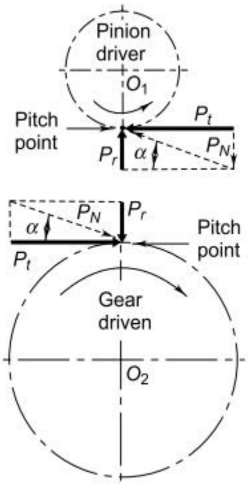
Spur Gear

Force analysis of gear:

Assumptions:

1. Changes of resultant force(P_N) in running condition is neglected.
2. Only one pair of teeth takes the entire load while analysis.
3. The effect of dynamic forces is neglected.



Transmitted torque by gear, $M_t = \frac{60(10^6)(kW)}{2\pi n}$ N-mm

Where,

kW = power transmitted by gear (kW)

n= speed of rotation (rpm)

Tangential component, $P_t = \frac{2M_t}{d_g}$ N

Where, d_g = gear diameter (driven)

Therefore,

Radial force, $P_r = P_t \tan \alpha$ N

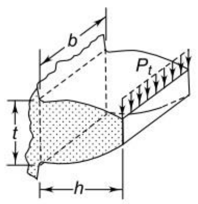
And Resultant force along pitch line, $P_N = {P_t}/{\cos \alpha}$

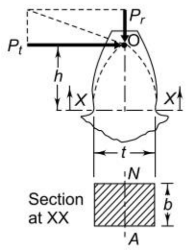
***<u>Beam strength of gear tooth</u>*:**

Lewis equation:

Assumption:

1. Effect of radial force (P_r) is neglected.
2. Tangential component(P_t) is uniformly distributed over the face width of the gear.
3. Stress-concentration effect is neglected.
4. Only one pair of teeth takes the load at a time.





At section XX,

$$\sigma_b = \frac{M_b y}{I} = \frac{(P_t x h)\left(\frac{t}{2}\right)}{\left[\left(\frac{1}{12}\right) b t^3\right]}$$

Rearranging,

$$P_t = m b \sigma_b Y$$

Where, Lewis form factor, $Y = \left(\frac{t^2}{6hm}\right)$

m = module

Lewis equation,

$$\boldsymbol{S_b = m b \sigma_b Y}$$

Where,

S_b = Beam strength (maximum tangential force) (N)

For design criteria, $S_b \geq P_{eff}$(Effective load in 'N')

⇨ $S_b = P_{eff}.(factor\ of\ safety)$

AGMA = American Gear Manufacturing Association

Berth equation for effective load (later modified by AGMA),

$$P_{eff} = \frac{C_s P_t}{C_v}$$

Where,

C_s = service factor = ${(M_t)_{max}}/{M_t} = {(P_t)_{max}}/{P_t} = {(Starting\ torque)}/{(Rated\ torque)}$

$$C_v = \text{velocity/dynamic factor} = \begin{cases} \left({3}/{3+v}\right) & , if\ v < 10\ m/s \\ \left({6}/{6+v}\right) & , if\ 10 \le v < 20\ m/s \\ \left({5.6}/{5.6+\sqrt{v}}\right) & , if\ v \ge 20 m/s \end{cases}$$

Where, $v = \frac{\pi d_g n}{(60\ x\ 10^3)}\ m/s$ (pitch line velocity)

d_g = pitch circle diameter of gear (mm)

n = speed of rotation (rpm)

Dynamics factor according to AGMA:

$$C_v = \left(\frac{A + \sqrt{200v}}{A}\right)^B\ (m/s)$$

Where, $A = 50 + 56(1 - B)$ and $B = 0.25(12 - Q_v)^{2/3}$

Q_v = transmission accuracy level number ≈ quality number (8-12 for precision quality)

Buckingham equation for effective load,

$$P_{eff} = (C_s P_t + P_i)$$

Where,

P_i= Incremental dynamic load = $\frac{21v(Ceb+P_t)}{21v+\sqrt{(Ceb+P_t)}}$

Where,

C = deformation factor (N/mm^2) = ${k}/{\left(\frac{1}{E_p}+\frac{1}{E_g}\right)}$

k = constant depending upon the form of tooth.

E_p = Elastic modulus of pinion

E_g = Elastic modulus of gear

And, e = sum of errors between two meshing teeth (mm)

Modified bending stress (AGMA):

$$\sigma_b = P_t C_o C_v K_s \left(\frac{1}{bm}\right)\left(\frac{K_H K_B}{Y_J}\right)\ (N/mm^2)$$

Where,

C_o = Overload factor

K_H = load distribution factor

K_B = Rim thickness factor

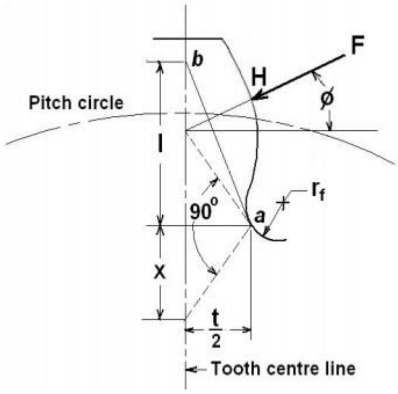
K_s = size factor

$Y_J = \frac{Y}{K_f}$ = modified Lewis form factor

Where,

k_f = stress concentration factor $= H + \left(\frac{t}{r}\right)^L \left(\frac{t}{l}\right)^M$

$$H = 0.340 - 0.4583662\emptyset$$
$$L = 0.316 - 0.4583662\emptyset$$
$$M = 0.290 + 0.4583662\emptyset$$
$$r = \frac{\left(b - r_f\right)^2}{(d/2) + b - r_f}$$



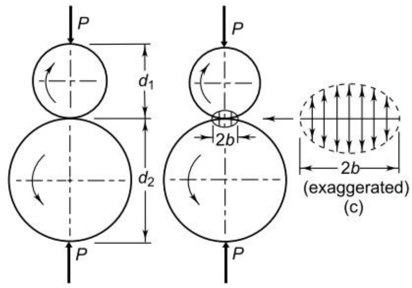
$\emptyset$ = pressure angle
r_f = fillet radius
b = dedendum
d = pitch diameter

***Wear strength of gear tooth*:**

Buckingham's equation based on Hertz contact theory.

Assumptions:

1. Cylinders are made of isotropic materials.
2. Elastic limit of the material is not exceeded.
3. Radius of pinion and gear are very large compared with contact width (2b).



Contact stress,

$$\sigma_c = \frac{2P_N}{\pi b l}\ (N/mm^2)$$

Where,

$$b = \left[\frac{2P\left(\frac{1-\mu_p^2}{E_p} + \frac{1-\mu_g^2}{E_g}\right)}{\pi l\left(\frac{1}{d_p} + \frac{1}{d_g}\right)}\right]^{1/2}\ (mm)$$

P = force pressing two cylinders (N)
b = half width of deformation (mm)
l = axial length of the cylinders (mm)
μ = Poisson's ratio

Substituting,

$$\mu = 0.3, l = b, P_N = {P_t}/{\cos\alpha}$$

Also,

$$Q = \frac{2d_g}{\left(d_g \pm d_g\right)}\ (ratio\ factor)$$

$$\ldots\ldots for\ external\ mesh\ and\ internal\ mesh\ respectively$$

The wear strength becomes,

$$S_w = bQd_pK$$

Where,

S_w = Wear strength (maximum tangential force) (N)

For design criteria, $S_w \geq P_{eff}$(Effective load in 'N')

⇨ $S_w = P_{eff}.(factor\ of\ safety)$

Where,

K =load-stress factor = $\frac{\sigma_c^2 \sin\alpha \cos\alpha\left(\frac{1}{E_p}+\frac{1}{E_g}\right)}{1.4} = 0.16\left(\frac{Brinell\ Hardness\ Number}{100}\right)^2$

Modified contact stress (AGMA):

$$\sigma_c = Z_E\sqrt{P_tC_oC_vK_s\left(\frac{K_H}{d_pb}\right)\left(\frac{Z_R}{Z_I}\right)} \quad (N/mm^2)$$

Where,

Z_E = Elastic co-efficient $\left(\sqrt{N/mm^2}\right)$

Z_R = Surface condition factor

Z_I = Geometry factor for pitting resistance.

For exercise, derivation and details study, please go through the source.

source: 1. Design of Machine Elements by V B Bhandari.

2. NPTEL : https://nptel.ac.in/courses/112/106/112106137/
3. Shigley's Mechanical Engineering Design (reference)

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