups of S^n. Simplicial Approximation, Induced Homomorphisms on the Homology Groups, Brouwer Fixed Point Theorem and Related Results. [24 – 30 Lectures]

References

[1] J.R.Munkres, Topology: A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.

[2] F.H. Croom, Basic Concepts of Algebraic Topology, Springer, NY, 1978.

[3] G.E. Bredon, Topology and Geometry, Springer, India, 2005.

[4] E.H. Spanier, Algebraic Topology, McGraw-Hill, 1966.

[5] I.M. Singer, J.A. Thorpe, Lecture Notes on Elementary Topology and Geometry, Springer, India, 2003.

[6] A. Hatcher, Algebraic Topology, Cambridge University Press, 2003.

[7] J. Dieudonné, A History of Algebraic and Differential Topology, 1900 - 1960, Birkhäuser, 1989.

MATPGDET03/ MATPGDET04 (Advanced Complex Analysis – II)

Learning Objectives

The student will –

- study homotopy of paths, fundamental groups and related topics.
- understand the functions of several complex variables.
- understand the analytic functions, Cauchy-Riemann equations and Cauchy's integral formula for several complex variables.
- gain knowledge on power series of a function of several complex variables.
- gain knowledge on the zeros and singularities of an analytic function of several complex variables and related theory.

Course Outcomes

Upon successful completion of this course, the students will –

- acquire necessary skills to explain the concepts, state and prove theorems and properties involving the above topic.
- develop a deep understanding of the concepts and theorems in Complex Analysis.
- acquire necessary skills to research on various topics of functions of several complex variables.

Course Details

Unit I

Riemann surfaces, Homotopy of paths, fundamental groups, Coverings, Analytic continuation, Branched meromorphic continuation, The Riemann surface of an algebraic function, Puiseux expansion, The Riemann sphere. [12 – 18 Lectures]

Unit II

Functions of several complex variables. Power series in several complex variables. Region of convergence of power series. Associated radii of convergence. [4 – 6 Lectures]

Unit III

Analytic functions. Cauchy-Riemann equations. Cauchy's integral formula. Taylor's expansion. Cauchy's inequalities. Zeros and Singularities of analytic functions. Maximum modulus theorem. Weierstrass preparation theorem, Analytic sets,. Analytic set germs, Regular and singular points of analytic sets. [14 – 18 Lectures]

Unit IV

Isolated singularities of holomorphic functions, Differentiable manifolds, Tangent bundles and vector fields, Transversality, Lie groups, Complex manifolds, Isolated critical points, The universal unfolding, Morsifications, Finitely determined function germs, Classification of simple singularities, Real morsifications of the simple curve singularities. [12 – 16 Lectures]

References

[1] L. Hörmander, An Introduction to Complex Analysis in Several Variables, 3rd Edition, North Holland Ed, 1988.

[2] M. Range, Holomorphic Functions and Integral Representations in Several Complex Variables, 1st Edition, Springer-Verlag Ed, 1986.

[3] S.G. Krantz, Function Theory of Several Complex Variables, 2nd Edition, AMS Chelsea Publishing, 2001.

[4] R. Narasimhan, Several Complex Variables, New Edition, Chicago University Press, 1974.

[5] M. Herve, Several Complex Variables, 2nd Edition, Oxford University Press, 1988.

[6] S.C. Chen, M.C. Shaw, Partial differential equations in several complex variables, New Edition, American Mathematical Society, Providence, RI, 2001.

[7] S. Bochner, W. T. Martin. Several Complex Variables, Princeton University Press,1948.

[8] R.C. Gunning, H. Rossi, Analytic Functions of Several Complex Variables, New Edition, AMS, 2009.

[9] B.A. Fuks. An Introduction to the Theory of Analytic Functions of Several Complex Variables, 1st Edition, Amer Mathematical Society, 1963.

[10] W. Ebeling, Functions of several complex variables and their singularities, New Edition, AMS, 2007.

MATPGDET03/ MATPGDET04 (Advanced Differential Geometry – II)

Learning Objectives

- This is a sequence of Advanced Differential Geometry – I in previous semester.

Course Outcomes

- After going through the course, a student will come to know what is meant by manifolds, submanifolds and Lie groups.
- They will also be able to manipulate with ease the basic operations on tangent vectors, differential forms and tensors both in a local coordinate description and a global coordinate free approach.
- In short, they will be matured enough to enter into the realm of Riemannian Geometry.

Course Details

Tangent Space to a Manifold
The Tangent Space to a Manifold at a Given Point, The Differential of a Map, The Chain Rule, Bases for a Tangent Space at a Po,int, A Local Expression for the Differential, Curves in a Manifold, Computing the Differential Using Curves, Immersions and Submersions, Rank, Critical and Regular Points. [10 – 12 Lectures]

Submanifolds of a Smooth Manifold
Submanifolds, Level Sets of a Function, The Regular Level Set Theorem, Examples of Regular Submanifolds. [8 – 10 Lectures]

The Rank of a Smooth Map
Constant Rank Theorem, The Immersion and Submersion Theorems, Images of Smooth Maps, Smooth Maps into a Submanifold, The Tangent Plane to a Submanifold in Three Dimensional Euclidean spaces. [6 Lectures]

Brief Introduction to Tangent Bundle
Topology of the Tangent Bundle, its Manifold Structure, Vector Bundles, Smooth Sections. [6 – 8 Lectures]

Vector Fields
C^∞ Bump Functions, Partitions of Unity, Smoothness of a Vector Field, Integral Curves, Local Flows, The Lie Bracket, Push-forward of Vector Fields, Related Vector Fields. [10 –12 Lectures]

Lie Groups
Lie Groups, Examples, The Matrix Exponential, Trace of a Matrix, Differential of det at Identity. 6 – 8 Lectures]

References

[1] Loring W. Tu, An Introduction to Manifolds, Springer, 2011 (This is the text book for this course).

[2] S. Kumaresan, A Course in Differential Geometry and Lie Groups, Hindusthan Book Agency (Text and Readings in Mathematics 22), 2002.

[3] John M. Lee, Introduction to Manifolds, 2nd edition, Springer, 2013.

MATPGDET03/ MATPGDET04 (Algebraic Graph Theory – II)

Learning Objectives

- This course is a continuation of the course Algebraic Graph Theory – I in the previous semester.
- Relation between groups and graphs in terms of Cayley graphs will be taught in depth.
- Graphs obtained from algebraic structures such as commutative rings, fields and vector spaces will also be introduced.

Course Outcomes

Upon successful completion of this course, the students will –

- develop knowledge about relation between groups and graphs.
- be able to construct Cayley graph from a given group and a generator of it.
- be able to obtain zero-divisor graphs, comaximal ideal graph, annihilator graph and some other graphs from a given commutative ring or field.
- be able to obtain graphs from a given vector space.
- be able to apply algebraic graph theoretical knowledge to real world graphs.

Course Details

Cayley graph from groups
Construction and Recognition, Isomorphism, Subgraphs, Factorization. [10 – 12 Lectures]

Graphs from commutative rings
Zero divisor graph, Cayley graph, Cayley sum graph, Total graph, Unit graph, Standard comaximal graph, Comaximal ideal, Annihilator graph, Generalized total graph, The Jacobson graph, Generalized unit and unitary Cayley graph. [20 – 24 Lectures]

Graphs over Semigroups
Power graph on finite groups and semigroups. [10 – 12 Lectures]

Graphs over vector spaces. [10 – 12 Lectures]

References

[1] A. White, Graphs of Groups on Surfaces, Mathematics Studies 188, North-Holland, Amsterdam, 2001.

[2] N. Biggs, Algebraic Graph Theory, Cambridge University Press, 1993.

[3] C. Godsil, G. Royle, Algebraic Graph Theory, Graduate Texts in Mathematics 207, Springer-Verlag, 2001.

MATPGDET03/ MATPGDET04 (Dynamical Systems – II)

Learning Objectives

- These correspond to qualitative changes in the predictions of the model.
- This course will study questions of stability and bifurcation for both systems of nonlinear dynamical systems and iterated nonlinear maps.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- use cobweb diagrams to illustrate stability of a fixed point or periodic orbit.
- use a Lyapunov function to investigate the stability of a fixed point. Use the Poincaré-Bendixson theorem and polar coordinates to investigate the existence of limit cycles.
- investigate bifurcations of one-dimensional dynamical systems in general; draw bifurcation diagrams. Find fixed points and periodic orbits of iterated one-dimensional mappings, including the logistic map in particular, and discuss their stability.
- understand the connection between the Lyapunov exponent and chaos.

Course Details

Unit I

Non-hyperbolic critical points in $\mathbb{R}^2$. Implicit function theorem and bifurcation theory. Structural stability and Peixoto's theorem. Bifurcation in one dimensional systems. Bifurcations at nonhyperbolic equilibrium points: saddle-node bifurcation: pitchfork bifurcation: transcritical bifurcation: imperfect bifurcation. Hopf bifurcations and bifurcations of limit cycles from a multiple focus. Bifurcations at non-hyperbolic periodic orbits. [8 – 10 Lectures]

Unit II

Higher codimension bifurcations at nonhyperbolic equilibrium points; Universal unfolding of normal form. Bifurcation of steady solution in $\mathbb{R}^2$: Hopf bifurcation; homoclinic and heteroclinic bifurcations; Lorenz system and its properties. One parameter family of Rotated vector fields, Homoclinic bifurcations; Melnikov's methods; The Takens-Bogdanov bifurcation. Finite codimension bifurcations in the class of Bounded Quadratic Systems (BQS). [12 – 14 Lectures]

Unit III

Chaos; chaotic orbits; graphical illustration using logistic map; Lyapunov exponent; Period-doubling bifurcation: period doubling route to chaos. Period 3 implies chaos. Universality in chaos. [10 – 12 Lectures]

Unit IV

Dynamics of maps; Logistic map; properties of logistic map for parameter value $> 2+\sqrt{5}$; Cantor set: Cantor set structure of logistic map: topological conjugacy of logistic and shift map: chaotic behavior of logistic map: Feigenbaum constants. Sharkowskii's theorem: Li-Yorke's theorem for 1D maps. Henon map; standard map, twist map. [10 – 14 Lectures]

Unit V

Poincare map; Smale horseshoe map; Smale-Birkhoff homoclinic theorem; Melnikov's method: damped forced pendulum: damped forced Duffing oscillator: Onset of chaos. Two dimensional attractors: Henon attractor: attractor of Lorenz type. Dimension of an attractor. [10 – 12 Lectures]

References

[1] M.W. Hirsch, S. Smale, R.L. Devaney, Differential Equations, Dynamical Systems and An Introduction to Chaos, Elsevier Academic Press, 2nd Edition, 2004.

[2] S. Wiggins, Introduction to Applied Nonlinear Dynamical Systems and Chaos, Springer, New York, 2013.

[3] L. Perko, Differential Equation and Dynamical Systems, Springer, New York, 3rd Eds, 2014.

[4] R.L. Devaney, An Introduction to Chaotic Dynamical System, Addison-Wesley, 1989.

[5] D.K. Arrowsmith, Introduction to Dynamical Systems, Cambridge Univ. Press, 1990.

[6] P.O. Drazin, Nonlinear Systems, Cambridge Univ. Press, 1993.

[7] P. Glendinning, Stability, Instability and Chaos, Cambridge Univ. Press, 1994.

[8] C. Robinson, Dynamical Systems: Stability, Symbolic Dynamics and Chaos, CRC Press, 1999.

[9] K.T. Alligood, T.D. Sauer, J.A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer, 1997.

[10] V.I. Arnold, Ordinary Differential Equations, M.I.T. Press, Cambridge, 1973.

[11] J. Guckenheimer, P. Holms, Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields, Springer, New York, 1983.

[12] J. Carr, Applications of Center Manifold Theory, Springer, New York, 1981.

[13] C. Sparrow, The Lorenz Equations: Bifurcations, Chaos and Strange Attractors, Springer, New York, 1982.

MATPGDET03/ MATPGDET04 (Solid Mechanics – II)

Learning Objectives

- To make students understand the principle of torsion, flexure of beams.
- Expose the students to vibration of elastic solids, propagation of waves in elastic solids.
- To develop concept of thermoelasticity, coupled thermoelasticity and Generalized thermoelasticity.
- To expose students to variational method in elasticity and its applications.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- formulate and solve different tortion, bending and flexure problems.
- understand radial, torsional, longitudinal, flexural, rotatory vibration of different elastic body and solve corresponding problem.
- learn about velocity of elastic waves in isotropic and anisotropic solid media and to develop concept on plane waves, spherical waves and surface waves.
- learn basic equations of classical, coupled and generalized thermoelasticity theories and as well as their application in solving thermoelastic problems.
- understand different variational theorem and solve Euler's equation using Ritz and Galerkin method.

Course Details

Torsion and flexure
Torsion of a prismatic rod with an arbitrary cross-section. Prandtl's stress function. Solution of torsion problem for simple cross sections. Torsion of a rod with variable circular cross-section. Flexure of prismatic rod by terminal load, Solution of flexure problem for circular, elliptic and rectangular rod. [10 – 12 Lectures]

Vibration of elastic solids
General Theorems on Vibration of Elastic Solids. Radial, Torsional vibrations of a circular cylinder. Radial and Rotatory vibrations of a sphere. Longitudinal, Torsional and Flexural vibrations of thin rod. Flexural, and extensional vibrations of a circular ring in a plane. [10 – 12 Lectures]

Elastic waves
Propagation of waves in an isotropic elastic solid body, Waves of dilatation and waves of distortion, Motion of a surface of discontinuity. Plane waves and spherical waves. Body waves. Wave propagation in two-dimensions. Surface waves. Free Rayleigh waves and free Love waves. [10 – 12 Lectures]

Thermoelasticity
Concept of thermoelasticity. Thermoelastic state of stress and strain. Thermoelastic equations of equilibrium and motion. Displacement equations for the stationary, quasi-static and dynamic problems. Coupling of temperature and strain fields. Simple applications. Concept of generalized thermoelasticity. [10 – 12 Lectures]

Variational methods
Theorems of Minimum Potential Energy. Theorems of Minimum Complementary Energy. Uniqueness of Solutions. Reciprocal theorem of Betti and Rayleigh – applications. Solution of Euler's equation by Ritz, Galerkin method. [10 – 12 Lectures]

References

[1] I.S. Sokolnikoff, Mathematical Theory of Elasticity. McGraw Hill, 1956.

[2] A.E.H. Love, A treatise on mathematical theory of elasticity, Dover, 1954.

[3] S.P. Timoshenko, J.N. Goodier, Theory of Elasticity, McGraw Hill, 1970.

[4] Y.A. Amenzade, Theory of Elasticity, Mir Publishers, Moscow, 1971.

[5] N.I . Muskhelisvili, Some basic problems on the theory of elasticity. Nordhoff, 1953.

[6] Y.C. Fung, Foundation of solid mechanics. Prentice Hall, 1965.

[7] Y.C. Fung, P. Tong, Classical and Computational Solid Mechanics, World Scientific, 2005.

[8] M.H. Sadd, Elasticity: Theory, Applications and Numerics, Academic Press, 2014.

[9] L.D. Landau, E. M. Lifshitz: Theory of Elasticity, Pergamon Press, 1989.

[10] A.C. Eringen, E.S. Suhubi, Elastodynamics, Vol.I and II, Academic Press, 1974.

[11] V.Z. Parton, P.I. Perlin, Mathematical Methods of the theory of elasticity, vol. I, II, Mir Publishers, 1984.

[12] A.E. Green, W. Zerna, Theoretical Elasticity, Dover, 2012.

[13] S.G. Lekhnitskii, Theory of elasticity of an anisotropic body, Mir Publisher, Moscow, 1981.

[14] J.D. Achenbach, Wave Propagation in Elastic Solids, North Holland Publishing Company, 1984.

[15] K.F. Graff, Wave Motion in Elastic Solids, Dover Publications Inc., 2003.

[16] W. Nowacki, Thermoelasticity, Addison-Wesley Publishing Co., 1962.

[17] W. Nowacki, Dynamic Problems of Thermoelasticity, Springer, 1975.

[18] J.L. Nowinski, Theory of thermoelasticity with applications. Sijthoff and Noardhaoff International Publishers, 1978.

[19] R.B. Hetnarski, M.R. Eslami, Thermal Stresses – Advanced Theory and Applications, Springer, 2008.

[20] M.R. Eslami, R.B. Hetnarski, J. Ignaczak, N. Noda, N. Sumi, Y. Tanigawa, Theory of Elasticity and Thermal Stresses, Springer, 2015.

[21] Ranjit S. Dhalwal, Avtar Singh, Dynamic Coupled Thermoelasticity, Hindustan Publishing Corporation, 1986.

MATPGDET03/ MATPGDET04 (Mathematical Biology – II)

Learning Objectives

- To study the discrete type system like delay system, plant herbivore system.
- To formulate bacterial growth in Chemostat and study for it qualitative behaviour.
- To learn the pattern formation of diffusive model.
- To learn the inheritance of genetic characteristic.
- To formulate infectious disease modelling and study.
- To formulate harvesting of biological population and determine the optimal harvesting policy.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- formulate discrete type system, bacterial growth in chemostat, diffusive model, inheritance of genetic characteristic, infectious disease modelling, harvesting policy of exploited population.
- have qualitative properties of the above systems.
- have prediction for future policy making.

Course Details

Stochastic Model

Pure Birth and Death Processes, models with variable age distributions. Linear birth, death and immigration. [6 – 8 Lectures]

Delay in population models

Introduction, 1-dimensional continuous and discrete models. Stability, Bifurcation. [6 – 8 Lectures]

Diffusion and Models for Developmental Pattern Formation

Background, The general balance law, Fick's law, diffusivity of motile bacteria, model formulation, spatially homogeneous and inhomogeneous solutions, Turing model, conditions for diffusive stability and instability, pattern generation with single species model. [8 – 10 Lectures]

Models for Population Genetics

Introduction, basic model for inheritance of genetic characteristic, Hardy-Wienberg law, models for genetic improvement, selection and mutation- steady state solution and stability theory. [10 – 12 Lectures]

Diseases modelling

Introduction, simple, general and recurring epidemic. Stochastic epidemic models: with/without removal, with carrier and/or infectives. [10 – 12 Lectures]

Multi-species models in fishery management

Combined harvesting of two ecologically independent fish species following logistic growth, bionomic equilibrium optimal harvest policy, combined harvesting of two competing fish species following logistic growth. [8 – 10 Lectures]

References

[1] J.D. Murray, Mathematical Biology, Springer and Verlag, 2002.

[2] N.T.J. Bailey, The Mathematical Theory of Infectious Diseases (Mathematics in Medicine series), Hodder Arnold, 2nd edition, 1975.

[3] L. Edelstein-Keshet, Mathematical Models in Biology, The Random House/ Birkhauser Mathematics Series, 1987.

[4] E. A. Allman, J. A. Rhodes, Mathematical Models in Biology: An Introduction, Cambridge University Press, 2004.

[5] T.C. Gard, Stochastic differential equation with application in population dynamics, Marcel- Dekker, New York, 1987.

[6] C.W. Gardiner, Stochastic Methods for Physics, Chemistry and Natural Sciences, Springer Verlag, 1983.

[7] L. Arnold, Stochastic Differential Equations: Theory and Applications, Wiley, 1974.

[8] L.A. Segel, Modelling Dynamical Phenomena in Molecular Biology, Cambridge University Press, 1983.

[9] J.M. Cushing, Integro-differential Equations and Delay Models in Population Dynamics, Springer, 1977.

[10] Y. Kuang, Delay Differential Equations with Applications in Population Dynamics, vol. 191 of Mathematics in Science and Engineering, Academic Press, Massachusetts, 1993.

[11] F. Brauer, P.v.d. Driessche, Jianhong Wu, Mathematical Models in Population Biology and Epidemiology, Springer, 2012.

[12] F.Brauer, P.v.d. Driessche, Jianhong Wu, Mathematical Models in Epidemiology, Springer, New York, NY, 2019.

[13] R.S. Cantrell, C. Cosner, Spatial Ecology via Reaction-Diffusion Equation, John Wiley & Sons, Ltd, 2004.

[14] C.W. Clark, Mathematical Bioeconomics: The optimal Management of Renewable Resources, John Wiley & Sons, New York, 1976.

[15] C.W. Clark, Bioeconomics Modeling and Fisheries Management, John Wiley & Sons, New York,1985.

[16] H. Malchow, S.V. Petrovskii, E. Venturino: Spatiotemporal Patterns in Ecology and Epidemiology: Theory, Models, and Simulation, Chapman and Hall/CRC, 2007.

[17] R.J. Hosking, E. Venturino, Aspects of Mathematical Modelling: Applications in Science, Medicine, Economics and Management, 2007.

MATPGDET03/ MATPGDET04 (Fluid Mechanics – II)

Learning Objectives

- To expose the students to basics of compressible fluid flow.
- To expose the students to the steady flow through De Laval nozzle.
- To study flow along a wavy boundary and flow past a slight corner.
- To solve the Chaplygin's equation.
- To learn Karman-Tsien approximation method, Prandtl-Glauert method and Jangen-rayleigh method of approximation.
- To study two dimensional steady flow and learn method of characteristic.
- To solve Euler's-Tricomi equation.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- derive governing equations of fluid motion, Bernoulli's integral for steady isentropic and irrotational motion.
- define polytropic gas, critical speed and determine equation satisfied by velocity potential, stream function.
- determine mach number and understand circulation theorem, permanence of irrotational motion.
- solve steady linearised subsonic and supersonic flows.
- distinguish between normal and oblique shock wave.
- solve Chaplygin's equation and Euler's-Tricomi equation.
- solve transonic flow and hypersonic flow problems.

Course Details

Unit I

Basic thermodynamics of compressible fluids: Six governing equations of fluid motion, crocco- vazsonyl equation. Propagation of small disturbances in a gas.mach number. Dynamics similarity of two flows.Circulation theorem. Permanence of irrotational motion. Bernoulli's integral for steady isentropic and irrotational motion. Polytropic gas. Critical speed. Equation satisfied velocity potential and stream functions. Prandtl-Mayer fluid past a convex corner. [18 – 20 Lectures]

Unit II

Steady flow through a De Laval nozzle. Normal and oblique shock wave shock polar diagram one dimensional similarity flow. Steady linearised subsonic and supersonic flows. Prandtl-Glauert transformation. Flow along a wavy boundary flow past a slight corner. Jangen-rayleigh method of approximation. Thin supersonic wind Ackeret's formula. [18 – 20 Lectures]

Unit III

Legendre and molenbroek transformations Chaplygin's equation for stream function. Solution of chaplygin's equation. Subsonic gas jet problem limiting line. Motion due to a two dimensional source and a vortex Karman-Tsien approximation. Two dimensional steady flow: Riemann invariants. Method of characteristic. Transonic flow. Law transonic similarity. Euler's-Tricomi equation and its fundamental solution. Hypersonic flow. [18 – 20 Lectures]

References

[1] A.S. Ramsay, Hydrodynamics, Bell & Sons Ltd, 1913.

[2] H. Lamb, Hydrodynamics, Cambridge University Press, 1932.

[3] L.D. Landou and E.M. Lifchiz, Fluid Mechanics, Pergamon Press, London, 1959.

[4] L.M. Thomson, Theoretical hydrodynamics, Dover Publication, 1938.

[5] L.M. Thomson, Theoretical Aerodynamics, Dover Publication, 1958.

[6] Shih-I. Pai, Introduction to the theory of compressible flow, Van Nostrand, 1959.

[7] P. Niyogi, Inviscid Gas Dynamics, Mcmillan, 1975.

[8] K. Oswatitsch, Gas Dynamics, Academic Press, 1956.

MATPGDET03/ MATPGDET04 (Operations Research – II)

Learning Objectives

- This course covers some advanced topics in inventory management system and queuing theory.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- understand probabilistic models of inventory management systems with probabilistic demand, probabilistic order level and that with instantaneous demand, uniform demand and lead time.
- model some given problems in both discrete and continuous probability versions.
- explain some particular problems such as newspaper boy problem, spare parts problem and baking company problem.
- understand different Markovian queuing models and steady-state solutions of those models.
- apply the knowledge of decision making in inventory management systems and queuing models to real life problems.

Course Details

Inventory management & Queueing Theory

Probabilistic demand models. Expected cost. Probabilistic order level systems. Probabilistic order level systems with instantaneous demand. Probabilistic order level systems with uniform demand. [12 Lectures]

Probabilistic order level systems with lead time. Discrete and continuous probability versions of the models. Problems on the two versions of the models. [12 Lectures]

Newspaper boy problem. Spare parts problem. Baking company problem.Equivalence of probabilistic order level systems. [10 Lectures]

Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1. [12 Lectures]

References

[1] E. Nadder. Inventory Systems, John Wiley, 1966.

[2] G. Hadley and T. M. Whitin Analysis of Inventory Systems, Prentice Hall, 1963.

[3] R. J. Tersine, M. Hays, Principles of Inventory and Material Management, Prentice Hall, 1994.

[4] K. Swarup, P.K. Gupta, Manmohan, Operations Research: Introduction to Management Science Operation, Sultan Chand Publishers, New Delhi, 1986.

[5] H.A. Taha, Operations Research: An Introduction, Pearson Education India; 9th edition, 2014.

MATPGDET03/ MATPGDET04 (Financial Mathematics – II)

Learning Objectives

- The students will learn: General Derivatives, Options, Forwards and Futures contract, Outright purchase, Implied repo-rate, Swaps, Hedging and Investment Strategies.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- acquire the knowledge of: forward contracts, futures contracts, swaps and options work, how they are traded and how they are priced.
- evaluate the price of options.
- demonstrate a clear understanding the financial research planning.

Course Details

Financial Economics

General Derivatives: derivative, underlying asset, over the counter market, short selling, short position, long position, ask price, bid price, bid-ask spread, lease rate, stock index, spot price, net profit, payoff, credit risk, dividends, margin, maintenance margin, margin call, mark to market, no-arbitrage, risk-averse. [10 – 12 Lectures]

Options

Call option, put option, expiration, expiration date, strike price/exercise price, European option, American option, Bermudan option, option writer, in-the-money, at-the-money, out-of-the- money, covered call, naked writing, put-call parity.

[10 – 12 Lectures]

Forwards and Futures

Forward contract, futures contract, outright purchase, fully leveraged purchase, prepaid forward contract, synthetic forwards, cost of carry, implied repo-rate. [10 – 12 Lectures]

Swaps

Swap, swap term, prepaid swap, notional amount, swap spread, deferred swap, simple commodity swap, interest rate swap.

[10 – 12 Lectures]

Hedging and Investment Strategies

Hedging, arbitrage, diversifiable risk, non-diversifiable risk, spreads (option, bull, bear, vertical, box, ratio), collar width, collared stock, zero-cost collar, straddle, strangle, written straddle, butterfly. [10 – 12 Lectures]

References

[1] S.A. Broverman, Mathematics of Investment and Credit (Fifth Edition), ACTEX Publications, 2010.

[2] J.W. Daniel, L.J.F. Vaaler, Mathematical Interest Theory (Second Edition), The Mathematical Association of America, 2009.

[3] S.G. Kellison, The Theory of Interest (Third Edition), McGraw-Hill, 2009.

[4] C. Ruckman, J. Francis, Financial Mathematics: A Practical Guide for Actuaries and other Business Professionals (Second Edition), BPP Professional Education, 2005.

[5] Wai-Sum. Chan, Yiu-Kuen. Tse, Financial Mathematics for Actuaries, Updated Edition, McGraw- Hill Education (Asia), 2013.

[6] R.L. McDonald, Derivatives Markets (Third Edition), Pearson, 2013.

MATPGDET03/ MATPGDET04 (Lie Theory of Ordinary and Partial Differential Equations)

Learning Objectives

- To know a new theory for solving more general types of Ordinary and Partial Differential Equations.
- To learn Lie's theory of differential equations which unifies many specific methods for solving differential equations and provides powerful new ways to find solutions.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- apply Lie's Theory of Ordinary and Partial Differential Equations as one of the powerful methods.
- apply Lie's theory to both linear and nonlinear Ordinary and Partial Differential Equations.

Course Details

Unit I

Lie Group of Transformations and infinitesimal Transformations: Introduction to Lie Group of Transformations, Infinitesimal Transformations, Point Transformations and Extended Transformations (Prolongations), Multi-parameter Lie Group of Transformations and Lie Algebras, Mappings of curves and surfaces, Local Transformations. [10 – 12 Lectures]

Unit II

Ordinary Differential Equations: Elementary Examples, First-Order ODEs, Invariance of Second and Higher Order ODEs Under Point Symmetries, Reduction of Order of ODEs under multi-parameter Lie Group of Point Transformations. [10 – 12 Lectures]

Unit III

Invariance of a PDE: Introduction, Determining Equations for symmetries of a kth-Order PDE, Invariance of Scalar PDE, Elementary Examples. [10 – 12 Lectures]

Unit IV

Invariant Solution of PDEs: Invariant Solutions, Example, Invariance for a system of PDEs. Determining Equations for symmetries of a system of PDEs, Examples. [10 – 12 Lectures]

Unit V

Application to Boundary Value Problems. Formulation of Invariance of a Boundary Value Problem for a Scalar PDE, Incomplete Invariance for a Linear Scalar PDE, Incomplete Invariance for a Linear system of PDEs. [10 – 12 Lectures]

References

[1] G.W. Bluman, S.C. Anco, Symmetry and integration methods for differential equations, Springer, 2002.

[2] P.J. Olver, Application of Lie Groups to Differential Equations, Springer, 2000.

[3] N.H. Ibragimov, Elementary Lie Group analysis and Ordinary differential equations, John Wiley & Sons, 1999.

[4] H. Stephani, Differential Equations: Their Solution Using Symmetries, Cambridge University Press, 1989.

[5] G. Baumann, Symmetry Analysis of Differential Equations With Mathematica, Springer, 2000.

Syllabus for

GE Courses Offered to Students of Other Master's Programme

Semester – III

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATPGGE01	Mathematical Analysis	GE	4	4	50	In Page 58

Semester – IV

Sl. No.	Course Code	Course Title	Course Type	Credit	No. of classes per week	Full Marks	Contents
1	MATPGGE02	Mathematical Methods	GE	4	4	50	In Page 59

MATPGGEC01 (Mathematical Analysis)

Learning Objectives

- The prerequisite for this course is a standard course of Calculus containing basic notions like sequences and their convergence, series, real valued functions of a real variable, their limit, continuity and differentiability, integration etc along with familiarity with complex numbers.
- Ideas regarding basic notions of several variable calculus like partial derivatives is a must.
- This course is designed for them who are likely to use concepts more as a tool to study other subjects like Physics and different branches of Engineering.

Course Outcomes

Upon successful completion of this course, the students will –

- feel more comfortable and confident to use/ apply concepts in Mathematics.

Course Details

Module A: Real Analysis

Unit I

Review of Single Variable Calculus: A quick review to the notions of Limit, Continuity and Differentiability. Riemann and Improper Integrals. Quick revision of Partial Derivatives. [10 Lectures]

Unit II

Multiple Integrals: Double and Triple Integrals, Evaluation by Repeated integrals. Fubini's Theorem (statement and Applications only). [10 Lectures]

Unit III

Line and Surface Integrals: Simple properties. Green's Theorem and Divergence Theorem. [12 Lectures]

Module B: Complex Analysis

Unit I

Limit, Continuity and Differentiability of Complex Functions of a Complex Variable. Holomorphic (Complex Analytic) Functions. Cauchy Riemann Equations. Harmonic Functions. Construction of Complex Analytic Functions. [12 Lectures]

Unit II

Contour Integration. Cauchy Goursat Theorem (Statement and Applications) (for both Simply Connected and Multiply Connected Regions). Cauchy's Integral Formula. Maximum Modulus Theorem. Taylor's and Laurent Series. Singularities and Pole. Residue Theorem. [16 Lectures]

References

[1] Tom M. Apostol, Mathematical Analysis, 2nd edition, Narosa Publishing House, 1996.

[2] Tom M. Apostol, Calculus, Volumes I and II, John Wiley and Sons, 2003.

[3] George B. Thomas and Ross L. Finey, Calculus, 9e, Pearson, 1996.

[4] Kenneth A. Ross, Elementary Real Analysis, Springer, 1980.

[5] W. Flemming, Functions of Several Variables, Addison-Wesley, 1991.

[6] S. Ponnusamy, Foundations of Complex Analysis, Narosa, 2005.

[7] H. S. Kasana, Complex Variables, Theory and Applications, 2e, PHI, 2005.

MATPGGEC02 (Mathematical Methods)

Learning Objectives

- To learn about the basics of ordinary differential equations.
- To learn about the basics of partial differential equations.
- To learn about Fourier series.
- To learn about the Fourier transforms.
- To learn about the Laplace transforms.
- To learn about the Integral transformation and integral equations.
- To learn about the Newton forward and backwards method.
- To learn about Lagrange's method.

Course Outcomes

Upon successful completion of this course, the students will be able to –

- know about the basics of ordinary differential equations.
- know about the basics of partial differential equations.
- know about the basics of Fourier series and Fourier transforms.
- know about the basics of Integral equations and Integral transforms
- know how to apply Newton forward and backward interpolation methods.
- know the Importance of Lagrange's method for unequally spaced data.

Course Details

Unit I

Ordinary Differential Equations: Linear differential equation with constant coefficients; Complementary function, Particular integral. Euler's homogeneous equation and deduction to an equation of constant coefficients. Second order linear equation with variable coefficients: exact equation: reduction of order; method of variation of parameters; reduction to normal form; change of independent variables. Riccati Equation, Simple eigenvalue problems. System of linear differential equations with constant coefficients. [12 Lectures]

Unit II

Partial Differential Equations: first order partial differential equation, geometrical interpretation, second order partial differential equations with constant coefficients and their classification into elliptical, parabolic and hyperbolic type, solution of one dimensional wave and diffusion equations and Laplace equation of dimension 2 by the method of separation of variables.

[12 Lectures]

Unit III

Fourier Series: periodic functions, trigonometric series of sines and cosines, Euler formulae, Fourier series in the interval $(-\pi,\pi)$, Dirichlet's conditions, even and odd functions, half range sine and cosine series, Fourier series in the intervals $(0,2\pi),(-a,a),(0,a)$ etc. [8 Lectures]

Unit IV

Integral Transforms: Definitions and properties of Laplace and Fourier transforms. Inverse transform, convolution theorem, application to determine the solution of linear ordinary and partial differential equations with constant coefficients. [6 Lectures]

Unit V

Integral Equations: Introductory concepts: Definition, Fredholm and Volterra integral equations. Integro-differential equations. Relations between differential and integral equations. [6 Lectures]

Fredholm integral equation: Equations of the first and second Kind. Solution with separable kernels. Volterra integral equation: First and second type. Solution with separable kernels.

[10 Lectures]

Unit VI

Interpolations: Newton's forward and backward difference formulae; Lagrange's method. [6 Lectures]

References

[1] J. C. Burkill,The Theory of Ordinary Differential Equations, Interscience, 1956.

[2] E.A. Codington , I. Levinson, Theory of Ordinary Differential Equations, McGraw-Hill, New York, 1955.

[3] G.F. Simmons, Differential Equations with Applications and Historical Notes, McGraw-Hill, 2010.

[4] M. Spigiel, Applied Differential Equations, Prentice Hall India, 1967.

[5] I.N. Sneddon, Elements of Partial Differential Equations, Mcgraw Hill, 1957.

[6] F. John, Partial Differential Equations, Springer, New York, 2012.

[7] T. Amarnath, An Elementary Course in Partial Differential Equations, Narosa Pub. House, 2003.

[8] K.S. Rao, Partial differential equations. PHI publishers, 2011.

[9] L Debnath, D. Bhatta, Integral Transforms and Their Applications, Wiley Interscience, 1989.

[10] B. Davies, Integral Transforms and Their Applications, Springer, 2012.

[11] S. M. Jemyan, The Classical Theory of Integral Equations: A Concise Treatment, Birkhauser, Boston, 2012.

[12] W.V. Lovitt, Linear Integral Equations, Dover, New York, 2014.

[13] F. G. Tricomi, Integral Equations, Dover, 1985.

[14] M.K. Jain, S.R.K. Iyenger, R.K. Jain, Numerical Methods for Scientific and Engineering Computation, New Age International, 2005.